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Adaptive diffusive time-step in conjugate heat transfer interface conditions for thermal-barrier-coated applications

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Abstract

High pressure turbines are subjected to high temperature flow exiting the combustion chamber. This paper presents a reliable method for a conjugate heat transfer (CHT) procedure using different interface treatments. Indeed, a coupled approach is used to properly capture the transient heat load variations at the interface and have a better estimation of the conduction within the solid. The numerical methods developed in this paper are derived from a 1D CHT model problem in which the thermal properties of each domain drive the numerical stability. Thus, from this model, two fundamental parameters are introduced : a "numerical" Biot number, Bi_ν , and an optimal coefficient. Even if the optimal coefficient is theoretically unconditionally stable, the stability zone is drastically reduced when the Biot number increases. In this paper, a unified approach applicable to a wide range of Biot numbers is proposed for the case of multiple materials involved in a coupling process. In order to properly estimate the transient variation, an adaptive diffusive time-step has been developed based on physical thermal properties. This time-step is able to capture the transient effects inside the thermal boundary layer. Coupled results demonstrate a fast and steady converging behavior. Special attention is given to properly define the variable relaxation parameter used in this algorithm. The gain in using an adaptive optimal coefficient is discussed.

Keywords: Conjugate heat transfer, Dirichlet, Robin, Thermal barrier coating

Nomenclature

α	Coupling coefficient [$W.m^{-2}.K^{-1}$]
Δt_c	Coupling time step [s]
Δt_D	Diffusive time step [s]
Δy	First cell size [m]
γ	ratio of specific heats
λ	Thermal conductivity [$W.m^{-1}.K^{-1}$]
Λ_s	Solid thickness [m]
μ	Dynamic viscosity [$kg.m^{-1}.s^{-1}$]
$\overline{D_f}$	Normalized Fourier number
ρ	Density [$kg.m^{-3}$]
a	Thermal diffusivity [$m^2.s^{-1}$]
Bi	Biot number
Bi^Δ	Mesh Biot number
Bi_ν	Numerical Biot number
CFL	Courant–Friedrichs–Lewy number
D_f	Fourier number
g	Temporal amplification factor
K	Thermal conductance [$W.m^{-2}.K^{-1}$]
q	Heat flux [$W.m^{-2}$]
T	Temperature [K]
t	Time [s]

1. Introduction

In many sectors, such as transportation, energy or defense, gas turbine engines generate hot air to improve efficiencies and power. This hot air needs to be properly estimated and controlled in order to avoid engine failure. This mainly relies on thermal barrier coatings (TBC) that enable modern engines to operate at significantly higher temperatures than the blade material can withstand. TBC provide thermal insulation to the metallic/super-alloy engine part which allow an increase in temperature and efficiency. The efficiency and core power of gas turbine engines, used to propel aircraft and to generate electricity, are directly related to the gas temperature entering the turbine section. Thus, efforts to increase aircraft engine efficiency have led to increasingly high temperatures at the inlet of high pressure (HP) turbines. Such high temperatures dramatically impact the life span of the turbine blades [1]. These temperatures combined with inlet non-uniformities challenge the design of modern HP turbine blade.

Improvements to TBC materials are required to further improve the thrust-to-weight ratio and durability of gas turbines [2]. Thus, a more precise determination of the heat load at the interface is needed and a coupling procedure is thus necessary to obtain more accurate information at the interface [3].

Moreover, turbine blades must also deal with azimuthal and radial non uniformity known as hot streak [4; 5]. This phenomenon focuses the heat load at certain areas of the blade. It is then important to correctly calculate the proper heat fluxes and wall temperatures to estimate the life span of the blade.

Perelman first proposed a formulation for conjugate heat transfer (CHT) [6; 7]. The approach used in this paper is a partitioned conjugate heat transfer computation wherein the fluid and the solid domain are resolved separately. A normal mode analysis has been previously studied by Giles [8] to enhance stability for fluid-structure interactions. He proposed a stability criterion for time synchronized coupled simulation based only on the first cell size of each domain. On the basis of the same approach, the behavior of interface conditions in CHT has also been studied using a normal mode analysis [9; 10; 11; 12]. This relies on the fact that the nature of instabilities obtained thanks to a mathematical model can give insight into the potential instabilities in realistic coupled computations. Note that other approaches exist such as the energy method [13], the multi-scale time in-

tegration for high fidelity computations [14] or a frequency-domain method for periodic unsteady flows [15]. Verstaete [16] proposed a stability criterion based on the Biot number for steady CHT. Indeed, at low Biot numbers, the Dirichlet-Neumann condition is imposed, whereas at high Biot numbers, the Neumann-Dirichlet condition is recommended. He also mentioned that in complex industrial configurations, the Biot number is not uniform over the surface, leading to possible instabilities. In this paper, a stability criterion is proposed to simultaneously compute different Biot numbers on the same coupled surface using a Dirichlet-Robin interface condition.

The critical parameter is the Biot number, i.e. the ratio of external to internal heat transfer coefficients. When TBC are employed, this Biot number can be very high. Based on the normal mode analysis of Godunov-Ryabenkii [17], an optimal relaxation parameter for the Robin condition can be expressed [18]. Errera and Duchaine [19] have then introduced a numerical Biot number. This number takes into account the fluid thermal properties, the time step, the solid conductance, and the mesh size, and is relevant to define the numerical stability zone. The one-dimensional analysis enables the stabilization of the computation at the interface by locally transferring the thermal information needed. This method has been shown to massively improve CPU cost [20; 21; 22].

Previous work [23] has shown that Dirichlet-Robin interface conditions based on an optimal relaxation coefficient performed well at low Biot numbers but can lead to instabilities at high Biot numbers. The Dirichlet-Robin and Neumann-Dirichlet conditions are usually used for separated solid materials. For instance, a Dirichlet-Robin condition is often used when a high thermal conductivity material is involved and a Neumann-Dirichlet condition is better suited to low conducting materials such as ceramics. However, it will be shown in this paper that applying separate interface conditions to different parts of the same non-homogeneous surface (implying a variable Biot number across the interface) may generate stability issues.

The main purpose of this paper is to provide a generalized interface treatment over a wide range of Biot numbers on the same coupled surface. This paper will focus on the Dirichlet-Robin boundary condition for stable partitioned CHT strategies based on an optimized coupling scheme. It will be shown that the transition between low and high Biot numbers can be easily treated without having to switch the boundary condition through the use of an adaptive interface condition.

2. Numerical investigations

2.1. CHT approaches

There are two main approaches to CHT computations: a monolithic approach, and a partitioned approach. In the monolithic approach, the equations are solved simultaneously [24; 25; 26]. This can be a good solution if both the solid and the fluid evolve simultaneously with similar time scales. The main advantage of a monolithic approach is that the mutual influence between the fluid and the solid is directly taken into account. It also has a positive effect on stability, and no coupling iterations within a time step are required.

In a partitioned approach, systems are spatially decomposed into different domains. Partitioned techniques are very popular [27; 28; 29], since they allow the direct use of specifically designed solvers for different fields. Each solver is dedicated to a specific field which contributes to improve efficiency over the monolithic approach. This approach is considered in this paper.

2.2. Fluid solver

The numerical simulations have been performed using the elsA software developed at ONERA and co-owned by AIRBUS, SAFRAN and ONERA. It is capable of solving both the Reynolds-Averaged Navier-Stokes (RANS) and Large Eddy Simulation (LES) on multi-block structured or unstructured meshes [30; 31]. The compressible 3D flows are solved using an upwind or central space discretisation and it can compute complex industrial applications taking advantage of modern massively parallel techniques.

The governing equations are the time-dependent Navier-Stokes equations, which express the conservation laws of mass, momentum, total energy and any other scalar quantities written in the conservative form :

$$\partial_t Q + \vec{\nabla} \cdot F(Q) = 0 \quad (1)$$

where Q represents the conservative quantities and F the flux, including inviscid and viscous terms. For this study, a RANS Spalart-Allmaras equation is used as the turbulence model to compute the eddy viscosity. The convective fluxes are computed using the Jameson scheme. For steady flow calculations, an implicit time integration is achieved through a backward Euler scheme combined with a scalar LU-SSOR method.

2.3. Solid solver

The solid solver is based on a finite element method. The solid software package, named Z-set [32], is a comprehensive suite of integrated analysis programs for general structural analysis. It is an advanced object-parallel solver for structural mechanics with many non-linear solution capabilities. It is capable of modeling the non-linear response of materials under various loads.

It can also solve many types of heat problems. Assuming that there are no heat sources, the temperature distribution within the solid body is modeled by the following conductive heat transfer equation

$$\nabla \cdot (\lambda \nabla T) = 0 \quad (2)$$

Where $T = T(x, y, z)$ is the unknown temperature field within the body. Since this paper focuses on steady conjugate heat transfer, the unsteady part of the equation 2 can be omitted. λ is the thermal conductivity, which in this study, is treated as a constant scalar.

2.4. Coupling library

The exchange of data between the two aforementioned solvers is carried out through the CWIPI (Coupling With Interpolation Parallel Interface) library [33]. This library takes into account the grids, as well as the processes in which the data is located.

CWIPI aims to provide a fully parallel communication layer for mesh based coupling between several solvers through MPI communications. The library takes into account all types of geometrical elements (polygon, polyhedral) with an unstructured description. CWIPI provides functionalities such as the construction of the communication graph between distributed geometric interfaces, interpolation non-coincident meshes, and exchange of coupled fields.

2.5. Temporal steady coupling strategy

The partitioned CHT strategy provides a fast and stable steady solution to fluid-solid thermal problems. The conventional serial staggered algorithm (CSS) is used to minimize the CPU cost since the temperature evolution in the fluid and solid have hugely distinct time-scales [28; 27]. It consist of 4 steps :

- Solve solid equations

- Transfer thermal data to the fluid boundary condition
- Solve fluid equations
- Transfer thermal data to the solid boundary condition

and this cycle is repeated until convergence. Since only the steady state is sought, and due to the aforementioned difference in time-scales, the transient computation of the solid diffusion is not required and thus only the Laplace equation needs to be solved. On the fluid side, the steady-state equations are solved by a pseudo-time marching scheme. Particular attention needs to be given to stability issues.

The coupling model used in this paper is based on a 1D analysis. Indeed, it is assumed that possible instabilities come from modes that are normal to the fluid-solid interface. Thus, the 1D model remains generally valid for 2D and 3D configurations.

2.6. Dirichlet-Robin procedure

Interface conditions are needed on either side of the interface, where coupling conditions are applied. Our goal is to ensure a stable CHT process and to avoid destabilizing effects [34; 20]. It is well known that Robin conditions have stabilizing features, and thus a Robin condition is applied on the solid side

$$\hat{q}_s + \alpha_f \hat{T}_s = q_f + \alpha_f T_f \quad (3)$$

The subscripts f and s denote the fluid and solid domains respectively and the $(\hat{\bullet})$ notation indicates the sought-after values. q is the interface heat flux ($W.m^{-2}$) and T is the interface temperature. The general Robin condition (3) introduces the numerical coupling parameter α_f ($W.m^{-2}.K^{-1}$), which influences both the stability of the coupling process and the precision of the heat flux exchanged. Thus, in order to improve the precision of the heat flux exchanged and therefore accelerate the computation, we can either improve the interpolation order to reduce $(T_f - \hat{T}_s)$ or we can decrease the value of the relaxation parameter α_f .

On the fluid side, a Dirichlet condition is imposed:

$$\hat{T}_f = T_s \quad (4)$$

2.7. Fundamental parameters

The mesh Fourier number is defined as follows :

$$D_f = a_f \frac{\Delta t_c}{\Delta y_f^2} \quad (5)$$

Where, a_f is the fluid thermal diffusivity, Δt_c the coupling time-step and Δy_f the fluid first cell size. At this point, it is interesting to introduce the function $\overline{D}_f$ representing a normalized Fourier number (see [18; 20] for details) in terms of D_f

$$\overline{D}_f = \frac{D_f}{(1 + D_f + \sqrt{1 + 2D_f})} \quad (6)$$

$\overline{D}_f$ is defined in the domain $[0, \infty[$ and varies in the range $[0, 1[$.

The thermal interaction at the fluid-solid interface between the first fluid cell and the steady solid medium is characterized by a local mesh Biot number, which represents the ratio between the conductance of the first fluid cell and the conductance of the solid domain:

$$Bi^\Delta = \frac{K_f}{K_s} \quad (7)$$

Where $K_f = 2\lambda_f/\Delta y_f$ and $K_s = \lambda_s/\Lambda_s$. The stabilizing effect of the coupling time step, represented by the Fourier number, is taken into account by the numerical Biot number defined as follows

$$Bi_\nu = Bi^\Delta(1 - \overline{D}_f) \quad (8)$$

This dimensionless number results from a balance between the unsteady fluid and steady solid domain properties. Bi_ν can be regarded as the "transient" thermal fluid-structure interaction. This number is a key factor in the stability analysis of the CHT procedure [18; 23].

2.8. Stability bounds of the Dirichlet-Robin procedure

The Godunov-Ryabenkii (G-R) stability analysis is very similar to the standard Fourier stability method except that the Fourier analysis ignores boundary conditions. A normal mode solution is applied to the model problem. The main results of this stability analysis are summarized in Table 1. At high Biot numbers, the amplification factor, $|g(\alpha_f)|$, is partly outside the

stability domain (first row in Table 1) and completely inside the stability domain for low numerical Biot numbers (second row in Table 1).

Table 1 shows that the coupling procedure without relaxation can be ensured by increasing D_f (and hence $\overline{D_f}$) such that condition $Bi^{(\Delta)}(1 - \overline{D_f}) < 1$ holds. Otherwise, a certain level of relaxation is needed and the optimal level is provided by the choice of the optimal coefficient given by

$$\alpha_f^{opt} = \frac{K_f}{2}(1 - \overline{D_f}) \quad (9)$$

The role of α_f^{opt} in controlling and guiding the behavior of the CHT process is fundamental.

α_f	0		$\alpha_f^{\min}$		α_f^{opt}		∞	condition
$ g $	g^0	UNSTABLE	1		g^{opt}		1	$Bi^{(\Delta)}(1 - \overline{D_f}) \geq 1$
$ g $	g^0				g^{opt}		1	$Bi^{(\Delta)}(1 - \overline{D_f}) \leq 1$
$g^0 = Bi_v \qquad g^{opt} = \frac{Bi_v}{2 + Bi_v}$								

Table 1: Numerical characteristics of Dirichlet-Robin procedure

3. Fluid-Solid thermal interaction

3.1. Numerical and conventional Biot numbers

The local Biot number Eq. 8, Bi_v , directly takes into account the thermal and dynamic response of the boundary layer and is directly involved in the stability of the coupled process. If a transient fluid flow is employed during the CHT computation, this local parameter will drive and control the stability of the CHT problem. On the other hand, the conventional Biot number measures the resistance of the heat flow within the solid relative to the resistance presented by the convection processes at the surface using a linear assumption. It is clear that this conventional Biot number, not easily defined during the transients, cannot be used to set up a numerical CHT procedure so long as the transient fluent state is involved.

3.2. The effect of the numerical Biot Number

The stability condition $|g(\alpha_f)| < 1$ leads to a lower stability bound α_f^{min} , defined as :

$$\alpha_f > \alpha_f^{min} = \frac{K_f}{2}(1 - \overline{D_f}) - \frac{K_s}{2} \quad (10)$$

The lower stability bound can also be defined using the numerical Biot number Bi_ν :

$$\alpha_f^{min} = \frac{K_s}{2}(Bi_\nu - 1) \quad (11)$$

In this case, the coupling procedure exhibits a lower stability bound α_f^{min} . This demonstrates how the stability relies not only on the ratio of thermal resistances, but also on the dynamics of the transient fluid system. The higher the local Biot number, the more difficult it will be to stabilize the coupling process.

3.3. Weak and strong fluid-structure interaction

Special attention must thus be given to the conditions provided by Table 1 from which one can define :

- Weak interaction: $Bi_\nu = Bi^\Delta(1 - \overline{D_f}) \leq 1$, if this condition holds, the "transient" thermal resistance of the fluid domain at the shared interface is greater than the resistance offered by the whole solid domain. A Dirichlet condition on the fluid side is therefore appropriate.
- Moderate interaction: $Bi_\nu = Bi^\Delta(1 - \overline{D_f}) > 1$, if on the contrary, this condition holds, then it implies either that the solid thermal gradients are not negligible or that the thermal fluid conductance is larger than that of the solid.
- Strong interaction: $Bi_\nu \gg 1$, in this case, a Dirichlet condition imposed on the fluid does not provide the most efficient solution. This situation is becoming more frequent in the aerospace industry since TBCs are increasingly used to manage increasing temperatures and higher efficiency.

3.4. *Instability, oscillations and low convergence*

Low values of α_f ($\alpha_f < \alpha_f^{opt}$) will result in rapid convergence, but $\overline{D_f}$ must be sufficiently large to allow heat diffusion on the solid side. If not, it will then be necessary to enhance stability by increasing the coupling parameter. However, large values of α_f ($\alpha_f \gg \alpha_f^{opt}$) will always lead to an extremely slow convergence since this corresponds to a slow diffusion of heat. The most striking point is that at the remarkable coefficient $\alpha_f = \alpha_f^{opt}$, the modulus of the amplification attains its absolute minimum value, always less than unity.

4. **Adaptive relaxation parameter based on the mesh Fourier number**

In most aerothermal computations, only the steady state is sought, and a relaxation parameter is often used to stabilize the coupling process. In this paper, the coupling procedure uses a first order interpolation at the fluid/solid interface. Thus, in order to have the most accurate prediction of the heat flux transferred, we can either implement a higher interpolation order at the interface or reduce the relaxation parameter. The coupling process introduced in [20; 23] is not compatible with a variable relaxation, especially at high fluid/solid interaction. However, the use of a variable relaxation parameter can result in increased convergence speed as well as better stability of the coupling process.

This section focuses on getting a reliable variable relaxation parameter in order to avoid unnecessary uncertainties caused by the low order interpolation method used. As we shall go on to examine, this is done by introducing the fluid diffusive time-step inside the mesh Fourier number D_f equation. This enables the progressive reduction of the optimal value of the relaxation parameter while maintaining stability.

4.1. *The diffusive time-step*

The total transport energy equation can be written as :

$$\frac{\partial \rho E}{\partial t} + \frac{\partial \rho E u_j}{\partial x_j} = \frac{\partial u_i \tau_{ij}}{\partial x_j} - \frac{\partial q_j}{\partial x_j} \quad (12)$$

Where E is the total energy, u is the velocity, τ is the stress tensor, q is the heat flux. They are expressed as :

$$E = e + \frac{u_i u_i}{2} \quad (13)$$

$$q_j = -\lambda \frac{\partial T}{\partial x_j} \quad (14)$$

$$\tau_{ij} = -p\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\mu\frac{\partial u_k}{\partial x_k}\delta_{ij} \quad (15)$$

Where e is the internal energy. When using the Favre weighted average, the total transport energy equation can be expressed as :

$$\frac{\partial \bar{\rho} \tilde{E}}{\partial t} + \frac{\partial}{\partial x_j}(\bar{\rho} \tilde{u}_j \tilde{E}) = -\frac{\partial}{\partial x_j}(\overline{\rho u_j'' E''}) + \frac{\partial \overline{\tau_{ij} u_i}}{\partial x_j} + \frac{\partial}{\partial x_j}(\lambda \frac{\partial \bar{T}}{\partial x_j}) \quad (16)$$

Where $(\tilde{\bullet})$ symbol refers to the Favre average and $(\bullet'')$ refers to the fluctuations. Assuming a Boussinesq-like approach, the turbulent heat flux can be modeled using a gradient approximation for the turbulent heat-flux :

$$\overline{\rho u_j'' E''} = -\frac{c_p \mu_t}{Pr_t} \frac{\partial T}{\partial x_j} \quad (17)$$

and the total heat flux is defined as :

$$\widetilde{q_j^{tot}} = \widetilde{q_j^{lam}} + \widetilde{q_j^{turb}} \quad (18)$$

Since we are interested in defining the diffusive time-step Δt_D near the wall for turbulent flows, the thermal diffusivity is then defined as :

$$a_f = \frac{\mu}{\rho} \frac{\gamma}{Pr} \left(1 + \frac{\mu_t}{\mu} \frac{Pr}{Pr_t}\right) \quad (19)$$

The following space discretization is used:

$$\frac{\partial^2 T}{\partial y^2} \simeq \frac{T_{i+1} - 2T_i + T_{i-1}}{\Delta y^2} \quad (20)$$

Using the Von Neumann stability analysis, the diffusive time-step in this paper is defined as follows:

$$\Delta t_D = C \Delta y^2 \frac{\rho}{\mu} \frac{Pr}{\gamma} \left(1 + \frac{\mu_t}{\mu} \frac{Pr}{Pr_t}\right)^{-1} \quad (21)$$

Where C is practically defined as $C = CFL/2$. Since the fluid solver used is fully implicit, accelerating methods can be used such as local time-step and CFL acceleration where the CFL number is locally defined using the local time-step and the local cell-size.

4.2. The variable relaxation parameter

The main focus of this paper is on very strong thermal interactions. As a result, the relaxation parameter can be very high. Thus, a variable relaxation parameter is needed to reduce the uncertainties at the interface as much as possible. The optimal relaxation parameter α^{opt} is defined using a mesh Fourier number, D_f , that appears naturally when solving the Godunov-Ryabenkii problem. In order to properly define the optimal value of this relaxation parameter, the value of the mesh Fourier number has to be correctly estimated. The mesh Fourier number is defined using a particular coupling time-step Δt_c and the first cell-size Δy_f .

Figure 1: Normalized Fourier number $\bar{D}_f$

Figure 1 represents the normalized mesh Fourier number (defined by Eq. 6). As can be seen, when D_f increases, $\bar{D}_f$ also increases, which leads to diminishing the relaxation parameter value as shown in eq. 9. Accordingly,

this leads to an increase of the computational speed. The coupling time-step used is defined as :

$$\Delta t_c = \sum_i^n \Delta t_i^f \quad (22)$$

Where Δt_c is the coupling time-step, Δt_i^f is the fluid time-step, n is the coupling iteration and i the current iteration. This means that the coupling frequency influences the stability of the coupling process. In order to avoid this frequency dependency, some basic assumptions are made in this paper in order to simplify the methodology so as to directly take into account the fluid-based time-step instead of a specified coupling time-step. In the current paper, we assume the following three assumptions :

- The unstable modes are considered normal to the wall. Thus a 1D approach can be used for 2D and 3D configurations.
- The near wall mesh is refined so no wall-laws are necessary.
- The diffusive time step is always considered as the fluid local time step.

Under those assumptions, it is reasonable to consider a new mesh Fourier number calculated on the basis of the diffusive time-step:

$$D_f = a_f \frac{\Delta t_D}{\Delta y_f^2} \quad (23)$$

Using this definition, the mesh Fourier number provides a local variation of the relaxation parameter based on the *CFL* number. Indeed, $\alpha_f^{opt} = f(K_f, \overline{D_f})$ (eq. 9) and since increasing the *CFL* number leads to an increase in D_f , the optimal relaxation parameter α_f^{opt} decreases, and a more accurate value of the heat flux can be locally transmitted.

5. Validation

It was recommended in a previous paper [23] to use a Dirichlet-Robin condition at low Biot numbers and a Neumann-Dirichlet for high Biot numbers. In this paper, a unified interface treatment is proposed in order to stabilize the computation when using different successive materials at the interface.

To validate this approach for a multi-conductivity coupled interface, an academic test case is used to demonstrate the ability to provide stable computations when different thermal properties are involved at the interface. The test case is a three-block flat plate : metal-TBC-metal. Although this is an academic configuration, it exhibits huge disparities in solid conductivities.

5.1. Flat plate with multiple coupled surface materials

The configuration presents two different materials at the coupled surface. A low conducting material, representing the thermal barrier coating, is placed alongside a more thermally conductive material (figure 2). Different conductivities have been studied to ensure the stability condition.

Figure 2: Solid (metal-TBC-metal) domain for the coupled heat transfer problem

5.1.1. Mesh and computation parameters

Figure 3: Mesh for the coupled fluid solid domain

The fluid domain is discretized with a multi-block structured mesh and is resolved with a finite volume method, while the solid domain is discretized using an unstructured mesh and resolved using a finite element method (Figure 3). In order to avoid uncertainties due to spatial interpolation, the solid domain is discretized using quadrilaterals cells with the same evolution ratio as the fluid domain. Indeed, this is done for validation purposes, however for more complex configurations, meshes are not coincident and the coupling library handles the transfer of physical quantities for different mesh densities

and element types at the interface. The fluid mesh domain is divided into 2 sub-domains, a coupled domain and a buffer domain. The fluid buffer domain avoids the instabilities generated by the inlet velocity and allows the fluid to stabilize before entering the coupled domain. This domain is $0.06m$ long and it is considered long enough to let the flow restabilize before reaching the coupled interface. In the buffer domain, the bottom wall is adiabatic so no heat is exchanged, whereas the bottom wall of the coupled domain exchanges heat with the solid. This wall is first initialized as Dirichlet condition. The total height of the fluid domain is $0.4m$ with a symmetry condition on the upper boundary. The fluid domain is composed of 11 000 elements and the first cell size is set at $\Delta y_f = 2.5 \cdot 10^{-5}m$ so as $y^+ < 1$ to properly capture all regions of both the momentum and thermal boundary layer without the use of wall-laws. An ideal gas is used with $\gamma = 1.4$ and the Sutherland's law for air is used to compute the dynamic viscosity μ . The fluid Mach number is $M = 0.1$ and the total temperature is $T_i = 1200K$. It should also be pointed out that if the boundary layer is properly captured, convection plays a minor role near the wall and all the heat generated by convection is transferred to the wall through diffusion. The CFL number in this case is 20.

For the solid, the top wall is coupled with the fluid whereas an isothermal condition is used at the bottom boundary and all other boundaries are adiabatic. The bottom wall of the solid is set at $1000K$. The solid domain is $0.3m$ long and $0.01m$ thick. The thermal barrier coating is $200\mu m$ thick. The thermal conductivity of the metal is $\lambda_{metal} = 20W \cdot m^{-1} \cdot K^{-1}$. The first cell size in the solid domain is $\Delta y_s = 10^{-4}m$. The TBC is directly taken into account by the solid solver where the conductivity matrix is decomposed into two sub-domains (the metal and the TBC). Since the steady conduction equation is solved inside the solid domain, the equation depends fully on the thermal boundary condition and thus few cells are required to properly capture the temperature gradient.

Moreover the computation stability is impacted by the coupling frequency which has been set to every 100 fluid time-steps. This value has been shown to lead to stable and quickly converged results.

5.1.2. Probes positions

The Dirichlet condition is imposed on the fluid side and the Robin condition is in return, imposed on the solid side. Different relaxation coefficient will be tested to ensure stability. Several probes have been set to evaluate the fluctuation of the solution as shown in figure 4.

Figure 4: Probe positions at the interface of the solid domain for the coupled heat transfer problem

The probes are positioned as follows :

- probe 1 : leading edge
- probe 2 : transition metal/TBC
- probe 3 : transition TBC/metal
- probe 4 : trailing edge

The fluid and the solid domains can interact in various ways from low fluid-structure interaction (FSI) characterized by low Biot numbers to high FSI (high Biot numbers). High numerical Biot numbers generally indicate that the solid thermal gradients are predominant and thus, a non-spatially-uniform temperature field within the solid body may be considered. As a result, on a practical level, a Dirichlet condition imposed on the fluid does not provide the most relevant interfacial scheme. It is thus essential to validate such an approach in very high thermal FSI characterized by high Biot numbers. This Biot number can be increased by either modifying fluid thermal properties or decreasing the thermal conductivity. In this paper the solid thermal properties have been decreased in order to increase this Biot number. Indeed, increasing the Biot number by modifying the fluid part leads either to modifying the size of the first cell, which is impractical, or modifying the fluid thermal properties, which is unacceptable. Thus three different thermal conductivities λ_{TBC} , representing three different Biot numbers, are compared: 0.1, 0.01, 0.001 W/m/K.

5.2. Representation of the stability zone

Figure 5: Plot of the maximum amplification factor ($\max|g|$) compared to the relaxation parameter α_f .

Figures 5 show the maximum amplification factor for different thermal conductivities of the TBC. As illustrated, when the thermal conductivity of the TBC decreases, it becomes increasingly difficult to stabilize the computation. The representation of the amplification factor based on the Godunov-Ryabenkii stability analysis exhibits a stable zone when $|g| < 1$, $\alpha_f > \alpha_f^{min}$. We can then define a minimum and a maximum value of α_f . As mentioned, $\alpha_f^{min} = K_s[Bi_\nu(1 - \overline{D_f}) - 1]/2$ and $\alpha_f^{max} = K_f/2$. Considering those minimum and maximum values of α_f , an optimal value can be found to obtain a good compromise between both stability and CPU cost.

The temporal amplification factor is plotted in Figure 5 as a function of the coupling coefficient α_f , for three different values of the solid conductivity. The vertical lines are defined by the equation $\alpha_f = \alpha_f^{min}$ where α_f^{min} is the lower stability bound (see Eq.11). In other words, CHT computations with coefficients lower than are theoretically unstable. As can be seen, the curves are composed of two half-lines and it has been shown [18] that the y-intercept ($\alpha_f = 0$) represents the strength of the thermal fluid-structure interaction. Its mathematical expression is given by (eq.8). This strength is also referred in this paper as to the numerical Biot number and, as can be seen, this number is an increasing function of the thermal conductivity. As a result, for large solid conductivities (metallic body), the temporal amplification is totally inside the stability domain.

As Bi_ν increases, the amplification factor increases and approaches $|g| < 1$ which leads to more complicated fluid/solid aerothermal computations. For very high Biot numbers, the optimal coefficient α_f^{opt} gets closer to the lower stability bound, α_f^{min} , and thus calculations become more prone to instabilities..

5.3. Several conditions at the interface

Previous works [23; 20] have shown that at low Biot numbers, the most suitable interface treatment is the Dirichlet-Robin condition, which means a temperature imposed on the fluid side and a Robin condition on the solid side, and at high Biot numbers, the most suitable treatment is the Neumann-Dirichlet condition, consisting of an imposed heat flux on the fluid side and a temperature imposed on the solid side. In the present case, it is complicated to choose a single interface condition since, in the metallic part, the Biot number is low, whereas at the TBC part, the Biot number becomes very large.

It is therefore particularly important and complex to set up a unified interface methodology on the basis of the Dirichlet-Robin interface condition that can be adapted to the requirement of a TBC. The first approach consists of imposing a Dirichlet-Robin condition where the Biot number is low and the Neumann-Dirichlet at the interface of the TBC. Different thermal conductivities have been studied.

5.3.1. CHT results

This CHT procedure converges poorly, especially due to the transition between the metal and TBC, as shown in Figures 6 and 7. In this configuration, instabilities are generated at the transition between metal and TBC that distort the results. This is thought to be due to the sudden change of the boundary treatment at the interface. Applying two different boundary conditions at the same coupled surface (Dirichlet-Robin at the metal part and Neumann-Dirichlet at the TBC part) seems to generate a non-physical solution. Having different successive materials makes this approach difficult to apply.

Figure 6: Temperature inside the solid, $\lambda_{TBC} = 0.001$

Figure 7: Convergence history, $\lambda_{TBC} = 0.001, Bi_\nu = 4000$

5.4. Dirichlet-Robin condition across the whole interface

In this section, several relaxation coefficients have been tested for various stability conditions. Such coefficients are often compared to the heat transfer coefficient. As it is known, the heat transfer coefficient is inversely related to the distance from the inlet and decreases very rapidly from an infinite value at the leading edge to a small reasonable value at the trailing edge ($1/\sqrt{x}$). A well-suited value of the relaxation coefficient needs to be defined to balance stability and CPU cost.

When the relaxation parameter is outside of the stability zone, oscillations might occur. Either these oscillations die out after some coupling iterations, or they increase, leading to instabilities. As it was pointed out, α_f^{min} is directly related to the numerical Biot number. This means that the stability zone gets narrower when the numerical Biot number increases.

In order to avoid instabilities, the Dirichlet-Robin condition is chosen throughout the coupled surface for all numerical Biot numbers. The metal conductivity is maintained at $20 \text{ W.m}^{-1}.\text{K}^{-1}$ but the thermal conductivity of the thermal barrier coating is modified and taken to extreme values to enhance the thermal interaction between the fluid and the solid.

5.4.1. Stability conditions

In this section, a thermal conductivity $\lambda_{TBC} = 0.1$ was adopted, and the maximum numerical Biot number is thus $Bi_\nu \simeq 52$. The maximum temporal amplification factor is represented in figure 8 while with the representation of the stability zone.

As mentioned, the computation is stable if $\max(|g|) < 1$ and this condition must be verified for both materials. This can be seen in Figure 8, where the coupled surface over the metallic part is stable if $\alpha_f > 9815$. This value is the minimal value that ensures stability for the metallic part. However, we need to take into account the multi-conductivity surface entirely and the minimum value of α_f is the one that ensures stability for both materials. In this case, the minimal value is $\alpha_f = 10314$ for the TBC. On the other hand, the theory predicts that the optimal coupling coefficient is $\alpha_f^{opt} = 10515$.

Figure 8: Maximum amplification factor compared to the relaxation parameter, $\lambda_{TBC} = 0.1$

λ_{TBC}	0.1	0.01	0.001
$Bi_\nu = Bi(1 - \overline{D_f})$	50	430	4000
$\alpha_f^{min} = K_s[Bi_\nu(1 - \overline{D_f}) - 1]/2$	10314	10490	10512
$\alpha_f^{opt} = K_f(1 - \overline{D_f})/2$	10515	10515	10515

Table 2: Stability criterion

Similarly, the α_f^{min} and α_f^{opt} values can be calculated for all Bi_ν to ensure smooth and fast convergence. It can be seen from Table 2 that the optimal value α_f^{opt} remains unchanged for all Bi_ν , while the α_f^{min} varies with the numerical Biot number. Thus the stability zone is increasingly restricted for larger Biot numbers.

5.4.2. Probes comparison

Figure 9: Variation of the temperature at the interface for each probe at $0.5\alpha_f^{opt}$, $\lambda_{TBC} = 0.1$, $Bi_\nu = 50$

Figure 10: Variation of the temperature at the interface for each probe at $0.5\alpha_f^{opt}$, $\lambda_{TBC} = 0.01$, $Bi_\nu = 430$

Figure 11: Variation of the temperature at the interface for each probe at $2\alpha_f^{opt}$, $\lambda_{TBC} = 0.1$, $Bi_\nu = 50$

Figure 12: Variation of the temperature at the interface for each probe at $2\alpha_f^{opt}$, $\lambda_{TBC} = 0.01$, $Bi_\nu = 430$

According to the theory coming from the 1D model, choosing α_f outside the stability zone might result in a solution that oscillates or diverges. Indeed, At a relatively low Biot number, the choice of the coefficient $\alpha_f = 0.5\alpha_f^{opt}$, which is outside of the stability zone, does not necessarily lead to oscillatory behavior (Figure 9). However, for a larger Biot number, oscillatory behavior is shown with the same coefficient, as shown in 10. On the other hand, for both $Bi_\nu = 50$ and $Bi_\nu = 430$ taking $\alpha_f = 2\alpha_f^{opt}$ produces, as expected, no oscillations since α_f is inside the stability zone, although this choice leads to a slower computation (Figures 11, 12).

Figures 13, 14, 15 point out the ability of the optimal relaxation parameter α_f^{opt} to stabilize a CHT computation for any Biot number at all probes. It can be also seen that fewer coupling iterations are required.

Figure 13: Variation of the temperature at the interface for each probe at $\alpha_f = \alpha_f^{opt}$, $\lambda_{TBC} = 0.1$, $Bi_\nu = 50$

Figure 14: Variation of the temperature at the interface for each probe at $\alpha_f = \alpha_f^{opt}$, $\lambda_{TBC} = 0.01$, $Bi_\nu = 430$

Figure 15: Variation of the temperature at the interface for each probe at $\alpha_f = \alpha_f^{opt}$, $\lambda_{TBC} = 0.001$, $Bi_\nu = 4000$

5.4.3. Convergence

In this section, particular attention is paid to the convergence history using different relaxation parameters. It is shown that using the optimal value α_f^{opt} is the most suitable condition in order to obtain a good compromise between CPU cost and stability. The simulation is considered converged when the temperature difference between the solid and the fluid is less than $\Delta T = (T_f - T_s) < 10^{-3}$ in L_∞ norm. Four different values of α_f are compared : $\alpha_f = 0.5\alpha_f^{opt}$, $\alpha_f = \alpha_f^{opt}$, $\alpha_f = 2\alpha_f^{opt}$ and $\alpha_f = 10\alpha_f^{opt}$.

Figure 16: Convergence of the temperature at the interface, $\lambda_{TBC} = 0.1$, $Bi_\nu = 50$

Figure 17: Convergence of the temperature at the interface, $\lambda_{TBC} = 0.01$, $Bi_\nu = 430$

Figure 18: Convergence of the temperature at the interface, $\lambda_{TBC} = 0.001$, $Bi_\nu = 4000$

Figures 16, 17, 18 show the convergence history for each of the values of thermal conductivity of the TBC, i.e. for different numerical Biot numbers. When Bi_ν is relatively low, all α_f can slightly be outside the stability zone ($\alpha_f = 0.5\alpha_f^{opt}$), as shown in Figure 16, but nonetheless, the CHT procedure converges. However, when Bi_ν increases, as shown in Figures 17 and 18, it is particularly important to stay inside the stability bound in order to avoid instabilities. Indeed, if $Bi_\nu \gg 1$, having a relaxation parameter $\alpha_f < \alpha_f^{min}$ results in a non-converging process. This can be seen when $\alpha_f = 0.5\alpha_f^{opt}$. On the other hand, if $\alpha_f > \alpha_f^{opt}$, the simulation converges, but at a slower rate than with the recommended value in the current paper. At such high Biot numbers, the stability criterion ensures both stability and fast convergence.

5.4.4. Variation of the CFL number

As previously mentioned, the new version of the optimal coupling coefficient α_f^{opt} depends now on the CFL number. Indeed, increasing the CFL number leads to an acceleration of the coupled simulation. The optimal value is further reduced while still remaining within the stability zone. Figure 19 shows the accelerating effect using the CFL number.

Figure 19: Accelerating effect with the CFL number at $\alpha_f^{opt}, \lambda_{TBC} = 0.1$

6. Conclusion

A generalized method has been presented for a multi-conductivity coupled interface. The Dirichlet-Robin condition was shown to be suitable for a wide range of Biot numbers based on a theoretical 1D model. This method depends on the use of an optimal coefficient coming from a stability analysis but this coefficient has been enriched by introducing the fluid diffusive-time step and the CFL number. The resulting physics-based approach has shown that unconditionally stable computations can be obtained with a unified interface treatment, suitable for both metallic or ceramic materials.

When different materials are involved, and thus different Biot numbers, the Dirichlet-Robin condition seems to be the most suitable interface treatment. This approach has the ability to stabilize any CHT computation, regardless of the thermal resistance of the material.

It was shown that the stability zone depends mainly on the numerical Biot number, which is a local representation of the strength of the thermal coupling along the fluid-solid interface. This number plays a key role in the stability analysis and has the great advantage of being defined during the fluid transients. Furthermore the relaxation parameter proposed in this paper becomes adaptive and always remains inside the stability zone. Indeed, the fluid diffusive time step gives an excellent approximation of the local time step that can be used in the transient phase of the coupled process. This procedure provides an accurate aerothermal solution while ensuring unconditionally stable computations.

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