A Multi-disciplinary Approach for Mechanical Metamaterial Synthesis: A Hierarchical Modular Multiscale Cellular Structure Paradigm

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Chapter 20
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Abstract Recent advanced manufacturing techniques such as 3D printing have prompted the need for designing new multiscale architectured materials for various industrial applications. These multiscale architectures are designed to obtain the desired macroscale behavior by activating interactions between different length scales and coupling different physical mechanisms. Although promising results have been recently obtained, the design of such systems still represents a challenge in terms of mathematical modeling, experimentation, and manufacturing. In this paper, some research perspectives are discussed aiming to determine the most efficient methodology needed to design novel metamaterials. A multidisciplinary approach based on Digital Image Correlation (DIC) techniques may be very effective. The main feature of the described DIC-based approach consists of the integration of different methodologies to create a synergistic relationship among the different steps

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from design to fabrication and validation. Experimental techniques and modeling approaches are envisioned to be combined in feedback loops whose objective is to determine the required multiscale architectures of newly designed metamaterials. Moreover, it is necessary to develop appropriate mathematical models to estimate the behavior of such metamaterials. Within this new design approach, the manufacturing process can be effectively guided by a precise theoretical and experimental framework. In order to show the applicability of the proposed approach, some preliminary results are provided for a particular type of mechanical metamaterial, namely, pantographic metamaterials. Lastly, the most relevant challenges are highlighted among those that must be addressed for future applications.

Keywords: Synthesis of metamaterials · Generalized models · Analog circuits · Pantographic structures · Digital image correlation · Homogenization

20.1 Introduction

It is possible to find natural materials that exhibit very exotic and unusual behavior due to their microstructures organized with complex hierarchies (Lakes, 1993). These hierarchical architectures consist of a combination of numerous structural patterns at different length scales, and each pattern is made of architectured microstructures characterized by lower length scales. Here, the overall response generated at the macroscale is related not only to each of the lower-scale microstructures but also to their interactions. The most common example of such natural materials is bone tissues (Maggi et al, 2017; Giorgio et al, 2017; Chia and Wu, 2015; Cima et al, 1994). In Fig. 20.1, their structural hierarchy is illustrated from macro- to nano-scales. The overall response of bone is obtained by the interactions of various features at different length scales. As can be seen from Fig. 20.1, the microstructure of bone also gives very inspirational ideas to design new metamaterials, namely, different parts of a material may have various microstructural patterns depending on the desired macroscale response. In this particular example, osseous tissues (i.e. cancellous and cortical bones) have different structural patterns at the microscale, lamellae are arranged in different manners to form trabeculae and ostea. Consequently, different responses are obtained at particular locations.

Plant stems are another example of natural multiscale materials. They need to resist both axial load from their own mass and bending moment from the wind. Fig. 20.2 shows an example of an internal microstructure enabling for such a strength. A scanning electron micrograph of a hawthorn stem reveals its foam-like interior structure. Gibson et al (1995) showed that this foam-like architecture improves the buckling resistance of the plant.

It can be noted that multiscale natural materials have been inherently optimized by natural selection through a very long process. For instance, bone tissues living now on Earth are the result of a very long (many million year) selection and adaptation process. During the so-called Cambrian explosion, the diversification of living species

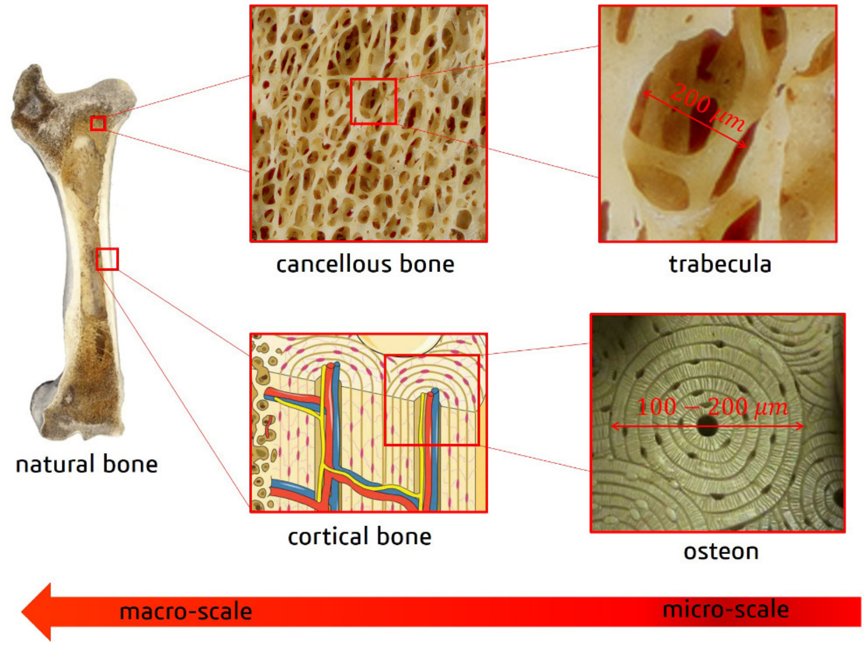

Fig. 20.1 Structural elements of bone at different length scales

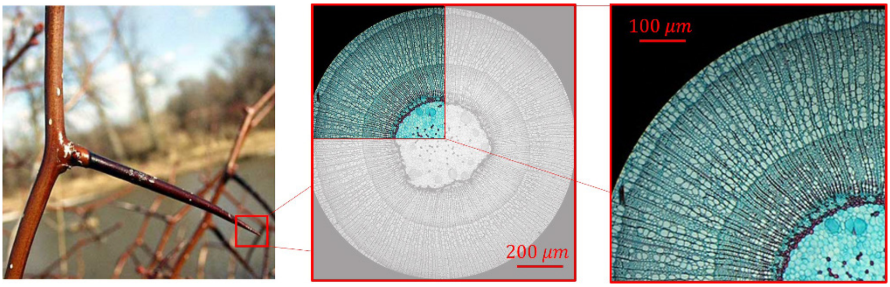

Fig. 20.2 Microstructure (right and center) of hawthorn stem (left)

experienced an exponential growth, and in the most recent taxonomy list, it is possible to find at least 69,276 different species. Therefore, many adaptations occurred in the evolution of bone tissues, and different structures at various length scales are observed nowadays. If enough time were given to natural selection, one would still discover new multiscale materials!

With the newest manufacturing techniques, in particular with 3D printing, many researchers are trying to design novel materials whose exotic macroscopic properties are obtained with suitably designed multiscale microstructures (Liu et al, 2013; Geers et al, 2003). Materials that do not exist in nature, and whose design is based on multiscale modeling to exhibit desired performances, are sometimes called metamaterials (Barchiesi et al, 2019; Gatt et al, 2015). The concept of metamaterials is becoming more and more popular, and their applications are garnering considerable academic and industrial interest (dell'Isola et al, 2019a,b). Therefore, the multiscale structures observed in nature may inspire the design of such materials for technological applications (Wegst et al, 2015). For instance, based on the multiscale structure of bones (Fig. 20.1), artificial bio-resorbable materials have been invented and produced for bone grafting processes (Fig. 20.3). Scaffolds used to favor bone reconstruction and remodeling have more chances to be effective if their internal microstructures have suitable bone-mimicking features. Further, trabecular metals are being used in bone reconstruction (Fig. 20.3). Moreover, the structure of bone has been the source of inspiration for light-weight structure applications such as aluminum foams (Fig. 20.3, Andrews et al, 1999).

In addition to biomechanical applications, a lot of attempts have been made to design multiscale architectured materials (e.g. metamaterials) inducing some specific types of overall behavior that is not observed in existing natural materials. In such designs, application-tailored responses are obtained by coupling different physical phenomena, and the interactions between different length scales. In general, metamaterials are categorized based on the main interaction phenomena occurring in their microstructures. Although electromagnetic interactions were first used to design optical metamaterials (Veselago, 1968), other important physical phenomena are currently exploited in their design. For example, metamaterials that are designed to control the propagation of acoustic (elastic) waves are referred to as acoustic metamaterials (di Cosmo et al, 2018). In such materials, an elementary cell is periodically repeated in the microstructure. In order to control wave propagation, the

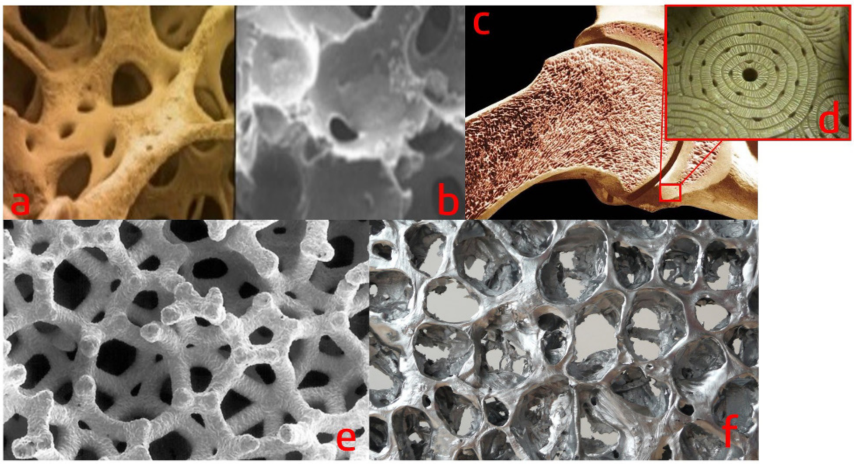

Fig. 20.3 Some multiscale materials. Example of bone tissue (a) and a bio-resorbable artificial graft (b) Giorgio et al (2016b). In (c) and (d) the multi-scale structure of bone is evident: from trabeculae to osteons. In (e) trabecular metal Andreykiv et al (2005) and in (f) aluminum foam are shown

elementary cell is designed with a smaller length scale compared to that involved in the targeted application. In the field of optical and acoustic metamaterials, many novel products have been designed. Typical examples are materials with negative index of refraction (Veselago, 1968, 1967), and those behaving like a low-density plasma with an effective dielectric constant that becomes negative below the effective plasma frequency (Pendry et al, 1996). Other important trends in metamaterial design are images focusing below the diffraction limit (Deng et al, 2009; Zhang et al, 2009; Ambati et al, 2007; Ao and Chan, 2008; Jia et al, 2010; Liu et al, 2007) (e.g. hyperlenses are able to transform evanescent waves into propagating waves, which can be detected at large distance, and superlenses amplify these evanescent waves) and metafluids (Norris, 2009).

This paper focuses on mechanical metamaterials, namely multiscale materials whose behavior is only determined in terms of mechanical interactions among different structures at different scales. Mechanical metamaterials have been investigated in a large number of different studies (e.g. see Kadic et al, 2012; Lee et al, 2012; dell'Isola et al, 2015c; Vangelatos et al, 2018, 2019; Barchiesi et al, 2018; Misra et al, 2018; Laudato et al, 2018; Del Vescovo and Giorgio, 2014a; Carcaterra et al, 2015; Turco et al, 2017a; Barchiesi and Placidi, 2017; Placidi et al, 2017b)). Auxetic structures (Lakes, 1987) (i.e. materials which have negative Poisson's ratio) and locally resonant microstructured materials (Liu et al, 2000) with negative refraction index are typical examples. There are many interesting mathematical problems to be solved in the design of such metamaterials. In reality, the long natural selection process that did manage to optimize the functionality of many natural materials has to be sped up because some applications cannot wait so long.

A main change in research paradigm is needed for the design of new metamaterials. Usually, in mathematical physics, a model is built by conjecturing some postulates assumed to be satisfied to model some specific aspects of the physical reality. For instance, if one wants to model a deformable body in the elastic regime, a time-dependent field of placement and an action functional (e.g. see Germain, 1973; Auffray et al, 2014)) in the set of admissible motions is introduced to describe its evolving shape. Once the postulated action functional is conjectured, the motions predicted by means of the Principle of Least Action can be compared with experimental evidence. If the material parameters appearing in the action functional are usually determined with a small set of measurements, and allow for the description of many more experiments, then one can say that the experimental evidence supports the validity of conjectured models. In this way, the mathematical model for a given class of phenomena is tailored to predict the overall performance of the given material under different design conditions.

Conversely, in the design of metamaterials, an approach that reverses the above-described conceptual order is followed (dell'Isola et al, 2016a). A mathematical model that a priori describes the desired overall behavior is first proposed. Then, the corresponding synthesis problem is solved, namely, finding a (possibly multiscale and/or multiphysics) micro-architecture whose overall behavior is modeled with the selected mathematical model. The synthesized multiscale structure is then fabricated, and its behavior experimentally tested. The final steps of the described "reversed order" process have been made possible with the recent developments in 3D printing technologies. A specific example of such novel metamaterials is given by the so-called pantographic sheets. In Alibert et al (2003); Seppecher et al (2011), a synthesis problem was solved to model this type of structures. In these studies, to find microstructures for one-dimensional and two-dimensional continua, the governing equations for the described microstructures were obtained by a Lagrangian whose potential energy depends on the second gradient of displacement fields at the macroscale in the case of plane motions.

The current increasing interest in metamaterials is mainly due to the availability of new advanced manufacturing techniques such as 3D printing (Rumpf et al, 2013), optical lithography (Madou, 2011), roll-to-roll processing (Ok et al, 2012), electrospinning (Teo and Ramakrishna, 2006), dry and wet etching(Pearton et al, 1993), micro-molding (Heckele and Schomburg, 2003), and micro-machining (Masuzawa et al, 1985). With this spectacular progress obtained in advanced manufacturing techniques in the past ten years, it is much easier to design and manufacture multiscale architectures performing desired overall responses in different industrial applications (Engheta and Ziolkowski, 2006). All these new manufacturing techniques are seen as solutions in a more and more complex manufacturing environment, specifically in terms of customization, multifunctionality, innovative design, and geometry. These new manufacturing technologies not only enable for accurate fabrication with characteristic lengths of the order of micrometers and even less but they are also getting less expensive and more reliable. Thus, with these new techniques, it is possible to manufacture multiscale materials obtained as a result of the solution to the synthesis problem mentioned above. As an example, the relevant length scales of a pantographic sheet that was designed and 3D printed for light-weight structural applications are shown in Fig. 20.4.

Furthermore, the reassessment of the existing mathematical models for the description of deformable bodies is unavoidable from theoretical points of view as another consequence of this progress in material technology. Since materials may have complex hierarchical architectures, the classical description of continuum mechanics is no longer applicable to model exotic responses (dell'Isola et al, 2017). Therefore, researchers have to develop and reformulate many well-known classical concepts such as stress, strain, strain energy, constitutive laws, and balance equations (Eugster and Glocker, 2017). The improvement of existing theoretical frameworks can be achieved with variational approaches and suitable homogenization techniques, which provide efficient micro-to-macro identification (Francfort and Murat, 1986; Abdoul-Anziz and Seppecher, 2018; dell'Isola et al, 2016b).

Although many results have been presented in the literature (and they are really promising), the design of new metamaterials still remains a formidable challenge. The main issue to overcome corresponds to the "complexity" that is involved at every stage of the process in terms of modeling, experimentation, and manufacturing. The sought description requires a robust design approach that creates a synergistic interplay among all involved highly complex design stages to provide an efficient feedback loop in data analysis. This kind of approach may provide the expected progress in the field of the design of novel metamaterials. In order to mitigate such

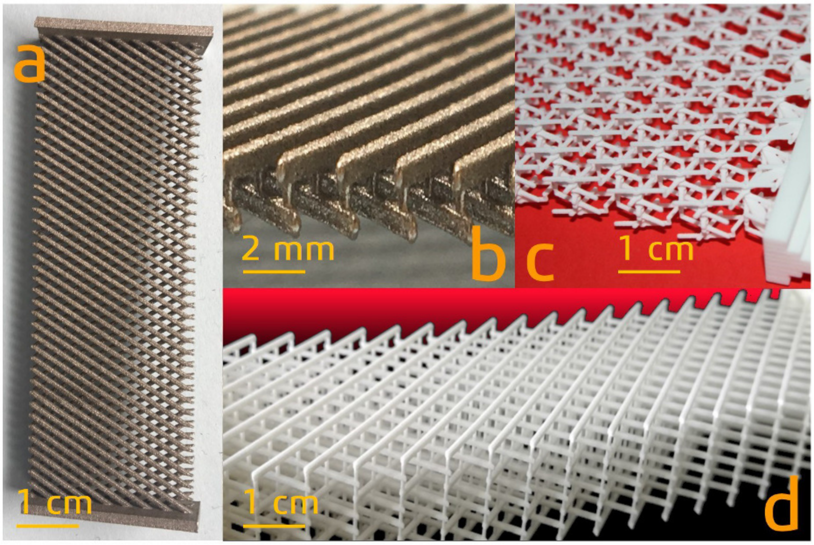

Fig. 20.4 Relevant scales from macro to micro (a-d) for pantographic sheets

challenge, it is suggested to focus, as a first stage, on the design of mechanical metamaterials. For such materials, the behavior at the macroscale is achieved with mechanical interactions of structural networks arranged at different length scales.

The design strategies need to be improved. A systematic methodology, which combines modeling, experimental and manufacturing points of view simultaneously, is called for. Instead of conjecturing metamaterial microstructures without any theoretical guidance, and then trying to experimentally investigate their mechanical properties, an *a priori* synthesis can precede any 3D printing activity, while the feedback from experiments allows for verifying the quality of the theoretical elaboration and, possibly, guide new theoretical investigations. Furthermore, using Lagrangian variational formalisms, one can carefully and efficiently study both static and dynamic responses of every type of materials and design new metamaterials for different industrial applications (Del Vescovo and Giorgio, 2014b; Placidi et al, 2014; Rosi et al, 2013).

The organization of the remainder of the paper is as follows. In Sect. 20.2, more details are given on the proposed synergistic design approach. Some conjectures about the steps required in the design approach are detailed in Sect. 20.3, and promising preliminary results are presented for an additively manufactured mechanical metamaterial in Sect. 20.4.

20.2 Synergistic Approach for Metamaterial Synthesis and Fabrication

The design and fabrication of new metamaterials is a challenging task. A new approach can be followed as the usual logical order is reversed. First, one has to start by characterizing, with suitable Lagrangians, the desired constitutive model. Then, the microstructure of the material whose macroscopic behavior is described by the *a priori* chosen Lagrangian is identified. Lastly, the designed metamaterial is manufactured for the targeted applications. From modeling, experimental and manufacturing standpoints, this procedure is challenging, namely, it consists of designing, fabricating and validating suitable multiscale architectures. In order to be successful, it is crucial to develop an efficient conceptual framework that integrates all the involved design steps by creating a synergistic feedback loop among different disciplines and techniques.

Among many different techniques, Digital Image Correlation (DIC) may have a very prominent role (Sutton et al, 2009; Grédiac and Hild, 2012) to create the envisioned synergistic approach. To check the validity of the design and synthesis of multiscale structures, refined and detailed measurements of material deformations is essential to guide the synthesis process and to validate its results. DIC is an (automatic) image analysis method that measures the deformation of tested specimens and generates displacement and strain fields at prescribed resolution. DIC is very popular in experimental mechanics (Sutton, 2013). This non-contact technique is carried out by using mathematical/numerical registration procedures to process digital images of specimens recorded during the experiment. Sophisticated DIC methods have been recently developed (Hild and Roux, 2012b; Sutton et al, 2009; Hild and Roux, 2012a; Tomičević et al, 2013), which are applicable to many mechanical situations, in particular, in the case of large deformations (Chevalier et al, 2001; Hild et al, 2002). The DIC techniques can efficiently enable the comparison between experimental evidence and theoretical models (Leclerc et al, 2009).

To transform digital images into data, experimental and numerical tools have to be used. The surface of the specimen has generally to be first prepared to make the motion of material points distinguishable for the DIC process. During the experiment, digital images are to be recorded with possibly high-definition cameras. At the beginning of the experiment, a reference digital image is recorded, to represent the reference configuration, and then the displacement field is calculated with a correlation between the reference image and subsequent images of the deformed configuration. The concepts at the basis of continuum mechanics, in particular, its kinematics and “deformatics”, play a central role in DIC (Sutton et al, 2009; Hild and Roux, 2012b).

The DIC techniques have proven to be effective in analyzing experimental results, and they can provide a rapid feedback to guide numerical and theoretical applications in metamaterial design (dell’Isola et al, 2019a,c). Due to the complexity of the considered mechanical systems, no closed-form solutions for their deformation problems are generally available. Hence, numerical simulations must be performed to predict deformation patterns. In general, these simulations must consider large deformation phenomena, and therefore sophisticated algorithms. By comparing the DIC results with numerical results (e.g. finite element simulations, see for example Niiranen et al (2017); Khakalo and Niiranen (2017); Niiranen et al (2016); Khakalo and Niiranen (2018); Eugster et al (2014); Cazzani et al (2016d); Turco et al (2016b); Cazzani et al (2016a,c,b); Grillanda et al (2019); Cazzani et al (2018b,a)), it is possible to validate the results of theoretical and practical syntheses. The detailed analysis of deformations made possible by DIC will point toward weak points in the process. DIC can also be used to analyze the image sets by using displacement fields generated via numerical simulations (i.e. via integrated frameworks (Leclerc et al, 2009; Mathieu et al, 2015)). Let us note that in some previous studies (Quiligotti et al, 2002), the calibration of material parameters was based on the choice of few geometrical properties of the specimen used in the experiments and in the analysis of the difference between these measured quantities and their predicted values. DIC analyses allow for more thorough and systematic comparisons between predicted and measured displacement fields.

Further, DIC techniques are also capable of measuring displacement fields at different length scales (Turco et al, 2018; dell'Isola et al, 2019a,b). This is another essential feature of DIC that will have to be exploited in a more extensive way in the present context as multiscale models are developed in the description and design of metamaterials. By using multiscale DIC analyses, and considering both the desired overall behavior and its microscopic features, the mathematical synthesis process and its transformation into 3D printed specimens can be modified or developed again and again based on the data provided by the DIC-based synergistic procedure.

The theoretical synthesis process of a specific metamaterial produces an architectured microstructure that is represented by the CAD modeler (e.g. standard tessellation language or STL file), and then used in the fabrication step. Within the described design framework, this file can be used in both numerical simulations and 3D printing processes. Among all the advanced manufacturing techniques, 3D printing is one of the most promising technologies for the fabrication of complex materials and geometries. It can be easily optimized to produce specimens made of multiscale architectured materials. Among its main features, 3D printing has a very significant advantage in comparison with conventional manufacturing techniques, namely, it can easily make complex 3D objects with its layer-wise approach, by eliminating the dependence on additional design constraints. One can easily deal with geometric complexity and control the microstructure of fabricated parts in detail. The process of 3D printing also enables for effective multimaterial fabrication. This feature will increase its range of applications in the design of metamaterials.

A further step in the development of the proposed methodology will consist in the formulation of a corrective algorithm, which must automatically modify the STL file once the results of some experiments are analyzed with DIC techniques. A DIC-based system, which couples DIC registration algorithms with synthesis and finite element procedures, may expedite and make effective the feedback redesigning action. In this way, one can automatically unify theoretical and experimental studies by integrating the whole design and verification processes.

20.3 Digital Image Correlation-based Metamaterial Design Process

In this section, the main steps of the DIC-based metamaterial design process is discussed and its main features are delineated. *In the first step of the process*, the required macroscopic behavior has to be identified carefully. At this step, it would be ideal to find out possible design constraints due to the applied manufacturing technology. One cannot 3D print any kind of designed microstructure because of the limits related to geometry, material and resolution of the printing device.

Many interesting macroscopic responses may be sought for different applications. For instance, one can

- require that the designed material remain elastic in large deformation regimes;
- demand the design of a material to exhibit wide frequency band gaps;
- look for an optimized bone scaffold, favoring the reconstruction and remodeling of bone tissues.

A clear understanding of all involved phenomena is an unavoidable prerequisite for this type of design processes, and a precise mathematical formulation is needed for the description of designed metamaterials. For example, in the design of bio-resorbable grafts (Madeo et al, 2011) for bone healing applications (Fig. 20.1), the resorption mechanism must be understood as the material is expected to have a successful and effective integration with the bone structure, biological activities and healthy tissues (Giorgio et al, 2016a; Eugster and Glocker, 2013). Therefore, it is important to understand the driving features of newly designed metamaterials and their compatibility with existing systems. All these phenomenological aspects of the designed metamaterial must be specified by means of Lagrangian action functionals (and possibly Rayleigh dissipation functionals), which are assumed *a priori* to govern the behavior of the designed metamaterials (dell'Isola and Placidi, 2011).

In the second step, the hierarchical architecture of the metamaterial is synthesized. From the theoretical standpoint, a mathematical model, which describes the desired behavior, has been already proposed at the previous step. Due to the hierarchical complexity of the material, a multiscale modeling procedure must be followed in the synthesis scheme. It has to be noted that only few materials (i.e. very restricted classes of Lagrangian and Rayleigh functionals) can be synthesized by using a single scale architecture. Instead of trying to implement ineffective trial and error computations between microscopic and macroscopic scales, an extra intermediate step may be included in the synthesis scheme. Different discrete mechanical systems at several intermediate scales are to be introduced.

The proposed process is very similar to that used in the theory of synthesis of analogue circuits (Giorgio et al, 2015). As every passive linear n-port circuit can be synthesized by using an algorithmically produced graph and by linking any pair of points of this graph with four specific circuital elements (i.e. resistances, inductances, capacitors and transformers), it is expected that the most general microstructures for mechanical metamaterials can be built by reproducing some basic microstructures at different length scales. Moreover, by introducing only discrete mesoscopic models, the numerical algorithms are implemented efficiently, and the micro-to-macro transition process can be performed more easily (Turco et al, 2016a). As a further perspective, deduced from the analogue circuits field, it would be interesting and useful to produce piezoelectromechanical microstructures to be controlled by means of piezoelectric actuators. Some relevant results already present in the literature about this perspective can be found in Casadei et al (2012); Bergamini et al (2006, 2015).

In the third step, the synthesis scheme previously obtained must be transformed into real-world specimens, for instance by means of 3D printing techniques. Every basic microstructure must be built by supplying a suitable STL file to the selected 3D printer. These files can efficiently be used as a basis for *a posteriori* finite elements analyses and, at the same time, as guide for DIC data collection. This step requires the development of innovative engineering solutions (Golaszewski et al, 2018; Turco et al, 2017b; Gunenthiram et al, 2017; Haboudou et al, 2003; Andreau et al, 2019).

In the fourth step, the validation of the synthesis and construction steps must be performed. This step requires the systematic use of DIC-based techniques. Due to the multiscale nature of the considered microstructures, some computational meshes must be generated at different length scales by using, for instance, the gray level images of the tested specimens.

We give here an example already available in dell'Isola et al (2019b). In Fig. 20.5, meshes at macroscopic and mesoscopic levels are shown in the case of a pantographic sheet. These meshes overlap with the gray images of the test specimen. In the analysis, a coarse discretization of the region of interest is first created with triangular elements independent from its mesostructure or microstructure (Fig. 20.5 a). Then, the mesh is successively refined to increase the accuracy of the results, and in this way, the convergence of the analysis is expected. From the multiscale standpoint, this step looks like the transitions from continuum to discrete models.

The final steps consist of going back, thanks to the experimental results as elaborated by DIC, to the synthesis step and/or to the construction step. The discrepancies between the desired and the measured responses, as revealed by multiscale DIC analyses, redirect both the synthesis process and the scheme of specimens production. This feedback loop is made easier by the fact that the DIC meshes are tailored for micro-to-macro model identifications (Grédiac and Hild, 2012).

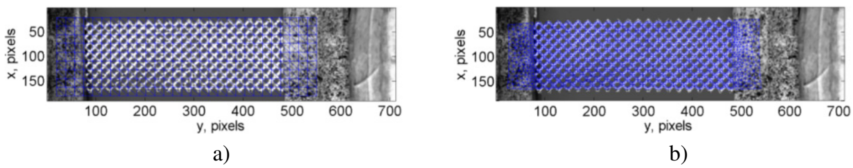

Fig. 20.5 Example of multiscale mesh applied to a pantographic structure

20.4 Preliminary Results

In this section, some preliminary results are presented to show the applicability of the proposed approach for the design of mechanical metamaterials. For this purpose, pantographic structures are considered. This is an example of a theoretical problem formulated as the result of experimental observations via DIC analyses. This multiscale design approach can be further utilized to develop more sophisticated DIC techniques to design and fabricate metamaterials (dell'Isola et al, 2019a,b).

The studies related to the design of higher gradient continua (Mindlin, 1965; dell'Isola et al, 2015a) would be addressed to show the potential of such a synergistic approach. For classical continuum media, the Cauchy theory is applied in terms of balance equations. This theory assumes that the strain energy is only a function of the first gradient of the displacement field. However, with the design of new advanced materials, it was shown that the strain energy can be a function of higher gradients of displacement fields (Seppecher et al, 2011; Alibert et al, 2003). Thus, higher gradient theories are developed to derive the macroscopic behavior of multiscale materials. In Fig. 20.6, the design of a beam whose strain energy depends on higher gradients of the displacement field in the axial direction is presented. A unit cell of the beam is arranged as shown in Fig. 20.6a), and the different levels of the structure in Fig. 20.6c). Then, by using appropriate homogenization techniques, it was proven that the strain energy depends on the second gradient of the displacement field in both vertical and horizontal directions (Seppecher et al, 2011). To simplify the micro-to-macro upscaling, a discrete model consisting of a network of mass particles connected with rotational and extensional springs was introduced (dell'Isola et al, 2016b). For the fabrication of the designed metamaterials, the theoretical data are transformed into a manufacturing process. For this particular structure, the following design has been proposed. Two layers of beams are oriented orthogonally and connected with a set of cylinders or joints allowing for the relative displacement of the beams (dell'Isola et al, 2015b). The alternation of empty and filled spaces enables DIC analyses for the resulting specimen (Turco et al, 2018). As seen from this example, one can extend

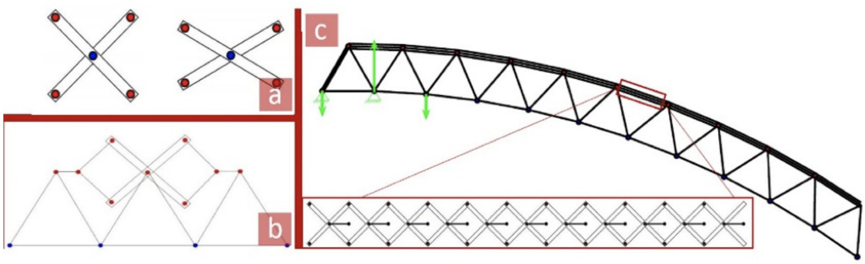

Fig. 20.6 Example of a multiscale scheme for higher gradient one-dimensional material (Seppecher et al, 2011)

this model and fabricate higher order gradient systems by exploiting this multiscale design approach (see Fig. 20.6b) for third gradient model).

The equilibrium shapes of the pantographic structures are shown in Fig. 20.7 for different cases. In Fig. 20.7a)-b), the experimental results are presented for shear and torsion loadings of a pantographic sheet. These pantographic sheets are made of aluminum alloy, and they are 3D printed and designed based on composition of elementary blocks. In Fig. 20.7c), the shear deformation is tracked by performing local DIC registrations.

The displacement fields of the macro- and meso-scale meshes (Fig. 20.5) are reported in Fig. 20.8. In this particular example, the results are presented for longitudinal displacement fields measured during a tensile (i.e. bias) test.

Regarding the experimental study, three major challenges were observed. First, the extension of this application to three dimensional problems might be difficult as the fabrication of beam lattices deforming in 3D is a more complex procedure. It is clearly more complicated to design a material exhibiting the desired overall behavior in three dimensional applications. Using ball joint links would be helpful to make this design possible. In Fig. 20.9, the design of a pivot/hinge link is illustrated. They can be fabricated with 3D printing technologies. Second, a multiscale architecture with nonlinear macroscale responses might be synthesized. This can be avoided by exploiting the synergistic nature of the design framework. Third, possible instability and buckling at the microscale may create a dramatic change in the macroscopic

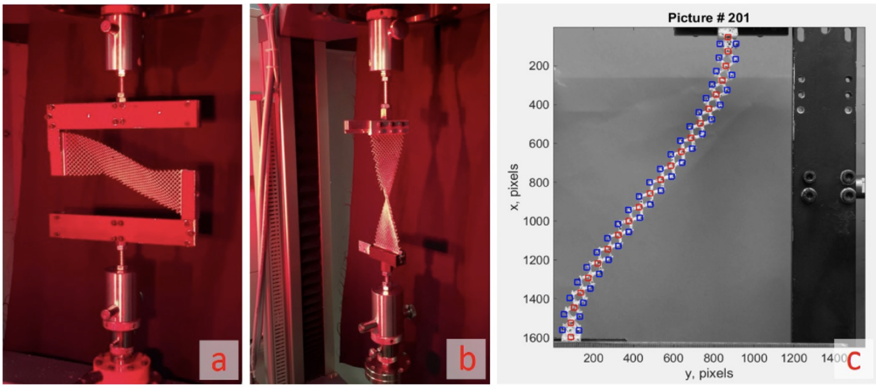

Fig. 20.7 Experiments on 3D printed pantographic sheet

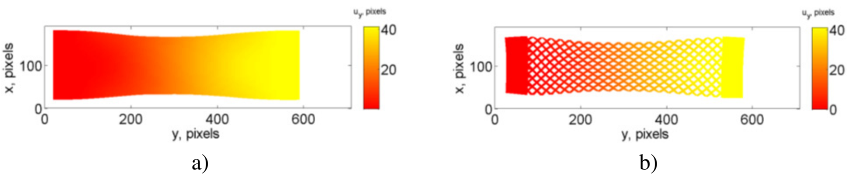

Fig. 20.8 Deformation of different DIC meshes in longitudinal direction

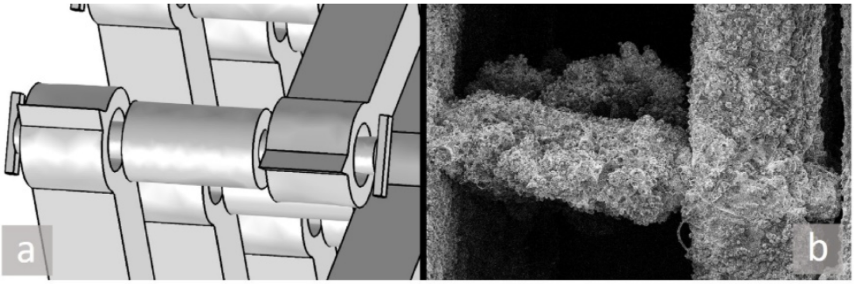

Fig. 20.9 Designed and printed pivot/hinge link dell'Isola et al (2019c)

response of the material. Hence, different critical phenomena must be taken into account to increase the reliability of the approach.

Moreover, from the manufacturing point of view, the structural pattern of the material must be arranged regarding the technological limits of the selected 3D printing technique. Some design rules must be standardized for 3D printing applications. Although it seems that 3D printing can easily deal with any geometric complexity, some important criteria have to be considered before fabricating the designed metamaterials. In general, these rules are applied for the design of supported/unsupported walls, overhangs, holes, connecting/moving parts, and engravings, and may vary for different 3D printing technologies. Further, in 3D printing applications, the overall quality of printed parts is highly dependent on the processing parameters. The latter ones may vary for different materials and applications. Therefore, it is crucial to investigate the behavior of printed materials with different processing parameters and their feasibility in metamaterial applications.

20.5 Conclusion

In this paper, in the process of synthesis and construction of novel metamaterials, it is proposed to systematically use DIC-based methodologies. Based on DIC output, the synthesis process of a specific metamaterial may be partially or totally automated by using algorithms similar to those utilized in structural optimization. In the short term, it is expected that by using DIC techniques to design, characterize and validate the overall properties of newly designed metamaterials, many interesting novel microstructures and useful exotic mechanisms may be invented. Another possible field of application for this techniques consists in the family of micropolar materials (Eremeyev and Pietraszkiewicz, 2016; Eremeyev and Lebedev, 2011) and elastic shells (Eremeyev and Zubov, 2007; Altenbach et al, 2015; Eremeyev and Lebedev, 2016). If one wants to address dynamical studies in the field of mechanical metamaterials, new methods and new approaches must be introduced. Some results useful for a future characterization of the dynamics in memamaterials can be found in Cazzani and

Ruge (2016, 2013); Piccardo et al (2014); Ferretti and Piccardo (2013); Luongo and Zulli (2012); Luongo et al (2008).

To show the applicability of the introduced approach, some preliminary results were presented, namely, those concerning so-called pantographic structures (see for example Placidi et al, 2017a; Scerrato et al, 2016; Boutin et al, 2017). Possible issues related to the design and manufacturing phases have been discussed and highlighted for the future applications.

It is envisioned that the proposed synergistic approach can be extended to the design of the following solutions:

1. metamaterials remaining in their elastic regime for large deformations,
2. metamaterials maintaining their mechanical properties under large temperature changes and experiencing only very limited creep phenomena,
3. metamaterials for bone scaffolds that are optimized for being bio-resorbable and bio-compatible with the host tissues.

Concerning this last class of metamaterials (e.g. see Madeo et al, 2012; Lekszycki and dell'Isola, 2012), the DIC-based framework may design bone scaffolds with adaptive optimal behavior. The latter is obtained when the metamaterial exhibits a proper response to a vast variety of external stimuli. Further, the desired overall response of biomechanical metamaterials can be achieved by enriching their microstructure with other exotic materials such as *shape memory alloys*.

References

Abdoul-Anziz H, Seppecher P (2018) Strain gradient and generalized continua obtained by homogenizing frame lattices. Mathematics and mechanics of complex systems 6(3):213–250

Alibert JJ, Seppecher P, dell'Isola F (2003) Truss modular beams with deformation energy depending on higher displacement gradients. Mathematics and Mechanics of Solids 8(1):51–73

Altenbach H, Eremeyev VA, Naumenko K (2015) On the use of the first order shear deformation plate theory for the analysis of three-layer plates with thin soft core layer. ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik 95(10):1004–1011

Ambati M, Fang N, Sun C, Zhang X (2007) Surface resonant states and superlensing in acoustic metamaterials. Physical Review B 75(19):195,447

Andreau O, Koutiri I, Peyre P, Penot JD, Saintier N, Pessard E, De Terris T, Dupuy C, Baudin T (2019) Texture control of 316l parts by modulation of the melt pool morphology in selective laser melting. Journal of Materials Processing Technology 264:21–31

Andrews E, Sanders W, Gibson LJ (1999) Compressive and tensile behaviour of aluminum foams. Materials Science and Engineering: A 270(2):113–124

Andreykiv A, Prendergast P, Van Keulen F, Swieszkowski W, Rozing P (2005) Bone ingrowth simulation for a concept glenoid component design. Journal of biomechanics 38(5):1023–1033

Ao X, Chan C (2008) Far-field image magnification for acoustic waves using anisotropic acoustic metamaterials. Physical Review E 77(2):025,601

Auffray N, dell'Isola F, Eremeyev V, Madeo A, Placidi L, Rosi G (2014) Least action principle for second gradient continua and capillary fluids: A lagrangian approach following piola's point of view. In: dell'Isola F, Maier G, Perego U, Andreaus U, Esposito R, Forest S (eds) The Complete

Works of Gabrio Piola: Volume I, Springer, Cham, Advanced Structured Materials, vol 38, pp 606–694
Barchiesi E, Placidi L (2017) A review on models for the 3d statics and 2d dynamics of pantographic fabrics. In: Wave Dynamics and Composite Mechanics for Microstructured Materials and Metamaterials, Springer, pp 239–258
Barchiesi E, Spagnuolo M, Placidi L (2018) Mechanical metamaterials: a state of the art. Mathematics and Mechanics of Solids 24(1):212–234, doi:10.1177/1081286517735695
Barchiesi E, Spagnuolo M, Placidi L (2019) Mechanical metamaterials: a state of the art. Mathematics and Mechanics of Solids 24(1):212–234, doi:10.1177/1081286517735695
Bergamini A, Christen R, Maag B, Motavalli M (2006) A sandwich beam with electrostatically tunable bending stiffness. Smart materials and structures 15(3):678
Bergamini AE, Zündel M, Flores Parra EA, Delpero T, Ruzzene M, Ermanni P (2015) Hybrid dispersive media with controllable wave propagation: A new take on smart materials. Journal of Applied Physics 118(15):154,310
Boutin C, Giorgio I, Placidi L, et al (2017) Linear pantographic sheets: Asymptotic micro-macro models identification. Mathematics and Mechanics of Complex Systems 5(2):127–162
Carcaterra A, dell'Isola F, Esposito R, Pulvirenti M (2015) Macroscopic description of microscopically strongly inhomogeneous systems: A mathematical basis for the synthesis of higher gradients metamaterials. Archive for Rational Mechanics and Analysis 218(3):1239–1262
Casadei F, Delpero T, Bergamini A, Ermanni P, Ruzzene M (2012) Piezoelectric resonator arrays for tunable acoustic waveguides and metamaterials. Journal of Applied Physics 112(6):064,902
Cazzani A, Ruge P (2013) Rotor platforms on pile-groups running through resonance: A comparison between unbounded soil and soil-layers resting on a rigid bedrock. Soil Dynamics and Earthquake Engineering 50:151–161, doi:10.1016/j.soildyn.2013.02.022
Cazzani A, Ruge P (2016) Stabilization by deflation for sparse dynamical systems without loss of sparsity. Mechanical Systems and Signal Processing 70–71:664–681, doi:10.1016/j.ymssp.2015.09.027
Cazzani A, Malagù M, Turco E (2016a) Isogeometric analysis: a powerful numerical tool for the elastic analysis of historical masonry arches. Continuum Mechanics and thermodynamics 28(1-2):139–156
Cazzani A, Malagù M, Turco E (2016b) Isogeometric analysis of plane-curved beams. Mathematics and Mechanics of Solids 21(5):562–577
Cazzani A, Malagù M, Turco E, Stochino F (2016c) Constitutive models for strongly curved beams in the frame of isogeometric analysis. Mathematics and Mechanics of Solids 21(2):182–209
Cazzani A, Stochino F, Turco E (2016d) An analytical assessment of finite element and isogeometric analyses of the whole spectrum of timoshenko beams. ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik 96(10):1220–1244
Cazzani A, Rizzi N, Stochino F, Turco E (2018a) Modal analysis of laminates by a mixed assumed-strain finite element model. Mathematics and Mechanics of Solids 23(1):99–119, doi:10.1177/1081286516666405
Cazzani A, Serra M, Stochino F, Turco E (2018b) A refined assumed strain finite element model for statics and dynamics of laminated plates. Continuum Mechanics and Thermodynamics pp 1–28, doi:10.1007/s00161-018-0707-x
Chevalier L, Calloch S, Hild F, Marco Y (2001) Digital image correlation used to analyze the multiaxial behavior of rubber-like materials. Eur J Mech A/Solids 20:169–187
Chia HN, Wu BM (2015) Recent advances in 3d printing of biomaterials. Journal of Biological Bngineering 9(1):4
Cima M, Sachs E, Cima L, Yoo J, Khanuja S, Borland S, Wu B, Giordano R (1994) Computer-derived microstructures by 3d printing: bio-and structural materials. In: Solid Freeform Fabr Symp Proc: DTIC Document, pp 181–90
di Cosmo F, Laudato M, Spagnuolo M (2018) Acoustic metamaterials based on local resonances: homogenization, optimization and applications. In: Generalized Models and Non-classical Approaches in Complex Materials 1, Springer, pp 247–274

Del Vescovo D, Giorgio I (2014a) Dynamic problems for metamaterials: review of existing models and ideas for further research. International Journal of Engineering Science 80:153–172

Del Vescovo D, Giorgio I (2014b) Dynamic problems for metamaterials: review of existing models and ideas for further research. International Journal of Engineering Science 80:153–172

dell'Isola F, Placidi L (2011) Variational principles are a powerful tool also for formulating field theories. In: Variational models and methods in solid and fluid mechanics, Springer, pp 1–15

dell'Isola F, Andreaus U, Placidi L (2015a) At the origins and in the vanguard of peridynamics, non-local and higher-gradient continuum mechanics: An underestimated and still topical contribution of gabrio piola. Mathematics and Mechanics of Solids 20(8):887–928

dell'Isola F, Lekszycki T, Pawlikowski M, Grygoruk R, Greco L (2015b) Designing a light fabric metamaterial being highly macroscopically tough under directional extension: first experimental evidence. Zeitschrift für angewandte Mathematik und Physik 66(6):3473–3498

dell'Isola F, Steigmann D, Della Corte A (2015c) Synthesis of fibrous complex structures: designing microstructure to deliver targeted macroscale response. Applied Mechanics Reviews 67(6):060,804

dell'Isola F, Bucci S, Battista A (2016a) Against the fragmentation of knowledge: The power of multidisciplinary research for the design of metamaterials. In: Advanced Methods of Continuum Mechanics for Materials and Structures, Springer, pp 523–545

dell'Isola F, Giorgio I, Pawlikowski M, Rizzi N (2016b) Large deformations of planar extensible beams and pantographic lattices: heuristic homogenization, experimental and numerical examples of equilibrium. Proc R Soc A 472(2185):20150,790

dell'Isola F, Della Corte A, Giorgio I (2017) Higher-gradient continua: The legacy of piola, mindlin, sedov and toupin and some future research perspectives. Mathematics and Mechanics of Solids 22(4):852–872

dell'Isola F, Seppecher P, Alibert JJ, Lekszycki T, Grygoruk R, Pawlikowski M, Steigmann D, Giorgio I, Andreaus U, Turco E, Golaszewski M, Rizzi N, Boutin C, Eremeyev VA, Misra A, Placidi L, Barchiesi E, Greco L, Cuomo M, Cazzani A, Corte AD, Battista A, Scerrato D, Zurba Eremeeva I, Rahali Y, Ganghoffer JF, Müller W, Ganzosch G, Spagnuolo M, Pfaff A, Barcz K, Hoschke K, Neggers J, Hild F (2019a) Pantographic metamaterials: an example of mathematically driven design and of its technological challenges. Continuum Mechanics and Thermodynamics doi:10.1007/s00161-018-0689-8

dell'Isola F, Seppecher P, Spagnuolo M, Barchiesi E, Hild F, Lekszycki T, Giorgio I, Placidi L, Andreaus U, Cuomo M, Eugster SR, Pfaff A, Hoschke K, Langkemper R, Turco E, Sarikaya R, MISRA A, DE ANGELO M, D'Annibale F, Bouterf A, Pinelli X, Misra A, Desmorat B, Pawlikowski M, Dupuy C, Scerrato D, Peyre P, Laudato M, Manzari L, Göransson P, Hesch C, Hesch S, Franciosi P, Dirrenberger J, Maurin F, Vangelatos Z, Grigoropoulos C, Melissinaki V, Farsari M, Muller W, Abali E, Liebold C, Ganzosch G, Harrison P, Drobnicki R, Igumnov LA, Alzahrani F, Hayat T (2019b) Advances in Pantographic Structures: Design, Manufacturing, Models, Experiments and Image Analyses. Continuum Mechanics and Thermodynamics doi:10.1007/s00161-019-00806-x

dell'Isola F, Seppecher P, Spagnuolo M, Barchiesi E, Hild F, Lekszycki T, Giorgio I, Placidi L, Andreaus U, Cuomo M, et al (2019c) Advances in pantographic structures: design, manufacturing, models, experiments and image analyses. Continuum Mechanics and Thermodynamics pp 1–52, doi:10.1007/s00161-019-00806-x

Deng K, Ding Y, He Z, Zhao H, Shi J, Liu Z (2009) Theoretical study of subwavelength imaging by acoustic metamaterial slabs. Journal of Applied Physics 105(12):124,909

Engheta N, Ziolkowski RW (2006) Metamaterials: physics and engineering explorations. John Wiley & Sons

Eremeyev V, Zubov L (2007) On constitutive inequalities in nonlinear theory of elastic shells. ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik: Applied Mathematics and Mechanics 87(2):94–101

Eremeyev VA, Lebedev LP (2011) Existence theorems in the linear theory of micropolar shells. ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik 91(6):468–476

Eremeyev VA, Lebedev LP (2016) Mathematical study of boundary-value problems within the framework of steigmann–ogden model of surface elasticity. Continuum Mechanics and Thermodynamics 28(1-2):407–422

Eremeyev VA, Pietraszkiewicz W (2016) Material symmetry group and constitutive equations of micropolar anisotropic elastic solids. Mathematics and Mechanics of Solids 21(2):210–221

Eugster S, Hesch C, Betsch P, Glocker C (2014) Director-based beam finite elements relying on the geometrically exact beam theory formulated in skew coordinates. International Journal for Numerical Methods in Engineering 97(2):111–129

Eugster SR, Glocker C (2013) Constraints in structural and rigid body mechanics: a frictional contact problem. Annals of solid and structural mechanics 5(1-2):1–13

Eugster SR, Glocker C (2017) On the notion of stress in classical continuum mechanics. Mathematics and Mechanics p 299

Ferretti M, Piccardo G (2013) Dynamic modeling of taut strings carrying a traveling mass. Continuum Mechanics and Thermodynamics 25(2-4):469–488

Francfort GA, Murat F (1986) Homogenization and optimal bounds in linear elasticity. Archive for Rational mechanics and Analysis 94(4):307–334

Gatt R, Mizzi L, Azzopardi JI, Azzopardi KM, Attard D, Casha A, Briffa J, Grima JN (2015) Hierarchical auxetic mechanical metamaterials. Scientific reports 5:8395

Geers M, Kouznetsova VG, Brekelmans W (2003) Multiscale first-order and second-order computational homogenization of microstructures towards continua. International Journal for Multiscale Computational Engineering 1(4)

Germain P (1973) The method of virtual power in continuum mechanics. part 2: Microstructure. SIAM Journal on Applied Mathematics 25:556–575, doi:10.1137/0125053, URL `doi.org/10.1137/0125053`

Gibson LJ, Ashby MF, Karam G, Wegst U, Shercliff H (1995) The mechanical properties of natural materials. ii. microstructures for mechanical efficiency. Proceedings of the Royal Society of London Series A: Mathematical and Physical Sciences 450(1938):141–162

Giorgio I, Galantucci L, Della Corte A, Del Vescovo D (2015) Piezo-electromechanical smart materials with distributed arrays of piezoelectric transducers: current and upcoming applications. International Journal of Applied Electromagnetics and Mechanics 47(4):1051–1084

Giorgio I, Andreaus U, Madeo A (2016a) The influence of different loads on the remodeling process of a bone and bioresorbable material mixture with voids. Continuum Mechanics and Thermodynamics 28(1-2):21–40

Giorgio I, Andreaus U, Scerrato D, dell'Isola F (2016b) A visco-poroelastic model of functional adaptation in bones reconstructed with bio-resorbable materials. Biomechanics and modeling in mechanobiology 15(5):1325–1343

Giorgio I, Andreaus U, Lekszycki T, Corte AD (2017) The influence of different geometries of matrix/scaffold on the remodeling process of a bone and bioresorbable material mixture with voids. Mathematics and Mechanics of Solids 22(5):969–987

Golaszewski M, Grygoruk R, Giorgio I, Laudato M, Di Cosmo F (2018) Metamaterials with relative displacements in their microstructure: technological challenges in 3d printing, experiments and numerical predictions. Continuum Mechanics and Thermodynamics pp 1–20, doi:10.1007/s00161-018-0692-0

Grédiac M, Hild F (2012) Full-field measurements and identification in solid mechanics. John Wiley & Sons

Grillanda N, Chiozzi A, Bondi F, Tralli A, Manconi F, Stochino F, Cazzani A (2019) Numerical insights on the structural assessment of historical masonry stellar vaults: the case of santa maria del monte in cagliari. Continuum Mechanics and Thermodynamics pp 1–24, doi:10.1007/s00161-019-00752-8

Gunenthiram V, Peyre P, Schneider M, Dal M, Coste F, Fabbro R (2017) Analysis of laser–melt pool–powder bed interaction during the selective laser melting of a stainless steel. Journal of Laser Applications 29(2):022,303

Haboudou A, Peyre P, Vannes A (2003) Study of keyhole and melt pool oscillations in dual beam welding of aluminum alloys: effect on porosity formation. In: First International Symposium on

High-Power Laser Macroprocessing, International Society for Optics and Photonics, vol 4831, pp 295–301
Heckele M, Schomburg W (2003) Review on micro molding of thermoplastic polymers. Journal of Micromechanics and Microengineering 14(3):R1
Hild F, Roux S (2012a) Comparison of local and global approaches to digital image correlation. Experimental Mechanics 52(9):1503–1519
Hild F, Roux S (2012b) Digital image correlation. In: Rastogi P, Hack E (eds) Optical Methods for Solid Mechanics. A Full-Field Approach, Wiley-VCH, Weinheim (Germany), pp 183–228
Hild F, Raka B, Baudequin M, Roux S, Cantelaube F (2002) Multi-scale displacement field measurements of compressed mineral wool samples by digital image correlation. Appl Optics IP 41(32):6815–6828
Jia H, Ke M, Hao R, Ye Y, Liu F, Liu Z (2010) Subwavelength imaging by a simple planar acoustic superlens. Applied Physics Letters 97(17):173,507
Kadic M, Bückmann T, Stenger N, Thiel M, Wegener M (2012) On the practicability of pentamode mechanical metamaterials. Applied Physics Letters 100(19):191,901
Khakalo S, Niiranen J (2017) Isogeometric analysis of higher-order gradient elasticity by user elements of a commercial finite element software. Computer-Aided Design 82:154–169
Khakalo S, Niiranen J (2018) Form ii of mindlin's second strain gradient theory of elasticity with a simplification: For materials and structures from nano-to macro-scales. European Journal of Mechanics-A/Solids 71:292–319
Lakes R (1987) Foam structures with a negative poisson's ratio. Science 235:1038–1041
Lakes R (1993) Materials with structural hierarchy. Nature 361(6412):511
Laudato M, Manzari L, Barchiesi E, Di Cosmo F, Göransson P (2018) First experimental observation of the dynamical behavior of a pantographic metamaterial. Mechanics Research Communications 94:125–127, doi:10.1016/j.mechrescom.2018.11.003
Leclerc H, Périé J, Roux S, Hild F (2009) Integrated digital image correlation for the identification of mechanical properties. In: Gagalowicz A, Philips W (eds) Computer Vision/Computer Graphics CollaborationTechniques. MIRAGE 2009, Springer, Berlin, Heidelberg, Lecture Notes in Computer Science, vol 5496, pp 161–171
Lee JH, Singer JP, Thomas EL (2012) Micro-/nanostructured mechanical metamaterials. Advanced materials 24(36):4782–4810
Lekszycki T, dell'Isola F (2012) A mixture model with evolving mass densities for describing synthesis and resorption phenomena in bones reconstructed with bio-resorbable materials. ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik 92(6):426–444
Liu Y, Greene MS, Chen W, Dikin DA, Liu WK (2013) Computational microstructure characterization and reconstruction for stochastic multiscale material design. Computer-Aided Design 45(1):65–76
Liu Z, Zhang X, Mao Y, Zhu Y, Yang Z, Chan CT, Sheng P (2000) Locally resonant sonic materials. science 289(5485):1734–1736
Liu Z, Lee H, Xiong Y, Sun C, Zhang X (2007) Far-field optical hyperlens magnifying sub-diffraction-limited objects. science 315(5819):1686–1686
Luongo A, Zulli D (2012) Dynamic instability of inclined cables under combined wind flow and support motion. Nonlinear Dynamics 67(1):71–87
Luongo A, Zulli D, Piccardo G (2008) Analytical and numerical approaches to nonlinear galloping of internally resonant suspended cables. Journal of Sound and Vibration 315(3):375–393
Madeo A, Lekszycki T, dell'Isola F (2011) A continuum model for the bio-mechanical interactions between living tissue and bio-resorbable graft after bone reconstructive surgery. Comptes Rendus Mécanique 339(10):625–640
Madeo A, George D, Lekszycki T, Nierenberger M, Rémond Y (2012) A second gradient continuum model accounting for some effects of micro-structure on reconstructed bone remodelling. Comptes Rendus Mécanique 340(8):575–589
Madou MJ (2011) Manufacturing techniques for microfabrication and nanotechnology, vol 2. CRC press

Maggi A, Li H, Greer JR (2017) Three-dimensional nano-architected scaffolds with tunable stiffness for efficient bone tissue growth. Acta Biomaterialia 63:294–305

Masuzawa T, Fujino M, Kobayashi K, Suzuki T, Kinoshita N (1985) Wire electro-discharge grinding for micro-machining. CIRP Annals 34(1):431–434

Mathieu F, Leclerc H, Hild F, Roux S (2015) Estimation of elastoplastic parameters via weighted FEMU and integrated-DIC. Exp Mech 55(1):105–119

Mindlin RD (1965) Second gradient of strain and surface-tension in linear elasticity. International Journal of Solids and Structures 1(4):417–438

Misra A, Lekszycki T, Giorgio I, Ganzosch G, Müller WH, dell'Isola F (2018) Pantographic metamaterials show atypical Poynting effect reversal. Mechanics Research Communications 89:6–10

Niiranen J, Khakalo S, Balobanov V, Niemi AH (2016) Variational formulation and isogeometric analysis for fourth-order boundary value problems of gradient-elastic bar and plane strain/stress problems. Computer Methods in Applied Mechanics and Engineering 308:182–211

Niiranen J, Kiendl J, Niemi AH, Reali A (2017) Isogeometric analysis for sixth-order boundary value problems of gradient-elastic kirchhoff plates. Computer Methods in Applied Mechanics and Engineering 316:328–348

Norris AN (2009) Acoustic metafluids. The Journal of the Acoustical Society of America 125(2):839–849

Ok JG, Seok Youn H, Kyu Kwak M, Lee KT, Jae Shin Y, Jay Guo L, Greenwald A, Liu Y (2012) Continuous and scalable fabrication of flexible metamaterial films via roll-to-roll nanoimprint process for broadband plasmonic infrared filters. Applied Physics Letters 101(22):223,102

Pearton S, Abernathy C, Ren F, Lothian J, Wisk P, Katz A (1993) Dry and wet etching characteristics of inn, aln, and gan deposited by electron cyclotron resonance metalorganic molecular beam epitaxy. Journal of Vacuum Science & Technology A: Vacuum, Surfaces, and Films 11(4):1772–1775

Pendry JB, Holden A, Stewart W, Youngs I (1996) Extremely low frequency plasmons in metallic mesostructures. Physical review letters 76(25):4773

Piccardo G, Ranzi G, Luongo A (2014) A complete dynamic approach to the generalized beam theory cross-section analysis including extension and shear modes. Mathematics and Mechanics of Solids 19(8):900–924

Placidi L, Rosi G, Giorgio I, Madeo A (2014) Reflection and transmission of plane waves at surfaces carrying material properties and embedded in second-gradient materials. Mathematics and Mechanics of Solids 19(5):555–578

Placidi L, Andreaus U, Giorgio I (2017a) Identification of two-dimensional pantographic structure via a linear D4 orthotropic second gradient elastic model. Journal of Engineering Mathematics 103(1):1–21

Placidi L, Barchiesi E, Battista A (2017b) An inverse method to get further analytical solutions for a class of metamaterials aimed to validate numerical integrations. In: Mathematical Modelling in Solid Mechanics, Springer, pp 193–210

Quiligotti S, Maugin GA, dell'Isola F (2002) Wave motions in unbounded poroelastic solids infused with compressible fluids. Zeitschrift für angewandte Mathematik und Physik ZAMP 53(6):1110–1138

Rosi G, Giorgio I, Eremeyev VA (2013) Propagation of linear compression waves through plane interfacial layers and mass adsorption in second gradient fluids. ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik 93(12):914–927

Rumpf RC, Pazos J, Garcia CR, Ochoa L, Wicker R (2013) 3d printed lattices with spatially variant self-collimation. Progress In Electromagnetics Research 139:1–14

Scerrato D, Giorgio I, Rizzi NL (2016) Three-dimensional instabilities of pantographic sheets with parabolic lattices: numerical investigations. Zeitschrift für angewandte Mathematik und Physik 67(3):53

Seppecher P, Alibert JJ, dell'Isola F (2011) Linear elastic trusses leading to continua with exotic mechanical interactions. Journal of Physics: Conference Series 319(1):012,018

Sutton M (2013) Computer vision-based, noncontacting deformation measurements in mechanics: A generational transformation. Appl Mech Rev 65(AMR-13-1009):050,802

Sutton M, Orteu J, Schreier H (2009) Image correlation for shape, motion and deformation measurements: basic concepts, theory and applications. Springer Science & Business Media

Teo WE, Ramakrishna S (2006) A review on electrospinning design and nanofibre assemblies. Nanotechnology 17(14):R89

Tomičević Z, Hild F, Roux S (2013) Mechanics-aided digital image correlation. The Journal of Strain Analysis for Engineering Design 48(5):330–343

Turco E, dell'Isola F, Cazzani A, Rizzi NL (2016a) Hencky-type discrete model for pantographic structures: numerical comparison with second gradient continuum models. Zeitschrift für angewandte Mathematik und Physik 67(4):85

Turco E, Golaszewski M, Cazzani A, Rizzi NL (2016b) Large deformations induced in planar pantographic sheets by loads applied on fibers: experimental validation of a discrete lagrangian model. Mechanics Research Communications 76:51–56

Turco E, Giorgio I, Misra A, dell'Isola F (2017a) King post truss as a motif for internal structure of (meta) material with controlled elastic properties. Royal Society open science 4(10):171,153

Turco E, Golaszewski M, Giorgio I, D'Annibale F (2017b) Pantographic lattices with non-orthogonal fibres: Experiments and their numerical simulations. Composites Part B: Engineering 118:1–14

Turco E, Misra A, Pawlikowski M, dell'Isola F, Hild F (2018) Enhanced piola–hencky discrete models for pantographic sheets with pivots without deformation energy: Numerics and experiments. International Journal of Solids and Structures

Vangelatos Z, Komvopoulos K, Grigoropoulos C (2018) Vacancies for controlling the behavior of microstructured three-dimensional mechanical metamaterials. Mathematics and Mechanics of Solids p 1081286518810739

Vangelatos Z, Melissinaki V, Farsari M, Komvopoulos K, Grigoropoulos C (2019) Intertwined microlattices greatly enhance the performance of mechanical metamaterials. Mathematics and Mechanics of Solids p 1081286519848041

Veselago V (1967) Properties of materials having simultaneously negative values of the dielectric and magnetic susceptibilities. Soviet Physics Solid State USSR 8:2854–2856

Veselago VG (1968) The electrodynamics of substances with simultaneously negative values of and μ. Soviet physics uspekhi 10(4):509

Wegst UG, Bai H, Saiz E, Tomsia AP, Ritchie RO (2015) Bioinspired structural materials. Nature materials 14(1):23

Zhang S, Yin L, Fang N (2009) Focusing ultrasound with an acoustic metamaterial network. Physical review letters 102(19):194,301