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Abstract. In this paper, continuing the line of our previous works, a nonlinear adaptive excitation control is designed for a synchronous generator modeled by a standard third order model on the basis of the physically available measurements of relative angular speed, active electric power and terminal voltage. The power angle, which is a crucial variable for the excitation control, is not assumed to be available for feedback, as mechanical power is also considered as an unknown variable. The feedback control is supposed to achieve transient stabilization and voltage regulation when faults occur to the turbines so that the mechanical power may permanently take any (unknown) value within its physical bounds. Transient stabilization and voltage regulation are achieved by a nonlinear adaptive controller, which generates both on-line converging estimates of the mechanical power and a trajectory to be followed by the power angle that converges to the new equilibrium point compatible with the required terminal voltage. The main contributions here, compared with our previous works, is the use of on-line computation and tracking of equilibrium power angle, and the proof of exponential stability of the closed loop system for states and parameter estimates, instead of the previous asymptotical one.

1 Introduction

The problem of stabilization of power generators is a classical power systems and control systems problem. It has been approached for some time now in many works initially by classic control and linear modern control techniques with good results, but only locally valid. Recently this problem has been treated by nonlinear methods as Lyapunov techniques (see for instance [12], [10], [7]).

Recently, feedback linearization techniques were proposed in [6], [4] and [13] to design stabilizing controls with the purpose of enlarging the stability region of the operating condition. Nonlinear adaptive controls are also proposed in [1] and [14].

The nonlinear feedback control algorithms so far proposed in the literature make use of power angle and mechanical power measurements which are physically not available and have the difficulty of determining the faulted equilibrium value which is compatible with the required terminal voltage once the fault (mechanical or electrical failure) has occurred.

Following the lines of our previous works [5] and [3] we make use of the standard third order model used in [14] (see [2] and [15]) to show that the terminal voltage, the relative angular speed and the active electric power (which are actually measurable and available for feedback) are state variables in the physical region of the state space

In this paper, continuing our previous results, we study the zero dynamics of the system, with respect to the terminal voltage, for typical values to show the existence of two equilibrium points, one stable and one unstable. This is a motivation for the use of nonlinear control instead of the classical one computed using the approximate linearized model around the stable point.

In our previous work [5], a nonlinear adaptive feedback control on the basis of physically available measurements (relative angular speed, active electric power and terminal voltage) was presented. There, when a perturbation occurred, the system was maintained in the old equilibrium point, no longer valid, causing wrong outputs, while the estimation of the new equilibrium point was made. Then, a trajectory to drive the system to the new equilibrium point was computed.

In the present work, estimation of the new equilibrium point and computation of the trajectory that drives the system there are done on-line. Global exponential stability is guaranteed for the whole closed loop system to this new previously unknown equilibrium point. There is a considerable improvement with respect to the output errors with the on-line procedure, and robustness is guaranteed by the exponential stability.

2 Dynamical model

As in [5], we consider the simplified mechanical model expressed in per unit as

$$\begin{aligned} \dot{\delta} &= \omega \\ \dot{\omega} &= -\frac{D}{H}\omega + \frac{\omega_s}{H}(P_m - P_e) \end{aligned} \tag{1}$$

where: δ(rad) is the power angle of the generator relative to the angle of the infinite bus rotating at synchronous speed ω_s; ω(rad/s) is the angular speed of the generator relative to the synchronous speed ω_s i.e. $\omega = \omega_g - \omega_s$ with ω_g being the generator angular speed; $H(s)$ is the per unit inertia constant; $D(p.u.)$ is the per unit damping constant; $P_m(p.u.)$ is the per unit mechanical input power; $P_e(p.u.)$ is the per unit active electric power delivered by the generator to the infinite bus. Note that the expression ω_s^2/ω_g is simplified as

$\omega_s^2/\omega_g \simeq \omega_s$ in the right-hand side of (1). The active and reactive powers are given by

$$P_e = \frac{V_s E_q}{X_{ds}} sin(\delta) \tag{2}$$

$$Q = \frac{V_s}{X_{ds}} E_q \cos(\delta) - \frac{V_s^2}{X_{ds}} \tag{3}$$

where: $E_q(p.u.)$ is the quadrature's EMF; $V_s(p.u.)$ is the voltage at the infinite bus; $X_{ds} = X_T + \frac{1}{2}X_L + X_d(p.u.)$ is the total reactance which takes into account $X_d(p.u.)$, the generator direct axis reactance, $X_L(p.u.)$, the transmission line reactance, and $X_T(p.u.)$, the reactance of the transformer, and the definition $X_S \triangleq X_T + \frac{1}{2}X_L$. The quadrature EMF, E_q, and the transient quadrature EMF, E'_q, are related by

$$E_q = \frac{X_{ds}}{X'_{ds}} E'_q - \frac{X_d - X'_d}{X'_{ds}} V_s cos(\delta) \tag{4}$$

while the dynamics of E'_q are given by

$$\frac{dE'_q}{dt} = \frac{1}{T_{d0}}(K_c u_f - E_q) \tag{5}$$

in which: $X'_{ds} = X_T + \frac{1}{2}X_L + X'_d(p.u.)$ with X'_d denoting the generator direct axis transient reactance; $u_f(p.u.)$ is the input to the (SCR) amplifier of the generator; K_c is the gain of the excitation amplifier; $T_{d0}(s)$ is the direct axis short circuit time constant.

Especially because P_e is measurable while E'_q is not, it is convenient to express the state space model using (δ, ω, P_e) as states, which are equivalent states as long as the power angle δ remains in the open set $0 < \delta < \pi$, as follows.

$$\begin{aligned}
\dot{\delta} &= \omega \\
\dot{\omega} &= -\frac{D}{H}\omega - \frac{\omega_s}{H}(P_e - P_m) \\
\dot{P}_e &= -\frac{1}{T'_{d0}}P_e + \frac{1}{T'_{d0}}\left\{\frac{V_s}{X_{ds}} sin(\delta)[K_c u_f + T'_{d0}(X_d - X'_d)\frac{V_s}{X'_{ds}}\omega sin(\delta)]\right. \\
&\quad \left. + \ T'_{d0} P_e \omega \cot(\delta)\right\}
\end{aligned} \tag{6}$$

in which (δ, ω, P_e) is the state and u_f is the control input.

Note that when δ is near 0 or near π the effect of the input u_f on the overall dynamics is greatly reduced. Note also that here we have introduced the notation

$$T'_{d0} = \frac{X'_{ds}}{X_{ds}} T_{d0}$$

The generator terminal voltage modulus is given by

$$V_t = \left(\frac{X_s^2 P_e^2}{V_s^2 sin^2(\delta)} + \frac{X_d^2 V_s^2}{X_{ds}^2} + \frac{2 X_s X_d}{X_{ds}} P_e cot(\delta) \right)^{\frac{1}{2}}$$

which is the output of the system to be regulated to its reference value $V_{tr} = 1(p.u.)$

2.1 Power Angle

The power angle is not measurable and is also not a physical variable to be regulated; the only physical variable to be regulated is the output V_t, while (V_t, ω, P_e) are measured and are available for feedback action.

As a matter of fact (V_t, ω, P_e) is an equivalent state for the model (6) (as proved in [5])

$$\delta = arccotg \left(\frac{V_s}{X_s P_e} \left(-\frac{X_d V_s}{X_{ds}} + \sqrt{V_t^2 - \frac{X_s^2}{V_s^2} P_e^2} \right) \right) \tag{7}$$

If the parameters (V_s, X_s, X_d, X_{ds}) are known, state measurements are available. From (7) it follows that in order to regulate the terminal voltage V_t to its reference value $(V_{tr} = 1(p.u.))$ δ should be regulated to

$$\delta_s = arccotg \left(\frac{V_s}{X_s P_m} \left(-\frac{X_d V_s}{X_{ds}} + \sqrt{V_{tr}^2 - \frac{X_s^2}{V_s^2} P_m^2} \right) \right) \tag{8}$$

From a physical viewpoint the natural choice of state variables is (V_t, ω, P_e) which are measurable. The state feedback control task is to make the stability region of the stable equilibrium point $(V_{tr}, 0, P_m)$ as large as possible. In fact the parameter P_m may abruptly change to an unknown faulted value P_{mf} due to turbine failures so that $(V_{tr}, 0, P_m)$ may not belong to the region of attraction of the faulted equilibrium point $(V_{tr}, 0, P_{mf})$. The state feedback control should be design so that typical turbine failures do not cause instabilities and consequently loss of synchronism and inability to achieve voltage regulation.

A reduction from P_m to $(P_m)_f$ of the mechanical power generated by the turbine, changes the operating condition: the new operating condition $(\delta_s)_f$ is the solution of

$$-\frac{(P_m)_f}{P_m} + \frac{sin(\delta)_f}{sin(\delta_s)} = 0$$

and since $(P_m)_f$ is typically unknown, the corresponding new stable operating condition $(\delta_s)_f$ is also unknown.

2.2 Zero Dynamics

If we regulate the voltage output (V_t) to its reference value (V_{tr}), the zero dynamics will be given by

$$\begin{aligned}
\dot{\delta} &= \omega \\
\dot{\omega} &= -\frac{D}{H}\omega + \frac{\omega_s}{H}(P_m + \frac{X_d}{X_s X_{ds}} V_s^2 \sin(\delta)\cos(\delta) \\
&- \frac{V_s}{X_s}\sin(\delta)\sqrt{V_{tr}^2 - \frac{X_d^2}{X_{ds}^2} V_s^2 \sin^2(\delta)}
\end{aligned}$$

which are very complex, and for some initial conditions or parameters values may become unstable.

If we use the values defined in [5] we may plot $\dot{\omega}$ as a function of ω and δ, there will then be two points of δ that satisfy the equilibrium of the zero dynamics. These points (the two real ones) are $\delta = 1.26$ and $\delta = 2.96$.

Linearizing the system around each one of these two points, we will have as eigenvalues the pairs $[-0.31 - 7.58I, -0.31 + 7.58I]$ and $[-13.86, +13.86]$ respectively.

Thus we have shown the existence of two equilibrium points, one stable and one unstable. There will then be an attraction region for the stable one. If one is driven out of this region (by initial conditions or by a fault), the controller will not act regulating ω and δ and the system will become unstable. This shows that using the output error voltage as the only error signal may be dangerous as one may regulate this voltage and loose stability.

3 Adaptive Controller and Main Result

In this section we present the calculation of the adaptive controller as in [5]. Our main result is then to prove the global exponential stability of the whole system, with parameter exponential convergence.

The model (6) is rewritten substituting P_m by $\theta(t)$ which is a possibly time-varying disturbance: this parameter is assumed to be unknown and to belong to the compact set $[\theta_m, \theta_M]$: where the lower and upper bounds θ_m, θ_M are known.

Let $f(\theta, x)$ be a C^3 reference signal to be tracked. Define $(\lambda_1 > 0)$

$$\begin{aligned}
\tilde{\delta}(t) &= \delta(t) - f(\theta, x) \\
\omega^* &= -\lambda_1 \tilde{\delta} + f(\dot{\theta}, x) \\
\tilde{\omega} &= \omega - \omega^* = \omega + \lambda_1 \tilde{\delta} - f(\dot{\theta}, x)
\end{aligned}$$

so that the first two equations in (6) are rewritten as

$$
\begin{aligned}
\dot{\tilde{\delta}} &= -\lambda_1\tilde{\delta} + \tilde{\omega} \\
\dot{\tilde{\omega}} &= -\frac{D}{H}\omega + \frac{\omega_s}{H}(\theta(t) - P_e) - \lambda_1^2\tilde{\delta} + \lambda_1\tilde{\omega} - f(\ddot{\theta}, x)
\end{aligned}
$$

Define ($\lambda_2 > 0, k > 0$) the reference signal for P_e as

$$
P_e^* = \frac{H}{\omega_s}\left\{-\frac{D}{H}\omega - \lambda_1^2\tilde{\delta} + \lambda_1\tilde{\omega} - f(\ddot{\theta}, x) + \lambda_2\tilde{\omega} + \tilde{\delta} + \frac{1}{4}k\left(\frac{\omega_s}{H}\right)^2\tilde{\omega}\right\} + \hat{\theta}
$$

while $\hat{\theta}$ is an estimate of $\theta = P_m$ and $\tilde{P}_e = P_e - P_e^*$ so that (6) may be rewritten as ($\tilde{\theta} = \theta - \hat{\theta}$)

$$
\begin{aligned}
\dot{\tilde{\delta}} &= -\lambda_1\tilde{\delta} + \tilde{\omega} \\
\dot{\tilde{\omega}} &= -\tilde{\delta} - \lambda_2\tilde{\omega} - \frac{\omega_s}{H}\tilde{P}_e - \frac{k}{4}\left(\frac{\omega_s}{H}\right)^2\tilde{\omega} + \frac{\omega_s}{H}\tilde{\theta} \\
\dot{\tilde{P}}_e &= -\frac{1}{T'_{d0}}P_e + \frac{V_s}{X_{ds}T'_{d0}}sin(\delta)K_c u_f + \frac{(X_d - X'_d)V_s^2}{X_{ds}X'_{ds}}\omega sin^2(\delta) + P_e\omega\cot(\delta) \\
&\quad - \frac{H}{\omega_s}\left\{\left(-\lambda_1^2 + 1 + \lambda_1\frac{D}{H}\right)(-\lambda_1\tilde{\delta} + \tilde{\omega})\right. \\
&\quad \left. + \left(-\frac{D}{H} + \lambda_1 + \lambda_2 + \frac{k}{4}\left(\frac{\omega_s}{H}\right)^2\right)\left(-\frac{D}{H}\omega - \lambda_1^2\tilde{\delta} + \lambda_1\tilde{\omega} - \frac{\omega_s}{H}P_e - f(\ddot{\theta}, x)\right)\right\} \\
&\quad - \left(-\frac{D}{H} + \lambda_1 + \lambda_2 + \frac{k}{4}\left(\frac{\omega_s}{H}\right)^2\right)\hat{\theta} - \dot{\hat{\theta}} \\
&\quad - \left(-\frac{D}{H} + \lambda_1 + \lambda_2 + \frac{k}{4}\left(\frac{\omega_s}{H}\right)^2\right)\tilde{\theta} + \frac{D}{\omega_s}f(\ddot{\theta}, x) + \frac{H}{\omega_s}f(\dot{\ddot{\theta}}, x)
\end{aligned}
$$

Defining $\lambda_3 > 0$, we then propose the control law

$$
\begin{aligned}
u_f &= \frac{T'_{d0}X_{ds}}{V_s K_c \sin(\delta)}\phi_0 \\
\phi_0 &= \frac{1}{T'_{d0}}P_e - \frac{(X_d - X'_d)}{X_{ds}X'_{ds}}V_s^2\omega\sin^2(\delta) - P_e\omega\cot(\delta) \\
&\quad + \frac{H}{\omega_s}\left\{\left(-\lambda_1^2 + 1 + \lambda_1\frac{D}{H}\right)(-\lambda_1\tilde{\delta} + \tilde{\omega})\right. \\
&\quad \left. + \left(-\frac{D}{H} + \lambda_1 + \lambda_2 + \frac{k}{4}\left(\frac{\omega_s}{H}\right)^2\right)\left(-\frac{D}{H}\omega - \lambda_1^2\tilde{\delta} + \lambda_1\tilde{\omega} - \frac{\omega_s}{H}P_e - f(\ddot{\theta}, x)\right)\right\} \\
&\quad + \left(-\frac{D}{H} + \lambda_1 + \lambda_2 + \frac{k}{4}\left(\frac{\omega_s}{H}\right)^2\right)\hat{\theta} + \dot{\hat{\theta}} \\
&\quad - \frac{k}{4}\left(-\frac{D}{H} + \lambda_1 + \lambda_2 + \frac{k}{4}\left(\frac{\omega_s}{H}\right)^2\right)^2\tilde{P}_e - \frac{D}{\omega_s}f(\ddot{\theta}, x) - \frac{H}{\omega_s}f(\dot{\ddot{\theta}}, x) - \lambda_3\tilde{P}_e + \frac{\omega_s}{H}\tilde{\omega}
\end{aligned}
$$

then, the closed loop system becomes

$$
\begin{aligned}
\dot{\tilde{\delta}} &= -\lambda_1\tilde{\delta} + \tilde{\omega} \\
\dot{\tilde{\omega}} &= -\tilde{\delta} - \lambda_2\tilde{\omega} - \frac{\omega_s}{H}\tilde{P}_e - \frac{k}{4}\left(\frac{\omega_s}{H}\right)^2\tilde{\omega} + \frac{\omega_s}{H}\tilde{\theta} \\
\dot{\tilde{P}}_e &= \frac{\omega_s}{H}\tilde{\omega} - \lambda_3\tilde{P}_e - \left(-\frac{D}{H} + \lambda_1 + \lambda_2 + \frac{k}{4}\left(\frac{\omega_s}{H}\right)^2\right)\tilde{\theta} \\
&- \frac{k}{4}\left(-\frac{D}{H} + \lambda_1 + \lambda_2 + \frac{k}{4}\left(\frac{\omega_s}{H}\right)^2\right)^2\tilde{P}_e
\end{aligned}
\tag{9}
$$

The adaptation law is (γ is a positive adaptation gain)

$$
\dot{\hat{\theta}} = \gamma Proj\left(\left(\tilde{P}_e\left(\frac{D}{H} - \lambda_1 - \lambda_2 - \frac{k}{4}\left(\frac{\omega_s}{H}\right)^2\right) + \tilde{\omega}\frac{\omega_s}{H}\right), \hat{\theta}\right)
\tag{10}
$$

where $Proj(y, \hat{\theta})$ is the smooth projection algorithm introduced in [11]

$$
\begin{aligned}
&Proj(y,\hat{\theta}) = y, && if\ p(\hat{\theta}) \leq 0 \\
&Proj(y,\hat{\theta}) = y, && if\ p(\hat{\theta}) \geq 0\ \ and\ \ \langle gradp(\hat{\theta}), y\rangle \leq 0 \\
&Proj(y,\hat{\theta}) = [1 - p(\hat{\theta})|grad\ p(\hat{\theta})|], && otherwise
\end{aligned}
\tag{11}
$$

with

$$
p(\theta) = \frac{(\theta - \frac{\theta_M + \theta_m}{2})^2 - (\frac{\theta_M - \theta_m}{2})}{\epsilon^2 + 2\epsilon(\frac{\theta_M - \theta_m}{2})}
$$

for ϵ an arbitrary positive constant which guarantees in particular that:

$$
\begin{aligned}
&i)\ \ \theta_m - \epsilon \leq \hat{\theta}(t) \leq \theta_M + \epsilon \\
&ii)\ \ |Proj(y,\hat{\theta})| \leq |y| \\
&iii)\ (\theta - \hat{\theta})Proj(y,\hat{\theta}) \geq (\theta - \hat{\theta})y
\end{aligned}
$$

Consider the function

$$
W = \frac{1}{2}(\tilde{\delta}^2 + \tilde{\omega}^2 + \tilde{P}_e{}^2)
\tag{12}
$$

whose time derivative, according to (9), is

$$
\begin{aligned}
\dot{W} &= -\lambda_1\tilde{\delta}^2 - \lambda_2\tilde{\omega}^2 - \lambda_3\tilde{P}_e{}^2 + \tilde{\omega}\frac{\omega_s}{H}\tilde{\theta} - \frac{k}{4}\left(\frac{\omega_s}{H}\right)^2\tilde{\omega}^2 \\
&- \left(-\frac{D}{H} + \lambda_1 + \lambda_2 + \frac{k}{4}\left(\frac{\omega_s}{H}\right)^2\right)\tilde{\theta}\tilde{P}_e - \frac{k}{4}\left(-\frac{D}{H} + \lambda_1 + \lambda_2 + \frac{k}{4}\left(\frac{\omega_s}{H}\right)^2\right)^2\tilde{P}_e{}^2
\end{aligned}
$$

Completing the squares, we obtain the inequality

$$\dot{W} \le -\lambda_1 \tilde{\delta}^2 - \lambda_2 \tilde{\omega}^2 - \lambda_3 \tilde{P_e}^2 + \frac{2}{k}\tilde{\theta}^2 \tag{13}$$

which guarantees arbitrary $\mathcal{L}_\infty$ robustness from the parameter error $\tilde{\theta}$ to the tracking errors $\tilde{\delta}, \tilde{\omega}, \tilde{P}_e$.

The projection algorithms (11) guarantee that $\tilde{\theta}$ is bounded, and, by virtue of (12) and (13), that $\tilde{\delta}$, $\tilde{\omega}$ and $\tilde{P_e}$ are bounded. Therefore, $\dot{\hat{\theta}}$ is bounded. Integrating (13), we have for every $t \ge t_0 \ge 0$

$$-\int_{t0}^{t} (\lambda_1 \tilde{\delta}^2 + \lambda_2 \tilde{\omega}^2 + \lambda_3 \tilde{P_e}^2) d\tau + \frac{2}{k}\int_{t_0}^{t} \tilde{\theta}^2 d\tau \ge W(t) - W(t_0)$$

Since $W(t) \ge 0$ and, by virtue of the projection algorithm (11),

$$\tilde{\theta}(t) \le \theta_M - \theta_m + \epsilon$$

it follows that

$$\int_{t0}^{t} (\lambda_1 \tilde{\delta}^2 + \lambda_2 \tilde{\omega}^2) d\tau \le W(t_0) + \frac{2}{k}(\theta_M - \theta_m + \epsilon)^2 (t - t_0)$$

which, if $W(t_0) = 0$ (i.e. t_0 is a time before the occurrence of the fault), implies arbitrary $\mathcal{L}_2$ attenuation (by a factor k) of the errors $\tilde{\delta}$ and $\tilde{\omega}$ caused by the fault. To analyze the asymptotic behavior of the adaptive control, we consider the function

$$V = \frac{1}{2}(\tilde{\delta}^2 + \tilde{\omega}^2 + \tilde{P_e}^2) + \frac{1}{2}\frac{1}{\gamma}\tilde{\theta}^2$$

The projection estimation algorithm (11) is designed so that the time derivative of V satisfies

$$\dot{V} \le -\lambda_1 \tilde{\delta}^2 - \lambda_2 \tilde{\omega}^2 - \lambda_3 \tilde{P_e}^2 \tag{14}$$

Integrating (14), we have

$$\lim_{t\to\infty} \int_{t0}^{t} (\lambda_1 \tilde{\delta}^2 + \lambda_2 \tilde{\omega}^2 + \lambda_3 \tilde{P_e}^2) d\tau \le V(0) - V(\infty) < \infty$$

From the boundedness of $\dot{\tilde{\delta}}, \dot{\tilde{\omega}}$ and $\dot{\tilde{P}}_e$, and Barbalat's Lemma (see [9], [8]) it follows that

$$\lim_{t\to\infty} \left\| \begin{bmatrix} \tilde{\delta}(t) \\ \tilde{\omega}(t) \\ \tilde{P}_e(t) \end{bmatrix} \right\| = 0$$

We may now rewrite the closed loop system following the normal form:

$$\dot{\tilde{x}} = A\tilde{x} + \Omega^T\tilde{\theta}$$
$$\dot{\tilde{\theta}} = -\Lambda\Omega\tilde{x}$$

which leads to:

$$\dot{\tilde{x}} = \begin{bmatrix} -\lambda_1 & 1 & 0 \\ -1 & -(\lambda_2 + c_2) & -\frac{\omega_s}{H} \\ 0 & \frac{\omega_s}{H} & -\left(\lambda_3 + \frac{k}{4}c_1^2\right) \end{bmatrix} \tilde{x} + \begin{bmatrix} 0 \\ \frac{\omega_s}{H} \\ -c_1 \end{bmatrix} \tilde{\theta} \tag{15}$$
$$\dot{\tilde{\theta}} = -\gamma \left[0 \ \frac{\omega_s}{H} \ -c_1 \right] \tilde{x}$$

where c_1 and c_2, as c_3 on next equation, are constants. And then computing:

$$\Omega\Omega^T = \frac{\omega_s^2}{H^2} + c_1^2 = c_3 > 0$$

We then may show by persistency of excitation that $\tilde{x}$ and $\tilde{\theta}$ will be globally (for the model validity region) exponentially stable, then all error signals go exponentially to zero, for all C^3 $f(\theta, x)$. This is also valid then for the particular case where $f(\theta, x) = \delta_r$ where δ_r is given by equation (8). But, since δ_r is a one-to-one smooth function of θ, it will converge to the correct equilibrium value δ_s as $\tilde{\theta}$ converges to 0, i.e. the reference trajectory will converge to the unknown equilibrium point and then $\lim_{t\to\infty}(\delta - \delta_s) = 0$.

4 Simulation results

In this section some simulation results are given with reference to the eight-machine power system network reported in [4] with the following data:

$\omega_s = 314.159$ rad/s $\quad D = 5$ p.u. $\quad H = 8s$
$T_{d0} = 6.9s \quad K_c = 1 \quad X_d = 1.863$ p.u.
$X_d' = 0.257$ p.u. $\quad X_T = 0.127$ p.u. $X_L = 0.4853$ p.u.

The operating point is $\delta_s = 72^o$, $P_m = 0.9$ p.u., $\omega_0 = 0$ to which corresponds $V_t = 1$ p.u., with $V_s = 1$ p.u..

It was considered a fast reduction of the mechanical input power, and simulated according to the following sequences

1. The system is in pre-faulted state.
2. At $t = 0.5s$ the mechanical input power begins to decrease.
3. At $t = 1.5s$ the mechanical input power is 50% of the initial value.

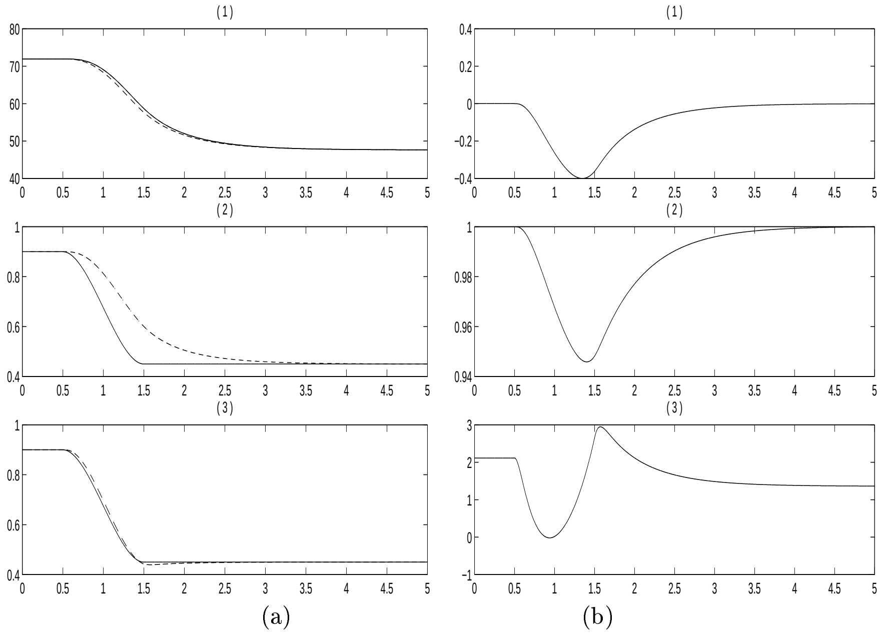

Fig.1. a1) Real δ (-), Calculated δ (-.), δ_r (- -) a2) P_m (-),$\hat{\theta}$ (- -) a3) P_m (-), P_e (- -) b1) ω b2) V_t b3) Control signal

The simulations were carried out using as control parameters
$\lambda_i = 20 \quad 1 \leq i \leq 3$
$\gamma = 1 \quad k = 0.1$

Fig. 1.a1) shows that the calculated power angle matches perfectly the real one. It also shows that the trajectory for the power angle (δ_r) goes smoothly to its final value, and that δ follows it perfectly.

In Fig. 1.a2) one may see that the estimation of the mechanical power is accurate, it may be very fast if we change the parameters, and specially if larger errors are accepted for the state and output variables. This may be understood by looking at equation (10).

The electrical power is also correctly driven to the mechanical one as we see in Fig. 1.a3). The same may be observed in Fig. 1.b1) for the rotor velocity.

Fig. 1.b2) shows how the output voltage drops during the fault, and goes to its correct value when the system is driven to the correct equilibrium point. If estimation was not correct, there would be a steady state error.

Finally, one can see in Fig. 1.b3) that the control signal is very smooth and is kept inside the prescribed bounds.

Note that during all time, errors are very small. They can be made even smaller by increasing the parameter k.

5 Conclusions

In a previous work we have computed the zero dynamics of the system with respect to the terminal voltage having then obtained a highly nonlinear second order dynamics. Based on typical values, we show here that there is one stable and one unstable points, and then, an attraction region for the stable one. This is a motivation to be concerned with all the state vector and not only with the output voltage since, even keeping it regulated to its reference value one may find instability for the whole system. It is also a motivation for the nonlinear control as the system may always be driven to an unstable point where a linear control, specially one designed using the linearized system around the stable point, will not be able to stabilize it.

Finally, using the same controller as in previous works, we prove the exponential stability of the closed loop system. We also prove that the estimate of the parameter converges exponentially to its true value. The system may be driven arbitrarily fast to the new equilibrium point. The only restriction will be the magnitude of the control signal and the accepted error signal.

Our present research includes the problem of transmission line failure. We have also started the procedure to do practical implementations to verify ours simulations. The multi-machine problem will then be the next step.

Acknowledgments

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