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An optimization model and genetic-based matheuristic for parking slot rent optimization to carsharing*

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Abstract

Carsharing represents a major example of smart mobility service that allows a customer to rent a vehicle for a limited amount of time paying a per-minute fee. It may relieve people of the costly and non-sustainable burden of owning a car, especially when residing in a city. Though the spread of carsharing may bring significative benefits to (smart) cities, its penetration can be obstructed by non-up-to-date regulations, which can be still tied to a non-smart vision of mobility. In this study, we provide an overview of remarkable city regulations for carsharing, particularly highlighting the importance that parking policies can have in favouring the diffusion and use of carsharing services. Given such importance, we characterize the optimization problem of a local government that wants to analytically choose the best subset of parking slots to rent to carsharing companies, in order to improve urban mobility. To model and solve the problem we propose a new Binary Linear Programming problem and genetic-based matheuristic. Finally, we present results from computational tests referring to realistic data of the Italian city of Rome,

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showing that our optimization approach can return a fair territorial distribution of the parking slots, satisfying various families of constraints limiting the distribution.

Keywords: Shared Mobility, Carsharing, Parking Slot Renting, Optimization, Matheuristic.

1. Introduction

In recent times, smart mobility systems have attracted a lot of attention, since they are considered a fundamental component of modern smart cities, as recognized by national and international establishments and by major companies active in the landscape of digital economy (e.g., Benevolo et al. 2016; EU-INEA 2017). A *Smart Mobility* (SMOB) system can be defined as a strongly Information and Communications Technology (ICT)-supported Transport System (e.g., GeSi-ACN 2015; Kenny 2013). ICT is a crucial component of SMOB systems, which enables a continuous connection between the system administrators, the customers/users and the mobile and fixed infrastructures. Furthermore, it represents a key building block for offering innovative trustable and sustainable ways to move in urban and extra-urban scenarios. A major example of SMOB is represented by *carsharing* services. Nowadays, carsharing is intended as a mobility service that allows a user to rent a car for very short period of times (e.g., a few minutes) using a smartphone application and paying a per-minute fee (Weikl and Bogenberger, 2013). Such services have contributed to revolutionize urban mobility in the last decade, basing their success also on the strong diffusion of smartphones.

It is now widely recognized that carsharing and other SMOB systems can have a (very) positive impact on the quality of life in urban and extra-urban scenarios, sensibly reducing the negative sides of transport systems (e.g., pollution caused by traffic, road congestion). For an overview of the benefits of SMOB, we refer the reader to (Benevolo et al., 2016; Bencardino and Greco, 2014; Carrese et al., 1996, 2017, 2020; Hessel, 2015; Lyons, 2016). Some studies have tried to precisely assess this positive impact. For example, Martin and Shaheen (2011) showed that the introduction of carsharing in North America led to a decrease in the average number of vehicles per family from 0.47 to 0.24 and that, on average, each shared car substituted from 9 to 12 private cars. Besides the positive impact on environment and economy, carsharing and other SMOB systems have a positive social impact too. Indeed, a considerable part of the costs that road transportation entails does not appear as internal costs borne by the drivers, but is taken into

33 account as external costs (e.g., environmental impact) borne by the collec-
34 tivity. Such external costs represent a third of the total and about 90% of
35 them is due to private car owners (Lombard et al., 2005). Within this con-
36 text, carsharing and other SMOB systems present features that can ease the
37 internalization of costs, since they support the passage from privately-owned
38 cars to *mobility as a service*, based on the concept of *pay-as-you-go*.

39 Though their benefits are pretty evident and clear, SMOB systems have
40 found difficulties in being implemented. This is due to various reasons, such
41 as: 1) the presence of regulatory frameworks that are often confused and
42 not up-to-date for welcoming new innovative SMOB digital platforms and
43 services; 2) the inertness of policy makers, which may ineffectively support
44 the expansion of SMOB services, thus dooming them to remain just at an
45 experimental and very low-scale level.

46 In recent years, among SMOB systems, carsharing has attracted partic-
47 ular attention and has been the subject of intense research from many point
48 of views. Recent research studies have been aimed at better quantifying
49 the benefits of carsharing, such as how it contributes to reduce the need for
50 a private car (Becker et al., 2018) and how it favours the penetration and
51 adoption of electric vehicles (see e.g., Meisel and Merfeld 2018). Other stud-
52 ies have evaluated how different reservation mechanisms of shared cars may
53 be used to influence user behaviour (e.g., Wu et al. 2019 analyze how users
54 respond to different reservation schemes adopted for tackling imbalances
55 between supply and demand in free-floating carsharing).

56 A major topic of investigations has been represented by the develop-
57 ment of methods for choosing where to locate charging stations for electric
58 carsharing. Limiting our attention to some more recent works, we recall
59 the studies: (Cheng et al., 2019), which proposes to combine statistical
60 models with machine learning techniques not only to identify sites for de-
61 ploying new charging stations, but also for adjusting the location of stations
62 already deployed; (Chen et al., 2018), which analyzes the performance of
63 regression models for evaluating how station features, city environment and
64 transportation facilities affect carsharing services and proposes a method
65 for deciding the location of stations; (Wang et al., 2019), which develops
66 a four-step method that first estimates and distributes charging demand of
67 electric vehicles in a city and then establishes the location of normal and
68 fast charging stations for both private and shared vehicles; (Biondi et al.,
69 2016), which proposes an optimization approach for establishing the cost-
70 optimal location and capacity of charging stations, using queueing theory
71 for expressing the demand of recharge; finally, (Li et al., 2017) proposes a
72 multi-criteria decision approach, including factors like travel purposes and

73 distance from existing stations, for optimally locating carsharing stations
74 and tests its performance on EVCARD, the electric carsharing system of
75 Shanghai.

76 Another major topic of investigations has been represented by the prob-
77 lem of relocating vehicles, which can sensibly increase the profit of car-
78 sharing, as highlighted in the recent survey (Illgen and Höck, 2019) and in
79 other works such as: 1) (Boyaci et al., 2017), which proposes a simulation-
80 optimization approach for a station-based one-way carsharing system that
81 provides for reservations and relocations; 2) (Bruglieri et al., 2018), which
82 proposes a two-phase heuristic for solving a 3-objective carsharing reloca-
83 tion problem, considering cost minimization, fair distribution maximization
84 and service level maximization; 3) (Zhao et al., 2018), which focuses on op-
85 timally managing vehicle rebalancing and staff relocation in a station-based
86 one-way carsharing system, proposing an innovative algorithm combining
87 Lagrangian relaxation and dynamic programming for its solution.

88 In the present work, we review some major city carsharing regulations, high-
89 lighting the importance of adopting reserved parking slots for carsharing ve-
90 hicles. Also, we propose a mathematical optimization model and algorithm
91 for establishing where to put reserved parking slots at disposal of carsharing
92 users. The reserved parking slots are simple parking spaces meant to be
93 spread around a city more pervasively of renting/charging stations. Their
94 essential purpose is to speed up the parking phase of a rent, reducing the
95 cruising time needed by a carsharing user for finding an empty slot.
96 Specifically, our major contributions are the following:

- 97 1. we provide an overview of the carsharing regulations of a number of
98 major cities, particularly highlighting the importance of parking poli-
99 cies in making carsharing a success and discussing the Italian case in
100 more detail;
- 101 2. given the crucial role of parking policies in carsharing, we character-
102 ize the optimization problem of a Local Government (for example the
103 council of a municipality) that must choose which parking slots to rent
104 to carsharing companies in a city, while finding an optimal balance be-
105 tween the interest of the population and those of the profit-oriented
106 companies. To model this problem, we propose to adopt mathematical
107 optimization techniques and we represent the problem as a Binary Lin-
108 ear Programming problem, which includes boolean variables to model
109 the possibility of renting or not a cluster of parking slots. The purpose
110 of this model, is to provide an easy-to-use mathematical tool for Local
111 Governments, which can be easily adopted and tuned.

To the best of our knowledge, such decision problem has never been considered in literature and addressed through optimization techniques as we do in the present paper. We thus remark that there is no related literature to review and compare with;

3. we prove that the considered optimization problem is NP-Hard and thus can prove difficult to solve even for state-of-the-art optimization software. To tackle the problem, we therefore propose a new matheuristic that combines a genetic algorithm with exact mathematical programming techniques adopted in large neighborhood searches;
4. we present the results of computational tests on realistic instances referring to the city of Rome. Such data are also defined on the basis of the experience gained within our collaborations with major carsharing companies active in Rome and with E-Go, a carsharing service launched at University Roma Tre with the support of the electric utility company Enel (see Carrese et al. 2017). They also take into account the current regulations of the City of Rome for carsharing (City of Rome - DGC136, 2016).

The remainder of the paper is organized as follows. In Sections 2, we overview city regulations, whereas in Section 3, we define the model for optimal parking slot renting. In Sections 4 and 5, we present the matheuristic and the results of computational tests, respectively. In Section 6, we discuss challenges related to the real-world adoption of the proposed optimization approach. Finally, in Section 7, we derive conclusions and discuss possible directions for future work.

2. A review of local regulatory situations for carsharing

As a major expression of SMOB, carsharing has widely spread especially in the USA and Europe, becoming one of the new important modes of urban transport (Pinna et al., 2017). Several studies pointed out that the use of privately-owned cars, though still very common, has experienced a significative decline in many countries since several years (see e.g., Millard-Ball and Schipper 2011; Newman and Kenworthy 2011). Furthermore, carsharing has gained a lot of popularity as a more sustainable way to reduce emissions of CO2 (Martin and Shaheen, 2011).

A good parking policy is indicated in literature as one of the most effective strategies that a local government can implement to stimulate carsharing (e.g., Rivasplata et al. 2013). A detailed study made in (Shaheen et al., 2010) highlights that in North America over 70 local governments

149 and municipalities, such as San Francisco and Vancouver, have adopted
150 specific policies to favour the parking of carsharing, also including the reser-
151 vation of parking slots for shared cars. This study also reports that a survey
152 among the San Francisco residents revealed that just 20% of the people were
153 against the reservation of slots for carsharing. Recently, also the New York
154 City Council has approved legislation for a pilot study aimed at evaluating
155 the reservation of parking slots to carsharing companies (see New York City
156 Council 2015).

157 Another very interesting study that has investigated how local govern-
158 ments can support and stimulate carsharing, in particular by reserving park-
159 ing slots is (Dowling and Kent, 2015). The authors focused on the case of
160 Sydney, which in 2013 offered 1000 shared cars managed by various compa-
161 nies, leading to an estimated benefit of more than 300 millions of dollars for
162 carsharing users. The study highlights that the carsharing was not taken
163 into consideration in the regional transport plans, but was instead consid-
164 ered at a local level, in the regulations of Sidney city and in that of 6 out of
165 8 the districts where carsharing operated.

166 In 2016, the City of Sydney has approved new regulations for managing
167 on-street carsharing parking slots with the aim of exploiting them more ef-
168 ficiently - see (City of Sydney, 2016). A first objective of these regulations
169 is to define the requirements and obligations of companies that apply for
170 reserved carsharing parking slots: for example, they require to deploy vehi-
171 cles that emit less than 175 g/km of CO₂ and to produce a report about
172 the use of each vehicle each month. A second objective is to clearly state
173 the rules by which a company may get parking slots: the rent must obtain
174 a preventive approval from the residents and retailers of the area and from
175 local committees for bicycle and pedestrian mobility. The number and po-
176 sition of the parking slots are defined on the basis of an evaluation of the
177 potential demand for carsharing and on the basis of the district features:
178 if in a district less than 3.5% of all the slots are reserved to carsharing,
179 then a company *already renting slots* may request at most 4 slots, if it can
180 prove that the 3 slots that are closest to those requested have been used
181 at least 18 times in a month; for districts exceeding the threshold of 3.5%,
182 the conditions are more stringent. In order to favour competition, a com-
183 pany without rented slots can rent without restrictions at most 3 slots in
184 a district with up to 900 total slots or at most 6 in a district with more
185 than 900 slots. Always in the metropolitan area of Sydney, specifically in
186 the Municipality of Ashfield, the local government has approved regulations
187 specific for carsharing (Municipality of Ashfield, 2010), which were aimed at
188 deeply involving the citizens in the process of assignment of parking slots.

189 Assigning slots to carsharing in an area required the approval of 75% of
190 the residents. Furthermore, the rules favoured the location of slots close to
191 parks and retailers. Last but not least, they provided for making pay the
192 cost of realization of the reserved slots to the carsharing companies, which
193 every year must submit a report on the usage of the slots.

194 In 2011, the Canadian city of Calgary approved regulations aimed at
195 favouring carsharing specifically impacting on parking policy (City of Cal-
196 gary, 2011). In particular, the carsharing companies are obliged to relocate
197 their vehicles so that the reserved slots have a car available for a minimum
198 number of hours everyday. Also, each company may receive at most 3%
199 of the total number of slots available in any area identified as commercial.
200 Finally, it is interesting to cite the case of the city of Vancouver, where the
201 price for renting parking slots differs depending upon the district (City of
202 Vancouver, 2016): more attractive districts in terms of business activities
203 are associated with higher prices.

204 Concerning Italy, urban carsharing is currently active in 29 cities and,
205 according to a 2016 survey, was counting 5764 vehicles. Concentrating the
206 attention on three of the largest cities (Milan, Rome and Turin), the first
207 very interesting observation to be made is that, though carsharing was in-
208 troduced in these cities many years ago, it has been interested by a limited
209 regulation and emission of policies, particularly those on parking, aimed at
210 improving its effectiveness and efficiency. The city of Milan offers the most
211 advanced urban carsharing system in Italy, with several companies operat-
212 ing in the city. In 2013, new regulations were introduced to allow the shared
213 cars to access and park for free in central restricted traffic zones and to park
214 in slots restricted to residents. The regulations also state that a carshar-
215 ing company must pay a yearly fixed price of 1100.00 euros for operating
216 a vehicle in the city and access the benefits stated above. The more re-
217 cent regulations (City of Milan, 2016) have introduced the obligation for the
218 carsharing company to regularly update their fleet, imposing that a vehicle
219 must be replaced once reached 4 years of service or 100,000 km.

220 The city of Rome has also been interested by regulations, similar to
221 those of Milan: first regulations from 2004 allowed free access and parking in
222 central restricted traffic zones; then, in 2010, it has been imposed that shared
223 vehicles must be at most 3 years old. Finally, in 2016, further regulations
224 - e.g., (City of Rome - DGC136, 2016) - have been aimed at strengthening
225 the penetration of electric vehicles, establishing the realization of additional
226 charging stations in central zones of the city.

227 In Turin, the regulations (City of Turin, 2015) not only allow the free
228 access and parking in central restricted traffic zones, but also grant the right

229 to drive on fast lanes reserved to bus and taxi services. A distinctive feature
230 of these regulations is that they have been especially aimed at containing
231 the air pollution, by imposing that the fleet of each carsharing company
232 must contain at least 30% of low-emission natural-powered vehicles (e.g.,
233 bi-fuel methane-gasoline) and at least 10 electric vehicles. Furthermore, the
234 regulations provide for that recharge stations of the company must be open
235 also to private electric vehicles and that each company must cover with
236 service an area of at least 40 square kilometers.

237 On the basis of the overview provided in this section, it can be seen that
238 parking policies constitute an important element of regulations adopted to
239 encourage the development and penetration of carsharing in cities. We can
240 distinguish two major parking solutions: 1) assigning specific parking slots
241 on the basis of a request made by a carsharing company; 2) renting a set
242 of slots to all companies without distinction (i.e., no slot is reserved to a
243 specific company). The chosen solution must be then accompanied by other
244 decisions, such as: 1) the total number of slots made available in the city
245 and in each zone; 2) the maximum number of slot reserved to each company;
246 3) the location of the slots. Furthermore, a parking policy should also fix
247 the price that a company must pay to gain the right of using a slot. This
248 price could be differentiated depending on the attractiveness of the area:
249 commercial and business districts could be more attractive than residential
250 districts and thus be associated with higher prices; also, larger groups of
251 contiguous parking slots should be associated with higher prices, given the
252 higher chances of attract carsharing vehicles and thus increasing availability
253 for the users.

254 In order to guarantee a full application of the policy, the local govern-
255 ments must keep watch on the rented slots, timely sanctioning abuses and
256 adopting ad-hoc fines for sanctioning people parking their private cars in re-
257 served slots. We note that this aspect is not so trivial: the first experiments
258 of carsharing directly managed by the City of Rome also failed because the
259 reserved slots were often occupied by private cars, which counted on the very
260 low chance of being fined by the local police. As found in several policies,
261 it is also important that the local governments provides for obtaining regu-
262 lar reports about the conditions of each carsharing company, in particular
263 defining some key performance indicators that allow to clearly evaluate the
264 impact of carsharing and of parking slot reservation on the administered
265 territory. Last but not least, the reservation of parking slots to carshar-
266 ing should be subordinated to a positive advice of the local residents, as
267 provided in the policies of several cities.

268 As it is evident, the problem that the local governments face when de-

269 ciding how to put parking slots at disposal of carsharing companies presents
 270 a lot of factors to be taken into account. This leads to a complex decision
 271 problem that typically pursues the maximization of a measure expressing
 272 the benefit for the city and the population to adopt carsharing and specific
 273 parking policies. This is actually an optimization problem where the local
 274 government wants to take the best decision on the basis of its interest. In
 275 the next section, we investigate the possibility of modelling and solving such
 276 optimization problem through a mathematical optimization approach.

277 **3. An optimization model for car slot renting**

278 We consider the problem of the Local Government (LG) of a city that
 279 needs to decide which parking slots must be reserved and rented to carshar-
 280 ing companies that are active in its territory. The overall aim is to favour
 281 the diffusion of such services among the population, in order to improve
 282 the penetration of smart mobility and thus traffic conditions. We propose
 283 to tackle such problem by means of Mathematical Programming techniques
 284 (Bertsimas and Tsitsiklis, 1997): we first translate the problem faced by the
 285 LG into mathematical terms, deriving a suitable mathematical optimization
 286 model, and then propose a matheuristic algorithm for its solution. By adopt-
 287 ing a Mathematical Programming approach, we allow to find a solution that
 288 grants the best performance according to an objective function that may
 289 include key performance indicators.

290 We recall here some fundamentals of mathematical optimization, refer-
 291 ring the reader to the book (Bertsimas and Tsitsiklis, 1997) for a more ex-
 292 haustive and detailed introduction. An optimization model, also commonly
 293 called *mathematical programming problem*, is a mathematical model that is
 294 made up of three major components: 1) a set of decision variables, which
 295 model the choices that the decision maker can take (for example, if we refer
 296 to the parking slot renting problem that we consider in this paper, a variable
 297 models whether the LG does or does not rent a slot); 2) a set of feasibility
 298 constraints that model limitations on the value that can be assigned to the
 299 variables (for example, the LG cannot rent more than a fixed number of
 300 parking slots); 3) an objective function, which guides the search for the best
 301 solution of the problem, evaluating how good is a complete assignment of
 302 values to the variables.

We can express more formally some basic mathematical programming

concepts by referring to the following problem denoted by MP:

$$\max c^T x \quad (MP) \quad (1)$$

$$Ax \geq b \quad (2)$$

$$x \in \{0, 1\}^n \quad (3)$$

where $c \in \mathbb{R}^n$, $A \in \mathbb{R}^{m \times n}$, and $b \in \mathbb{R}^m$. We remark that the problem MP we consider is a *Binary Linear Programming* problem, since it only includes binary decision variables, assuming either 0 or 1 values, and linear constraints and objective function.

If we assign a value in $\{0, 1\}$ to each decision variable x_j , $\forall j = 1, 2, \dots, n$ of MP, we obtain a *solution*. A solution is either *feasible*, if it satisfies all the constraints (2), or *infeasible*, if it violates at least one of the constraints (2). If we denote by $\mathcal{F}$ the set of all feasible solutions, a feasible solution $x^* \in \mathcal{F}$ is said *optimal* if it offers the best objective value among all the feasible solutions, i.e. it maximizes the objective function (1) (formally, $c^T x^* \geq c^T x$, $\forall x \in \mathcal{F}$).

3.1. System elements

The optimization problem faced by the LG can be described as follows (we anticipate that in Figure 1, we provide a small example aimed at clarifying the notation introduced in the remainder of this section). The LG administers a city made up of a *set of districts* denoted by D . Each district $d \in D$ includes a set of subdistricts, denoted by $S(d)$. In each subdistrict $s \in S(d)$ with $d \in D$, the LG has identified a number of *parking slot clusters* available for renting to carsharing companies: a parking slot cluster (or briefly, *cluster*) is a set of parking slots that is reserved for parking carsharing cars. Formally, for each district $d \in D$ and subdistrict $s \in S(d)$, we denote by $C(s, d)$ the set of clusters available in s . For each cluster $c \in C(s, d)$, we denote by n_c the number of parking slots composing the cluster. A cluster must be rented as a whole, i.e. it is not possible to just rent a part of its slots. As in real-world studies, we assume that the LG has identified a profit measure π_c that quantifies the benefits of renting a cluster c on the basis of preliminary studies (see, for example, the regulation of the City of Rome - DGC136 2016). Such measure may take into account several distinct factors, such as the revenue obtained renting the cluster, the cost associated with maintaining the cluster, the economical benefits of having carsharing services in an area (e.g., financial, environmental and social).

In line with policies overviewed in the previous section, we strongly believe that the renting of parking slots should be coordinated with local residents. To this end, we consider very important to include a limit on the

Figure 1: An example of target area partitioned into districts and subdistricts, containing parking slot clusters. The districts are separated by thick continuous lines, whereas the subdistricts of a district are separated by dotted thinner lines. Parking slots clusters are represented by groups of small rectangles placed side by side and each rectangle visualizes one slot. Referring to the notation introduced in Subsection 3.1, this example of target area contains three districts, so $D = \{d_1, d_2, d_3\}$. The subdistricts of each districts are: $S(d_1) = \{s_1, s_4\}$, $S(d_2) = \{s_2, s_3\}$ and $S(d_3) = \{s_5, s_6, s_7\}$. Concerning the parking slot clusters, we note that subdistricts s_2 and s_6 do not contain clusters available for renting inside them (so, $C(s_2, d_2) = C(s_6, d_3) = \emptyset$). Instead, all the other subdistricts contains at least one cluster (for example, s_1 has 2 clusters, $C(s_1, d_1) = \{c_1, c_2\}$, both containing 3 parking slots (i.e., $n_{c_1} = n_{c_2} = 3$), whereas s_4 contains only cluster c_3 including 6 slots (i.e, $C(s_4, d_1) = \{c_3\}$, with $n_{c_3} = 6$).

total number of parking slots that can be rented in each district. This is done for avoiding to arouse widespread discontent among local residents, which typically wants to have a consistent fraction of the parking slots in a district to be (freely) available to car owners. For example, the number of rented slots could be required to not exceed a fraction of the total number of parking slots available in the district. Furthermore, it is also important to include a minimum number of slots that must be rented, to favour the diffusion of carsharing; this could reflect the dimension of the fleet maintained by the companies. For each sub-district $s \in S(d)$ with $d \in D$, we denote such lower and upper limits on the number of rented parking slots by $\eta_s^{\min}$ and $\eta_s^{\max}$, respectively. Furthermore, we assume that the LG also wants to include a lower and upper limit $\gamma_s^{\min}, \gamma_s^{\max}$ on the number of clusters that may be rented in each subdistrict.

Another important aspect that we want to model is the possibility for the LG to classify the clusters per types and to include a minimum and maximum number of clusters for each type in each district. For example, clusters could be distinguished per type by the number of parking of slots that they include. On the basis of our direct experience with the creation of a new carsharing service in Rome, we expect that an LG wants to have a good balance between clusters of bigger and smaller dimensions. Another example of type distinction is that between shopping, business and residential clusters, depending in which zone of a district they are located. From a modelling point of view, we introduce a set T to denote the cluster types and we denote by $t(c) \in T$ the type of a cluster c . For each district $d \in D$, we denote by $\tau_{dt}^{\min}$ and $\tau_{dt}^{\max}$ the minimum and maximum number of clusters of type t allowed in district d . In Figure 1, we provide an example to clarify the notation introduced.

3.2. Modelling the optimization problem

After having introduced all the relevant system elements and the related notation, we can state the optimal decision problem that the LG faces as follows.

Optimal Carsharing Parking Slot Renting Problem (Opt-ParkRent): given the set of districts D , the corresponding sets of subdistricts $S(d) \forall d \in D$ and sets of parking slot clusters $C(s, d) \forall d \in D, s \in S(d)$, the profit π_c and number of slots n_c of each cluster $c \in C(s, d)$, the upper and lower bounds $\eta_s^{\min}, \eta_s^{\max}, \gamma_s^{\min}, \gamma_s^{\max}, \tau_{dt}^{\min}$ and $\tau_{dt}^{\max}$ on the number of rentable slots, rentable clusters and rentable cluster types, Opt-ParkRent consists of choosing which clusters to rent to carsharing services, so that all lower and

upper bounds on rentable slots, rentable clusters and rentable cluster types are satisfied and the total profit is maximized.

We model the optimization problem Opt-ParkRent as a *Binary Linear Programming* problem. In order to model the decision of renting or not a parking slot cluster, we introduce a binary decision variable $x_{dsc} \in \{0, 1\}$ for each district $d \in D$, subdistrict $s \in S(d)$ and cluster $c \in C(s, d)$, defined as follows:

$$x_{dsc} = \begin{cases} 1 & \text{if cluster } c \text{ in subdistrict } s \text{ of district } d \text{ is rented} \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

These decision variables are employed in the following constraints defining the set of feasible solutions of the optimization problem. First, we need a set of constraints to express that, for each subdistrict, the limits on the number of rented parking slots cannot be exceeded:

$$\eta_s^{\min} \leq \sum_{c \in C(s, d)} n_c \cdot x_{dsc} \leq \eta_s^{\max} \quad \forall d \in D, s \in S(d) \quad (5)$$

We remark that here the decision variable is multiplied by the number n_c of slots in a cluster.

Then, we must express the limits on the number of clusters that can be rented in each subdistrict:

$$\gamma_s^{\min} \leq \sum_{c \in C(s, d)} x_{dsc} \leq \gamma_s^{\max} \quad \forall d \in D, s \in S(d) \quad (6)$$

Finally, we need constraints to express the limits on the number of cluster types that can be rented in each district:

$$\tau_{dt}^{\min} \leq \sum_{s \in S(d)} \sum_{c \in C(s, d): t(c)=t} x_{dsc} \leq \tau_{dt}^{\max} \quad \forall d \in D, t \in T \quad (7)$$

We note that in these constraints the two summations involve the decision variables of all the clusters located in sub-districts of the district d that are of cluster type t .

The objective is to maximize the total profit, expressed as the summation of the decision variables over all districts, subdistricts and clusters:

$$\max \sum_{d \in D} \sum_{s \in S(d)} \sum_{c \in C(s, d)} \pi_c \cdot x_{dsc} \quad (8)$$

397 By joining (4) - (8), we obtain the overall Binary Linear Programming prob-
 398 lem presented below, denoted by BLP-PS, which we use as basis to model
 399 and solve the problem of renting parking clusters Opt-ParkRent.

$$\begin{aligned}
 & \max \sum_{d \in D} \sum_{s \in S(d)} \sum_{c \in C(s,d)} \pi_c \cdot x_{dsc} & (\text{BLP-PS}) \\
 & \eta_s^{\min} \leq \sum_{c \in C(s,d)} n_c \cdot x_{dsc} \leq \eta_s^{\max} & \forall d \in D, s \in S(d) \\
 & \gamma_s^{\min} \leq \sum_{c \in C(s,d)} x_{dsc} \leq \gamma_s^{\max} & \forall d \in D, s \in S(d) \\
 & \tau_{dt}^{\min} \leq \sum_{s \in S(d)} \sum_{c \in C(s,d): t(c)=t} x_{dsc} \leq \tau_{dt}^{\max} & \forall d \in D, t \in T \\
 & x_{dsc} \in \{0, 1\} & d \in D, s \in S(d), c \in C(s, d)
 \end{aligned}$$

400 Concerning the computational complexity of our problem Opt-ParkRent
 401 associated with parking slot optimization and modelled as BLP-PS, we now
 402 show that it is NP-Hard. To this end, we state and prove the following
 403 proposition.

404 **Proposition 1.** *Opt-ParkRent is NP-Hard.*

405 PROOF. In order to prove the result, we refer to the optimization model
 406 BLP-PS that we proposed to model Opt-ParkRent. We can first note that
 407 all the feasibility constraints (5), (6) and (7) present the following general
 408 form:

$$LB_i \leq \sum_{j \in J} a_{ij} \cdot x_j \leq UB_i \quad (9)$$

Here, i is the index of a generic constraint (we denote by I the set of constraint indices) and j is the index of a generic decision variable (we denote by J the set of variable indices). Each constraint contains a summation over all binary decision variables x_j multiplied by suitable non-negative coefficients (in particular, we note that we have $a_{ij} = 0$ when x_j is not included in a constraint, $a_i = 1$ when x_i appears in constraints (6) and (7) and $a_i = n$, with n representing the number of parking slots associated with x_i when x_i appears in a constraint (5)). Each summation is then bounded below by a

value $LB_i > 0$ and above by $UB_i > 0$. The overall problem then reads as:

$$\begin{aligned} \max \quad & \sum_{j \in J} \pi_j \cdot x_j \\ LB_i \leq \quad & \sum_{j \in J} a_{ij} \cdot x_j \leq UB_i & \forall i \in I \\ x_j \in \{0, 1\} \quad & & \forall j \in J \end{aligned}$$

409 The previous problem constitutes a *multidimensional* generalization of the
 410 *knapsack problem with minimum filling constraints*, introduced and proved
 411 to be NP-Hard in (Xu, 2011). As a consequence, also the problem Opt-
 412 ParkRent is NP-Hard. $\square$

413 4. A Genetic-based Matheuristic for solving BLP-PS

414 Because of its NP-Hard nature, the parking slot optimization problem
 415 Opt-ParkRent, modelled by BLP-PS, can be hard to solve even for state-of-
 416 the-art optimization solvers, such as IBM ILOG CPLEX. We therefore pro-
 417 pose to solve BLP-PS by a matheuristic that combines a genetic algorithm
 418 with solution generation and improvement phases based on the execution of
 419 suitable large neighborhood searches. Matheuristics are a relatively recent
 420 trend in metaheuristic methods that tries to better exploit the potential-
 421 ities of exact mathematical optimization techniques through hybridization
 422 with heuristic methods (see e.g., Maniezzo et al. 2009). Concerning the use
 423 of large neighborhood searches, we stress that we rely on *exact* searches,
 424 namely the search is not a simple local search based on heuristic rules of
 425 exploration, but a large exploration that is formulated as an optimization
 426 problem solved exactly by a state-of-the-art solver (see e.g., Blum et al.
 427 2011 for an introduction). The rationale at the basis of exact searches is
 428 that, while a state-of-the-art solver may find big difficulties in solving a full
 429 problem, it may instead be able to effectively and efficiently solve suitable
 430 subproblems obtained from it and associated with the large neighborhood
 431 searches.

432 Genetic Algorithms (GAs) are well-known heuristic algorithms inspired
 433 from the evolution dynamics of a population of individuals. For an exhaus-
 434 tive introduction to theory and applications of GAs, we refer the reader to
 435 (Golberg, 1988; Karakatic et al., 2015; Srinivas et al., 1994). Focusing on
 436 its essential features, a GA manages a population of individuals, in which
 437 each individual represents a feasible solution for the optimization problem at
 438 hand. The *chromosome* of an individual specifies the value that is assumed

439 by each decision variable in the solution associated with the individual. The
 440 *genetic strength* of an individual is evaluated by a *fitness function*, which
 441 assigns a value representing a measures of quality for the solution/individual.

442 In our case, we strengthen the performance of a GA by combining it
 443 with *relaxation-based variable fixing*, in which the value of decision variables
 444 is set a-priori considering the value of variables in an optimal solution to
 445 a suitable (tight) linear relaxation of the problem. The GA-based solution
 446 construction is then followed by an *exact large neighborhood search*, aimed
 447 at finding solutions of improved quality by exploring portions of the solution
 448 space obtained by fixing the value of a subset of decision variables of the
 449 complete problem.

450 More in detail, the general structure of the GA that we use as basis for
 the matheuristic is depicted in Algorithm 1. We now proceed to discuss in

Algorithm 1 General Genetic Algorithm (GA-alg)

- 1: Creation of the initial population
 - 2: **while** an arrest condition is *not* satisfied **do**
 - 3: Selection of individuals who generate the offspring
 - 4: Generation of the offspring by crossover
 - 5: Mutation of part of the population
 - 6: Death of part of the population
 - 7: **end while**
 - 8: Improvement by Exact Large Neighborhood Search
-

451 detail the features of all the steps of the algorithm.
 452

453 *4.1. Characteristics of the GA population*

454 *4.1.1. Representation of the individuals and fitness function*

455 In the case of our problem, a natural modelling choice within the GA-
 456 based matheuristic is to let coincide the chromosome of an individual with
 457 the binary vector x specifying the value of each variable x_{dsc} with $d \in D, s \in$
 458 $S(d), c \in C(s, d)$ included in BLP-PS.

It is also natural to adopt the objective function (8) of BLP-PS as fitness
 function. For a given individual associated with a chromosome x , the fitness
 of the corresponding individual is thus equal to the revenue of the rented

clusters, namely:

$$\pi(x) = \sum_{d \in D} \sum_{s \in S(d)} \sum_{c \in C(s,d)} \pi_c \cdot x_{dsc} .$$

4.1.2. Initial population

In order to define the individuals composing the initial population, we rely on both a deterministic and a probabilistic individual generation procedures.

The deterministic generation is based on a procedure that follows the principles of RINS (*Relaxation Induced Neighborhood Search*) (Danna et al., 2005), a well-known heuristic for Mixed Integer Linear Programming. Specifically, let $\text{BLP-PS}^{\text{RLX}}$ be a linear relaxation of BLP-PS (i.e., a version of BLP-PS obtained by substituting the integrality requirement $x_{dsc} \in \{0, 1\}$ on each decision variable with its continuous relaxation $0 \leq x_{dsc} \leq 1$) and let x^{RLX} be its optimal solution. The optimal solution x^{RLX} can be used as basis to fix a-priori the value of some variables in BLP-PS, thus obtaining a smaller and easier-to-solve problem that can be tackled by a state-of-the-solver. We denote by $\text{BLP-PS}^{\text{FIX-RINS}}$ the problem where some variables have their value fixed a-priori and the strategy for fixing is the following:

$$\begin{aligned} \text{IF} \quad x_{dsc}^{\text{RLX}} &\leq 0 + \epsilon \quad \text{THEN} \quad x_{dsc} = 0 \\ \text{IF} \quad x_{dsc}^{\text{RLX}} &\geq 1 - \epsilon \quad \text{THEN} \quad x_{dsc} = 1 \end{aligned}$$

with $\epsilon > 0$. In other words, the rule provides for fixing to either 0 or 1 a variable that in x_{dsc}^{RLX} is sufficiently close to 0 or 1. Indeed, such closeness can be considered a good indication that the variable should be fixed to that value in a solution of good quality (see Danna et al. 2005 for an exhaustive discussion of the principles of the RINS algorithm). The problem $\text{BLP-PS}^{\text{FIX-RINS}}$ that we obtain by the RINS-fixing procedure is passed to a state-of-the-art solver and we add all the feasible solutions found during the solution procedure to the initial population of the GA algorithm.

We also generate solutions through a partial probabilistic variable fixing procedure that uses again the optimal solution x^{RLX} of the linear relaxation, but in a different way. To this end, we can first note that a fractional optimal value x_j^{RLX} can be interpreted as the probability of fixing to 1 the corresponding variable x_j in a good feasible solution (we refer the reader to the book by Motwani and Raghavan (1995) and to the papers (Maniezzo, 1999; D’Andreagiovanni et al., 2015; D’Andreagiovanni and Nardin, 2015) for a discussion about the interpretation of fractional binary solutions as

479 probability in randomized rounding algorithms in a general and in a meta-
480 heuristic context). We thus create a number of solutions using x_j^{RLX} as
481 probabilities and confronting them with randomly generated numbers, simi-
482 larly to Ant Colony Optimization procedures illustrated in (Maniezzo, 1999;
483 D’Andreagiovanni et al., 2015; D’Andreagiovanni and Nardin, 2015). This
484 probabilistic generation procedure is operated for a part of all the variables
485 and the resulting partial fixing is passed to a state-of-the-art solver for solv-
486 ing the resulting problem of reduced size (again, we add all the feasible
487 solutions found during the solution procedure to the initial population of
488 the GA algorithm).

489 4.2. Evolution of the population

490 4.2.1. Selection

491 In order to select the individuals that combine their chromosomes for
492 generating the new population, we execute a *tournament selection*: if $\mathcal{P}$ is
493 the current population of individuals, as first step we define $k > 0$ subgroups
494 of individuals through random selection of a number $\lfloor \alpha \cdot |\mathcal{P}| \rfloor$ of individuals
495 from $\mathcal{P}$, with $\alpha \in (0, 1)$. Then we select the $m < \lfloor \alpha \cdot |\mathcal{P}| \rfloor$ individuals that
496 offer the best fitness function value in each group. We extract individuals
497 from this restricted pool for giving raise to the new generation by means of
498 crossover.

499 4.2.2. Crossover

500 Individuals belonging to the restricted pool defined in the previous step
501 are randomly paired so to constitute $\lfloor k \cdot m / 2 \rfloor$ couples. From each couple, we
502 generate one offspring through crossover of chromosomes. Specifically, given
503 a couple of individuals (the *parents*) associated with two solution vectors
504 x^1, x^2 of the model BLP-PS, the crossover mixes the values of variables
505 in the same position of the parents, so to give birth to one offspring x^{off} ,
506 hopefully associated with higher fitness value.
507 Specifically, x^{off} is defined according to two rules:

- 508 1. if the parents present the same binary value in a position j , then the
509 offspring inherits this value in the same position j (i.e., if $x_j^1 = x_j^2$ then
510 $x_j^{\text{off}} = x_j^1$);
- 511 2. if the parents have distinct binary values in a position j (i.e., $x_j^1 \neq x_j^2$),
512 then the variable x_j^{off} is not fixed and its value is decided by running an
513 optimization solver. Specifically, let BLP-PS^{FIX} be the subproblem of
514 BLP-PS obtained by fixing a subset of variables according to the pre-
515 vious rule 1. Then we set as offspring the best feasible solution found

516 by a state-of-the-art solver used to solve BLP-PS^{FIX} within a time
 517 limit (so, again we exploit the solver to tackle a suitable subproblem
 518 of the complete hard-to-solve problem BLP-PS).

519 The rationale at the basis of these two crossover rules is that two so-
 520 lutions presenting the same valorization of a variable constitute a good
 521 indication that such valorization should be kept in the offspring, whereas
 522 for deciding how to valorize variables with conflicting values in the par-
 523 ent solutions, we exploit the solver to tackle a subproblem of the complete
 524 hard-to-solve problem BLP-PS.

525 4.2.3. *Mutation*

526 After having created the new generation of individuals, we provide for
 527 trying to obtain diversified solutions and possibly escaping from local op-
 528 tima by applying the *mutation* technique: we randomly choose a number of
 529 individuals $\lfloor \gamma \cdot |\mathcal{P}| \rfloor$ with $0 < \gamma < 1$ and we randomly choose a number of
 530 positions in the chromosome whose binary value is inverted. If this leads to
 531 make the solution associated with the individual infeasible, then the individ-
 532 ual is discarded and is included in the count of the death process described
 533 below.

534 4.2.4. *Death*

535 After the execution of crossover and mutation operations, we attempt at
 536 removing the weakest individuals in the population, mimicking a process of
 537 death. To this end, $\lfloor k \cdot m/2 \rfloor$ individuals presenting the lowest fitness value
 538 are discarded.

539 4.3. *Improvement by Exact Large Neighborhood Search*

Once that the GA algorithm has been run iteration after iteration, reach-
 ing the time limit for its execution, we use the best feasible solution found
 as basis for implementing an exact large neighborhood search. Specifically,
 let x^{BEST} be such best feasible solution, we try to improve it searching a
 better solution in the neighborhood that includes all the feasible solutions
 obtainable by inverting the binary value of at most $\Gamma > 0$ entries of x^{BEST} .
 Since we implement an exact search, as explained in previous subsections, we
 need to formulate the search as an optimization problem solved by a state-
 of-the-art-solver. We can accomplish this by adding the following *hamming*
distance constraint to BLP-PS:

$$\sum_{j \in J: x_j^{\text{BEST}}=0} x_j + \sum_{j \in J: x_j^{\text{BEST}}=1} (1 - x_j) \leq \Gamma \quad (HD)$$

540 Indeed, this constraint counts the number of binary variables that change
541 values with respect to the best feasible solution found x^{BEST} and such num-
542 ber must be not larger than Γ .

543 We denote by BLP-PS^{HD} the modified problem BLP-PS that includes the
544 hamming distance constraint (HD), which we then solve by a state-of-the-
545 art-solver with a time limit.

546 An overview of the phases of the matheuristic algorithm for solving BLP-
PS is presented in Algorithm 2.

Algorithm 2 GA-based Matheuristic for BLP-PS

- 1: Solve the linear relaxation BLP-PS^{RLX} of BLP-PS with optimal solution x^{RLX}
 - 2: Generate the initial population exploiting x^{RLX} as specified in Subsect. 4.1.2
 - 3: **while** a global time limit is not reached **do**
 - 4: Select individuals for crossover as specified in Subsect.4.2.1
 - 5: Execute the crossover and generate the offspring as specified in Subsect.4.2.2
 - 6: Mutate part of the individuals as specified in Subsect.4.2.3
 - 7: Discard part of the individuals as specified in Subsect.4.2.4
 - 8: **end while**
 - 9: Solve BLP-PS^{HD} for finding an improvement of x^{BEST} , the best solution found
by the GA
 - 10: Return the best feasible solution found x^{BEST}
-

547

548 5. Computational results

549 We employed the optimization model BLP-PS and the GA-based Matheuris-
550 tic defined in the previous sections to solve Opt-ParkRent - the Optimal
551 Carsharing Parking Slot Renting Problem introduced in Section 3.2 - us-
552 ing realistic data referring to the Italian city of Rome. These data were
553 defined on the basis of the specific regulations of the city (City of Rome -
554 DGC136, 2016) and of our experience in collaborating with professionals of
555 major carsharing companies active in Rome and with E-Go, a carsharing
556 service launched at University Roma Tre with the support of the electric
557 utility company Enel (see Carrese et al. 2017).

558 Inside each subdistrict, we identified a number of parking slot clusters,
559 on the basis of the features of the subdistrict (size, road configuration, im-
560 portance in the Roman urban mobility system). Concerning the cluster
561 types, we identified 5 types on the basis of the number of parking slots in-
562 cluded in a cluster. Each cluster presents a number of slots ranging from
563 1 to 12, with bigger clusters located close to important landmarks (for ex-
564 ample, train stations). The lower and upper bound on the number of slots

rentable in each subdistrict, those on the number of clusters rentable in each subdistrict and those on the number of types rentable for each type in each district (i.e., $\eta_s^{\min}, \eta_s^{\max}, \gamma_s^{\min}, \gamma_s^{\max}, \tau_{dt}^{\min}$ and $\tau_{dt}^{\max}$), are defined considering the features of the resident population, business activities, available public transportation and urban fabric.

The matheuristic solution algorithm was run on a 2.70 GHz Windows machine with 8 GB of RAM and using IBM ILOG CPLEX 12.5 as software for solving the optimization subproblems (CPLEX, 2020). The code was written in C/C++ and interfaced with CPLEX through Concert Technology.

The matheuristic ran with a total time limit of 3600 seconds: 3000 seconds were granted to the construction phase, while 600 seconds were granted to the improvement phase. The parameters of the GA algorithm were the following: the tournament selection involves $k = 10$ groups, in which a fraction $\alpha = 0.1$ of individuals of the population $\mathcal{P}$ are included; the crossover is operated on the $m = 10$ individuals presenting the highest fitness value in each group. After having created new individuals, a fraction $\gamma = 0.05$ of the population undergoes mutation. Finally, we set $\epsilon = 0.1$ in the RINS search and Γ equal to 10% of the total number of decision variables in the hamming distance constraint employed in the exact search for improved solutions.

As first test, we considered a quite small instance, in terms of number of clusters, made up of 5 districts and including 27 subdistricts in total, where we identified 191 potentially rentable clusters. Each subdistrict contains from 2 to 22 clusters, whose number of slots ranges from 1 to 10 (bigger clusters are located close to important landmarks, for example, train stations). The results associated with the optimal solution of the problem are visualized in Figures 2a and 2b. The solution specifies the clusters that must be rented in each subdistricts, in order to maximize the total profit while respecting renting constraints.

As a comment to the results, we can first note that, if the problem contained just one single constraint imposing a maximum number of slots over all the subdistricts, then we could adopt a simple and fast heuristic solution approach consisting of: 1) computing the profit per parking slot of each cluster; 2) sorting the clusters from the highest to the lowest in terms of such profit; 3) select one by one the clusters, following the sorting order, until the maximum number of rentable slots is exceeded. However, this simple solution approach would possibly lead to a solution of quite low quality offering an unfair distribution of the clusters over the territory. To avoid this, the additional constraints that limit the number of slots, the number of clusters and the number of types of clusters are needed. This results into a more complex structure of the problem, where the connections

Figure 2: Parking clusters for the City of Rome

605 between districts, subdistricts and cluster types discourage the application
 606 of the previously cited simple heuristic solution and requires instead the
 607 application of a more sophisticated solution algorithm like our matheuristic.

608 Analyzing the optimal solution of this small instance and putting it into
 609 relation with the input data of the optimization problem, we can note that
 610 there is a tendency to push the number of rented clusters to its imposed
 611 minimum $\eta_s^{\min}$ in less profitable subdistricts. In contrast, the number of
 612 activated clusters tends to be closer to the allowed maximum $\eta_s^{\max}$ in more
 613 attractive subdistricts, where clusters containing a higher number of parking
 614 slots are present. Going more into details, in one of the largest downtown
 615 subdistrict, which hosts important business and administrative activities
 616 and is close to important public transportation nodes, it is activated one of
 617 the largest clusters plus a combination of well-spaced clusters with 5 and 7
 618 slots. As it can be observed in the figure, the presence of the constraints ex-
 619 pressing limitations on the rentable clusters allow to have a fair distribution
 620 of the rented clusters over all the districts and subdistricts, thus contributing
 621 to pursue a fundamental objective of the LG.

622 After this test, we considered an instance of larger size, containing a
 623 sensibly higher number of parking slot clusters, namely 500 in total, and we
 624 studied the effect of varying the upper bounds $\eta_s^{\max}$ on the total number
 625 of rentable slots and $\gamma_s^{\max}$ on the total number of rentable clusters. The

Table 1: Results for varying upper bounds $\gamma_s^{\max}$ and $\eta_s^{\max}$ for a 500 cluster instance. For each combination of the two bounds, the table reports two values: 1) the number of activated clusters; 2) the total profit associated with the solution (in EUR).

		$\gamma^{\max}\%$							
		5		10		15		20	
$\eta^{\max}\%$	5	28	802935	31	815747	31	815747	31	815747
	10	32	1516350	43	1605688	46	1607896	46	1607896
	15	32	1944942	53	2300368	62	2351909	65	2352546
	20	32	2065162	56	2830020	73	2973650	78	2997582
	25	32	2086103	57	3183516	77	3545895	93	3687400
	30	32	2086103	57	3398607	81	4148823	100	4317304

626 results are presented in Table 1, where $\eta_s^{\max}$ can vary from 5 to 30% of all
627 slots available in a subdistrict s and $\gamma_s^{\max}$ may vary from 5 to 20% of the
628 total number clusters in a subdistrict s (note that in the table we express
629 such values as fraction of decimal numbers). Each entry of the table reports
630 two values for the best solution: the first is the number of activated clusters,
631 whereas the second is the total profit granted by the solution (reported in
632 EUR).

633 In the solutions obtained for the model BLP-PS for this instance, we
634 can note that there is a tendency to push the number of rented clusters to
635 the dominating upper bound $\eta_s^{\max}$ or $\gamma_s^{\max}$: the objective function pursues
636 the activation of as many large clusters as possible, presenting the highest
637 profit value, and this is jointly upper bounded by $\gamma_s^{\max}$, the bound on the
638 number of clusters, and by $\eta_s^{\max}$, the bound on the total number of parking
639 slots $\eta_s^{\max}$. So, for example, when considering an upper bound of 5% on
640 the number of activated slots, the associated profit remains the same for
641 an upper bound on the number of activated clusters higher or equal than
642 10%, indicating that in this case is the bound on the number of clusters
643 to dominate the other. In contrast, for example, an upper bound on the
644 number of activated clusters equal to 5% dominates that on the number of
645 slots for a percentage higher or equal than 10%.

646 Looking at Table 1, we can notice that the number of activated clusters
647 can sensibly increase as the upper bounds on the number of rentable clusters
648 and slots increase. In particular, we can pass from a minimum of 28 activated
649 clusters granting a profit of about 800k EUR, in the case of the lowest

combination of bounds, to a maximum of 102 clusters associated with a profit that is more than 5 times higher, namely about 4300k EUR, in the case of the highest combination of bounds. For intermediate bound combinations, the number of activated clusters and associated profit vary quite regularly, thus giving the possibility to the LG to adjust the bounds in order to pursue the target of cluster activation that better suits its objective for renting to carsharing services.

Finally, we conducted computational tests on a set of 20 data instances presenting an increasing number of districts, subdistricts and clusters, with the number of clusters ranging from 478 to 3428 and the number of total available slots ranging from 1578 to 17140. We compared the performance of our GA-based matheuristic to that of the solver CPLEX, used to solve the model BLP-PS with a time limit of 3600 seconds (the time budget assigned to CPLEX is thus equal to the total time budget assigned to the matheuristic). As upper bounds $\gamma_s^{\max}$ and $\eta_s^{\max}$, we adopted 20% of the total number of available slots and 15% of the total number of available clusters.

The results of our tests are reported in Table 2, where: a) ID is the instance identification; 2) Σs is the total number of available clusters in the instance; 3) Σn is the total number of available parking slots in the instance; 4) Σn^{MH} is the total number of activated parking slots in the best solution found by our MatHeuristic (denoted by the acronym MH) within the time limit; 5) π^{MH} is the total profit (expressed in EUR) associated with the best solution found by MH within the time limit; 6) t^{MH} is the time (expressed in seconds) required by MH to find the best solution within the time limit; 7) Σn^{CPLEX} is the total number of activated parking slots in the best solution found by CPLEX within the time limit; 8) π^{CPLEX} is the total profit (expressed in EUR) associated with the best solution found by CPLEX within the time limit; 9) t^{CPLEX} is the time (expressed in seconds) required by CPLEX to find an optimal solution (when an optimal solution is not found within the time limit, we insert the time limit of 3600 seconds as value); 10) $\Delta\pi\%$ is the percentage of profit increase that MH grants with respect to CPLEX. In Table 2, the values in the columns π^{MH} and π^{CPLEX} include an asterisk (*) when the value is optimal for the instance.

Comparing the performance of our matheuristic MH and CPLEX, we can subdivide the instances in three subsets on the basis of the results obtained:

- in the case of the smaller instances from I1 to I6, both MH and CPLEX find the optimal solution. However, CPLEX is faster and MH takes on average 18% more time to identify the best solution;
- also in the case of the instances from I7 to I14, both MH and CPLEX

689 find the optimal solution, but MH is faster and reduces the computa-
690 tional time of about 12% on average;

- 691 • in the case of the larger instances from I15 to I20, CPLEX does not
692 identify an optimal solution within the time limit. Moreover, for all
693 these instances, the value of the best solution found by CPLEX is
694 lower than that of the best solution found by MH. Specifically, the
695 best solutions found by MH offer an increased profit that is about 10%
696 higher on average. Finding such better solutions requires a consistent
697 fraction of the available time budget.

698 For all the instances, it is interesting to note that, in general, it is the
699 bound on the number of available slots to lead the cluster activation, pushing
700 towards the selection of a subset of clusters that respect the bound $\eta_s^{\max}$
701 and that are as large as possible, so to grant the highest profit for single slot
702 (we recall indeed that larger clusters are preferred by carsharing companies,
703 since they grant more visibility to the service, and are thus associated with
704 a higher renting price per single slot composing the cluster). It is also
705 interesting to note that, for all instances, while the number of activated
706 slots remains around 20% of the total number of available slots, the profit
707 sensibly increases passing from the smallest to the largest instance: this is
708 due to the fact that in larger instances there is the possibility of activating
709 a higher number of clusters of larger size, which, as discussed, are more
710 attractive and their rent grants a remarkably higher profit per single slot
711 than clusters of smaller size.

712 6. Discussion

713 Concerning the real-world adoption and implementation of the original
714 optimization approach we proposed, we believe that a crucial first challenge
715 is represented by convincing local governments of the advantages that a
716 rigorous mathematical optimization approach may offer with respect to trial-
717 and-error approaches that we have often observed among professionals in the
718 field of urban mobility and transportation systems. Several recent works
719 have clearly shown the advantages of adopting mathematical optimization
720 to solve real-world optimization problems on behalf of companies, also in
721 the transportation field (see e.g., Borndörfer et al. 2018; Fujisawa et al.
722 2016). Referring in particular to the case of carsharing, reserved parking
723 slots are not a novelty, but, to the best of our knowledge, a mathematical
724 optimization model has never been proposed for the selection of the slots
725 among a set of candidate sites. In most of the cases, the reserved slots are

Table 2: Results for instances of increasing size

ID	Σc	Σn	Σn^{MH}	π^{MH}	t^{MH}	Σn^{CPLEX}	π^{CPLEX}	t^{CPLEX}	$\Delta\pi\%$
I1	478	1578	307	1350349*	343	307	1350349*	259	0.00
I2	562	1967	392	2085164*	468	392	2085164*	406	0.00
I3	601	2164	422	2701938*	650	422	2701938*	513	0.00
I4	639	2109	423	2164170*	602	423	2164170*	538	0.00
I5	776	2484	492	3094116*	764	492	3094116*	676	0.00
I6	802	2807	551	3910623*	681	551	3910623*	629	0.00
I7	932	3542	684	4705261*	613	684	4705261*	680	0.00
I8	949	3702	737	5326327*	756	737	5326327*	846	0.00
I9	1047	4188	837	5391653*	852	837	5391653*	892	0.00
I10	1122	4713	942	5654585*	877	942	5654585*	1007	0.00
I11	1367	5742	1117	6527808*	1492	1117	6527808*	1873	0.00
I12	1492	6267	1235	7911311*	2285	1235	7911311*	2642	0.00
I13	1643	7394	1465	10138284*	1964	1465	10138284*	2365	0.00
I14	1828	8226	1642	10691682*	2554	1642	10691682*	2911	0.00
I15	2156	9702	1849	11444247	3126	1720	10647578	3600	6.96
I16	2382	10958	2072	13704218	3295	1896	12545038	3600	8.45
I17	2505	11774	2326	14696307	3332	2140	13524295	3600	7.97
I18	2891	13588	2638	16095856	3483	2387	14571878	3600	9.46
I19	3125	14375	2681	16892588	3490	2386	15034753	3600	10.99
I20	3428	17140	3233	21079811	3204	2861	18657918	3600	11.48

726 simply placed close to major landmarks, like airports and railway stations,
727 without an analytical approach supporting their selection and without in
728 particular pursuing the fair distribution principle that we have formalized
729 through the feasibility constraints (5)-(7).

730 Once the mathematical optimization approach is accepted as effective
731 and efficient, the mobility officers of the local governments should then
732 closely work together with the optimization experts: the aim would be to
733 properly define a data input of the model that is meaningful and valid for the
734 districts subject to carsharing parking planning. In this phase, the officers
735 could also suggest additional (soft) constraints to be taken into account for
736 defining a model and generating a data input that allow to better represent
737 specific features of the considered districts. Within this phase, the risk of
738 miscommunication between officers and experts is concrete, especially when
739 the officers do not have an education or background that allow them to
740 understand and express system modelling needs. In our recent experience,
741 it seems that the situation is getting better year after year, thanks to the
742 tendency to hire data science experts in various departments of local govern-
743 ments, allowing the optimization consultants to interface with officers who
744 are more familiar with mathematical modelling and algorithmic approaches.

745 After having defined the input data, solved the model and obtained a
746 carsharing parking slot plan, the local governments could still face conflicts
747 during the implementation of the plan. In particular, referring to the Italian
748 case that we better know also thanks to our direct consulting experience,
749 the parking slots reserved to carsharing would further increase the number
750 of slots that are subtracted to toll free parking and to the parking of district
751 residents and this could lead to legal appeals by residents. In major Italian
752 cities, substantial modifications to parking plans have regularly generated
753 appeals, even reaching the Supreme Court of Cassation, which has ruled
754 against them. To avoid this, we think that it is fundamental to orchestrate
755 relevant changes to parking plans with local residents, involving them in the
756 decision process. From the point of view of our optimization model, this
757 would imply to involve the residents in defining the location and size of the
758 candidate clusters and the values of the bounds that appear in constraints
759 (5)-(7), which limit the number of slots that can be reserved to carsharing
760 in districts and subdistricts.

761 7. Conclusions and Future Work

762 In this paper, we have characterized the optimization problem of a local
763 government that wants to analytically choose the best subset of parking

764 slots to rent to carsharing companies, with the aim of reducing cruising for
 765 finding parking and improve urban mobility. We proved that the problem is
 766 NP-Hard and we proposed a new Binary Linear Programming problem to
 767 model it. Given the NP-Hard nature of the problem, which can make it hard
 768 to solve even for state-of-the-art solvers, we proposed a new genetic-based
 769 matheuristic for its solution. The matheuristic combines a genetic algorithm
 770 with solution construction and improvement phases that are based on the
 771 execution of exact large neighborhood searches. We tested the performance
 772 of our new optimization approach on realistic instances referring to the city
 773 of Rome, showing that it can offer a better computational performance of a
 774 state-of-the-art solver and that it can return a fair distribution of clusters
 775 over the territory. As future work, we intend to define a multiperiod version
 776 of the model, to consider parking slot renting over long-term periods and
 777 include variations of demand for carsharing over time, also extending the
 778 computational tests to other cities. In this context, we would expect to
 779 face demand uncertainty, not knowing in advance the precise demand for
 780 carsharing services in the future, and thus it would be necessary to adopt
 781 optimization under data uncertainty methods, such as (Multiband) Robust
 782 Optimization (Büsing and D’Andreagiovanni, 2012). Furthermore, another
 783 topic that we consider worth of being investigated is how the clusters selected
 784 for renting should be assigned to carsharing companies. We expect that
 785 this should be done by means of public tenders, also taking into account
 786 fair competition criteria that favour the entry of new carsharing companies,
 787 similarly to the rules adopted by cities like Sydney - see (City of Sydney,
 788 2016).

789 References

- 790 Becker, H., Ciari, F., Axhausen, K.W., 2018. Measuring the car ownership
 791 impact of free-floating car-sharing - a case study in Basel, Switzerland.
 792 Transport. Res. D 65, 51-62.
- 793 Bencardino, M., Greco, I., 2014. Smart Communities. Social Innovation at
 794 the Service of the Smart Cities. TeMA. J. Land Use Mob. Environ., IN-
 795 PUT 2014, 39-51.
- 796 Benevolo, C., Dameri, R.P., D’Auria, B., 2016. Smart Mobility in Smart
 797 City - Action Taxonomy, ICT Intensity and Public Benefits. In: T. Torre
 798 et al. (Eds.), Empowering Organizations, Lecture Notes in Information
 799 Systems and Organisation 11, Springer.

- 800 Bertsimas, D., Tsitsiklis, J.N., 1997. Introduction to Linear Optimization.
801 Athena Scientific, Belmont.
- 802 Biondi, E., Boldrini, C., Bruno, R., 2016. Optimal deployment of stations
803 for a car sharing system with stochastic demands: A queueing theoretical
804 perspective. IEEE 19th International Conference on Intelligent Trans-
805 portation Systems (ITSC 2016), 1089-1095.
- 806 R. Borndörfer, T. Klug, L. Lamorgese, C. Mannino, M. Reuther, T.
807 Schlechte: Handbook of Optimization in the Railway Industry. Springer,
808 Cham (2018)
- 809 Boyaci, B., Zografos, K.G., Geroliminis, N., 2017. An integrated
810 optimization-simulation framework for vehicle and personnel relocations
811 of electric carsharing systems with reservations, Transport. Res. B 95,
812 214-237.
- 813 Blum, C., Puchinger, J., Raidl, G.R., Roli, A., 2011. Hybrid metaheuristics
814 in combinatorial optimization: a survey. Applied Soft Computing 11(6),
815 4135-4151.
- 816 Bruglieri, M., Pezzella, F., Pisacane, O., 2018. A two-phase optimization
817 method for a multiobjective vehicle relocation problem in electric car-
818 sharing systems. J. Comb. Optim. 36, 162-193.
- 819 Büsing, C., D’Andreagiovanni, F.: New results about Multi-band Uncer-
820 tainty in Robust Optimization. In: Klasing, R. (ed.) Experimental Algo-
821 rithms - SEA 2012, Lecture Notes in Computer Science, vol. 7276, pp.
822 63-74. Springer, Heidelberg (2012)
- 823 Carrese, S., D’Andreagiovanni, F., Giacchetti, T., Nardin, A., Zamberlan, L.
824 2020. An optimization model for renting public parking slots to carsharing
825 services. Transportation Research Procedia 45, 499-506.
- 826 Carrese, S., Giacchetti, T., Nigro, M., Patella, S.M., 2017. An innovative car
827 sharing electric vehicle system: an italian experience. WIT Transactions
828 on The Built Environment 176, 245-252.
- 829 Carrese, S., Gori, S., Picano, T., 1996. Relationship between parking loca-
830 tion and traffic flows in urban areas. Advanced Methods in Transportation
831 Analysis, 183-214, Springer.

832 Chen, X., Cheng, J., Ye, J., Jin, Y., Li, X., Zhang, F., 2018. Locating Station
833 of One-Way Carsharing Based on Spatial Demand Characteristics. J. Adv.
834 Transport. 2018, ID 5493632.

835 Cheng, Y., Chen, Ding, X., Zeng, L., 2019. Optimizing Location of Car-
836 Sharing Stations Based on Potential Travel Demand and Present Opera-
837 tion Characteristics: The Case of Chengdu. J. Adv. Transport. 2019, ID
838 7546303.

839 City of Ashfield, 2010. Sustainability Action Table.
840 [http://www.ashfield.nsw.gov.au/files//your_council/policies_plans_and_reports/](http://www.ashfield.nsw.gov.au/files//your_council/policies_plans_and_reports/policies/Ashfield_Sustainability_Action_Table_2010.pdf)
841 [policies/Ashfield_Sustainability_Action_Table_2010.pdf](http://www.ashfield.nsw.gov.au/files//your_council/policies_plans_and_reports/policies/Ashfield_Sustainability_Action_Table_2010.pdf)

842 City of Calgary. 2011. Bill number TP017 (A Parking Policy Framework for
843 Calgary). [http://www.calgary.ca/CA/city-clerks/Documents/Council-](http://www.calgary.ca/CA/city-clerks/Documents/Council-policy-library/TP017-ParkingPolicyFramework.pdf)
844 [policy-library/TP017-ParkingPolicyFramework.pdf](http://www.calgary.ca/CA/city-clerks/Documents/Council-policy-library/TP017-ParkingPolicyFramework.pdf)

845 City of Milan, 2016. Deliberazione Giunta Comunale n.55 (in Italian).
846 [https://www.comune.milano.it/dseserver/webcity/garecontratti.nsf/](https://www.comune.milano.it/dseserver/webcity/garecontratti.nsf/51607b595b240841c1256c4500569c90/fb7227ebfe458120c1257fa900593003/$FILE/ATTS0RDL.pdf/Determina%20-%2020160427_55%20-%20Bando%20CarSharing%202016.pdf)
847 [51607b595b240841c1256c4500569c90/fb7227ebfe458120c1257fa900593003/\\$FILE](https://www.comune.milano.it/dseserver/webcity/garecontratti.nsf/51607b595b240841c1256c4500569c90/fb7227ebfe458120c1257fa900593003/$FILE/ATTS0RDL.pdf/Determina%20-%2020160427_55%20-%20Bando%20CarSharing%202016.pdf)
848 [/ATTS0RDL.pdf/Determina%20-%2020160427_55%20-](https://www.comune.milano.it/dseserver/webcity/garecontratti.nsf/51607b595b240841c1256c4500569c90/fb7227ebfe458120c1257fa900593003/$FILE/ATTS0RDL.pdf/Determina%20-%2020160427_55%20-%20Bando%20CarSharing%202016.pdf)
849 [%20Bando%20CarSharing%202016.pdf](https://www.comune.milano.it/dseserver/webcity/garecontratti.nsf/51607b595b240841c1256c4500569c90/fb7227ebfe458120c1257fa900593003/$FILE/ATTS0RDL.pdf/Determina%20-%2020160427_55%20-%20Bando%20CarSharing%202016.pdf)

850 City of Rome, 2016. Deliberazione Giunta Comunale n.136 (in Italian).
851 <http://www.comune.roma.it/servizi/DeliberazioniAttiWeb/>

852 City of Sidney, 2016. Council Meeting Notice No. 10/1579
853 [http://www.cityofsydney.nsw.gov.au/_data/assets/pdf_file/0007/277738/161121](http://www.cityofsydney.nsw.gov.au/_data/assets/pdf_file/0007/277738/161121_COUNCIL_MINUTES.pdf)
854 [_COUNCIL_MINUTES.pdf](http://www.cityofsydney.nsw.gov.au/_data/assets/pdf_file/0007/277738/161121_COUNCIL_MINUTES.pdf).

855 City of Turin, 2015. Determinazione Dirigenziale n.46 (in Italian)
856 http://www.comune.torino.it/giunta_comune/intracom/htdocs/2015/2015_00248.pdf.

857 City of Vancouver, 2016. Bill number 11591 (Street and traffic regulation in
858 the city of Vancouver) <http://bylaws.vancouver.ca/2849c.pdf>

859 D’Andreagiovanni, F., Krolikowski, J., Pulaj, J., 2015. J. A fast hybrid
860 primal heuristic for Multiband Robust Capacitated Network Design with
861 multiple time periods. App. Soft Comp. 26, 497-507.

862 D’Andreagiovanni, F., Nardin, A., 2015. J. Towards the fast and robust
863 optimal design of wireless body area networks. App. Soft Comp. 37, 971-
864 982.

865 Danna, E., Rothberg, E., Le Pape, C. 2005. Exploring relaxation induced
866 neighborhoods to improve MIP solutions. *Math. Program.* **102**, 71-90.

867 Dowling, R., Kent, J., 2015. Practice and public-private partnerships in sus-
868 tainable transport governance: The case of car sharing in Sydney, Aus-
869 tralia. *Transport Policy* 40.3, 58-64.

870 European Union Innovation and Networks Executive
871 Agency (EU-INEA), 2017. Smart Cities & Communities .
872 <https://ec.europa.eu/inea/en/horizon-2020/smart-cities-communities>

873 K. Fujisawa, Y. Shinano, H. Waki: Optimization in the Real World.
874 Springer, Cham (2016)

875 Global e-Sustainability Initiative (GeSI) and Accenture Strategy, 2015.
876 #SMARTer2030 ICT Solutions for 21 st Century Challenges.
877 http://smarter2030.gesi.org/downloads/Chapter_Mobility.pdf

878 Hessel, V., 2015. Smart Mobility - A tool to
879 achieve sustainable cities. Siemens Mobility Division.
880 http://www.vt.bgu.tum.de/fileadmin/w00bnf/www/VKA/2014_15/150212_Smart_Mobility_v5_TUM

881 Goldberg, D.E., 1988. Genetic Algorithms in Search, Optimization & Ma-
882 chine Learning. Addison-Wesley, Reading.

883 IBM ILOG CPLEX, [http://www-01.ibm.com/software/integration/optimization/cplex-](http://www-01.ibm.com/software/integration/optimization/cplex-optimizer)
884 [optimizer](http://www-01.ibm.com/software/integration/optimization/cplex-optimizer).

885 Illgen, S., Höck, M., 2019. Literature review of the vehicle relocation problem
886 in one-way car sharing networks, *Transport. Res. B*, 120, 193-204.

887 Karakatic, S., Podgorelec, V., 2015. A survey of genetic algorithms for solv-
888 ing multi depot vehicle routing problem. *Applied Soft Computing* 27, 519
889 -532.

890 Kenny, P., 2013. Smart Mobility for Smart Cities Eu-
891 ropean Policy and Practice. IBTTA Global Summit,
892 https://ibtta.org/sites/default/files/documents/2014/14Prague/Kenny_Padraig.pdf

893 Li, W., Li, Y., Fan, J., Deng, H., 2017. Siting of carsharing stations based
894 on spatial multi-criteria evaluation: A case study of Shanghai EVCARD.
895 *Sustainability* 9(1), 152.

- 896 Lombard, P.L., Molocchi, A., Buscema, I., Molinario, G.,
897 2005. I costi ambientali e sociali della mobilità in Italia
898 - Quinto Rapporto (in Italian). Ferrovie dello Stato,
899 http://www.emrg.it/Lezioni_Energia_Rinnovabile/V%20Rapporto%20italiano.pdf
- 900 Lyons, G., 2016. Getting smart about urban mobility - Aligning the
901 paradigms of smart and sustainable. *Transport. Res. A.*, 115, 4-14.
- 902 Maniezzo, V., 1999. Exact and Approximate Nondeterministic Tree-Search
903 Procedures for the Quadratic Assignment Problem. *INFORMS J. Comp.*
904 11 (4) 358-369.
- 905 Maniezzo, V., Stützle, T., Voß, S., 2009. *Matheuristics: Hybridizing Meta-*
906 *heuristics and Mathematical Programming*. Springer, Heidelberg.
- 907 Martin, E.W., Shaheen, S.A., 2011. Greenhouse gas emission impacts of
908 carsharing in North America. *IEEE Trans. Intell. Transp. Syst.* 12.4, 1074-
909 1086.
- 910 Meisel, S., Merfeld, T., 2018. Economic incentives for the adoption of electric
911 vehicles: A classification and review of e-vehicle services. *Transport. Res.*
912 D 65, 264-287.
- 913 Millard-Ball, A., Schipper, L., 2011. Are we reaching peak travel? Trends in
914 passenger transport in eight industrialized countries. *Transport Reviews*
915 31, 357-378.
- 916 Motwani, R., Raghavan, P., 1995. *Randomized algorithms*. Cambridge Uni-
917 versity Press, New York.
- 918 New York City Council, 2015. A Local Law to amend the
919 administrative code of the city of New York, in rela-
920 tion to establishing a carsharing parking pilot program.
921 [http://legistar.council.nyc.gov/LegislationDetail.aspx?ID=2404408&GUID=090E4AA8-](http://legistar.council.nyc.gov/LegislationDetail.aspx?ID=2404408&GUID=090E4AA8-3B4E-4B66-95B5-DB3B5B69C1D1)
922 [3B4E-4B66-95B5-DB3B5B69C1D1](http://legistar.council.nyc.gov/LegislationDetail.aspx?ID=2404408&GUID=090E4AA8-3B4E-4B66-95B5-DB3B5B69C1D1)
- 923 Newman, P., Kenworthy, J., 2011. Peak car use: Understanding the demise
924 of automobile dependence. *World Transp. Policy & Prac.* 17, 31-39.
- 925 Pinna, F., Masala, F., Garau, C., 2017. Urban Policies and Mobility Trends
926 in Italian Smart Cities. *Sustainability* 9(4), 1-21.
- 927 Rivasplata, C., Guo, Z., Lee, R., Keyon, D., 2013. Residential On-Site Car-
928 sharing and Off-Street Parking in the San Francisco Bay Area, California.
929 *Transp. Res. Record* 2359, 68-75.

- 930 Shaheen, S., Cohen, A., Martin, E., 2010. Carsharing Parking Policy - Re-
 931 view of North American Practices and San Francisco, California, Bay Area
 932 Case Study. *Transp. Res. Record* 2187, 146-156.
- 933 Srinivas, M., Patnaik, L.M, 1994. Genetic algorithms: a survey. *Computer*
 934 27(6), 17-26.
- 935 Wang, H., Zhao, D., Meng, Q., Ong, G.P., Lee, D., 2019. A four-step method
 936 for electric-vehicle charging facility deployment in a dense city: An em-
 937 pirical study in Singapore, *Transport. Res. A* 119, 224-237.
- 938 Weikl, S., Bogenberger, K., 2013. Relocation Strategies and Algorithms for
 939 Free-Floating Car Sharing Systems. *IEEE Intell. Transp. Syst. Mag.* 5.4,
 940 100-111.
- 941 Wu, C., Le Vine, S., Sivakumar, A., Polak, J., 2019. Traveller preferences
 942 for free-floating carsharing vehicle allocation mechanisms. *Transport. Res.*
 943 C 102, 1-19.
- 944 Xu , Z., 2011. The knapsack problem with a minimum filling constraint.
 945 *Naval Research Logistics* 60(1), 56-63.
- 946 Zhao, M., Li, X., Yin, J., Cui, J., Yang, L., An, S., 2018. An integrated
 947 framework for electric vehicle rebalancing and staff relocation in one-way
 948 carsharing systems: Model formulation and Lagrangian relaxation-based
 949 solution approach, *Transport. Res. B* 117, 542-572.