Dealing with uncertainty in ATM - the Flight Level Assignment problem

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Akli FUNDO[1] Jean-Benoist LEGER[2] Dritan NACE[2]
Chenghao WANG[2]

[1] University Polytechnic of Tirana, Sheshi Nene Tereza, Tirana, Albania
[2] Université de technologie de Compiègne, Heudiasyc, Compiègne, France
{jbleger, dnace, wachengh}@hds.utc.fr

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1 Introduction

The aircraft will be required to follow a negotiated conflict-free trajectory in the future ATM (Air Traffic Management) system, accurately defined in 4 dimensions (Waypoint composed of longitude and latitude, Altitude and Time) in order to reduce the need of controller's intervention during the tactical phase. Instead of rerouting aircraft's trajectory (new waypoints may be required) in a resolution of potential conflicts, the FLA (Flight Level Assignment) problem aims at optimally assigning flight level to reduce the total number of potential en-route conflicts and most importantly the induced cost (generally expressed as delays). In practice, flights are continously impacted by delays in departure time as well as en-route delays. Unfortunately, a very small deviation of flight departure time may lead to important adittional en-route delays due to unpredicted conflicts. Therefore, leveraging the tractable algorithms proposed in [1], a first robust approach to solve the FLA is proposed to tackle the issue of data uncertainty in [2]. Optimization problems in ATM have been widely studied these last decades. We focus on some works related to a certain extent to the flight level assignment problem. Let us firstly refer to [3], Cook et al. have shown how the uncertainty affects the ATM system and its importance in building a proper model, control it and improve its performance. We focus on an important facet of ATM, namely the flight level assignment, and report some new considerations on this issue and more generally on robust optimization. This work is in continuation of [2, 4].

2 Dealing with uncertainty in FLA

Let be given a general problem : $\max Cx$ such that $Ax \leq b$ and x is binary. A typical constraint in A is expressed as : $\sum_j x_j a_{i,j} \leq a_{i,0}$ and we consider the probabilistic separated constraint problem which gives constraints as : $Pr(\sum_j x_j a_{i,j} \leq a_{i,0}) \geq 1 - \varepsilon$. Following the idea of Klopfenstein in [1], one may relax the initial set of uncertain variables into a subset of variables (I_u) that we intend to control while the other variables are assumed to take their worst-case values. The corresponding constraint will be : $Pr(\sum_{j\in I_u} a_{ij}x_j + \sum_{j\notin I_u} a_{ij}x_j) \geq 1-\varepsilon$. Comming to our FLA problem, all above is written as :

$$Pr\left(\sum_{j\in S_{il}} p_{ij}x_j^l \leq P_i^l\right) \geq 1-\epsilon \Leftrightarrow Pr\left(\sum_{j\in S_{il}\cap I_u} p_{ij}x_j^l + \sum_{j\in S_{il}\setminus I_u} \bar{p}_{ij}x_j^l \leq P_i^l\right) \geq 1-\varepsilon, \quad (1)$$

where x_j^l is a binary variable taking 1 if an aircraft j is assigned at level l, 0 otherwise. S_{il} denotes a set of flights having a potential conflict with flight i at level l. P_i^l specifies the admissible induced cost for a given flight i at level l. The induced cost (is a random value due to uncertainty of flight departure times) of flight i when resolving a potential conflict (a conflict happens when two flights at some moment have a separation distance less to a security given distance) with j is denoted by p_{ij}. This random variable takes its values in $[0, \bar{p}_{ij}]$, where $\bar{p}_{ij}$ takes the worst-case value in case of conflict.

The main difficulty of handling this kind of constraints stands in measuring the probability of sum of independent induced cost p_{ij}, knowing that they follow a mixture Gaussian distribution (as shown in [5]) and are censored in $[0, \bar{p}_{ij}]$. More precisely, the distribution of p_{ij} can be represented by a linear mixture of dirac distribution and a set of truncated Gaussian distributions which are outcome of mixture Gaussian distribution of flight departure time.

There may be several ways to deal with the model above. A general method is to treat the induced cost as independent random variables and compute a lower bound probability of their sum in (1) by using Hoeffding's inequalities [6]. However, this method provides quite weak bounds as shown in [2]. Another common method is using Monte-Carlo Simulation method as shown also in [2]. MC simulation gives very good results but it scales not well with large instances. Inspired of closed-form proposed by Beauchamp [7] for the pdf of the sum of independent Gaussian random variables which are censored in $[0, \infty)$, we are working to adapt this model to Robust FLA. The first drawn conclusions show that from the problem becomes computationally intractable. We are working on approximation of this form through 1) approximating mixture Gaussian model and 2) approximating convolution of truncated Gaussian distribution.

3 Conclusions and perspectives

In this work, we try to conduct a tractable and analytical form to evaluate the probability of feasibility of given robust solution, however such evaluation may have exponential number of items to compute. A first approximation herein is made to reduce the number of item during evaluation. A more tight approximation will be investigated in the future work. Some numerical results will be shown in trhe conference. This work is in progress.

Références

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