Studying the Curvature and Determining the Tangential Velocity profile of a Rotating Fluid using Navier–Stokes equations

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Abstract. The shape of the liquid surface for a fluid present in a uniformly rotating cylinder was previously determined by making a Tangential velocity gradient along the radius of the rotating cylindrical container. A very similar principle can be applied if the direction of the produced velocity gradient is reversed, for which the source of rotation will be present at the central axis of the cylindrical vessel in which the liquid is present. Now if the described system is completely closed, the angular velocity will decrease as a function of time. But when the surface of the rotating fluid is kept free, then the Tangential velocity profile would be similar to that of the Taylor-Couette Flow, with a modification that; due to formation of a curvature at the surface, the Navier-Stokes law is to be modified. Now the final equation may not seem to have a proper general solution, but can be approximated to certain solvable expressions for specific cases of angular velocity.

1. INTRODUCTION

The mechanics of rotating fluids is an important part of the analysis of numerous scientific and engineering problems like Centrifuges, Turbomachinery, etc. In our paper we analyse and numerically determine the Curvature of fluid placed in a fixed cylindrical vessel with a rotating thin rod along its axis. This is one of the most interesting and conventional topics in the field of Fluid Mechanics. In our system we involve the rotation of the axis rather than the more generalised condition of rotating the vessel.

When we consider the condition of the vessel being rotated, then the shape of the liquid surface for the fluid was determined by making a velocity gradient along the radius of the cylindrical vessel (i.e. from the container's inner surface to the axis) and we also had the viscous force proportional to the velocity gradient which in-turn was one of the major causes for the liquid surface's curvature.

But in our case we will be keeping the cylindrical vessel fixed and place a very thin rod along the axis of the system and rotate it with a fixed angular velocity (ω) and in this condition the direction of the velocity gradient will get reversed (i.e. from the axis to the container's inner surface). Also in our case, we initially consider our system to be analogous to a closed system and using that we determine the time dependent angular velocity (ω) and after that we derive the expression of the curvature of our system. Finally, after deriving the expressions for the curvature, we try to generate the tangential velocity profile for the rotating fluid using modified version of **Navier-Stokes law** which further incorporates the idea of **Taylor-Couette flow** and following it; we try to find the vorticity and stream function for specific conditions of ω .

2. FLUID IN A CYLINDRICAL VESSEL AS A CLOSED SYSTEM

Consider a system, where a cylindrical vessel of height H and radius R is completely filled with a fluid of viscosity η . The container is closed from both sides. Now through an orifice on the upper cover, a very thin rod of density ρ_m , with radius r is inserted within the container touching the bottom of the container. Now the rod is being rotated with an angular velocity ω_0 , along the central axis of rotation. Due to the viscous force acting on the surface of the rod by the adjacent layers of fluid, if no further energy is provided to the rod, the angular velocity of the rod will decrease w.r.t. to time (t).

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Figure 1.: System containing fluid with a rotating rod present in a closed container

Now the velocity gradient is given by $\frac{dv}{ds} = \frac{r\omega}{R-r}$ where $r\omega$ is the tangential velocity of the fluid layer adjacent to the rod. Now the viscous force acting on the surface of the rod (with surface area A), by the fluid layer is given by:

$$F_{viscous} = -\eta A \frac{dv}{ds}$$

$$\Rightarrow F_{viscous} = -\eta(2\pi r H) \left(\frac{r\omega}{R-r} \right)$$

Now, net torque acting on the rod by the fluid is $\tau_{net} = r F_{viscous}$

Let the Moment of Inertia of the rod, submerged into the liquid is given by $I = \rho_m \pi r^2 H$.

$$I \frac{d\omega}{dt} = -\frac{2\pi\eta r^3 H \omega}{R-r}$$

$$\Rightarrow \int_{\omega_0}^{\omega_t} \frac{d\omega}{\omega} = -\frac{2\eta r}{\rho_m(R-r)} \int_0^t dt$$

$$\Rightarrow \boxed{\omega_t = \omega_0 e^{-\frac{2\eta r t}{\rho_m(R-r)}}} \quad (1)$$

3. CURVATURE OF THE FLUID IN A CYLINDRICAL VESSEL AS AN OPEN SYSTEM

Now with almost all the conditions of our system are kept as such, we can now consider that instead of being a completely closed system, we had removed the top of the container, with the fluid surface open to air, placed in the cylindrical container. Now it is obvious that since fluid is a non-rigid material, the overall shape of the fluid will change, which will depend upon how much angular velocity is provided by the rod.

3.1 Equation of the curve formed by surface of fluid

For understanding the shape of the surface of the fluid, we have to take an arbitrary curve formed by two-dimensional cross section of the fluid layer along X-axis and Z-axis respectively. Now as shown in the free body diagram (Figure 2) for a particle of mass δm present on the surface curve at co-ordinates (x, z) respectively, the particle is taken to be at rest with respect to the given cross-section.

Figure 2.: Free Body Diagram of any arbitrary fluid particle on the fluid surface

Hence the equation of forces are:

$$\begin{aligned}\delta mg \sin \theta &= \delta m \omega^2 x \cos \theta \\ \Rightarrow \delta m g \left(\frac{\sin \theta}{\cos \theta} \right) &= \delta m \omega^2 x \\ \Rightarrow \tan \theta &= \frac{\omega^2 x}{g}\end{aligned}$$

Where $\tan \theta$ is the slope at that point, now we can write:

$$\begin{aligned}\frac{dz}{dx} &= \tan \theta \\ \Rightarrow dz &= \tan \theta dx = \frac{\omega^2 x}{g} dx \\ \Rightarrow z &= \int_0^z dz = \frac{\omega^2}{g} \int_0^x x dx \\ \Rightarrow z &= \frac{\omega^2}{g} \left[\frac{x^2}{2} \right]_0^x \\ \Rightarrow z &= \frac{\omega^2 x^2}{2g}\end{aligned}$$

The above equation is that of a parabola. Now for complete three-dimensional surface, this complete equation can be transformed into a circular Paraboloid with the equation:

$$\boxed{z = \left(\frac{\omega^2}{2g} \right) (x^2 + y^2)} \quad (2)$$

Figure 3.: 3-Dimensional view of the Curvature of fluid surface

3.2 Expression for height of the surface from the base

Now since we got the type of surface to be formed when the fluid is rotated with an angular velocity ω , we have to locate at what height the surface curvature of the fluid will form from the base of the container. In other words, we have to find the function of height of each cylindrical layer of fluid w.r.t. angular velocity ω and radial distance s respectively.

Suppose the initial height of the water level is H and let the height of the vortex in paraboloid form is H_1 .

Now as we know that the volume of the paraboloid of height H_1 is given by $\frac{\pi R^2 H_1}{2}$, where as, the volume of the corresponding cylinder of Height and Radius same as that of the paraboloid is given by $\pi R^2 H_1$. As we can say that the volume of the paraboloid cavity formed due to the surface of the fluid is half of that of the corresponding cylinder, so the rise in the water level along the walls of the container (h) will be equal to the dip in the water level (h) along the central axis of rotation.

As shown in Figure 4, we can say that:

Figure 4.: Change in the Height of the fluid surface due to rotation

$$z = H - h + z'$$

Where:

$$z' = \frac{\omega^2 s^2}{2g} \quad \text{and} \quad h = \frac{\omega^2 R^2}{4g}$$

Hence, we can write, $z = H - \frac{\omega^2 R^2}{4g} + \frac{\omega^2 s^2}{2g}$ (3)

Now, we have further assumptions that the height of the container is such that the fluid does not spill out of the Container. The angular velocity is limited to the magnitude, such that the flow of the liquid is assumed to be in a steady condition, and the surface does not touch the bottom of the container.

4. TANGENTIAL VELOCITY PROFILE FOR THE ROTATING FLUID

The system we took into consideration is nothing but the flow of liquid of density ρ between two concentric cylinders, where the outer cylinder is static. We can say that there is no relative motion between the layers

adjacent to the inner cylinder and outer cylinder respectively. Now we have to consider the cylindrical co-ordinate system for the flow of fluid. Let V_z , V_s and V_θ be the velocity of fluid flow along Z-axis, radial 's' direction and azimuthal ' θ ' direction respectively. Now since the angular velocity of the rod ω is fixed, the curvature of the surface is fixed, due to which there is no flow of fluid along Z-direction. Therefore, we can say that $V_z = 0$.

The final form of continuity equation in cylindrical co-ordinate system becomes;

$$\frac{\partial \rho}{\partial t} + \frac{1}{s} \frac{\partial}{\partial s}(\rho s V_s) + \frac{1}{s} \frac{\partial}{\partial \theta}(\rho V_\theta) + \frac{\partial}{\partial z}(\rho V_z) = 0$$

Now, as the fluid is assumed to be incompressible [1] in nature, so $\frac{\partial \rho}{\partial t} = 0$. We know that the flow is steady and continuous in θ direction so $\frac{\partial}{\partial \theta}(\rho V_\theta) = 0$. Also $\frac{\partial}{\partial z}(\rho V_z) = 0$ as $V_z = 0$. Therefore the equation becomes;

$$\begin{aligned} \frac{1}{s} \frac{\partial}{\partial s}(s V_s) &= 0 \\ \Rightarrow s V_s &= \text{Constant } (C_1) \\ \Rightarrow V_s &= \frac{C_1}{s} \end{aligned} \quad (4)$$

Now at $s = r$, $V_s = 0$ and at $s = R$, $V_s = 0 \Rightarrow V_s = 0 \quad \forall s$

Now using **Navier-Stokes Equation**[2] in cylindrical co-ordinates, we can conserve momentum in s , θ and z direction.

By conserving momentum in z - direction, we get:

$$\rho \left(\frac{\partial V_z}{\partial t} + V_s \frac{\partial V_z}{\partial s} + \frac{V_\theta}{s} \frac{\partial V_z}{\partial \theta} + V_z \frac{\partial V_z}{\partial z} \right) = -\frac{\partial P}{\partial z} + \rho g + \eta \left[\frac{1}{s} \frac{\partial}{\partial s} \left(s \frac{\partial V_z}{\partial s} \right) + \frac{1}{s^2} \left(\frac{\partial^2 V_z}{\partial \theta^2} \right) + \frac{\partial^2 V_z}{\partial z^2} \right]$$

As $V_z = 0$; we get $\frac{\partial P}{\partial z} = \rho g$, which is the Pressure gradient along z -direction.

Now, by conserving momentum in s - direction, we get:

$$\rho \left(\frac{\partial V_s}{\partial t} + V_s \frac{\partial V_s}{\partial s} + \frac{V_\theta}{s} \frac{\partial V_s}{\partial \theta} - \frac{V_\theta^2}{s} + V_z \frac{\partial V_s}{\partial z} \right) = -\frac{\partial P}{\partial s} + \eta \left[\nabla^2 V_s - \frac{V_s}{s^2} - \frac{2}{s^2} \frac{\partial V_\theta}{\partial \theta} \right]$$

As $V_s = 0$; we get $\frac{\partial P}{\partial s} = \frac{\rho V_\theta^2}{s}$, which implies centrifugal force is balanced with the radial pressure gradient along s - direction.

Finally, by conserving momentum in θ -direction, we get:

$$\rho \left(\frac{\partial V_\theta}{\partial t} + V_s \frac{\partial V_\theta}{\partial s} + \frac{1}{s} \frac{\partial V_\theta}{\partial \theta} + V_z \frac{\partial V_\theta}{\partial z} + \frac{1}{s} V_s V_\theta \right) = -\frac{1}{\rho} \frac{\partial P}{\partial \theta} + \eta \left[\nabla^2 V_\theta - \frac{V_\theta}{s^2} + \frac{2}{s^2} \frac{\partial V_s}{\partial \theta} \right]$$

As the flow is uniform in azimuthal direction i.e. $\frac{\partial V_\theta}{\partial t} = 0$ and also there is no additional force in θ - direction i.e. $\frac{\partial P}{\partial \theta} = 0$, so the equation reduces to:

$$\begin{aligned} \nabla^2 V_\theta - \frac{V_\theta}{s^2} &= 0 \\ \Rightarrow \frac{1}{s} \frac{\partial}{\partial s} \left(s \frac{\partial V_\theta}{\partial s} \right) + \frac{1}{s^2} \frac{\partial^2 V_\theta}{\partial \theta^2} + \frac{\partial^2 V_\theta}{\partial z^2} - \frac{V_\theta}{s^2} &= 0 \\ \text{As, } \frac{\partial^2 V_\theta}{\partial \theta^2} &= 0 \\ \Rightarrow \left[\frac{1}{s} \frac{\partial}{\partial s} \left(s \frac{\partial V_\theta}{\partial s} \right) + \frac{\partial^2 V_\theta}{\partial z^2} - \frac{V_\theta}{s^2} \right] &= 0 \end{aligned} \quad (5)$$

In **Taylor-Couette Flow** [3], we assume that the cylindrical rod and container are infinitely long along z -direction, which means there is no contribution from the bottom of the container and surface of the fluid.

Hence, we have $\frac{\partial^2 V_\theta}{\partial z^2} = 0$, for **Taylor-Couette Flow** and the differential equation reduces to:

$$\frac{1}{s} \frac{\partial}{\partial s} \left(s \frac{\partial V_\theta}{\partial s} \right) - \frac{V_\theta}{s^2} = 0, \quad \text{and it's solution is given by, } V_\theta = r\omega \left(\frac{R/s - s/R}{R/r - r/R} \right).$$

But in our case, we are considering the curvature of the surface, as mentioned in Section 3.2. So, we have $\frac{\partial^2 V_\theta}{\partial z^2} \neq 0$ and this is a partial differential equation (PDE) but by using Equation 3 in Section 3.2, we can turn it into an ordinary differential equation (ODE).

$$\begin{aligned} \text{Using Equation 3 we have; } z &= H - \frac{\omega^2 R^2}{4g} + \frac{\omega^2 s^2}{2g} \\ \Rightarrow \frac{\partial z}{\partial s} &= \frac{\omega^2 s}{g} \\ \frac{\partial}{\partial s} \left(\frac{\frac{\partial V_\theta}{\partial s}}{\frac{\partial z}{\partial s}} \right) &= \frac{\partial^2 V_\theta}{\partial z^2} = \frac{g^2}{\omega^4} \cdot \frac{1}{s} \cdot \frac{\partial}{\partial s} \cdot \left(\frac{1}{s} \frac{\partial V_\theta}{\partial s} \right) \end{aligned}$$

So now equation 5 can be modified as:

$$\frac{1}{s} \frac{\partial}{\partial s} \left(s \frac{\partial V_\theta}{\partial s} \right) + \frac{g^2}{\omega^4 s} \frac{\partial}{\partial s} \left(\frac{1}{s} \frac{\partial V_\theta}{\partial s} \right) - \frac{V_\theta}{s^2} = 0$$

So the final equation will be,

$$\left(s^2 + \frac{g^2}{\omega^4} \right) \frac{d^2 V_\theta}{ds^2} + \left(s - \frac{g^2}{\omega^4 s} \right) \frac{dV_\theta}{ds} - V_\theta = 0 \quad (6)$$

Since, we are not able to find the general solution of this 2nd order ODE so, we have plotted it numerically as shown in the Figure 5, using specified boundary conditions at $s = r$, $V_\theta = r\omega$ and at $s = R$, $V_\theta = 0$.

Figure 5.: Numerically Plotted Graph of V_θ vs. s

The 3D graph between (V_θ, s, ω) is given in Figure 6 on the next page:

Figure 6.: 3-Dimensional Plot between V_θ , s , ω

5. EXPRESSION FOR TANGENTIAL VELOCITY FOR SPECIFIC CONDITIONS OF ANGULAR VELOCITY

5.1 Expression for very low value of ω

Now, in the 3-Dimensional plot (Figure 6), the plot is able to show the behaviour of V_θ for any values of ω and s respectively. But as we don't have any specific expression for V_θ , we can approximate equation 6 as follows:

$$\begin{aligned}
 & \frac{d^2 V_\theta}{ds^2} + \lim_{\omega \rightarrow 0} \left(\frac{s - \frac{g^2}{\omega^4 s}}{s^2 + \frac{g^2}{\omega^4}} \right) \frac{dV_\theta}{ds} - \lim_{\omega \rightarrow 0} \frac{V_\theta}{\left(s^2 + \frac{g^2}{\omega^4} \right)} = 0 \\
 \Rightarrow & \frac{d^2 V_\theta}{ds^2} - \frac{1}{s} \cdot \frac{dV_\theta}{ds} = 0 \\
 \Rightarrow & \boxed{V_\theta = \frac{r\omega(R^2 - s^2)}{R^2 - r^2}} \quad (7)
 \end{aligned}$$

Now, for very small value of ω ; the expression for tangential velocity V_θ can be plotted w.r.t. radial distance s as shown in Figure 7:

Figure 7.: Graph of V_θ vs. s for low ω

5.2 Expression for very high value of ω

Similarly for very high values of ω Equation 6 can be approximated as follows:

$$\begin{aligned}
 & \frac{d^2 V_\theta}{ds^2} + \lim_{\omega \rightarrow \infty} \frac{\left(s - \frac{g^2}{\omega^4 s}\right)}{\left(s^2 + \frac{g^2}{\omega^4}\right)} \frac{dV_\theta}{ds} - \lim_{\omega \rightarrow \infty} \frac{V_\theta}{\left(s^2 + \frac{g^2}{\omega^4}\right)} = 0 \\
 \Rightarrow & s^2 \frac{d^2 V_\theta}{ds^2} + s \frac{dV_\theta}{ds} - V_\theta = 0 \\
 \Rightarrow & \boxed{V_\theta = \frac{r^2 \omega (R^2 - s^2)}{s(R^2 - r^2)}} \quad (8)
 \end{aligned}$$

Now, for very high value of ω ; the expression for tangential velocity V_θ can be plotted w.r.t. radial distance s as shown in Figure 8: In order to accommodate very high value of ω within the container, it is necessary to take

Figure 8.: Graph of V_θ vs. s for high ω

the container of very large radius R as well as large height H . Otherwise our most preliminary assumption for the liquid not to spill out of the container, or the curvature should not touch the bottom of the container cannot be validated.

6. VORTICITY AND STREAM FUNCTION FOR SPECIFIC CONDITIONS OF ANGULAR VELOCITY

Vorticity ($\vec{K}$) is known to be a vector field, or more precisely, a pseudo vector field, which provides a local measure of the instantaneous rotation of a fluid section. Its significance in fluid dynamics or continuum mechanics is analogous to that of angular velocity in solid body mechanics.

By definition,

$$\vec{K} \equiv \nabla \times \vec{V}$$

Where; $\vec{V}$ is the velocity vector of the fluid.

The stream function (ψ) is defined for incompressible (divergence-free) flows in two dimensions – as well as in three dimensions with axisymmetry. The stream function can be found from vorticity using the following

Poisson's equation, $\nabla^2 \psi = -K_z$, where K_z is the z -component of the vorticity given as; $K_z = \vec{e}_z \left[\frac{1}{s} \frac{\partial}{\partial s} (sV_\theta) - \frac{\partial V_s}{\partial \theta} \right]$.

Since $V_s = 0$, the final differential equation is:

$$\frac{\partial^2 \psi}{\partial s^2} + \frac{1}{s} \frac{\partial \psi}{\partial s} + \frac{1}{s^2} \frac{\partial^2 \psi}{\partial \theta^2} + \frac{\partial^2 \psi}{\partial z^2} = -\frac{1}{s} \frac{\partial (sV_\theta)}{\partial s}$$

Since, there is uniform flow along azimuthal direction so the contribution of azimuthal flow to the overall divergence will be zero. Hence $\frac{\partial^2 \psi}{\partial \theta^2} = 0$. Now we have:

$$\frac{\partial^2 \psi}{\partial z^2} = \frac{g^2}{\omega^4} \cdot \frac{1}{s} \cdot \frac{\partial}{\partial s} \cdot \left(\frac{1}{s} \frac{\partial \psi}{\partial s} \right)$$

So the final differential equation will be:

$$\boxed{\frac{d^2 \psi}{ds^2} \left(s + \frac{g^2}{\omega^4 s} \right) + \frac{d\psi}{ds} \left(1 - \frac{g^2}{s^2 \omega^4} \right) + \frac{d(sV_\theta)}{ds} = 0} \quad (9)$$

6.1 Stream function for low value of ω

For low values of ω we can approximate equation 9 as:

$$\begin{aligned} \frac{d^2 \psi}{ds^2} - \frac{1}{s} \frac{d\psi}{ds} &= 0 \\ \Rightarrow \psi &= A_1 s^2 + B_1 \quad [\text{where } A_1 \text{ and } B_1 \text{ depend on specific boundary conditions}] \end{aligned} \quad (10)$$

6.2 Stream function for high value of ω

Similarly for high values of ω by modifying equation 9 we can obtain the stream function as:

$$\begin{aligned} \frac{d^2 \psi}{ds^2} + \frac{1}{s} \frac{d\psi}{ds} - \frac{2r^2 \omega}{R^2 - r^2} &= 0 \\ \Rightarrow \psi &= C_1 \log(s) + D_1 + \frac{r^2 s^2 \omega}{2(R^2 - r^2)} \quad [\text{where } C_1 \text{ and } D_1 \text{ depend on specific boundary conditions}] \end{aligned} \quad (11)$$

Figure 9.: Contour representing the Streamline flow in Fluid

The above figure (Figure 9) shows the streamline profiling of rotating fluid in the system, for any arbitrary value of ω . The boundary or limiting conditions for the stream function can be specifically constructed to determine the exact stream function from the family of stream functions shown in Equation 10 and 11.

7. CONCLUSION

We were able to get a lot of insights about how our system will behave with the rotational flow of fluid inside a fixed cylinder. We started with the system in which the liquid is placed in a closed cylindrical vessel, filled completely up to the brim. Now, although the angular velocity ω is dependent upon time, density of the rod and viscosity η of the fluid, it is independent of the density of the fluid. The conclusion seems obvious since if we keep mercury as fluid and iron as the rod, the system is expected to behave indifferently. With all our conditions introduced for our system kept as it is, we were able to find the profiling of the tangential velocity V_θ using the **Navier-Stokes Equation**, and were able to modify the **Taylor-Couette flow** in order to accommodate the effect of curvature in the profiling. Thus, finding the differential form for V_θ , for which we further have to use approximations in values of angular velocity ω . Now, moving a step further provided information about the vorticity as well as the stream function associated with it, relating how core aspects of continuum mechanics fits with the formulations we used for the profiling.

Now, referring to the prospective ahead, the core mathematical challenges solving the **Navier-Stokes equation** are still visible. We tried to find an explicit form for tangential velocity for any arbitrary value of ω , but our effort provided no definitive conclusion about the expression for V_θ . Hence we can say that still we have to be equipped with more mathematical methods to solve that differential equation. Also, exactness of V_θ if we try to reduce the radial dimension of the cylindrical rod to zero is also questionable. In **Taylor-Couette flow**, this exactness does not exist, since the above approximation on the inner rod cannot be defined. But in that case, we can see another prospective of transfer of rotational energy from a system with negligible dimension (for example, the rod been replaced by a string). Considering our system, we have not taken either Reynolds number or Taylor number in our calculations, which provide crucial information about how the flow is changing from laminar to turbulent, but any kind of approach can be made from the stream function itself.

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References

- [1] Stokes, G.G. (1842), "On the steady motion of incompressible fluids", Transactions of the Cambridge Philosophical Society, 7: 439–453, <https://archive.org/details/mathphyspapers01stokrich>
- [2] Fefferman, Charles L. "Existence and smoothness of the Navier–Stokes equation" <http://www.claymath.org/sites/default/files/navierstokes.pdf>
- [3] Taylor, G. I. "Stability of a Viscous Liquid Contained between Two Rotating Cylinders." Philosophical Transactions of the Royal Society of London. Series A, Containing Papers of a Mathematical or Physical Character, vol. 223, 1923, pp. 289–343. JSTOR, <https://www.jstor.org/stable/91148>.