Kinetic Modelling of Individual Starch Granules Swelling

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Abstract

A novel kinetic model is proposed for individual starch granule swelling based on swelling kinetics time series data. The data was acquired from time-lapse microscopy observations of starch granules in a dilute suspension and subsequent image analysis. Chemically modified waxy-maize starch was used. The size evolution of many granules when subjected to various hydro-thermal treatments on a temperature-controlled stage was determined. Observations showed that swelling is not an instantaneous process and varies from a granule to another. A model has been developed which takes into account the effect of temperature evolution and variability between the granules on their swelling. This model can contribute to robust multi-scale simulation of industrial processes involving starch suspensions.

Keywords: Starch Granule, Image analysis, Swelling kinetics, Time-series data, Modelling

1. Introduction

Starch is the predominant food reserve in plants and provides 70-80% of the calories consumed by humans worldwide in various forms of food products (Bertolini, 2010). In 2000, the world starch production was estimated to be around 48.5 million tons, including native and modified starches. Hence, starch has been a perennial subject of research through decades. Starch granules majorly consists of two polymers: amylose and amylopectin. Both amylose and amylopectin have α -glucose as the monomer, however, their difference arises

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from their linkages. Amylose has $\alpha(1-4)$ and is linear whereas amylopectin has $\alpha(1-6)$ and is highly branched. The ratio of amylose and amylopectin in the starch granule depends on its source and botanical origin. This in turn has drastic consequences on the physical properties such as rheology (Banks et al., 1974; Hoover, 2001; Srichuwong et al., 2005; Visser et al., 1997). Molecular properties such as molecular weight distribution and degree of amylopectin branching are also known to impact the functional attributes at the macro-scale (Blennow et al., 2001; Fredriksson et al., 1998; Singh et al., 2007).

Gelatinization is the process in which starch granules undergo an irreversible transformation when subjected to thermomechanical treatment in the presence of water. During this process starch granules swell by imbibing water and increase the overall viscosity of the suspension. It is also sometimes referred to as starch pasting in literature. Starch granules are semi-crystalline with concentric rings of crystalline and amorphous phases (Ratnayake and Jackson, 2008). On the microscopic scale, the process corresponds to the melting of crystallites, loss of birefringence, leaching of amylose into the continuous phase and sometimes rupture of the granular structure (Atwell, 1988; Biliarderis, 1992). During gelatinization, melting of crystalline region in the granule occurs and depending on the origin of the starch, the range for this transition is between 60 and 70 °C (Slade and Levine, 1995). The evolution of viscosity during gelatinization is also influenced by the process parameters such as concentration, shear rate, thermal history, pH and presence of other ingredients (Thomas and Atwell, 1999; Malumba et al., 2018). For high amylopectin cereal starches such as waxy maize starch, the granules tend to hydrate with ease, swell rapidly and rupture to a great extent which results in loss of paste viscosity. The rupture of the granules can be avoided by chemically modifying the starch. This involves cross-linking via chemical bonds that act as bridges between the two macromolecules. The introduction of functional groups into the starch molecules such as acetylation results in lower amylose leaching properties of starch. The resulting modified starches have markedly altered physicochemical properties (Ashogbon and Akintayo, 2014). Waxy maize starch is used in a wide variety of applications in the food industry owing to its stability (Tattiyakul and Rao, 2000).

The sensory perception of viscosity is an important mouthfeel sensation (Szczesniak,

1963; Wood, 1968). Starch is used as a viscosity enhancer in many food formulations. Hence, it is critical to accurately predict viscosity changes both from a product development standpoint and an engineering design standpoint. During the thermal processing of liquid food product containing starch, viscosity changes must be known to accurately estimate process variables as the process is highly coupled in terms of heat transfer, momentum transfer and transformation (Plana-Fattori et al., 2016). This warranted decades of research into the sole aspect of viscosity predictions and significant literature exist regarding the chief aspects which dictate the same. They are (a) the suspending medium’s viscosity η_0 , (b) the granule’s volume fraction ϕ , (c) the granule’s deformability, and (d) the imposed shear rate (Chen et al., 2007). Nayouf et al. (2003) indicated that volume fraction of granules can be estimated if the mean granule size (D_t) can be accurately determined.

Increasingly researchers started defining progress of gelatinization in terms of the increase in diameter of granules (Anuntagool et al., 2017; Okechukwu and Rao, 1996; Bagley and Christianson, 1982; Bakshi and Singh, 1980; Suzuki et al., 1976; Doublier et al., 1987; Kokini et al., 1992). Alternate methods used to define gelatinization are based on DSC, rheology or birefringence (Cui et al., 2014; Choi and Kerr, 2004; Dolan and Steffe, 1990; Biliaderis and Prokopowich, 1994; Chang and Liu, 1991; I’anson et al., 1990; Kohyama and Nishinari, 1991; Miles et al., 1985; Slade and Levine, 1991). However these are all equivalent indications of the same process. Granule swelling occurs in two phases, first is the hydration phase where there is a slight reversible increase in the diameter of the granules typically occurring in the temperature range 30–60 °C and a second irreversible phase where there is a large increase in diameter occurring at temperatures above 60 °C (Okechukwu and Rao, 1996). Some authors reported that mean size and size distribution of swollen granules were correlated with suspension consistency and flow behavior index (Okechukwu and Rao, 1996; Steeneken, 1989; Evans and Lips, 1990). Thus, there have been multiple attempts at kinetic modelling this phenomena to predict the particle size evolution.

Kubota et al. (1979) attempted to model this phenomena with Arrhenius law and proposed a first order kinetic equation with respect to non-dimensionalised consistency index. Okechukwu and Rao (1995) introduced a similar rate equation with another dimensionless

variable called extent of gelatinization based on mean particle diameter of the population. Lagarrigue et al. (2008) used the same formulation but argued that the reaction is second order. Krüger et al. (2003) proposed a kinetic model which suggest the rate of increase in diameter is proportional to the inverse of the mean diameter of the population diameter with Arrhenius behavior and reaching a plateau at maximum diameter. Some authors also suggested a set of reactions to describe gelatinization (Kokini et al., 1992). Malumba et al. (2013) compared a wide range of rate expressions (Nth order kinetic, Diffusional, Phase boundary and Avrami-Erofeev) to predict the isothermal swelling kinetics at different temperatures and showed that the predictability was best for Nth order reaction models especially second order and third order. They concluded that starch granule gelatinization is not a simple water diffusion or a boundary phenomenon. The heat transferred inside the granules induces phenomena such as melting of the crystallite, interaction between carbohydrate polymers. This includes inter-chain and intra-chain polymer folding and hydrogen bonding proceeding with semi-cooperative swelling of granules. They also argued that activation energy reported in literature must be called apparent activation energy as the same, changed based on the form and order of the rate equation assumed. Plana-Fattori et al. (2016) have used a modified form of rate constant with a cutoff minimum temperature instead of the traditional Arrhenius form, which they substantiated with experiments.

All the above-mentioned works have always been attempts to model the swelling kinetics of the population mean but not at the individual granule scale. Desam et al. (2018) tried to model the swelling of a single granule based on diffusion kinetics and Flory-Huggins theory of polymer mixing, but the model assumes all granules to have similar cross-links, tortuosity, and elastic restoring force. In this work, we propose a novel kinetic modeling at the individual granule scale rather than the population mean. Time-series data obtained from image processing of hot stage coupled microscope time-lapse images was used for building and fitting the model. This model captures the inter granule variability of starch swelling in water. This variability has not been quantified and addressed adequately in the previous modeling works. We try to capture these nuances in this work albeit it empirically and thus the parameters obtained are valid for chemically stabilized cross-linked waxy maize

95 starch (acetylated distarch adipate, C*Tex 06205), provided by Cargill (Baupite, France).
96 The method developed here enables modeling at the granular scale allowing one to capture
97 the dynamics better for multi-scale modeling translating into wide-reaching consequences
98 in plant equipment design, optimization, differentiated product development, simulation,
99 robust process development and robust model based process control systems.

100 **2. Materials and methods**

101 *2.1. Experimental method*

102 Chemically stabilized cross-linked waxy maize starch (C*Tex 06205), provided by Cargill
103 (Baupite, France), cross-linked via adipates, was used in $0.5g.kg^{-1}$ suspensions of starch in
104 water. This starch is composed of 99 % of amylopectin and less than 1% of amylose. This
105 type of starch does not lead to disruption and no release of amylose content is expected.
106 In terms of functionality, it has notably a good resistance to shear forces, a good storage
107 stability and a high paste clarity. The suspension was preheated to 50°C under gentle
108 stirring with a magnetic pellet. Using a dropper sampling was performed and sample was
109 transferred to the microscopic glass slides with adhesive spacers of 250 μm . Samples were
110 observed under 50X magnification using an Olympus BX-51 microscope (Olympus Optical
111 Co. Ltd., Tokyo, Japan). Images were acquired using a Basler A102fc digital camera (Basler
112 AG, Ahrensburg, Germany). Each observation is an 8 bit image of 1388×1038 pixels which
113 represents a $361 \times 270 \mu m^2$ area. Microscope glass slides with samples were placed on a
114 Linkam LTS120 stage (Linkam Scientific Instruments, Surrey, UK). The temperature was
115 controlled with the Linksys32 software. The samples were subjected to different temperature
116 profiles, starting with a holding at 50 °C for 1 min (figure 1a). Some samples were observed
117 under the microscope at different constant temperatures from 55 °C to 62 °C for 1 hour; no
118 image processing was carried out for these experiments as the objective of the same was only
119 to observe if the granules began to swell and to estimate the latency time before swelling.

2.2. Image data analysis and notation

The Java plugin for ImageJ software developed by Deslandes et al. (2019) was used to process the images. Ferret diameter is a length of an object measured along a specified direction. Thus, along different direction we have different Ferret lengths. We note the minimum and maximum Ferret diameter of a granule (μm), m and M respectively. The diameter of the granule is taken as the geometric mean of the Ferret lengths:

$$D = \sqrt{Mm}. \quad (1)$$

Figure 1

Image analysis and subsequent sigmoid curve fitting for the resulting time series data of diameter as a function of time was done following the procedure outlined in Deslandes et al. (2019). The parameters D_i , D_f , δ and $t_{1/2}$ denote the initial diameter (μm), the final diameter (μm), the swelling rate (s^{-1}) and the half-swelling time (s) of the granule respectively. These morphological and kinetics parameters are estimated by performing least-squares fitting on the size evolution data obtained from the image processing onto the sigmoid curve given by equation 2 (Deslandes et al., 2019). A typical diameter evolution of a granule with time along the least square fitting of the sigmoid curve is depicted in figure 1b.

$$\tilde{D}(t) = \frac{D_f - D_i}{1 + e^{-\delta(t-t_{1/2})}} + D_i \quad (2)$$

The beginning of the 1 minute hold time at 50 °C is taken as the origin of time and different temperature profiles as depicted in the figure 1a are imposed on the sample. The individual granule swelling duration (Δt) is estimated by extrapolation of the tangent at the half-swelling time. It can be calculated from the parameter δ using the following equation:

$$\Delta t = \frac{4}{\delta}. \quad (3)$$

We then define t_{onset} , the individual swelling onset time by

$$t_{onset} = t_{1/2} - \frac{\Delta t}{2}. \quad (4)$$

3. Experimental results and observation

3.1. Observation at constant temperature between 55 °C and 62 °C

Observing the samples under the microscope at different constant temperatures starting from 55 °C to 62 °C for 1 hour was the objective to determine the minimum temperature at which swelling starts. It was observed as shown in images of figure 2, the gradual increase in fraction of granules swollen as temperature increases from 57 °C to 62 °C. We can safely say that it required at least around 60 °C to observe swelling in majority of the granules, this is consistent with the literature reported values (Slade and Levine, 1995). However, this may not be entirely true as we saw some granules swell at 58 °C and some granules not swollen at 62 °C even after a duration of 1 hour. Nevertheless, in industrial processes, the maximal temperature is always much greater than 70 °C and residence time in heat exchangers is of some minutes only. In light of the practicality and the relevance to the industrial process, we make this assumption that the minimum swelling temperature is of 60 °C for all the granules.

For the experiments carried for 1 hour (1 image per 5 seconds) at 62 °C, the latency time in the onset of swelling was manually noted from the diameter vs time data obtained after post processing using the plugin developed by Deslandes et al. (2019). Figure 3 shows the experimental cumulative distribution (47 granules) and an exponential curve fit for the same.

Figure 2: Starch granules at the beginning and after 1 hour at different temperatures

We use this information in the section 4.1 to justify the choice of mathematical formulation.

3.2. Diameter increase during temperature ramp

The results of the image processing and data analysis during different temperature ramps are summarized in the table 1. Note that for the case E with maximum temperature 65°C only the completely swollen granules were used to compute the sigmoid curve parameters.

Figure 3: Experimental delay times for swelling onset at 62 °C

163 All the reported cases had a lapse rate of 1 image per second. The onset times reported
 164 here are from the beginning of the 1 min hold time at 50 °C.

Case	$T_{max}(^{\circ}C)$	ramp rate ($^{\circ}C/min$)	hold time (min)	Number of gran- ules	t_{onset} (s)		Δt (s)		Df/Di	
					mean	std	mean	std	mean	std
A	90	10	2	163	180	18	29	21	2.30	0.32
B	90	5	2	142	276	34	65	46	2.36	0.33
C	70	5	2	48	274	44	51	34	2.38	0.43
D	68	5	15	41	250	37	98	92	2.35	0.25
E	65	5	15	42	293	122	300	169	2.07	0.49

Table 1: Summary of Experimental conditions and swelling data with sigmoid fits

165 Equation 5 defines the degree of swelling parameter which is used to track the swelling
 166 of an individual granule.

$$S(t) = \frac{D(t) - D_i}{D_f - D_i} \quad (5)$$

167 Thus, $S(t)$ ranges from 0 to 1, where 0 corresponds to no swelling and 1 corresponds to
 168 complete swelling. To obtain the dimensionless variable S as function of time for each
 169 granule, D_i and D_f obtained from the sigmoid fitting are used. Thus one can obtain cloud
 170 plots of the non-dimensional diameter evolution for each of the experimental cases shown
 171 in figure 1a. The noise observed in the plots of dimensionless variable S is the direct result

of noise in the diameter of granule estimated through image processing (Deslandes et al., 2019).

From the figure 4 and table 1 it can be concluded that for a given temperature evolution all granules do not begin to swell at the same time. In fact, there is a latency time or delay in onset time involved in the process. The value of this latency time decreases with increase in temperature and varies from granule to granule as seen from the cloud plots in figure 4. Latency time does not depend on the diameter of the granules (Deslandes et al., 2019) and it approximately follows an exponential distribution at a constant temperature as seen in figure 3. It is also observed from figure 4 that granules with high latency time swell less rapidly than the ones with low latency times.

Granule diameter reaches a constant value given sufficient time and temperature. Individual granule characteristics and temperature profile determine the magnitude of time required to achieve this constant value. The ratio of the final diameter to the initial diameter or simply known as the ratio of swelling (D_f/D_i) varies from granule to granule. This is also reported by Deslandes et al. (2019). The mean ratio of swelling practically is almost independent of the temperature history and initial diameter. This is in accordance with mechanistic models (e.g. (Desam et al., 2020; Briffaz et al., 2014; Watanabe et al., 2001, 2007)) in which the final state corresponds to an equilibrium in which the water demand reaches zero. We also did not observe any significant correlation between latency times and swelling ratio(D_f/D_i) based on our data. From table 1 for temperatures above 68 °C we have nearly constant ratio of swelling around 2.35 but for lower temperature of 65°C, the ratio decreases suggesting partial swelling. However, as mentioned in industrial process the temperature used is higher to ensure complete swelling state i.e. swelling ratio of 2.35 in this case. As reported by Deslandes et al. (2019) the ratio of swelling approximately follows a truncated normal distribution.

These observations were used to develop a novel model that captures most of the essential features, which is discussed in detail in the next section.

(a) Case A

(b) Case B

(c) Case C

(d) Case D

(e) Case E

Figure 4: Non dimensional diameter evolution

4. Model Formulation and Results

4.1. Kinetic model development

As discussed in the previous section we observed that most granules do not swell at 60 °C even after 1 hour, so we set the minimum temperature of swelling at $T_{min} = 60$ °C (i.e. below 60 °C there is no swelling of granules). Based on the microscopic images we observed that each granule has a variable time delay in the onset of swelling which decreases with increase in temperature. To mathematical formulate the same, let at any instant the additional fraction (denoted by dF) of granules beginning to swell in next time unit be proportional to the fraction of granules that did not begin to swell yet (denoted by $1 - F$). In equation form it can be expressed as

$$dF = \frac{(1 - F)}{\tau} dt \quad (6)$$

$$\ln(1 - F) = - \int_0^{t_{onset}} \frac{dt}{\tau} \quad (7)$$

Integrating the above requires the knowledge about the characteristic latency time τ which depends on temperature T . When temperature is constant

$$F = 1 - \exp\left(\frac{-t_{onset}}{\tau}\right). \quad (8)$$

Hence in this mathematical formulation we have cumulative density function vs latency time following the exponential function as was observed in figure 3.

We propose that the characteristic latency time in function of temperature follows:

$$\tau = \max\left(0, \left(\frac{T_{ref} - T_{min}}{T - T_{min}}\right)\right) \tau_{ref} \quad (9)$$

where T_{ref} is arbitrary fixed at $T_{ref} = 62^\circ C$ and $T_{min} = 60^\circ C$ as indicated previously.

Since there is variability of swelling onset between granules, we introduce a parameter called "difficulty of swelling" parameter $\alpha \in [0, 1]$. For a granule, α is selected at random from uniform distribution. Thus during the ramp step of the heat treatment for a granule

219 whose difficulty of swelling is α , t_{onset} is given by equation 10.

$$\ln(1 - \alpha) = - \int_0^{t_{onset}} \frac{dt}{\tau} = - \int_0^{t_{onset}} \frac{1}{\tau_{ref}} \max \left(0, \left(\frac{T - T_{min}}{T_{ref} - T_{min}} \right) \right) dt \quad (10)$$

220 During the temperature ramp $T = T_{min} + \beta \times t$, where β is the ramp rate ($^{\circ}Cs^{-1}$). Solving
221 equation 10 results in

$$t_{onset} - t_{min} = \left(\frac{-2\ln(1 - \alpha)(T_{ref} - T_{min})\tau_{ref}}{\beta} \right)^{1/2} \quad (11)$$

222 where t_{min} is the time at which $T = T_{min} = 60^{\circ}C$.

223 Once the granule began to swell ($t > t_{onset}$), its diameter increases. A first-order reaction
224 rate is assumed inspired from Kubota et al. (1979), Lund and Lorenz (1984) and Okechukwu
225 and Rao (1995).

$$\frac{dS}{dt} = K(1 - S) \quad (12)$$

226 It is assumed the reaction does not proceed below the cut off temperature T_{min} similar to
227 the previous work by Plana-Fattori et al. (2016).

228 Kinetics for swelling also did speed up as temperature increased. Hence we propose the
229 following equation 13:

$$K = K_{ref}(1 - \alpha)^{0.5} \left(\frac{T - T_{min}}{T_{ref} - T_{min}} \right)^2 \quad (13)$$

230 where K_{ref} is the rate constant at T_{ref} for a granule which has no difficulty to swell ($\alpha = 0$).

231 Thus now one can solve for S of a granule given the temperature profile. Moreover from
232 Deslandes et al. (2019) we know that D_0 and D_{final}/D_0 follow normal distribution.

233 4.2. Calibration of model parameters and model predictions

234 Identification of swelling parameters τ_{ref} and K_{ref} was carried out using the S_{50} of the
235 data obtained from Case A described in table 1 and figure 4. Note that for a median granule
236 by definition the value of α is 0.5. Non-linear fitting of predicted by the model S_{50} with
237 the experimental S_{50} using “fitnlm” function of MATLAB (2018) for the Case A resulted
238 in values $\tau_{ref} = 190$ s and $K_0 = 0.0023$ s $^{-1}$. This means the median latency time at the

239 reference temperature of 62 °C is 132 seconds i.e. nearly 2 minutes.

Figure 5: Model prediction of non dimensional diameter evolution

Thus now we have a calibrated model. Therefore S_{10} , S_{50} and S_{90} for all the cases can be predicted and compared with the experimental results. Note that the values of alpha for S_{10} and S_{90} are 0.1 and 0.9 respectively as previously explained. In the figure 5 the predictions of S_{10} , S_{50} and S_{90} are shown with respect to the experimental data.

It should be noted that due to the noise in the measurement and the value of D_i derived from fitting the sigmoid curve we have S not beginning exactly at 0 for all the granules at the initial time points. We see the increase in the dispersion between S_{90} and S_{10} progressively from cases A to E. These observations are in-line with the model assumptions that rate constant increase with temperature and latency time decreases with temperature. From the plots in figure 5, we observe that model predictions overall are close approximations of the experimental data. The discrepancy for S_{90} in case of D and E could be due to the limited number of observed granules in this case.

4.3. Monte Carlo Simulation and results

Now that sufficient confidence has been established in the kinetic model, we can perform Monte Carlo simulations and predict the population D_{10} , D_{50} and D_{90} evolution. The following parameters listed here were used for the simulation

$$T_{ref} = 62^\circ C, T_{min} = 60^\circ C,$$

$$\tau_{ref} = 190s, K_{ref} = 0.0023s^{-1}, \text{ number of granules } I = 1000,$$

$$D_i \subset \mathcal{N}(14.77\mu m, 4.44\mu m), D_f/D_i \subset \mathcal{N}(2.34, 0.33),$$

The distributions of D_i and D_f/D_i were calculated from the pool experimental data of cases A to D. Case E was not considered as we see complete swelling did not always occur in this case. For granule initial diameter the normal distribution was used with truncated tails with minimum diameter as $0.25 \times$ mean diameter and maximum value $3 \times$ mean diameter. The ramp rates and hold times were feed into the simulation as per the individual cases as tabulated in table 1. A MATLAB script was written to execute the model, 4th order Range Kutta method was used for integration.

It can be observed in figure 6 that the model predictions are close to the experimental data and the model sufficiently captures the nuances of the process. The slope at the midpoint

Case A

Case B

Case C

Case D

Figure 6: Model prediction of diameter evolution

of granule swelling in case A is twice compared to the same in case B. The rate of swelling is slower progressively with D_{50} and D_{10} compared to D_{90} and this is more pronounced in the lower temperature cases C and D. The deviation in case D can be attributed to the lower sample size and higher dispersion at lower temperatures. The latency time is lower for the higher ramp rate case A compared to the rest of the cases. Finally, both in the model and the experimental data the dispersion in size is higher after swelling compared to initial population.

Figure 7 presents the cumulative particle size distribution evolution at different time points as the starch granule swelling proceeds. The dashed curves and continuous are the

Particle size evolution with time Case A

Particle size evolution with time Case B

Particle size evolution with time Case C

Particle size evolution with time Case D

Figure 7: Cumulative particle size distribution of different cases at different times and temperatures

277 experimentally observed and model predicted particle size distribution respectively. For
 278 both the simulation and experiment the curve at 68 °C in case A is much closer to the initial
 279 one compared to the same temperature in case B. The difference can be attributed to the
 280 difference in latency times (median of 134 seconds vs 197 seconds). This means that not
 281 only temperature is important but also the temperature history together dictate the swelling
 282 of the starch granules. The relative good agreement between predicted and measured size
 283 distributions at intermediate starch swelling for different temperature evolution validate the
 284 applicability of the model.

4.4. Model application and validation

In this section we make predictions using the developed model for the population swelling of the same starch using a completely different setup. Plana-Fattori et al. (2016) observed the volume size distributions of starch suspensions after different time-temperature histories using Malvern Mastersizer. The thermal treatment consisted of a series of three water baths, First the sample was immersed in a 50° C water bath until equilibrium was reached, then it was immersed into a water bath maintained at 92° C. After specified “heating time” the sample was shifted to the third water bath maintained at 4° C. All this while the bulk temperature of the sample was measured and noted at regular time intervals of 15 seconds.

For the sake of simplicity we only consider two cases with maximum temperature of 73° C and 92° C respectively (figure 8a). For simulation we only need the bulk temperature profiles above the cut off temperature of 60 ° C. One should note that swelling can go on even during the cooling step as long as the temperature is above 60° C.

Monte Carlo simulations using the same parameters mentioned in section 4.3 were performed. The difficulty of swelling parameter (α) and the maximum swelling ratio (D_f/D_i) are first randomly chosen for each of 100,000 granules. Then the onset times are calculated according to equation (10) using piece-wise linear interpolation of the experimental temperature evolution. The swelling degree (S) evolution are then calculated. The results of such simulations are shown in figure 8b for the first case. Note that because of the decreasing temperature after about 90 s in the first case ($T_{max}=73$ ° C) the rate at which the granules swell decreases in figure 8b. This phenomenon is more evident for α between 0.75 and 0.99. Thus some of the granules are in a frozen state of partial swelling in line with the interpretations of Plana-Fattori et al. (2016) in this case.

From the diameters of the 1000 granules at the end of the heat treatment for both the cases, the volume size distributions were computed to be compared with results from Malvern Mastersizer assuming spherical particles. Initial diameters from the Malvern Mastersizer were used to perform the simulation. The experimental values and model predictions are overlaid in figure 8c. Overall a fairly good prediction is obtained by the model. However, there is discrepancy at lower particle sizes for $T_{max}=73^\circ\text{C}$. Considering the differences in the

(a) Bulk Temperature profiles recorded for heating times of 2 minutes and 8 minutes (Courtesy of Plana-Fattori et al. (2016))

(b) Non-dimensional diameter evolution temperature profile 1

(c) Cumulative volume distribution

Figure 8

314 measuring techniques (microscopy for model fitting, light diffraction for Malvern) and various
 315 experimental and model assumptions (spherical particles, temperature uniform across the
 316 sample, $T_{min} = 60^{\circ} \text{C}$) the performance of the model on a whole is good, notably to predict
 317 the size distribution of partially gelatinized starch which is of practical interest (Hickman
 318 et al., 2008).

4.5. Possible explanation of variability and model limitations

The presented empirical model is based on individual granules observation and does not explain why different granules submitted to the same temperature evolution begin to swell at different instants (t_{onset}) and with different swelling duration (Δt). Literature suggests that various mechanisms are involved during starch swelling : melting of crystallites, moisture sorption, water diffusion, fusion of amylose/lipid complexes, leaching of amylose into the continuous phase, rupture of the granular structure (Briffaz et al., 2013, 2014; Chapwanya and Misra, 2015; Ozturk and Takhar, 2018). The three last phenomena do not occur in our case as waxy cross-linked maize starch was used for this work. In addition since we consider a very dilute system, there is no water availability limitation as discussed by Fukuoka et al. (2002).

Debet and Gidley (2006) identified three classes of starch granule swelling: (i) rapid swelling, (ii) slow swelling, (iii) limited swelling. For waxy maize starch there is no interaction between amylose, surface proteins and lipids and therefore rapid one-step swelling is observed. For other types of starch, the maximal swelling ratio can be function of the highest temperature reached. Indeed after a first swelling step (typically between 60 and 70 °C), a second step (typically between 80 and 100 °C) is observed, related to amylose/lipid/protein interaction for other types of starches (Briffaz et al., 2013). In our case, D_f/D_i was found to be independent on the maximal temperature (for $T_{max} > 65^\circ C$).

Melting of crystallites and moisture sorption can be affected by different parameters which can vary between granules such as crystallinity, branching of amylopectin, degree of cross linking, etc. (Debet and Gidley, 2006). This can explain variability in t_{onset} and Δt which was taken into account by a random parameter α (difficulty of swelling). One could expect that the 'difficulty of swelling' also influences the swelling ratio D_f/D_i , but no correlation was observed in our case between D_f/D_i and t_{onset} (p value $\sim 0.95 \gg 0.05$).

Diffusion can be assumed as Fickian (based on water concentration gradient), non-Fickian (based on water demand) (Watanabe et al., 2001) or following Flory-Huggins theory (Desam et al., 2018, 2020). Tortuosity can be considered as function of aqueous volume fraction and can vary between granules depending on the amorphous/crystalline shell structure. If

water availability is limited due to diffusion, the smallest granules should swell more rapidly. However, this was not observed in our case, there is no correlation between D_i and t_{onset} , Δt or D_f/D_i . It seems that in our case diffusion is not the limiting factor; different models were tested previously which confirmed that a diffusion based model is not pertinent for this kind of starch (Almeida et al., 2015).

To summarize, in our case (waxy cross-linked maize starch) variability in swelling onset and swelling duration could be attributed to the variability between granules in terms of crystallinity/structure, branching of amylopectin, cross linking. This variability comes perhaps from the different positions in the corn, positions on the cob, maturity, intra-granular heterogeneity, etc. These parameters influence the melting of the crystallites and the water diffusion. However, in our case the diffusion seems to not be the limiting phenomenon since the smallest granules do not swell faster than the biggest ones. It could be interesting to introduce variability for water activity curves and tortuosity in mechanistic swelling models such as the one developed by Desam et al. (2020). This approach could help one determine which of these parameters might potentially explain the swelling kinetics variability. For more complex starches (non-waxy or non-cross-linked) the presented model is perhaps not adapted and additional experimental efforts are required because additional phenomena occur.

5. Conclusion

A novel individual granule scale model was developed to predict the swelling kinetics of modified waxy maize starch granules under heat treatment. The model parameters were calibrated with data from one ramp rate and hold temperature and validated with data from other cases. From the experiments it is shown that there is granule to granule variability not only in terms of ratio of swelling but also in terms of latency to swell when a critical temperature is reached. These phenomena are independent of the diameter of the granules. For practical time scales we can say the swelling reaction does not initiate below 60 °C. The latency time is random with a characteristic time constant decreasing as temperature is increased. Also the kinetics of the reaction is enhanced at higher temperatures. A granule

with high latency time is also shown to have slower swelling rate compared to granules with low latency time.

The rheology of the suspension results from the hydrodynamic interactions and collisions between the granules which is a function of the volume fraction of granules and granule PSD. To design, control, scale up and operate complex processes for liquid food products one needs to leverage simulation techniques such as Computational Fluid Dynamics (fluid flow, heat transfer) and Discrete Element Method (particle evolution and interaction). This granule scale model can represent the commencement of such techniques for multi-scale modelling and simulation of modified starch suspension undergoing thermal treatment.

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