Simulation of Active Reflection Coefficient Phenomena of Large Antenna Array using FEM Domain Decomposition Methods

A. Barka, A. Jouadé, D. Jacquinot

Abstract—This paper proposes the application of the Finite Element Tearing and Interconnecting method (FETI-2LM) a Domain Decomposition Method (DDM) [1], [2] for the full-wave simulation of large antenna array (few thousands of radiating elements). The proposed methodology makes it possible to efficiently calculate in a post-processing step the antenna patterns for steering directions within the desired field of view while taking into account the active reflection coefficients. The methodology is based on an efficient extraction of the embedded radiating elements far field radiation patterns as well as the Generalized Scattering Matrix (GSM) [3] of the array by implementing massively parallel computers. The beauty of the GSM is its capability to handle the active S-parameters independently of the steering direction. This permits to generate the angular-dependent active reflection coefficients by post-processing. This implementation is a step forward to provide us with a tool that permits to evaluate by simulation large antenna array while taking into account all the phenomena and interactions that may degrade the key performances that are required at a higher level, to specify a global system before fabrication.

Index Terms—Finite element, Domain Decomposition Method, Parallel machines, Finite Element Tearing and Interconnecting (FETI), Active Reflection Coefficient, Antenna Array

I. INTRODUCTION

THE mutual couplings between the radiating elements need to be taken into account in order to achieve a proper modeling of an antenna array. These couplings can be responsible of blind directions for which the reflection coefficient is close to 1 causing a drop in the gain of the antenna in these singular directions. Taking into account the effects of active reflection coefficients during simulations of electromagnetic performances of an antenna array is therefore essential. This indicator allows to optimize either the antenna element itself but also the geometry of the array to reduce blind zone effects. These simulations performed with full-wave methods based on integral equations (EFIE, CFIE, MLFMM) [4] or partial differential equations (FEM finite element method) [5] can be extremely tedious and sometimes require the availability of very large computational resources. Several published works have demonstrated the effectiveness of exact or approximate numerical methods during parametric studies, based on domain decomposition approaches to characterize the antenna array as

well as all its electronic components (amplifiers, attenuators, power splitters and phase shifters) and S parameters effects on the active reflection coefficient. For example ONERA [6], [7] and XLIM University of Limoges [8]–[11] have shown the advantage of characterizing an antenna array by on the one hand the set of embedded radiation diagrams and on the other hand the Generalized Scattering Matrix (GSM) of the array. This then makes it possible to calculate by linear recombinations the antenna diagrams for scanning in azimuth and elevation without having to repeat the 3D simulations. These techniques make it possible to take into account finely the effect of the active reflection coefficients during the beam steering.

The method proposed in [10], [11], by taking advantage of existing formalisms on periodic arrays [3], [12], [13] and by using appropriate methods to efficiently exploit such formulations, accurately defines the performances of all building blocks, from the feeding circuits to the radiating elements, over a frequency range. However, the methodology to extract the scattering matrix of the array is based on a mix of periodic approximations (for the center elements) and partial local embedded zones for the border elements of the array.

Fig. 1: Back-fire antenna geometry.

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In this work, so as to efficiently calculate the embedded radiation patterns of all the radiating elements (internal and

boundary radiating elements) as well as the complete Scattering Matrix [6] of the array, we discuss the interest of the implementation of the FETI-2LM domain decomposition methods taking into account the finitude of the array on massively parallel computers initially developed in [14] and optimized in [15] and [16] for large size sparse arrays. The main goal is to implement a method allowing in a first step to calculate the radiating elements radiation patterns and the GSM of the array, and in a second post processing step to calculate the gain patterns of the antenna array for azimuth and elevation scanning directions while taking into account the active reflection coefficient. These diagrams can be quickly obtained with a very reduced computational effort, while offering an exact solution of the complete problem. The DDM method will be fully described in Section II, while Section III contains the obtained results from a 1600-elements antenna array.

II. DOMAIN DECOMPOSITION FOR GSM EXTRACTION

A. Step by step description

1) Field expansion on the n^{th} TEM ports of the array:

On each antenna port, the electric and magnetic fields (E_n and H_n) are expanding using incoming (e_n, h_n) and outgoing ($e_n, -h_n$) TEM electromagnetic modal waves [17] of amplitudes a_n and b_n .

$$\mathbf{E}_n = (\mathbf{a}_n + \mathbf{b}_n) \cdot \mathbf{e}_n \quad (1)$$

$$\mathbf{H}_n = (\mathbf{a}_n - \mathbf{b}_n) \cdot \mathbf{h}_n \quad (2)$$

Fig. 2: Regular array of 1600 elements.

2) *Solving the FEM problem:* The Generalized Admittance Matrix (GAM) of the array is first FEM-computed with the FETI-2LM domain decomposition method [14] and converted to a Generalized Scattering Matrix (GSM) 'S' as proposed in [6].

$$\mathbf{b} = \mathbf{S} \cdot \mathbf{a} \quad (3)$$

This step is time and memory consuming and is performed on parallel clusters. The antenna array ports are fed and the

simulation outputs are the GAM of the array and the complex radiation patterns $\tilde{\mathbf{e}}(\theta, \phi)$ of each antenna surrounded by the others.

3) *Beamsteering:* The beam-steering of the main lobe in a direction (θ_p, ϕ_p) is performed by defining an amplitude and phase distribution $a_n(\theta_p, \phi_p)$ on each port of the array. The amplitudes of the outgoing waves b_n re-entering in the antenna array due to mutual coupling are expressed through the GSM and equation 3. Finally, the electric farfield radiation pattern of an array of N elements for a (θ_p, ϕ_p) beam direction is expressed by:

$$\tilde{\mathbf{E}}_{(\theta, \phi)} = \sum_{n=1}^N (\mathbf{a}_n(\theta_p, \phi_p) - \mathbf{S} \times \mathbf{a}_n(\theta_p, \phi_p)) \tilde{\mathbf{e}}_n(\theta, \phi) \quad (4)$$

B. Active reflection coefficients

Hence, this calculation gives us the $n \times n$ scattering matrix of the radiating array. The active reflection coefficients of the elements located in this finite array can then be computed

$$\mathbf{S}_{i,i}^{\text{active}} = \frac{1}{\mathbf{a}_i} \sum_{j=1}^n \mathbf{S}_{ij} \cdot \mathbf{a}_j \quad (5)$$

and the global active reflection coefficient of the array is:

$$\mathbf{S}_{\text{array}}^{\text{active}} = \frac{1}{N} \sum_{i=1}^N \mathbf{S}_{i,i}^{\text{active}} \quad (6)$$

C. GSM and Gain parallel simulation on HPC clusters

We used the strategy proposed in a recent work [15] for sparse arrays, here by populating our regular grid of regular periodicity with two types of sub-domains (antenna and PEC extension of the arrays as shown on Fig. 2). Since the method is implemented on parallel machines, each computing core with at least 4 GB of memory is allocated exclusively to a sub-domain (elementary cell of the dense array). During the simulation we used 1764 sub-domains (1600 antenna sub-domains and 164 border PEC extension of the array). Each of the 1600 radiating elements is excited from its TEM port and the nearfield solution is exported on all the 1600 TEM ports of the array. The simulation is speed up by using the block Krylov recycling algorithms developed in [14]. Finally the GAM and the GSM are calculated in a post processing step following the methodology of [6]. By denoted by Z_0 the free space wave impedance and by P_r the radiated power of the antenna, the absolute gain of the array including the active reflection coefficients is obtained from the electric farfield defined by equation 4 and:

$$\mathbf{G}(\theta, \phi) = \frac{1}{2 \cdot Z_0 \cdot P_r} \tilde{\mathbf{E}}_{(\theta, \phi)} \cdot \tilde{\mathbf{E}}_{(\theta, \phi)}^* \quad (7)$$

III. RESULTS

As an example, an array of 1600 elements covering 420 m^2 in a regular grid is shown in Fig. 2. The computational statistics are indicated in Table I. The full wave simulation is performed with the FACTOPO code using 1764 Intel Xeon

Broadwell E5-2680v4 computing cores. During the simulation, each of the 1600 radiating elements is assigned to a core processor following the strategy proposed in [15]. Note that the simulation time for one embedded antenna is 9 minutes and the total simulation time to extract the complete GSM of the array (Figure 3) and the 1600 embedded diagrams is 240 hours. The antenna element is a back-fire antenna (Fig. 1) with 3 circular patches along the z-direction working at around 440 MHz and backed by a conducting ground plane. Such an antenna is a nice candidate for antenna array because it permits to have a high gain while having a low footprint along the array thanks to the use of the 3rd dimension (z-direction). This antenna permits to reduce coupling effects with the neighbours to avoid strong blind zones thanks to lower active reflection coefficients.

TABLE I: Computational statistics

Mesh parameters (unknowns)	Previous strategy
Antenna sub-domain	586,461
Skeleton interface problem	13 millions
Total unknowns of the array	1 billion
Simulation parameters	
Number of cores	1,764
Number of iterations	99
Stopping criterion	$\epsilon = 10^{-2}$
Elapse time (per antenna)	9 minutes
Elapse time (complete 1600x1600 GSM)	240 hours
Memory peak (per core)	2.3 Gb

Fig. 3: Simulated GSM of the 1600x1600 antenna array

Fig. 4 shows the simulated reflection coefficient of the isolated antenna element using the FACTOPO code and in Figure 5, the simulated radiation patterns are shown along $\phi = 0^\circ$ and $\phi = 90^\circ$ respectively. Such an antenna has a frequency bandwidth of about 35 MHz around a center frequency of 440 MHz. At the center frequency, the gain of this antenna is 10.5 dBi with side lobe level of -20 dB from the main lobe. The half-power beamwidths are 45° along $\phi = 0^\circ$ and 40° along $\phi = 90^\circ$.

Fig. 6 shows the active reflection coefficient of each element of the radiating panel. These coefficients are computed for a

Fig. 4: Simulated reflection coefficient of the isolated element with its surrounded metallic ground.

Fig. 5: Simulated radiation pattern of the isolated back-fire antenna along $\phi = 0^\circ$ and $\phi = 90^\circ$.

uniform amplitude excitation and a linear phase distribution corresponding to a specific beam steering (Azimuth= 15° and Elevation= 0°), while in Fig. 7 is shown the radiation pattern of the array for the considered beam steering taking into account the active reflection coefficient. Finally, it is shown the variability of the active reflection coefficients of the global panel depending on the beamsteering considered. It can be shown an increase of the active reflection coefficients for steering angles higher than 25° in Azimuth and 25° in Elevation.

IV. CONCLUSION

In this paper, the capability of calculating the active reflection coefficients of large arrays over a global field of view, and

Fig. 6: Active reflection coefficients of each element of the array at one steering angle.

Fig. 7: Global active reflection coefficient of the array for all steering angles.

in a postprocessing step, has been demonstrated. Associated with the FETI-2LM Domain Decomposition Methods, this strategy permits to take into account all the phenomena and interactions that may degrade the key performances of antenna array. This development is a valuable step forward for future developments in antenna and array designs.

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Fig. 8: Radiation pattern of the array in a specific direction.

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