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Users' Power Multiplexing Limitations in NOMA System over Gaussian Channel

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Abstract—**Non-Orthogonal Multiple Access (NOMA) is one of the promising techniques to ensure very high spectral efficiency in 5G mobile communications and beyond.In contrast to the orthogonal multiple access (OMA) technique, the NOMA shows outstanding performances in terms of throughput, user fairness, low latency and compatibility with the current and future communication systems. In this paper, we analyze the capacity region in NOMA system and the limited number of multiplexed users under a given power allocation vector (i.e. symmetric/asymmetric channel). Moreover, we have investigated the effect of large constellation order on the power allocation and bit error rate (BER). Comparisons between the coded and uncoded schemes are also presented.**

I. INTRODUCTION

5G communication systems use the most promising techniques that would guarantee the best performance (KPI- Key Performance Indicator) in terms of throughput, energy efficiency, and also the best low latency indicators [1], [2]. Two principal features of 5G are required: the latency of 1ms and the capacity of supporting for 10 Gb/s throughput [3]. NOMA is one of the highly recommended technologies to be used in future communication systems. Especially for its capacity to improve the efficiency use of the resources by exploiting channel quality differences among users and the use of advanced successive interference cancellation (SIC) receivers [4], [5]. The NOMA technique maintains a good backward compatibility when combined with orthogonal frequency-division multiple access (OFDMA) [6]. Moreover, NOMA is highly expected to increase system throughput due to the fact that each user uses all the bandwidth resource. To ensure this performance, NOMA scheme superposes multiple users in the same radio resource with different transmission power

There are different types of NOMA techniques, including code-domain and power-domain. In the code domain, NOMA scheme uses a specific spreading sequence for each user in order to share the entire resource with the rest of the active users while in the power domain, NOMA uses the power as a mode of sharing the spectral resource with different users [7]. In the latter, NOMA exploits the differences in channel gains between the users in order to multiplex through the power allocation.

In in this paper, we are interested into the NOMA downlink scenario. The base station adds up the users signals in a linear way under certain power partitions to balance the sum rate of all multiplexed users and the throughput fairness among individual users [8]. However, at the receivers level, users which benefit from the strong channels conditions decode and cancel successively the messages of the weak users to be able to decode their signals. As the authors confirm in [9], SIC is the heart of NOMA. It allows to separate the superimposed signals at the receiver [11].

In this paper, we first focus on the fundamentals of downlink NOMA scheme and we emphasize their keys: coded or uncoded signals superposition (SC) at the BS and the SIC at the receivers. Then, we discusses the channel capacity comparison between the OMA with the NOMA overs simulations in the downlink communication. Accordingly, we highlight and discuss the simulation results of downlink NOMA chain And analyze the multiplexing level of the users and its limits over AWGN channel. Finally, we draw the main conclusions and we give some insights.

II. SYSTEM MODEL

In this section, we explain the concept of single-cell downlink scenario, Let assume that the baste station (BS) and each user are equipped by a single antenna. The overall transmission bandwidth is normalized to the unit for ease of computation. The system model contains a base station, and N users. Let $\|h_1\|^2$; $i = 1,2,\ldots,N$ be the square of channel gain amplitude, between the BS and the i^{th} user, arranged in the ascending order, i.e., $\|h_1\|^2$ is the smallest and $\|h_N\|^2$ is the greatest. Therefore, the first user U_1 is the weakest user in terms of the channel conditions while the U_N is the strongest one. In what follows, we assume a Gaussian channel between BS transmitter and different receivers with a fixed gain $h_1, h_2, \ldots, h_N$. Fig. 1 represents a downlink NOMA scheme with N users.

The BS transmits the signal of N users simultaneously by multiplexing their signals using the SC. The Data is sent for each user over the entire bandwidth while the power resource is shared between them. We assume that the channel state information (CSI) of different users is available at the base station. The BS allocates the i^{th} lowest power P_i to the i^{th} user U_i , $P_i = P_t * \alpha_i$ with P_t is the total available power at the BS, α_i is the power allocation factor of U_i. Note that $\sum_{i=1}^{N} \alpha_i = 1$. So, without limiting the general scope of the other users, the weakest user U_1 benefits from the highest power allocation,

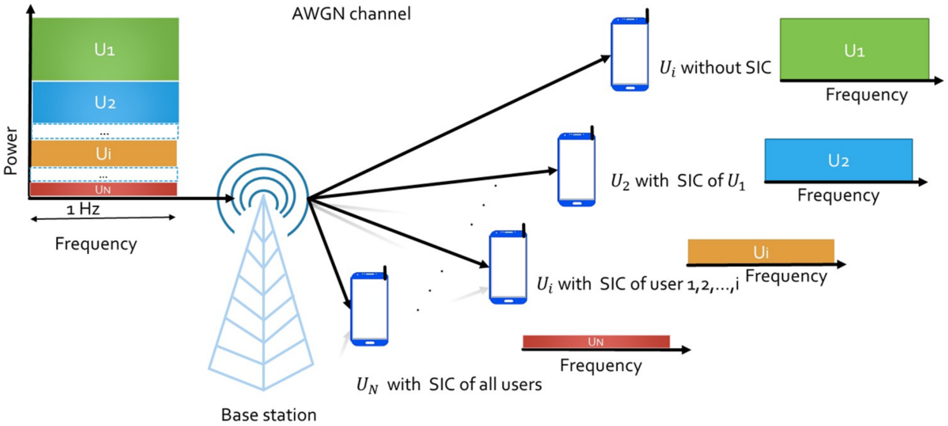

Fig. 1. NOMA downlink scenario scheme with N users

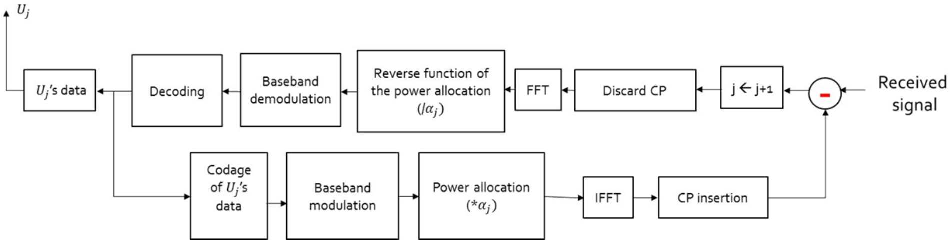

Fig. 2. The basic principle of SIC

while the lowest power is given to the strongest user U_N. On the receiving end, each user receives the signal transmitted by the BS which includes its signal and the signals of other users. All these signals are multiplexed with different power ratios. The integrated SIC algorithm at the users terminals level allows that the strongest users eliminate the weak users signals successively .Thus, each user U_i, $i = 1, 2, \ldots, N$ decodes all the signals of users U_k with $k < i$. Unlike the strong users, the weak users consider stronger user signals as interference. Fig. 2 details the principle of SIC based on the orthogonal frequency-division multiplexing (OFDM).

The j^{th} user's data U_j can be obtained in the j^{th} iteration where $j \in \{0, 1, ..., N-1\}$. The received signal at the i^{th} receiver U_i can be expressed as

$$y_i = h_i * X_t + w_i \tag{1}$$

with

$$X_t = \sum_{i=1}^{N} \sqrt{P_i} * x_i \tag{2}$$

where w_i is complex-Gaussian distributed additive noise with zero mean and variance σ_i^2, and X_t is the transmitted signal from the BS, and x_i is the signal for the user U_i.

According to the NOMA principle, the SC process is implemented at the BS while the SIC process is implemented at the receiver of strong users. We assume that these process are perfectly implemented and there are no error propagation. So, all the strong users can decode correctly their signals while the weakest users remove the inter-user interference. The throughput of U_i, R_i is represented as

$$R_i = \log_2 (1 + \frac{P_i |h_i|^2}{|h_i|^2 \sum_{m=i+1}^{N} P_m + \sigma_i^2}) \tag{3}$$

The strong user has a better channel condition, but its power is the lowest one compared to the other users. Thus the strong user receives high interference from weak users. However, the weak user is assigned biggest power allocation. Note that the achievable rate of user N is given by

$$R_N = \log_2 (1 + \frac{P_N |h_N|^2}{\sigma_i^2}) \tag{4}$$

III. DOWNLINK CHANNEL CAPACITY COMPARISON OMA/NOMA

In this section we compare the capacity region of NOMA and OMA. In downlink NOMA, users use the same entire 1 Hz bandwidth and the same time recourse. we assume that there is a single-cell with two users. we assume also that the signals pass by AWGN channel and $\|h_2\|^2 > \|h_1\|^2$. Using (3) and (4), the NOMA throughput of the two users is given successively by

$$R_1 = \log_2 (1 + \frac{P_1 |h_1|^2}{|h_1|^2 P_2 + \sigma_1^2}) \tag{5}$$

$$R_2 = \log_2 (1 + \frac{P_2 |h_2|^2}{\sigma_2^2}) \tag{6}$$

However in downlink OMA, the bandwidth is shared between the users. Thus the assigned bandwidth to the first user is β Hz while the second user obtains $(1-\beta)$ Hz with $(0<\beta<1)$. The achievable data rate by user 1 and user 2 are, respectively, expressed as

$$R_{O1} = \beta \log_2 (1 + \frac{P_1|h_1|^2}{|h_1|^2 P_2 + \sigma_1^2}) \tag{7}$$

$$R_{O2} = (1-\beta) \log_2 (1 + \frac{P_2|h_2|^2}{\sigma_2^2}) \tag{8}$$

NOMA scheme controls the rate of the two users one and two by adjusting the power resource allocation given in (5) and (6). So, if the power ratio P_1/P_2 is well chosen the users of NOMA can benefit from throughput fairness. According to [10], [11], if an asymmetric channels, where the signal-to-noise ratios (SNRs) of the two users are different, is considered, it can be numerically shown that the values of R_{O1} and R_{O2}, given, respectively, (5) and (6), are considerably higher than those of R_{O1}.

In the downlink, Fig. 3 shows the worst-case situation where the users are under a symmetric channel scenario conditions $\|h_1\| = \|h_2\|$. It is clear that the rate fairness is identical for OMA and NOMA for a different signal to noise ratios (SNR), $\eta_i = P_t\|h_i\|^2/\sigma_i^2$. The sum rates of the users are also equal for the NOMA and the OMA schemes in the 5 configurations given in Fig. 3. On the other hand, the best scenario for NOMA is when $\|h_1\|^2 < \|h_2\|^2$. Thus mean the users have asymmetric conditions channels, Fig. 4 represents this scenario for two users under asymmetric channels.

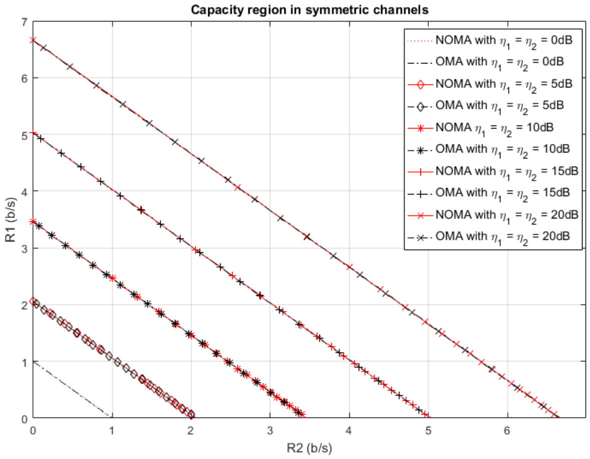

Fig. 3. Channel capacity of OMA and NOMA in symmetric channels

The SNR of the first user is varied while the SNR of the second user remains fixed at 20 dB (without loss of generality when $\|h_1\|^2 < \|h_2\|^2$). The overall throughput and the rate fairness of user 1 and user 2 in NOMA are superior to those of OMA. In fact, this knowledge of the NOMA channel capacity allows to assign an optimal power ratio between the users in order to provide the highest possible throughput for users while respecting the rate fairness. So the NOMA system ,in asymmetric channels, can achieve higher sum ergodic capacity compared to the OMA system. Therefore, the NOMA system is superior in terms of the capacity region than OMA.

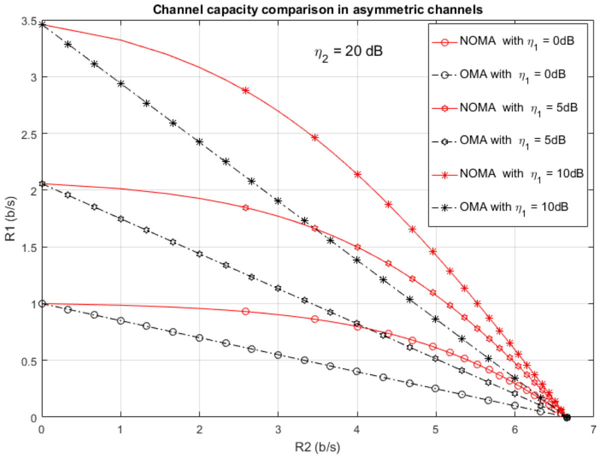

Fig. 4. Channel capacity of OMA and NOMA in asymmetric channels

IV. SIMULATION RESULTS

In this section we analyze the BER performances of the NOMA technique under Gaussian channel for large constellation order Section IV-1 and for three users Section IV-2. In first section, the analysis of a system with two-users under 4QAM and 16QAM is presented, then the NOMA with multiple users system under BPSK is simulated. Highlighting the effect of using a large modulation order and the effect of increasing the users number. In the simulations, we mean by coded data that the user use a channel coding while in the case of free channel coding we use the uncoded data therm. For each test we analyze the scenario in two cases: the case where the users data is uncoded, then the case where the data is coded with a convolutional code based on a rate of $1/2$ and constraint length of 7. In the latter case, the data at the receiver side are decoded using a hard viterbi decoder. All the tables Results are given for a bit error rate BER $=10^{-4}$ in the case of users with coded data.

1) Performance Analysis under large constellation sizes:

In this subsection, we analyze the NOMA chain under 4-QAM and 16-QAM in order to highlight the modulation order effect.

Tables I and II represent respectively , NOMA-4-QAM E_b/N_0 and NOMA-16QAM E_b/N_0 for two users $(N=2)$ with different power allocation factors. In the case of the 4-QAM, the value of α_1 assigned to the weakest user is varied from $\alpha_1 = 0.55$ to $\alpha_1 = 0.95$ with a step of 0.05. However, the value of α_1 in the 16-QAM case is varied from $\alpha_1 = 0.91$ to $\alpha_1 = 0.99$ with a step of 0.01. The α_2 is calculated as $\alpha_2 = 1-\alpha_1$. Note that the choice of α_i depends on the values of E_b/N_{0i} obtained during the simulations of

TABLE I

ACHIEVED PERFORMANCES FOR A 4-QAM NOMA SYSTEM WITH TWO USERS AT A BER=10^{-4} AND DIFFERENT VALUES OF E_b/N_0, α_1 AND α_2

4-QAM	α_1	0.95	0.90	0.85	**0.80**	0.75	0.70	0.65	0.60	0.55
	α_2	0.05	0.10	0.15	**0.20**	0.25	0.30	0.35	0.40	0.45
E_b/N_{01}(dB)		3,81	5,01	6,44	**7,99**	9,76	11,7	14,3	17,9	24
E_b/N_{02}(dB)		15,8	12,7	10,8	**9,75**	9,76	11,7	14,3	17,9	24
$E_b/N_{01}-E_b/N_{02}$(dB)		11,99	7,69	4,36	**1,76**	0	0	0	0	0

TABLE II

ACHIEVED PERFORMANCES FOR A 16-QAM NOMA SYSTEM WITH TWO USERS AT A BER=10^{-4} AND DIFFERENT VALUES OF E_b/N_0, α_1 AND α_2

16-QAM	α_1	0.99	0.98	0.97	0.96	0.95	0.94	**0.93**	0.92	0.91
	α_2	0.01	0.02	0.03	0.04	0.05	0.06	**0.07**	0.08	0.09
E_b/N_{01}(dB)		7,62	7,58	8,56	9,87	11,6	13,7	**16,4**	20	26,2
E_b/N_{02}(dB)		26	22,9	21,1	20	18,9	18,2	**17,5**	20	26,2
$E_b/N_{01}-E_b/N_{02}$(dB)		18,38	15,32	12,54	10,13	7,3	4,5	**1,1**	0	0

the different modulation orders. The best configuration are represented in bold. The best value for the pair (α_1, α_2) for the 4-QAM is ($\alpha_1 = 0.80$, $\alpha_2 = 0.20$) and for 16-QAM is ($\alpha_1 = 0.93$, $\alpha_2 = 0.07$). These pairs are chosen in terms of the lower E_b/N_{0i} possible with ($E_b/N_{01} - E_b/N_{02} < 2$dB). We notice that the 16-QAM gives the worst results with $E_b/N_{01} = 16.4$ dB and $E_b/N_{01} = 17.5$ dB. We notice also that the power allocation factor for U_2 is very low in the 16-QAM case ($\alpha_2 = 0.07$). In the other hand, the 4-QAM modulation guarantees a $E_b/N_{01} < 9.75$ dB for the two users. We note that a 64-QAM simulations are done for different values of α_1 and α_2, but they have not achieved the bit error rate of 10^{-4}. All the simulation have given a bit error rate higher than 10^{-2}.

Figures 5 and 6 give the simulation results of BER in terms of E_b/N_0 for for the best pair of power allocation factors chosen from Tables I and II respectively. Lines with star(*) depicts coded signal, however, lines with circle (o) the uncoded signal. In the first scenario, the BS multiplexes the uncoded data and send them to two users U_1 ans U_2 while in the second scenario the BS encodes the users data and multiplexes them, then it sends the multiplexed signal to the users.

In both scenarios the U_1 without SIC uses directly the received signal considering the U_2 signal as interference. However, the U_2 uses the *SIC* algorithm to obtain its data. We observe in the figures that the U_1 and U_2 with coded data are better in terms of BER from a threshold E_b/N_0 special for each user and for each modulation size. For the BPSK from $E_b/N_0 < 5B$, the coded data scenario BER is less good than the uncoded data scenario for U_1 and U_2. For the 4QAM the E_b/N_0 threshold is 5.8 dB while for the NOMA-16QAM the E_b/N_0 threshold goes up to 13.1 dB.

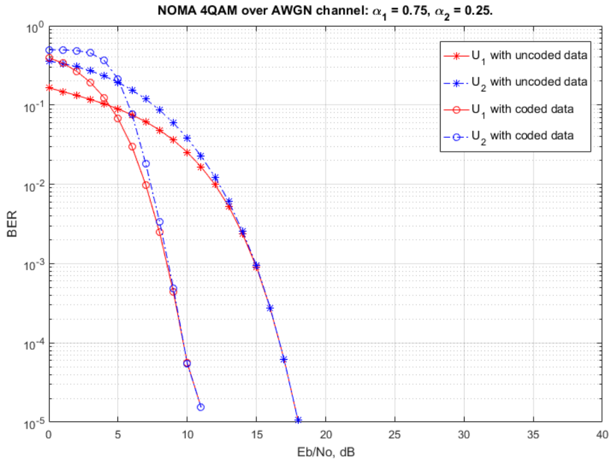

Fig. 5. NOMA scheme using 4-QAM modulation with coded and uncoded BER signal performances

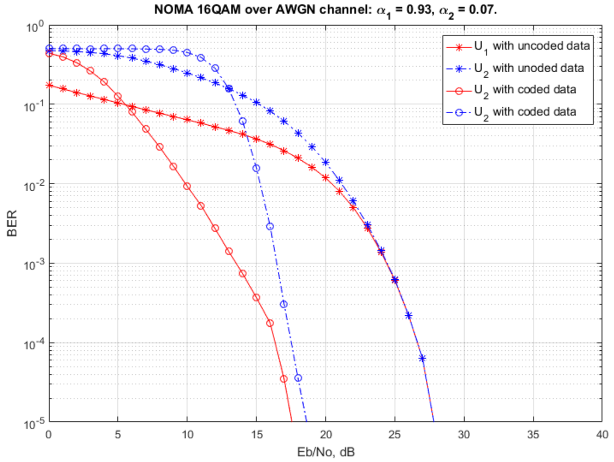

Fig. 6. NOMA scheme using 16QAM modulation with coded and uncoded BER signal performances

All these simulation results confirm that the small modulation size are more efficient in the NOMA communication system. Therefore, the large modulation size decreases the NOMA performances. Therefor, we can affirm that the small modulation order is the most suitable modulation type for the NOMA systems and this is due to the difficulty of reconstructing the multiplexed signals with close constellations as in the case of 64QAM and 16QAM. However, the case of 4QAM is easier to distinguish the modulation symbols.

2) Performances analysis under multiple users: To evaluate the maximum number of users in the NOMA system, the users number is varied from 2 to 4 users. The BPSK modulation is chosen following to the results of the previous section. Since the small modulation order is most suited to NOMA. Table III and table IV shows E_b/N_0 results for BER$= 10^{-4}$ in BPSK for 2 and 3 users respectively. The best configuration of power allocation is given in bold. For 4 users all the configurations do not allow that all the users achieve the bit error rate of 10^{-4}.

TABLE III
ACHIEVED PERFORMANCES FOR A BPSK NOMA SYSTEM WITH TWO USERS AT A BER=10^{-4} AND DIFFERENT VALUES OF E_b/N_0, α_1 AND α_2

BPSK	α_1	0.95	0.90	0.85	0.80	**0.75**	0.70	0.65	0.60	0.55
	α_2	0.05	0.10	0.15	0.20	**0.25**	0.30	0.35	0.40	0.45
E_b/N_{01} (dB)		3.92	5.06	6.61	8.1	**9.94**	12	14.6	18.1	22.8
E_b/N_{02} (dB)		19.5	14.6	12.2	10.6	**10.1**	12	14.6	18.1	22.8
$E_b/N_{01} - E_b/N_{02}$(dB)		15.58	9.54	5.59	2.5	**0.16**	0	0	0	0

TABLE IV
ACHIEVED PERFORMANCES FOR A BPSK NOMA SYSTEM WITH THREE USERS AT A BER=10^{-4} AND DIFFERENT VALUES OF E_b/N_0, α_1, α_2 AND α_3

BPSK	α_1	0,74	0,72	**0,71**	0,73	0,7	0,71	0,7	0,69	0,74	0,69	0,7
	α_2	0,2	0,21	**0,22**	0,2	0,23	0,21	0,22	0,25	0,19	0,24	0,25
	α_3	0,06	0,07	**0,07**	0,07	0,07	0,08	0,08	0,06	0,07	0,07	0,05
E_b/N_{01} (dB)		14,3	16,6	**17,9**	15,5	19,5	18,6	20,2	20,1	14,5	21,1	17,5
E_b/N_{02} (dB)		17,7	18,4	**18,4**	19,3	19,5	20	20,3	20,1	20,5	21,1	17,5
E_b/N_{03} (dB)		18,9	18,4	**18,4**	19,3	19,5	20	20,3	20,1	20,5	21,1	20,5
E_b/N_{02}-E_b/N_{01}(dB)		3,4	1,8	**0,5**	3,8	0	1,4	0,1	0	6	0	0
E_b/N_{03}-E_b/N_{02}(dB)		1,2	0	**0**	0	0	0	0	0	0	0	3
E_b/N_{03}-E_b/N_{01}(dB)		4,6	1,8	**0,5**	3,8	0	1,4	0,1	0	6	0	3

The best configuration of power allocation factors for NOMA using BPSK modulation under 2 users is ($\alpha_1 = 0.75, \alpha_2 = 0.25$) with a maximum of $E_b/N_{0max} = 10.1$ dB, while under 3 users the best configuration is ($\alpha_1 = 0.71, \alpha_2 = 0.22, \alpha_3 = 0.07$) with a maximum of $E_b/N_{0max} = 18.4$ dB. We notice that the NOMA systems with small user number allow to achieve a bit error rate of 10^{-4} in worse channel conditions compared to NOMA system with big user number.

The best configuration are shown in Fig. 7 and Fig. 8. They represents system with 2 and 3 active users respectively while Fig.9 represents a system with 4 users with ($\alpha_1 = 0.74$ dB, $\alpha_2 = 0.2$dB, $\alpha_3 = 0.04$dB, $\alpha_4 = 0.02$ dB). This simulation is chosen because it is among the best simulation under 4 users that two users with coded data at least achieve the bit error rate of 10^{-4}. We note that, lines with star(*) depicts coded signal, and lines with circle (o) the uncoded signal.

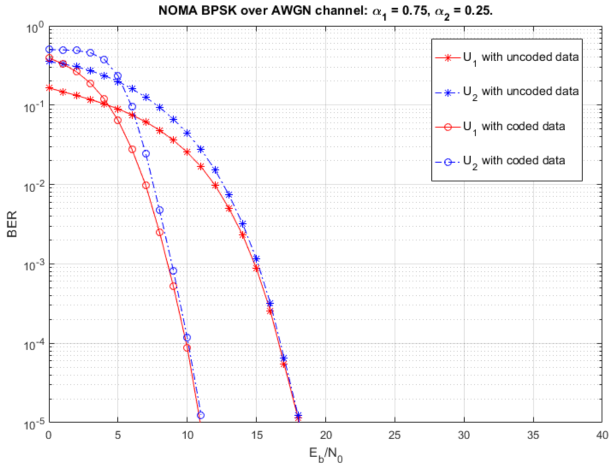

Fig. 7. NOMA scheme with coded and uncoded BER signal performances under 2 users

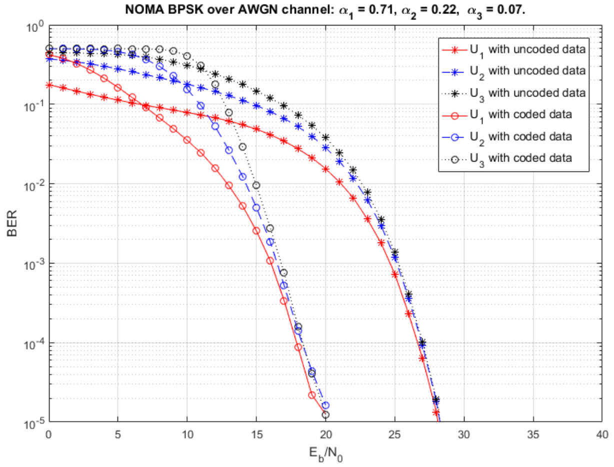

Fig. 8. NOMA scheme coded and uncoded BER signal performances under 3 users

The scenario with coded data is always less good in the worse channel conditions. The E_b/N_0 threshold for the BPSK with 2 users is 5.1 dB and 11.3 dB for 3 users. Thus the coded data scenario is better than the uncoded data one when the E_b/N_0 is bigger than these values. We notice that this E_b/N_0 threshold is smaller in the case of small users number. We notice also that NOMA under two users is the best configuration. However, in the case of 4 users we have just one user with uncoded data which achieve small bit error rate and 2 user in the case of coded data provided that they have good channel conditions. Therefore, we affirm that NOMA with small number in a signle-cell is more efficient. However, more the users number increases more the SIC is complex and the errors propagation increase fast. In addition when the number of users increases, the power allocation is became very

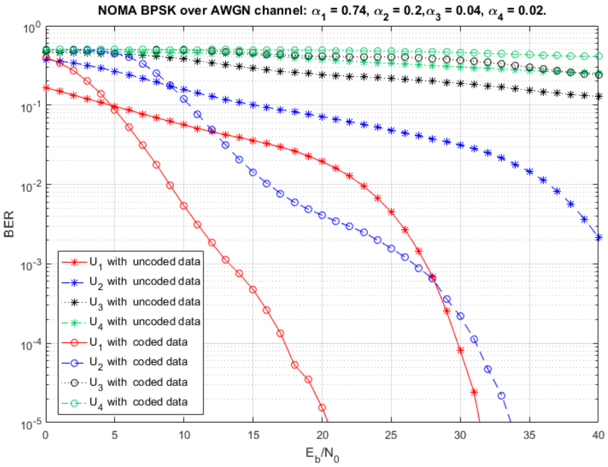

Fig. 9. NOMA scheme coded and uncoded BER signal performances under 4 users

complicated to assign to the users.

V. Conclusion

In this paper, the NOMA theory in downlink scenario for a single-cell with multiple users is described. The channel capacity of NOMA and OMA for asymmetric and symmetric users channels conditions is compared. Perforamnce Analysis under small and large constellation order are highlighted, tested and compared for two, three and four NOMA multiplexed users. The simulation is carried on the superiority of NOMA with crypted and uncrypted data respectively. As a result, the complexity of NOMA system is confirmed when the number of users or the modulation order increase.

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