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# 1 Experimental and theoretical study of density, potential and 2 current structures of a helium plasma in front of a RF antenna 3 tilted with respect to the magnetic field lines

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Potential and density structures in the vicinity of a RF electrode/antenna in a magnetized plasma are investigated using a RF compensated cylindrical Langmuir probe. These measurements were performed in the ALINE plasma device in which only electrons can be considered as well magnetized. Very precise 2-D maps of the plasma parameters are drawn thanks to a 3-D automatic manipulator on which the probe is mounted. The effect of the tilted magnetic angle between the RF biased surface and the magnetic lines is also studied thanks to a tilting electrode. Comparison of several simplistic models with the experiments proved the reliability of simple Langmuir probe measurements in such a RF and magnetized environment (space potential v.s. tilting angle of the antenna with respect to magnetic field lines, and recovering of the floating potential structure using measured currents). A fluid model based on total current density, and ion diffusion equations over the biased flux tube, provides the same density structures in front of the electrode than the measurements. Those density structures display a “bunny ears” shape, and can be explained using transverse RF and collisional current behaviour: in front of the antenna the transverse ion currents deplete the magnetized flux tube, while at the edge of the biased flux tube, the same currents rise the density.

## 9 I. INTRODUCTION

10 Potential structures in a magnetized plasma have  
11 been investigated for a long time<sup>1,2</sup>, in order to  
12 understand plasma fluxes and to study the edge of  
13 Tokamak plasmas responsible for heat fluxes at  
14 the wall<sup>3</sup>. A DC or RF biased source is able to  
15 raise a strong potential structure in a magnetized  
16 plasma due to RF sheath rectification<sup>4</sup>, and to in-  
17 duce convective cells all around the structure<sup>5,6</sup>.  
18 This phenomena are present in classical magne-  
19 tized discharges (e.g. magnetron discharges) and  
20 in the scrape off layer (SOL) of tokamaks in which  
21 ICRH (Ion Cyclotron Resonant Heating) antennas<sup>7</sup>  
22 are able to generate such potential structures .

23 The comprehension of density and potential  
24 structures in front of these antennas/electrodes is  
25 really important to evaluate the heat fluxes respon-  
26 sible for hot spot on their surfaces, and to prevent  
27 the sputtering, heating and destruction of connected  
28 parts at the wall or on the RF antenna. In addi-  
29 tion, the magnetic configuration is such that the sur-  
30 face of the RF antennae is not parallel nor perpen-  
31 dicular to magnetic field lines, but a little tilted<sup>8</sup> .  
32 Hence, the influence of the tilted angle of a biased  
33 electrode or antenna is treated here in the ALINE  
34 plasma device<sup>9</sup> in which the relative low RF power  
35 allows to make noiseless measurements.

36 To diagnose these potential structures, Langmuir  
37 probes can be used to get the plasma’s main pa-  
38 rameters, and probe measurements are technically  
39 speaking relatively easy to perform. There is al-  
40 ready a large documentation on this subject<sup>10-16</sup>  
41 especially in a steady and non magnetized plasma  
42 where a Langmuir probe measurement can provide  
43 a robust estimation of the whole bulk plasma den-  
44 sity, temperature and potentials (floating  $V_{fl}$  and  
45 plasma  $\phi_p$  ones).

46 But the problem is more challenging when  
47 performing these measurements in a magnetized  
48 plasma<sup>17-23</sup> and even more when connected to a  
49 radio-frequency (RF) antenna/electrode<sup>24-30</sup>. In-  
50 deed, turning on the magnetic field breaks down  
51 isotropy and the probe measurement is then only  
52 defined locally (measured parameters can drasti-  
53 cally change when moving across magnetic field  
54 lines). It is then needed to perform measurements  
55 into several areas of the plasma in order to access  
56 to the full structure of plasma parameters. But the  
57 most difficult issue is to understand how the current  
58 is collected by a cylindrical probe when it is aligned  
59 with magnetic field. This topic has been addressed  
60 by several authors in the case of strongly magne-  
61 tized electrons<sup>22,31</sup>. But in the case of weakly mag-  
62 netized ions, such as in small plasma discharges,  
63 the ion part in the  $I(V)$  characteristics remains the  
64 best way to deduce the plasma density, particularly  
65 in RF environment because ions are less sensitive  
66 to RF oscillations<sup>24</sup>. On the electrons part of the  
67 characteristics, cylindrical probes exhibits a strong

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68 slope break in the region of the plasma potential, so  
 69 that it is much more convenient to locate  $\phi_p$  than for  
 70 a planar or spherical probes. In some cases, a bump  
 71 rises on the  $I(V)$  in this potential range, due to a  
 72 density depletion of the biased plasma connected to  
 73 the probe during a voltage ramp<sup>21,32</sup>. All of these  
 74 knowledge of numerous cylindrical probe theories  
 75 have been reviewed to interpret probe data and  
 76 to draw potential, density and temperature contour  
 77 plots all around a RF electrode.

78 Two dimensional contour plots of plasma param-  
 79 eters have been measured experimentally in the 1  
 80 m long linear plasma device ALINE<sup>9,32,33</sup>. The an-  
 81 tenna (also called electrode) was tilted with respect  
 82 to the magnetic field lines. The measurements are  
 83 faced to some models to check the reliability of the  
 84 measurements in these specific conditions (RF +  
 85 magnetic field). The protocol is described in [sec-](#)  
 86 [tion II](#). In [Section III](#), the 2-D potentials and density  
 87 maps are plotted and discussed. A first model pre-  
 88 dicting the spacial evolution of the plasma potential  
 89 with respect to the inclination angle of the electrode  
 90 is provided. A second one recovers the measured  
 91 floating potential structure in front of the electrode  
 92 as function of the measured density and plasma po-  
 93 tential structures. Finally, [section IV](#) introduces a  
 94 fluid model using the total current and ion flux con-  
 95 servation equations. The numerical solution of this  
 96 model allows to recover the measured density pro-  
 97 files and to provide a physical explanation of the  
 98 pumping phenomenon enhanced by transverse cur-  
 99 rents all along the RF biased plasma column.

## 100 II. EXPERIMENTAL DESIGN

### 101 A. Setup and protocol

102 All the measurements were carried out in the  
 103 ALINE plasma device<sup>9,33</sup> (see scheme in Fig.3): a  
 104 cylindrical vessel (1 m long and 30 cm diameter).  
 105 Six coils are placed equidistantly along the cham-  
 106 ber to generate a magnetic field up to 100 mT at  
 107 the center of the cylinder (the input current goes up to  
 108 220 A). In the working volume, i.e. near the center  
 109 of the vessel, magnetic field lines are along the  
 110  $z$  axis (i.e. the axis of the vessel) – as they would  
 111 be in an infinite solenoid approximation – and the  
 112 magnetic field amplitude can be supposed homoge-  
 113 neous, i.e.  $\mathbf{B}(\mathbf{r}) = B_0 \mathbf{e}_z$ . See ref.<sup>32</sup> for more infor-  
 114 mations on this device.

115 A circular RF antenna (8 cm diameter and 1  
 116 cm thick) is placed at the center of the vessel (see  
 117 photograph in Fig.1). In the following work, the  
 118 antenna is directly connected to the RF generator  
 119 (without matching box). Ergo, the mean voltage on  
 120 the antenna is 0 and the plasma potential is oscil-  
 121 lating at the RF frequency  $\nu_{RF}$ . The antenna can

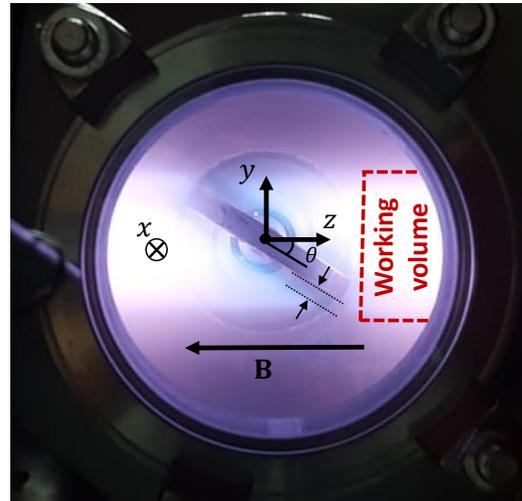


FIG. 1. Photograph of the electrode inclined with respect to the magnetic field/horizontal/ $z$  axis at 20 W, 94 mT and 3 Pa. The  $\sim 1$  cm-thick sheath is distinguishable: less shiny plasma due to less electrons in it since they are repelled by the electrode. Note: sheaths appear around all biased surfaces with respect to the plasma, including the probe.

122 be tilted with respect to the magnetic field lines at  
 123 any angle  $\theta \in [0, 360]^\circ$ . Note that the antenna plays  
 124 also the role of the electrode, and the rest of the  
 125 vessel is the anode (thus  $S_{\text{electrode}} \ll S_{\text{anode}}$ ). The  
 126 input power can go from 20 up to 200 W, and the  
 127 frequency range goes from 10 kHz up to 250 MHz.  
 128 In the frame of this work, the RF frequency was set  
 129 to 25 MHz.

130 The probe measurements obtained in this study  
 131 are performed in a Helium plasma ( $n \simeq 10^{16} \text{ m}^{-3}$ ).  
 132 To get the plasma parameters, cylindrical Langmuir  
 133 probe measurements were done inside the working  
 134 volume, in front of the electrode. The choice of a  
 135 cylindrical and thin probe (length  $L_p = 1$  cm and  
 136 radius  $r_p = 75$  microns) was motivated because of  
 137 a better spacial resolution: the measurement step in  
 138 the  $y$  and  $z$  directions,  $\Delta y$  and  $\Delta z$  respectively., can-  
 139 not be less than the probe dimensions (i.e.  $\Delta y \geq r_p$   
 140 and  $\Delta z \geq L_p$ ). To obtain a probe characteristic, a  
 141 voltage ramp from  $-100$  to  $100$  V at a frequency  
 142 of 65 kHz was applied on the probe tip and the col-  
 143 lected current is measured. The final  $I(V)$  curve  
 144 was the average over 20 successive voltage ramps.  
 145 The probe tip is RF compensated to allow mea-  
 146 surements in a radio-frequency discharge<sup>24,34,35</sup>.  
 147 Since the voltage ramp rate is lower than  $\omega_{RF}$  (78  
 148 Mrad/s), and both  $\omega_{pi}$  (66 Mrad/s) and  $\omega_{pe}$  (5.6  
 149 Grad/s), the measurement can be seen as stationary.

150 The Langmuir probe (also see ref.<sup>32</sup> for more  
 151 informations) is held by a manipulator which al-  
 152 lows us to perform measurements in a 3-D vol-  
 153 ume. The center of the electrode lays at the coordi-

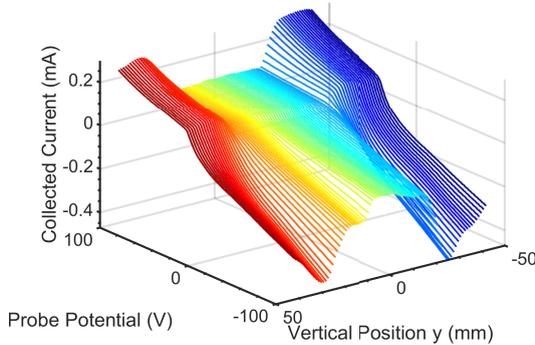


FIG. 2. The procedure applied for building the contour plot is the following: the probe is first fixed at a constant  $z$  position, and then moved in the  $y$  direction. Finally, the  $z$  position is changed. In this figure is depicted a  $y$ -profile of  $I(V)$  characteristics at  $z = 150$  mm, 94 mT, 3 Pa 25 MHz and 20 W, where the antenna is tilted by  $\theta = 25^\circ$ .

154 nates  $(0, 0, 0)$ . In the scope of this paper, the prob-  
 155 ing volume is the surface  $\{x = 0, y \in [-42, 42], z \in$   
 156  $[20, 200]\}$  mm with  $\Delta y = 1$  mm and  $\Delta z = 10$  mm  
 157 (note that the  $z$  range is adjusted regarding the electro-  
 158 de inclination, and that the ion larmor radius is of  
 159 the order of  $400 \mu\text{m}$  at 94 mT). The  $y$  range is lim-  
 160 ited by the manipulator that can not go further away  
 161 from its axis. A scan in the  $(x, y)$  is also performed  
 162 with a thicker spatial resolution of  $\Delta x = \Delta y = 2$  mm  
 163 (around 1800 characteristics) in order to perform  
 164 the measurement in a reasonable amount of time.

165 To obtain a 2-D contour plots of plasma param-  
 166 eters in front of the electrode,  $I(V)$  curve measure-  
 167 ments were performed at each node of the work-  
 168 ing volume grid and exploited using a simple and  
 169 flexible – but reliable fitting method of the ion sat-  
 170 uration current<sup>36–38</sup>, and presented in section II C.  
 171 Therefore one 2-D map needed over a thousand of  
 172 Langmuir probe characteristics and several hours  
 173 of acquisition (the ALINE plasma is stable enough  
 174 to make this possible). A  $y$ -scan is depicted in  
 175 Fig. 2: one position  $z$  is associated to 82 characteris-  
 176 tics. The gas pressure, the magnetic field amplitude,  
 177 and the electrode inclination were changed during  
 178 our experimental work to track their effects on the  
 179 plasma parameters. However, the RF power is fixed  
 180 at 20 W. The choice of low RF power is motivated  
 181 by the fact that at high power (*i*) the RF oscillations  
 182 can lead to bad estimation of plasma parameters<sup>24</sup>,  
 183 (*ii*) the characteristic can display a bump between  
 184 the exponential and the electron part<sup>32</sup> meaning that  
 185 the measurement disturbs the plasma, and (*iii*) the  
 186 long discharge time causes the electrode to heat up,  
 187 changing the plasma conditions between the begin-  
 188 ning and the end of the scan.

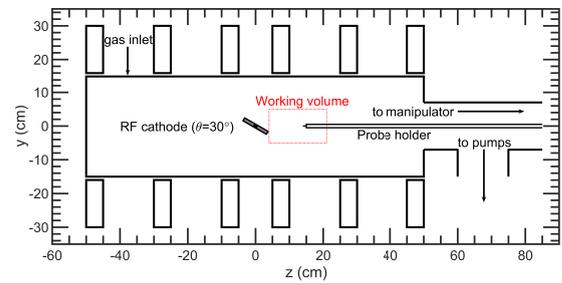


FIG. 3. The drawing of the ALINE plasma device (to scale, side view). Where the 12 rectangles on both sides of the chamber represent the coils. Magnetic field lines are parallel to the  $z$  direction, and its magnitude is constant in the working volume.

## 189 B. Dependency on potential and magnetic field

190 The effective collection surface of a Langmuir  
 191 probe is dependant of the applied voltage on the  
 192 probe tip, but also on the applied magnetic field.  
 193 Indeed the existence of a magnetic field changes  
 194 the motion of the particles: due to Lorentz force,  
 195 they follow a cyclotron motion and turn around the  
 196 magnetic field lines. The radius of charged parti-  
 197 cles around magnetic field lines – called Larmor ra-  
 198 dius – is smaller for electrons than for ions. Indeed,  
 199  $\rho_c = v_\perp / \omega_c \approx \sqrt{k_B T_e m} / eB$  (where  $\omega_c = eB/m$  is  
 200 the cyclotron frequency,  $B$  the magnetic field ampli-  
 201 tude,  $m$  the mass, and  $k_B = 1.38$  J/K the Boltzmann  
 202 constant). Therefore electrons are more confined  
 203 than ions. This sensitivity to the magnetic field also  
 204 changes the collection surface for each species<sup>22,23</sup>.  
 205 In some cases the electron and ion current growth  
 206 can even be similar in amplitude<sup>39</sup>. As it was shown  
 207 in a previous paper, using a thin cylindrical probe in  
 208 a magnetized plasma can lead to wrong estimation  
 209 of the plasma parameters due to electron density  
 210 depletion<sup>18–21,39,40</sup> once the applied voltage over-  
 211 comes the plasma potential,  $V > \phi_p$ . Therefore, it  
 212 is more convenient and reliable to work with the ion  
 213 part of the  $I(V)$  than with the electron part in mag-  
 214 netized conditions<sup>41</sup>. Anyway, the quasi-neutrality  
 215 approximation allows us to state that ion density ob-  
 216 tained with the ion part is close to the electron den-  
 217 sity, i.e.  $n_i \approx n_e$  for single charged ions (which is  
 218 the case in the ALINE experiments).

219 Since parallel to  $\mathbf{B}$  transport is much higher than  
 220 perpendicular transport, due to magnetic confine-  
 221 ment, we can talk about “magnetic flux tubes”.  
 222 Indeed, perpendicular transport can be driven by  
 223 collisions<sup>42</sup>, biasing effects<sup>43</sup> or anomalous trans-  
 224 port. In this work the “flux tubes” term references  
 225 to the magnetic channel connected to the electrode  
 226 on one end, and to the grounded wall on the other.  
 227 Since the probe holder is made of ceramics, its po-

228 tential is floating and at low RF power, we assume  
229 that its presence does not disturb the flux tube.

230 Both, voltage and magnetic field dependency  
231 make it very difficult to extract plasma parameters  
232 from the  $I(V)$ . Finally, the RF fluctuations also distort  
233 the shape of the characteristics<sup>24,27</sup>. Indeed,  
234 having a RF power supply leads to oscillations of  
235 the plasma potential due to the auto-polarisation  
236 of plasma<sup>29,30,44–48</sup>, moving the  $I(V)$  “back and  
237 forth”. And since the measurement is an average  
238 of this phenomenon, it leads to overestimation of  
239 the electron temperature (the exponential growth is  
240 wider). But the ion part remains more or less un-  
241 changed in the saturation region. This second argu-  
242 ment explains our choice of using the ion part of the  
243  $I(V)$ .

### 244 C. IV analysis

245 Theoretical processing of probe characteristics in  
246 magnetized plasmas is still an open research field.  
247 Lots of authors worked on this subject (Langmuir<sup>15</sup>,  
248 Bohm<sup>13</sup>, Laframboise<sup>31,49</sup>, Chen<sup>50</sup>, etc.<sup>10,11,51–53</sup>).  
249 A global overview on major theories is available  
250 herein<sup>12</sup>. Langmuir’s Orbital Motion Limit (OML)  
251 model is simple to use and to apply because, as-  
252 suming that there is no sheath, it states – under cold  
253 ions approximation – that  $I_i^2(V)$  is a linear function,  
254 whose slope is proportional to the density squared  
255 and independent from the temperature. However  
256 our characteristics do not fit with this model (the  
257 RF has nothing to do with that though<sup>25</sup>). Using  
258 ABR<sup>10,11</sup> or BRL-Chen<sup>52</sup> theories would be a good  
259 option, but it is recommended to check if the out-  
260 put parameters and the fitting are correct for all  
261 characteristics ; nevertheless our cumbersome maps  
262 (containing over 1000  $I(V)$  each) do not allow us  
263 to do so. Laframboise<sup>31,49</sup> did a very deep work  
264 on the collection of charged particle by a cylin-  
265 drical Langmuir probe in magnetized plasma. He  
266 computed the collected current, and his curves are  
267 universal because they only depend on the probe  
268 radius-to-electron Debye length ratio,  $r_p/\lambda_{De}$ . Un-  
269 fortunately, there is no analytical expression avail-  
270 able, but only numerical results. Chen did a param-  
271 eterization of these curves, but the algorithm still  
272 needs a supervisor to check the fitting of the mea-  
273 sured  $I(V)$  because those method need some initial  
274 input parameters. Mausbach and Steinbrüchel<sup>37,38</sup>  
275 proposed to fit the ion current with the function  
276  $a(b + \chi)^c$ , where  $a, b$  and  $c$  are fitting parameters  
277 and  $\chi$  the normalized potential drop between the  
278 plasma and the probe,  $e(\phi_p - V)/k_B T_e$ . In our ex-  
279 periment we assume that the ion current is given by  
280 the relation<sup>36</sup>,

$$280 \quad I_i^{\text{theo}} = I_{\text{isat}}(1 + \chi)^k, \quad (1)$$

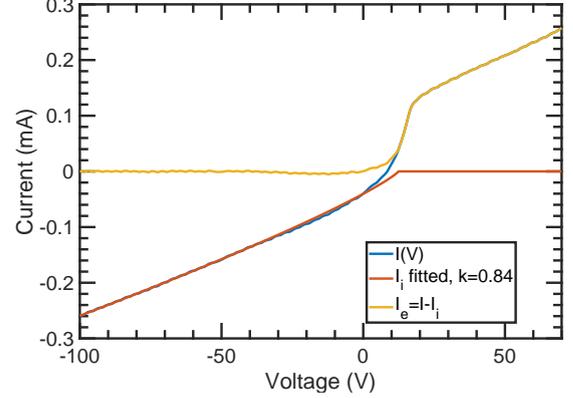


FIG. 4. Result of the numerical and automatic fitting of  
the ion current, allowing the determination of the plasma  
density  $n = 3.7 \times 10^{15} \text{ m}^{-3}$  and electron temperature  
 $T_e = 4.9 \text{ eV}$  after two iterations.

281 with  $I_{\text{isat}} = -0.61 \times enc_s S_p$  (assuming  $T_i \approx 0$  and  
282 thus  $c_s = \sqrt{k_B T_e / M}$  with  $M$  the ion mass) the  
283 ion saturation current and  $S_p = 2\pi r_p L_p$  the probe  
284 area. This parametrization allows us to ensure  
285  $I_i^{\text{theo}}(\phi_p) = I_{\text{isat}}$ .

### 286 The Algorithm

287 This algorithm was used to extract all parameters  
288 automatically, without input or guessed parameters,  
289 from the numerous characteristics measured: density  $n$ ,  
290 plasma potential  $\phi_p$ , probe floating potential  
291  $V_{\text{fl}}$  and plasma temperature  $T_e$ .

292 *a. Initialization:* First of all, one needs to  
293 determine both plasma and floating potentials.  
294 The  $I(V)$  is smoothed with a Savitzky-Golay  
295 algorithm<sup>54</sup> and the floating potential  $V_{\text{fl}} = V(I =$   
296  $0)$  is determined. The first derivative is com-  
297 puted numerically and smoothed again. The first  
298 maximum after the floating potential gives the  
299 position of the plasma potential  $\phi_p$ ,  $dI/dV|_{\phi_p} =$   
300  $\max(dI/dV)$ . Note: in the literature, the plasma  
301 potential is also called “space potential” in a mag-  
302 netized plasma, because each magnetic flux tube  
303 can have its own potential. Nevertheless the RF  
304 distortions and the noisy second derivative makes  
305 the use of Druyvesteyn’s theory<sup>55</sup> inapplicable (i.e.  
306  $d^2 I/dV^2 \propto \text{EEDF}(V)$ , the electron energy distribu-  
307 tion function) even after another smoothing.

308 *b. First fitting of  $I_i$ :* Now, one needs to esti-  
309 mate the electron temperature. To do so, the ion  
310 current must be removed from the characteristic. In  
311 the range  $V \in [-100, V_{\text{fl}} - 20] \text{ V}$ , a linear fit<sup>56</sup> of the  
312 curve is performed:  $I_i = C_1 V + C_2$ .

313 *c. First determination of  $T_e$ :* Once  $I_i$  is  
314 known, it is possible to compute  $I_e = I - I_i$ . A linear

315 fit of  $\ln I_e$  vs  $V$  is done in the range  $V \in [V_{\text{fit}}, \phi_p]$  V, 366  
 316 the slope being  $e/k_B T_e$ .

317 *d. Better fitting of  $I_i$ :* With the last computed 368  
 318 value of  $T_e$ , one can fit the ion current from the  $I(V)$  369  
 319 in the same range as in *b.*, but this time using equa- 370  
 320 tion (1).

321 *e. Better determination of  $T_e$ :* With this new 372  
 322 fit of  $I_i$ , the electron current can be evaluated, and 373  
 323  $T_e$  can be determined the same way as presented in 374  
 324 *c.*

325 *f. Loop and convergence:* The new value of 376  
 326  $T_e$  is compared to the old one, and if  $|T_e^{\text{new}} - T_e^{\text{old}}| >$  377  
 327  $\mathcal{E}$ , the algorithm starts again at point *d.* The conver- 378  
 328 gence is achieved within less than 5 loops generally 379  
 329 ( $\mathcal{E} = 0.2$  eV is chosen). The last value of  $I_{\text{isat}}$  ob- 380  
 330 tained in the loop is used to estimate the density:

$$n_{\text{fit}} = -\frac{I_{\text{isat}}}{0.61 \times e S_p \sqrt{k_B T_e^{\text{new}} / M}}, \quad (2)$$

331 The result of a fit is depicted in Fig.4. The discontinuity is due to the fact that the model do not take into account the region above plasma potential. We assume that, if this method do not give the precise estimate of  $n$  and  $T_e$  it can provide, at least, their spatial evolution.

### 337 III. TWO-DIMENSIONAL CONTOUR PLOTS OF PLASMA PARAMETERS 338

339 The data will be presented on 2-D heat maps, so 340  
 341 that every map represents the spacial structure of 342  
 343 a given quantity,  $Q(x = 0, y, z)$ . All measurements 344  
 345 were obtained in a single plane in front of the electrode in one discharge (i.e. it is the same plasma from the first point until the last one). The dashed lines on the following graphs delimits the “top” of the electrode (which also coincides with the top of the bright plasma, see Fig.1). For safety reasons, measurements are done with spatial margins (avoiding the probe to hit the electrode), thus inducing different probing areas in the  $z$  direction ( $\parallel B$ ).

351 Since heat maps plots are cumbersome, it is not 352  
 353 possible, in the scope of this paper, to show all of 354  
 355 them. We have chosen to depict the spatial evolution of the plasma parameters with respect to the inclination of the antenna/electrode for only one discharge condition: 20 W input RF-power, 3 Pa He pressure and 94 mT magnetic field. This gives six plots for each parameter and provides a good overview on the structure evolution with respect to the tilting angle. To compare with what happens at low field or at low pressure, two more plots are added at the inclination of  $45^\circ$  (one at 1.5 Pa, the other at 47 mT) and still with 20 W RF-input power.

364 This section will mainly yield a qualitative 365  
 366 overview of several plasma parameters evolution

367 with respect to the position, the magnetic field amplitude, the pressure and the inclination of the electrode. An attempt of understanding quantitatively the experimental outputs will be done in the next section.

371 However, the plasma temperature will not be presented in this study because of the big uncertainty in its determination: magnetic field and RF fields makes it very difficult to get reliable values due to the distortion of the characteristics in the exponential part. Nevertheless, measurements showed that the electron temperature is rather constant inside the flux tube, few centimetres away from the electrode. Therefore,  $T_e$  is assumed to be equal to 5 eV and constant in the rest of the paper.

### 381 A. Plasma potential

382 This subsection refers to figures 5(a) to 5(h). It is clear that the plasma channel in front of the electrode is biased with respect to the the “bulk” plasma (outside the channel). In addition, the bigger the section of the flux tube, the higher is the biasing<sup>29</sup>: at  $0^\circ$  (Fig.5(a)) the channel diameter is  $d_c = 1$  cm and the potential difference between inside and outside the magnetic channel is  $\Delta\phi_p = \phi_p^{\text{in}} - \phi_p^{\text{out}} \approx 5$  V, whereas at  $90^\circ$  (Fig.5(f)) the channel diameter  $d_c = 8$  cm and  $\Delta\phi_p = \phi_p^{\text{in}} - \phi_p^{\text{out}} \approx 25$  V.

392 For lower magnetic field (Fig.5(g)), the plasma is less confined, and thus, the perpendicular transport is higher: the plasma potential structure is shorter than with a 94 mT magnetic field amplitude. In the opposite, at low pressure (Fig.5(h)), the structure is longer, because there are less collisions, and each flux tube remains disconnected from each other.

399 Moreover, these maps reveal the existence of a transverse electric field,  $E_y = -\partial_y \phi_p$ , at the edge of the magnetic flux tube. This electric field also generates a drift of charged particles in the  $x$ -direction due to the  $\mathbf{E} \times \mathbf{B}$  drift:  $u_x = E_y / B$  (see Fig.6). The drift velocity in the Hall direction approaches 10 km/s at the edge of the potential structure.

406 According to the measurements, the plasma potential displays a plateau in the middle of the structure in the  $z$  direction. We were able to measure the evolution of this plateau amplitude with respect to the tilting angle  $\theta$ . This measurement was achieved at 3 Pa, for  $B_0 = 94$  mT and for several input RF power (from 13 to 79 W) at the location  $x = 0$  mm,  $y = 0$  mm and averaged over the range of  $80 \leq z \leq 110$  mm. The plot of  $\phi_p(\theta)$  are alike for all powers: similar shape, but an up-shift with increasing power. Therefore, all plasma potential vs.  $\theta$  is normalized to their value at  $\theta = 90^\circ$  (light coloured curves in Fig.7) and the averaged curved was computed (red curve with errorbars). All curves superimpose very well.

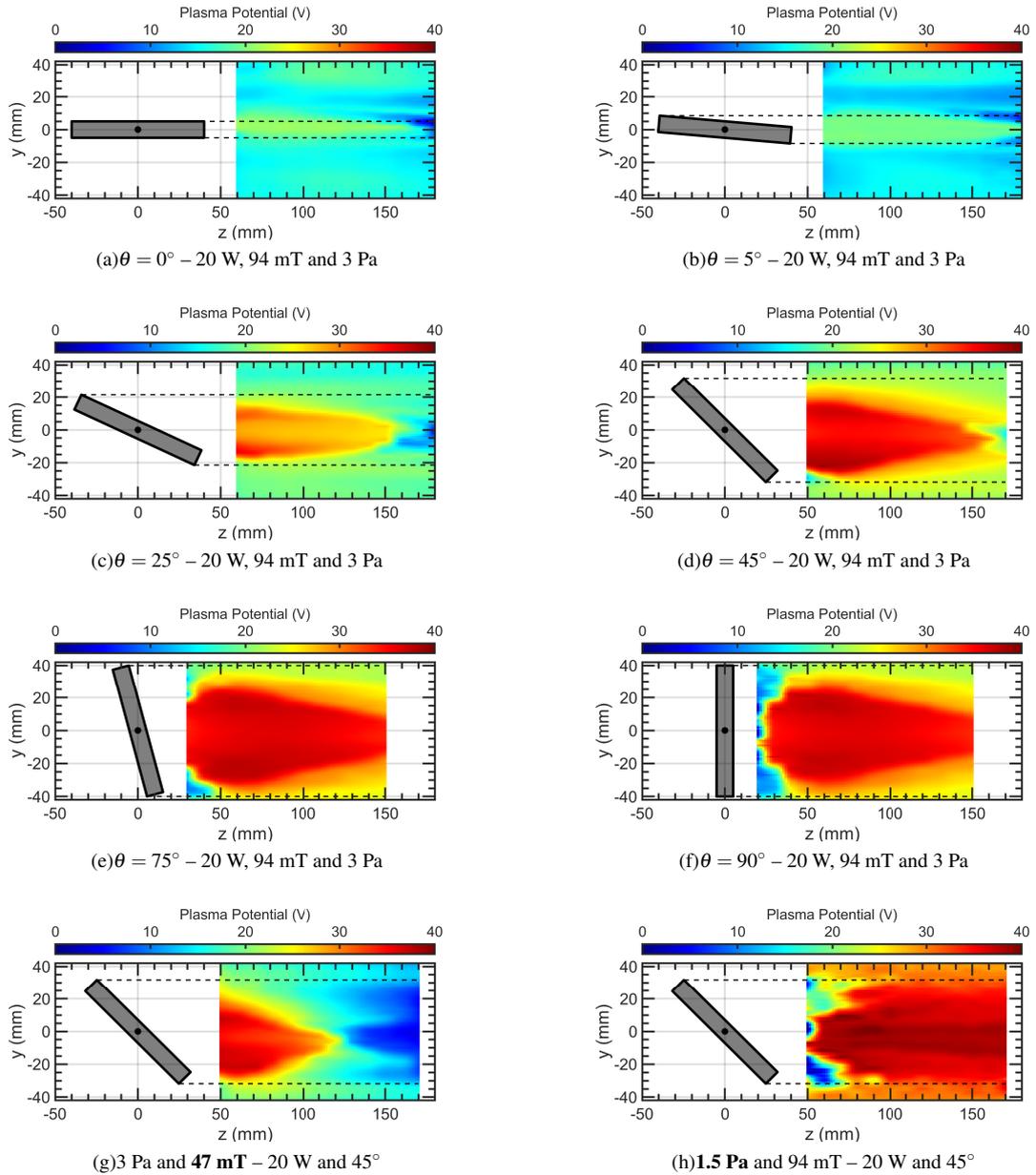


FIG. 5. Plasma potential contour plots in front of the RF antenna, for  $v_{\text{RF}} = 25 \text{ MHz}$ . Conditions are labelled below each plot. For (a) to (f): 1 discharge, all inclination. For (h) & (g) comparisons at  $45^\circ$  for low field and low pressure condition respectively. Note that the potential axis do not have the same range as the  $V_{\text{fl}}$  plots. (a)  $\theta = 0^\circ - 20 \text{ W, } 94 \text{ mT and } 3 \text{ Pa}$ , (b)  $\theta = 5^\circ - 20 \text{ W, } 94 \text{ mT and } 3 \text{ Pa}$ , (c)  $\theta = 25^\circ - 20 \text{ W, } 94 \text{ mT and } 3 \text{ Pa}$ , (d)  $\theta = 45^\circ - 20 \text{ W, } 94 \text{ mT and } 3 \text{ Pa}$ , (e)  $\theta = 75^\circ - 20 \text{ W, } 94 \text{ mT and } 3 \text{ Pa}$ , (f)  $\theta = 90^\circ - 20 \text{ W, } 94 \text{ mT and } 3 \text{ Pa}$ , (g)  $\theta = 45^\circ - 20 \text{ W, } 47 \text{ mT and } 3 \text{ Pa}$  and (h)  $\theta = 45^\circ - 20 \text{ W, } 94 \text{ mT and } 1.5 \text{ Pa}$ .

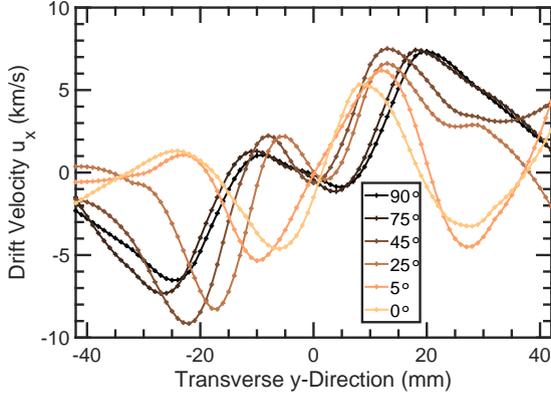


FIG. 6. Spatial evolution of the drift velocity  $u_x = \partial_y \phi_p / B$  at 94 mT, 3 Pa, obtained by taking  $\phi_p(y, z = 100 \text{ mm})$  from the 2-D maps, in the direction  $\perp \mathbf{B}$ .

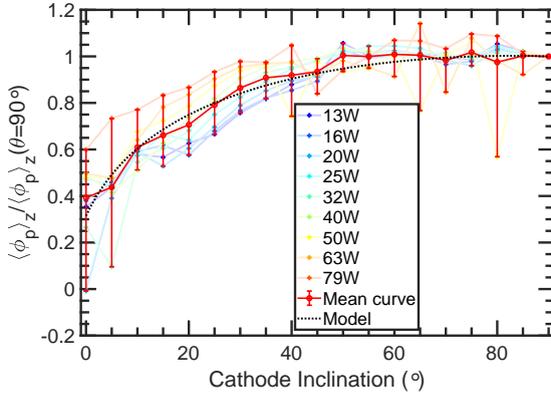


FIG. 7. Evolution of the plateau amplitude of the plasma potential structure with respect to the inclination of the electrode with respect to the magnetic field lines.

421 A simple flux tube model was built to understand  
422 this averaged curve. First of all, we assume that  
423 the section of the flux tube in front of the electrode  
424 is a thick disk of diameter  $d_c$ , and depends on the  
425 inclination of the probe:

$$d_c \approx 2R_c \sin \theta + T_c \cos \theta \quad (3)$$

426 where  $R_c = 4 \text{ cm}$  and  $T_c = 1 \text{ cm}$  are the electrode  
427 radius and thickness respectively. The frontal sur-  
428 face of the tube is then  $S_F = \pi d_c^2 / 4$ , and the lat-  
429 eral surface is  $S_L = \pi d_c L_{\parallel}$  ( $L_{\parallel}$  is the length of the  
430 magnetic flux tube, i.e. the distance electrode –  
431 grounded wall). We then assume perfectly confined  
432 electrons, and unmagnetized ions. The discharge  
433 model seen in the point of view of electrons and  
434 ions are given in figures 8(a) and 8(b) respectively.  
435 Next, one assumes the current density conserva-  
436 tion all over the flux tube boundaries. Recall that  
437  $J_{e,\text{sat}} = 0.25en\langle v_e \rangle$  the electron saturation current  
438 density in the approximation of Maxwellian elec-

439 trons ( $\langle v_e \rangle = (8k_B T_e / \pi m)^{1/2}$ ), and  $J_{i,\text{sat}} = 0.61enc_s$   
440 the cold ion saturation current density. Current den-  
441 sity conservation of the magnetic flux tube, consid-  
442 ering collisionless sheath and the absence of sec-  
443 ondary electrons, writes:

$$0 = \sum_{k=i,e} J_k \times S_k$$

$$\Leftrightarrow 0 = J_{i,\text{sat}}(2S_F + S_L)$$

$$- J_{e,\text{sat}} S_F (e^{\phi_{\text{RF}}} + e^{-\phi_p}) \quad (4)$$

444 were  $\phi$  are normalized potentials to  $e/k_B T_e$ . Let us  
445 now define the plasma floating potential (different  
446 from the probe one) to lighten the equations as the  
447 ratio :

$$\phi_{\text{fl}} \equiv \ln \left( \frac{J_{e,\text{sat}}}{J_{i,\text{sat}}} \right) \approx 4.02. \quad (5)$$

448 Equation (4) gives the expression of the plasma-  
449 or-space potential in the magnetic flux tube,

$$\phi_p = \phi_{\text{fl}} + \ln(1 + e^{\phi_{\text{RF}}}) - \ln \left( 1 + 2 \frac{L_{\parallel}}{d_c} \right) - \ln 2 \quad (6)$$

450 Averaging it over time, the second term can be  
451 approximated<sup>4,47</sup> by  $\tilde{\phi}_{\text{RF}} / \pi$ . Therefore, the mea-  
452 sured plasma potential can be approached by the  
453 equation:

$$\langle \phi_p \rangle_t = \phi_0 - \ln \left( 1 + \frac{2L_{\parallel}}{2R_c \sin \theta + T_c \cos \theta} \right) \quad (7)$$

454 for  $\phi_0 \sim \phi_{\text{fl}} - \ln 2 + \tilde{\phi}_{\text{RF}} / \pi$ . This equation is also  
455 plotted in Fig.7 in black dashed line. Since experi-  
456 ment and model are in good agreement, the hypoth-  
457 esis of unmagnetized ions is verified, i.e. ions un-  
458 dergo to one or more collisions during a cyclotron  
459 period ( $v_{iN} > \omega_{ci}$ ). In addition to that, if ions were  
460 magnetized, the third term in Eq.(6) would vanish  
461 together with the angle dependence and the model  
462 would not be able to reproduce the trend of the mea-  
463 surements. Moreover, this suggests that the mea-  
464 surements done in magnetized and RF conditions  
465 seem reliable and exploitable, which is encourag-  
466 ing. This result also highlights the fact that for a  
467 parallel antenna, the “active” surface is rather small,  
468 and thus, the self-biasing of the facing plasma is  
469 also small. On the opposite, as the antenna is tilted,  
470 the “active” surface of the antenna increases and  
471 thus the self bias becomes significant<sup>29</sup>.

472 Finally, the spacial shape of the plasma poten-  
473 tial structure appearing in figures 5(a) to 5(h), i.e.  
474  $\phi_p(y)$ , has a more complicated structure to under-  
475 stand and is coupled with the density structure.  
476 That is why the next section is devoted to this study,  
477 using a fluid model.

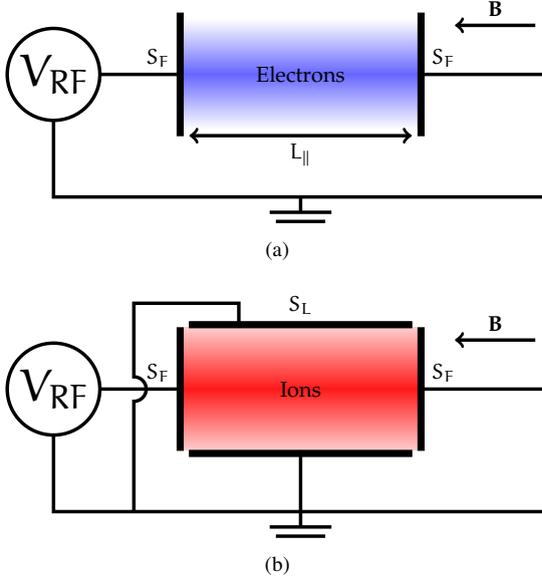


FIG. 8. Sketch of the used model. (a) Electron collection area and (b) Ion collection area.

#### 478 B. Plasma density

479 This subsection refers to figures 9(a) to 9(h).  
 480 The spatial structure of the plasma density shows a  
 481 plasma density depletion in the magnetic flux tube  
 482 connected to the electrode. This was already visible  
 483 on the raw  $I(V)$  in Fig.2 where the collected  
 484 current is lower inside than outside the electrode  
 485 flux tube. This is due in part by transverse RF  
 486 currents<sup>57</sup> and by DC transverse currents<sup>43</sup>. The  
 487 biased flux tube being positive compared to the sur-  
 488 rounding plasma potential, ion current tend to leave  
 489 the biased structure, while electrons are longitudi-  
 490 nally expelled from the flux tube to maintain the  
 491 quasi-neutrality. This mechanism explains the cen-  
 492 tral plasma depletion and why the border of the bi-  
 493 ased channel is highlighted by a peak of density.  
 494 The next section will provide a model to understand  
 495 this observation properly.

496 Density structures also generate currents, called  
 497 diffusion currents density  $\mathbf{J}_\perp \propto -qnD_\perp \nabla_\perp n$ ,  
 498 which in turn leads to electron drift around mag-  
 499 netic field lines (ions do not drift because they are  
 500 un-magnetized). This is commonly known as the  
 501 electron diamagnetic current density<sup>6</sup>,

$$\mathbf{J}_{De} = \frac{\mathbf{B} \times \nabla p}{B^2} \quad (8)$$

502 where  $p = nk_B T_e$  the electron pressure in the ap-  
 503 proximation of a perfect gas.

504 At low magnetic field, Fig.9(g), the lower con-  
 505 finement allows to fill in the depleted channel at  
 506 higher rate. That is, there is almost no clear dif-  
 507 ference between the inside and the outside in terms

508 of density number except at the edge of the biased  
 509 flux tube, where transverse ion currents occur.

510 Finally, at low pressure, collisions are less impor-  
 511 tant and the confinement is enhanced. This is per-  
 512 fectly demonstrated by the mesh plot in Fig.9(h):  
 513 the inside of the channel depicts a density plateau  
 514 at  $5 \times 10^{15} \text{ m}^{-3}$  whereas the bulk plasma rather be  
 515 at  $12 \times 10^{15} \text{ m}^{-3}$  and the transition is done abruptly  
 516 within 5 mm across magnetic field lines. In this  
 517 regime the transverse currents are the most impor-  
 518 tant.

#### 519 C. Floating potential

520 This subsection refers to the plots 10(a) and  
 521 10(b). Unfortunately, the floating potential is the  
 522 consequence of current balance on the probe ( $I_e =$   
 523  $|I_i|$ ) and depends implicitly on the collection sur-  
 524 faces, but since electrons and ions do not reach  
 525 the probe in the same way (effective collection sur-  
 526 faces are different<sup>22,23,32</sup>) it is more complicated to  
 527 evaluate on which part of the probe the collection  
 528 is done. Moreover, the floating potential depends  
 529 more on transverse electron fluxes than others: un-  
 530 magnetized ion flux collected by all the probe area  
 531 are low due to low ion velocity and the longitudinal  
 532 electron flux magnetically connected to the probe  
 533 is only collected on the front area of the probe de-  
 534 pending on the probe radius and electron Larmor ra-  
 535 dius which are very small as well ( $\pi r_p^2 \ll 2\pi L_p r_p$ ).  
 536 Outside the channel the floating potential stays  
 537 homogeneous, while inside, near the electrode, the  
 538 floating potential increases, and then decreases with  
 539  $z$ .

540 Nevertheless, since floating potential is a result  
 541 of all other plasma parameters, because it describes  
 542  $|I_e| = I_i$  at the probe: it depends on  $n$ ,  $T_e$  and  $\phi_p$   
 543 and especially on perpendicular fluxes, as explained  
 544 above. Hence, it is more consistent to work in a  
 545  $(x, y)$  plane, perpendicular to magnetic field lines.  
 546 The purpose of that kind of plot is to track the trans-  
 547 verse currents.

548 To access to the transverse currents densities ( $\perp$   
 549  $\mathbf{B}$ ), a 2-D scan is then performed in the  $(x, y)$  plane  
 550 at 20 W, 1.5 Pa and 94 mT (with these conditions  
 551 the magnetic channel is almost perfectly confined –  
 552 as discussed in previous subsections). This scan is a  
 553 result of over 1800  $I(V)$ , in front of the electrode, at  
 554  $z = 15 \text{ cm}$  (far enough from the electrode so that it  
 555 does not perturb the  $V_{fl}$  value). The same routine is  
 556 adopted to compute all plasma parameters, but only  
 557 the floating potential is plotted in Fig.11(b).

558 According to equation (8), knowing plasma den-  
 559 sity and average temperature ( $\sim 5 \text{ eV}$ ), one is able  
 560 to compute the electron diamagnetic current density

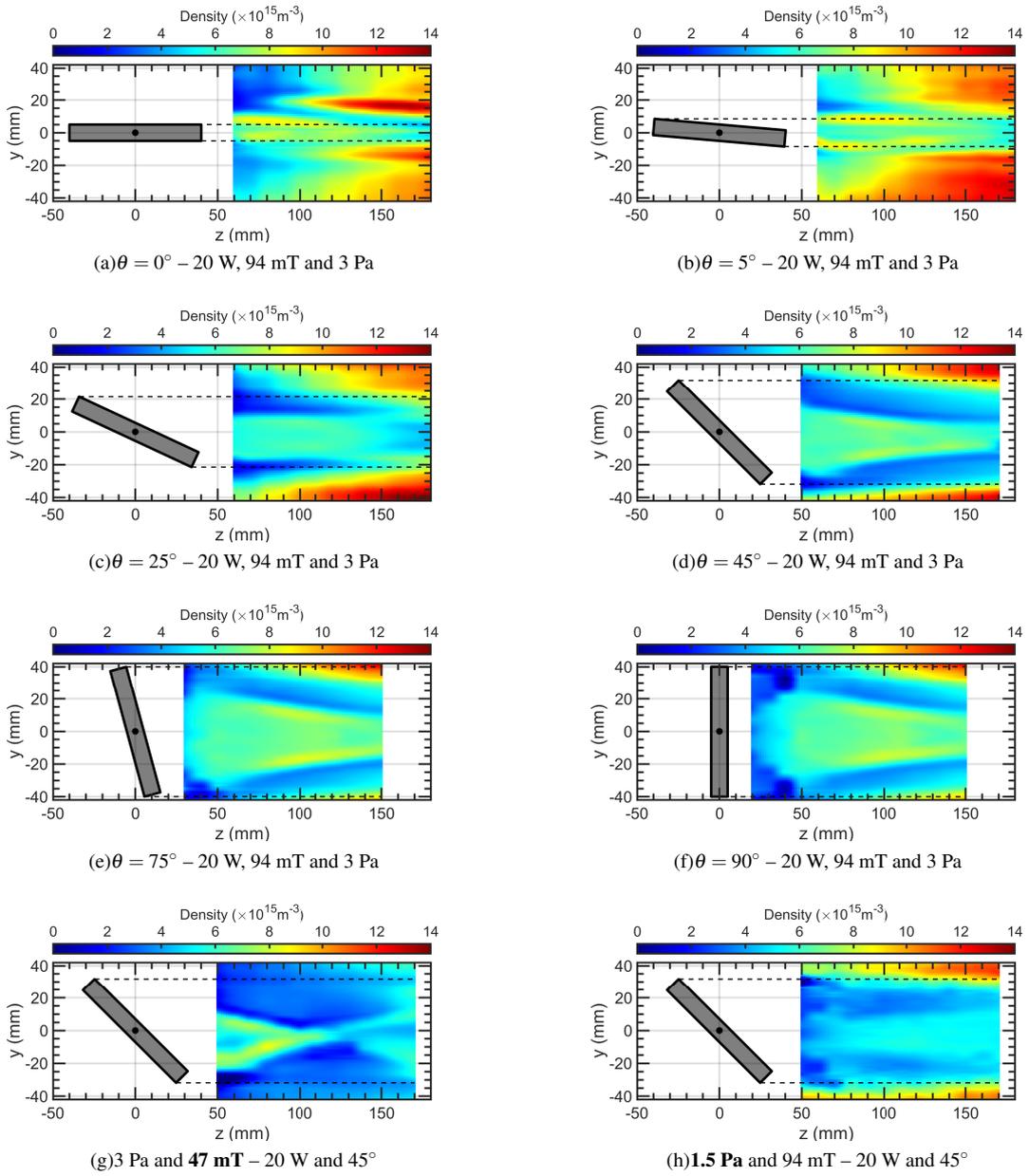


FIG. 9. Plasma density contour plots in front of the RF antenna, for  $\nu_{\text{RF}} = 25 \text{ MHz}$ . Conditions are labelled below each plot. For (a) to (f): 1 discharge, all inclination. For (h) & (g) comparisons at  $45^\circ$  for low field and low pressure condition respectively. (a)  $\theta = 0^\circ - 20 \text{ W, } 94 \text{ mT and } 3 \text{ Pa}$ , (b)  $\theta = 5^\circ - 20 \text{ W, } 94 \text{ mT and } 3 \text{ Pa}$ , (c)  $\theta = 25^\circ - 20 \text{ W, } 94 \text{ mT and } 3 \text{ Pa}$ , (d)  $\theta = 45^\circ - 20 \text{ W, } 94 \text{ mT and } 3 \text{ Pa}$ , (e)  $\theta = 75^\circ - 20 \text{ W, } 94 \text{ mT and } 3 \text{ Pa}$ , (f)  $\theta = 90^\circ - 20 \text{ W, } 94 \text{ mT and } 3 \text{ Pa}$ , (g)  $\theta = 45^\circ - 20 \text{ W, } 47 \text{ mT and } 3 \text{ Pa}$  and (h)  $\theta = 45^\circ - 20 \text{ W, } 94 \text{ mT and } 1.5 \text{ Pa}$ .

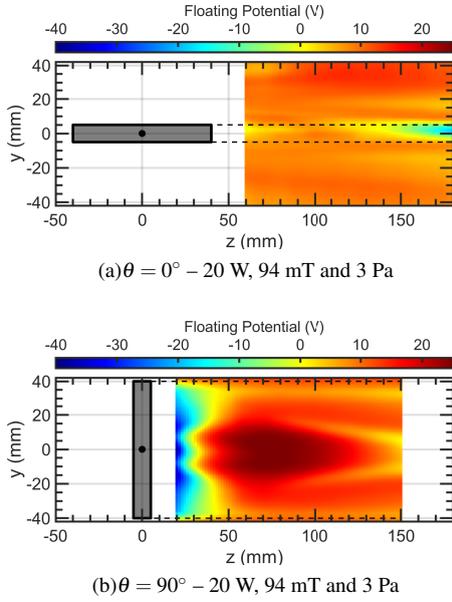


FIG. 10. Floating potential contour plots in front of the RF antenna, for  $\nu_{RF} = 25$  MHz, of aligned and normal electrode with magnetic field lines. (a)  $\theta = 0^\circ - 20$  W, 94 mT and 3 Pa, (b)  $\theta = 90^\circ - 20$  W, 94 mT and 3 Pa.

561 field,

$$\mathbf{J}_{De} = \frac{k_B T_e}{B} \mathbf{e}_z \times \nabla n, \quad (9)$$

562 but also the drift current density,

$$\mathbf{J}_D = en \frac{\mathbf{e}_z}{B} \times \nabla \phi_p. \quad (10)$$

563 The sum of all transverse current density field,  $\mathbf{J}_\perp =$   
 564  $\mathbf{J}_D + \mathbf{J}_{De}$ , is plotted on the measured floating poten-  
 565 tial map (arrows on Fig.11(b)). The amplitude  
 566 of the diamagnetic drift velocity,  $u_{De} = J_{De}/en$  is  
 567 within the range of 10 to 50 km/s (for recall,  $u_{e\parallel} \simeq$   
 568  $\langle v_e \rangle \sim 10^6$  m/s is still much larger). One can see  
 569 that the stream lines follows the  $V_{fl}$  structure, and  
 570 that electrons streams around the antenna's channel  
 571 anticlockwise. One can also notice that the electron  
 572 transverse flux drifts around the magnetic channel  
 573 connected to the antenna, and that there is no net  
 574 perpendicular flux contributing to a depleting or re-  
 575 fueling of the core plasma connected to the antenna.  
 576 The assumption  $\mathbf{J}_e = \mathbf{J}_{e\parallel}$  is verified.

577 Finally, in order to compute the probe floating  
 578 potential using only measured quantities, we as-  
 579 sume unmagnetized cold ions which are collected  
 580 by the whole probe and magnetized and maxwellian  
 581 electrons which are collected by only a small frac-  
 582 tion of the probe, that we assume to represent ap-  
 583 proximately 10% of the probe<sup>22,23</sup>, which is actu-  
 584 ally the effective probe collecting area for the par-  
 585 allel flux at such magnetic field magnitudes (50-  
 586 100 mT). In addition to those magnetized electrons

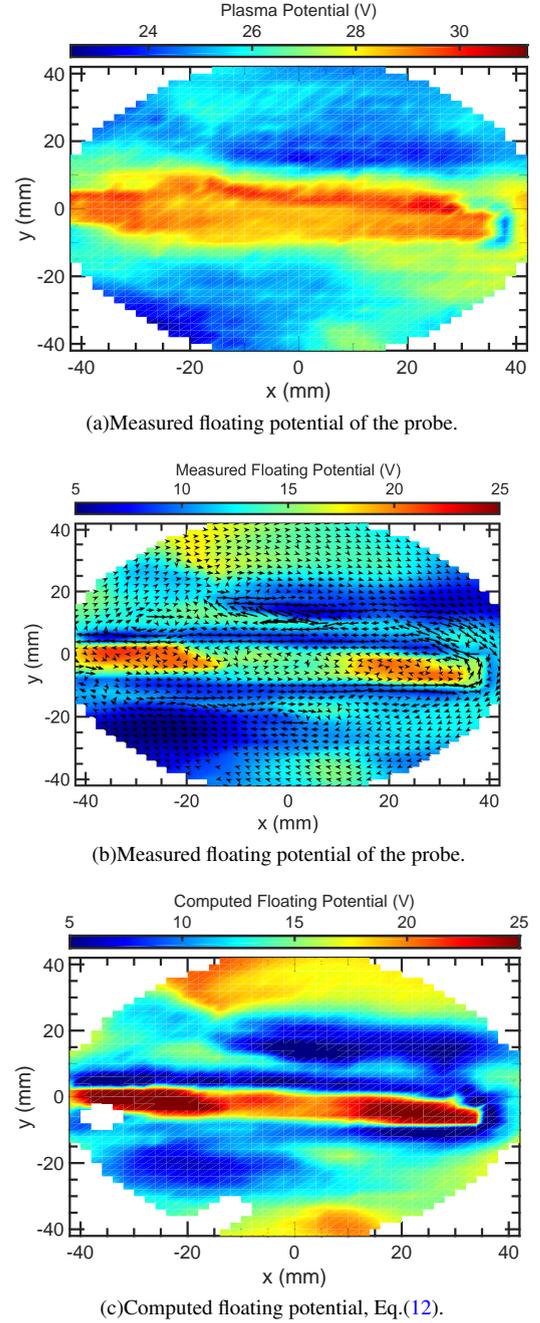


FIG. 11. Plasma and floating potential contour plots in front of the RF antenna. The electrode has its center at (0,0) and is 1 cm thick and 8 cm diameter, with tilt angle equal to  $0^\circ$ . The magnetic field points towards the reader. Conditions: 1.5 Pa, 94 mT,  $0^\circ$ ,  $z = 15$  cm. (a) Measured plasma potential, (b) Measured floating potential of the probe. The electron transverse current density field is plotted, (c) Reconstruction of the probe floating potential according to our model in Eq.(12). White regions are location where imaginary numbers were returned by the calculation.

587 we must add the transverse electron current density  
588 which we assume to be collected by only a portion  
589  $\kappa$  of the probe. Therefore, floating condition im-  
590 plies:

$$J_{i,\text{sat}}S_p = \left( \frac{J_{e,\text{sat}}}{10} e^{e(V_{\text{fl}} - \phi_p)/k_B T_e} + \kappa J_{\perp} \right) S_p, \quad (11)$$

leading to the modified floating potential formula,

$$V_{\text{fl}} = \phi_p - \frac{k_B T_e}{e} \ln \left( \frac{J_{e,\text{sat}}}{10 J_{i,\text{sat}}} \right) + \frac{k_B T_e}{e} \ln \left( 1 - \frac{\kappa J_{\perp}}{J_{i,\text{sat}}} \right). \quad (12)$$

591 The result is plotted in figure 11(c), where we have  
592 taken  $\kappa = 0.25$ . For a directed flux,  $\kappa$  should have  
593 been equal to 0.5 (the flux reaches the probe fol-  
594 lowing only one direction). However, 0.25 gives a  
595 better fitting on the experimental curves, and can  
596 be due to the fact that the measured currents are not  
597 perfectly perpendicular: there is a parallel compo-  
598 nent which is not taken into account in the model  
599 (since the frontal surface is smaller than the lateral  
600 one).

601 The computed floating potential matches pretty  
602 well with the measured data. The comparison of  
603 the floating potential map with the plasma poten-  
604 tial one, Fig.11(a), shows that  $V_{\text{fl}}(x, y)$  is not only a  
605 shifted value of  $\phi_p(x, y)$  (as suggested by the ‘‘clas-  
606 sical’’ probe theory,  $\phi_p - V_{\text{fl}} \propto T_e$ ). This observation  
607 seems to indicate that (i) transverse currents do ex-  
608 ist (ii) transverse currents drive the evolution of the  
609 measured probe floating potential (iii) the measure-  
610 ments done using a cylindrical Langmuir probe are  
611 reliable.

#### 612 D. Summary of the observations

613 In this section we have qualitatively described  
614 the evolution of the several plasma parameters in  
615 front of a tilted electrode in magnetized condi-  
616 tions by the mean of numerous cylindrical Lang-  
617 muir probe measurements. The measurements evi-  
618 denced the existence of a magnetic flux tube con-  
619 nected to the electrode, and following magnetic  
620 field lines. This channel is different from the bulk  
621 plasma, because its potential and temperature are  
622 higher, whereas its density is lower. The larger po-  
623 tential is explained by self biasing of the plasma in  
624 front of a RF biased surface. And the density de-  
625 pletion is due to perpendicular currents that pushes  
626 out ions from the channel<sup>29,43</sup>, which is enhanced  
627 by RF transverse currents<sup>58</sup>. Along with this, the  
628 magnetic confinement is visibly improved when in-  
629 creasing the magnetic field, and when decreasing  
630 the pressure (leading to less collisions and less per-  
631 pendicular diffusion). However, 1.5 Pa would be a

632 good compromise to develop a fluid model based on  
633 the flux tube approximation because electrons and  
634 ions can both be seen as magnetized at low pres-  
635 sure.

636 The measurement allowed us to understand a bit  
637 more the behaviour of a biased and magnetized  
638 plasma channel in ALINE. First, a simple model  
639 was developed to prove that ions are not magnetized  
640 (using  $\phi_p(\theta)$ , Fig.7 and Eq.(6)). In addition, it was  
641 shown, by making a scan on a plane normal to the  
642 magnetic field lines, that electron diamagnetic cur-  
643 rents are important in the building of the structures  
644 in front of the electrode (figure 11(c)).

645 However, the physical models in this section are  
646 quite simple, and not able to explain the particular  
647 shape of the plasma potential and density structures.  
648 That is why a more complex model is required, as  
649 depicted in the next section.

#### 650 IV. FLUID MODEL

651 This quasy-neutral slab model is able to com-  
652 pute the potential and the density profile in front  
653 of the antenna,  $\phi(t, y)$  and  $n(y)$ , using the to-  
654 tal current density continuity and the ion flux  
655 conservation<sup>27,28,57</sup> with sheath boundary condi-  
656 tions. However, due the non-linearity of the equa-  
657 tions it is not possible to provide an analytical so-  
658 lution, therefore the resolution will be implemented  
659 numerically. The following model solve the evolu-  
660 tion of density  $n$  and the space potential  $\phi$  in the  $y$   
661 (or  $\perp$ ) direction.

662 First, we are going to solve the space potential  
663 profile in front of the electrode using the total cur-  
664 rent density conservation:

$$\nabla \cdot \mathbf{J}_{\text{Tot}} = \nabla \cdot (\mathbf{J}_e + \mathbf{J}_i) = 0 \quad (13)$$

665 The space potential solution of this equation will be  
666 used to solve the density profile using the conserva-  
667 tion of the steady state ion flux,

$$\nabla \cdot (n\mathbf{u}_i) = S \quad (14)$$

668 with  $S$  the ion source term. Solving both equa-  
669 tions at once is in general numerically unstable  
670 (because of an explicit scheme), therefore (13) is  
671 solved using a constant density profile as a guess:  
672 we therefore use an iterative method: inserting the  
673 obtained density profile from Eq.(14) into Eq.(13)  
674 again, and the newly obtained density profile is un-  
675 changed from the prior one, therefore one itera-  
676 tion is enough. The current density conservation,  
677 Eq.(13), depends both on space and time in order to  
678 solve the RF fluctuations. Its solution is then aver-  
679 aged over an RF period to be injected in the second  
680 equation, Eq.(14), which is steady state.

We are assuming a magnetic flux tube with per-  
fectly magnetized electrons (i.e.  $\mathbf{J}_{e\perp} = \mathbf{0}$ ), using

the same approach as described in Fig.8(a) and Fig.8(b). The channel is connected on the one side to the RF antenna, and on the other side to the grounded wall. The ions are weakly magnetized, and can move across magnetic field lines. The ions perpendicular current density is given by:

$$\begin{aligned} \mathbf{J}_{i\perp} &= \mathbf{J}_{\text{pol}} + \mathbf{J}_{\text{mob}} + \mathbf{J}_{\text{diff}} \\ &= \frac{Mn}{B^2} \frac{\partial \mathbf{E}}{\partial t} + en\mu_{i\perp} \mathbf{E} - eD_{i\perp} \nabla n \end{aligned} \quad (15)$$

681 which are respectively the currents densities of,  
682 polarization<sup>27</sup>, mobility and diffusion<sup>59</sup>.  $M$  is the  
683 ion mass,  $\mu_{i\perp}$  is the ion transverse mobility coefficient,  
684 and  $D_{i\perp}$  is the ion transverse diffusion coefficient.  
685

### 686 A. Solving the potential profile

First, we focus on solving the equation (13) to find a solution for the space potential  $\phi$  in front of the electrode. As we assume constant density in this first step of calculation, the ion diffusion current density is neglected. Integrating (13) along the magnetic flux tube of length  $L_{\parallel}$ , and assuming homogeneous currents densities along the flux tube, leads to:

$$\begin{aligned} 0 &= \int_0^{L_{\parallel}} \nabla \cdot \mathbf{J}_{\text{Tot}} dz = \int_0^{L_{\parallel}} \frac{\partial J_{\parallel}}{\partial z} + \nabla_{\perp} \cdot \mathbf{J}_{i\perp} dz \\ &= 2J_{i,\text{sat}} - J_{e,\text{sat}} (1 + e^{\phi_{\text{RF}}}) e^{-\phi} + L_{\parallel} \nabla_{\perp} \cdot \mathbf{J}_{i\perp} \end{aligned} \quad (16)$$

687 where potentials are normalized to  $e/k_B T_e$ . The parallel  
688 current density is only defined by the sheath  
689 boundary conditions, assuming the parallel electric  
690 field is equal to zero into the neutral plasma (no pre-  
691 sheath in this model<sup>47</sup>). Injecting now expression  
692 (15) into Eq.(16), one will get

$$1 - \frac{e^{\phi_{\text{RF}} - \phi}}{2} (1 + e^{\phi_{\text{RF}}}) = C_{\perp} \eta \rho_{ci} \frac{\partial \Delta \phi}{\partial t} + \beta \Lambda^2 \Delta_{\perp} \phi \quad (17)$$

693 where  $C_{\perp} = \epsilon_0 L_{\parallel} \omega_{pi}^2 / \omega_{ci}^2$  is the perpendicular capac-  
694 itance of the flux tube<sup>27</sup>,  $\eta = M \omega_{ci} / (2 \times$   
695  $0.61 n e^2)$  is the resistivity (which depends on  $n$ ) and  
696  $\Lambda^2 = L_{\parallel} D_{i\perp} / (2 \times 0.61 c_s)$  is the typical squared dif-  
697 fusion length. A modified Einstein's relation is used  
698 here,  $\mu \equiv \beta |q| D / k_B T_e$  to add a last degree of free-  
699 dom to the system ( $\beta = 1$  do not provide a good  
700 final result). In this equation we can see that the RF  
701 leads to a capacitive behaviour of the flux tube.

702 To be able to solve the problem, we assume a  
703 known profile shape for the RF potential  $\phi_{\text{RF}}(y, t) =$   
704  $\tilde{\phi}_{\text{RF}} f(y) \sin(\omega t)$ . A Gaussian profile of amplitude  
705  $\tilde{\phi}_{\text{RF}}$  is not adequate regarding to the potential struc-  
706 tures obtained experimentally (plotted in figures  
707 5(a) to 5(h)) which depict a plateau in front of the

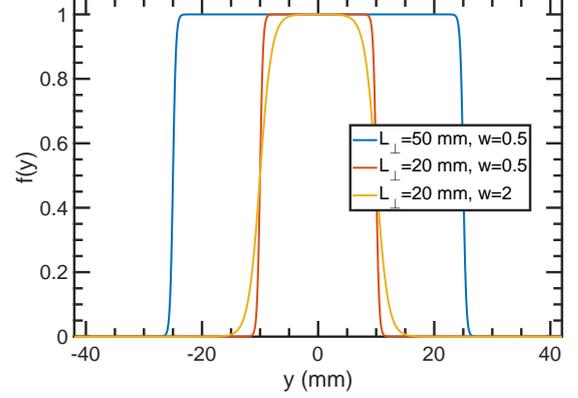


FIG. 12. Ad hoc spatial structure used for the initial RF potential.

708 antenna. The following profile is therefore chosen,  
709 being a compromise between a Gaussian and a gate  
710 function (avoiding discontinuities on the edge of a  
711 pure square shaped function):

$$f(y) = \frac{\tanh\left(\frac{y + L_{\perp}/2}{w}\right) - \tanh\left(\frac{y - L_{\perp}/2}{w}\right)}{2 \tanh(L_{\perp}/2w)} \quad (18)$$

712 where  $L_{\perp}$  is the diameter of the flux tube, and  $w$   
713 the typical potential gradient width of our structure  
714 at  $t = 0$ : when  $w \rightarrow 0$ ,  $f$  turns into a gate  
715 function, whereas  $f$  turns into a Gaussian as  $w$  increases.  
716 Some examples are plotted in Fig.12. Now, equation  
717 (17) can be solved numerically using finite el-  
718 ement methods (see appendix A). The resolution  
719 provides the time evolution potential profile in front  
720 of the cathode,  $\phi(y, t)$ .

### 721 B. Solving the steady state density profile

722 Now that we know  $\phi(y, t)$  it is possible to com-  
723 pute  $n(y)$ . However, to simplify the problem, we do  
724 consider the steady state of the density profile (i.e.  
725  $\partial_t n = 0$ ). We define the RF currents densities as,

$$J_{\text{RF}} = -\frac{L_{\parallel}}{T_{\text{RF}}} \int_0^{T_{\text{RF}}} \nabla_{\perp} \cdot \mathbf{J}_{i\perp} dt \quad (19)$$

726 from equation (16), averaged over one RF period.  
727 Therefore, inserting this into the ion flux conserva-  
728 tion equation (14) but this time taking into account  
729 the diffusion flux, one will get

$$S = \frac{\partial \Gamma_{i\parallel}}{\partial z} - \frac{J_{\text{RF}}}{e L_{\parallel}} - D_{i\perp} \Delta n \quad (20)$$

730 After integration along the flux tube of length  $L_{\parallel}$ ,  
731 the first term of the r.h.s. is a loss term equal to

732 the outwards ion flux,  $2\alpha n c_s$ , while l.h.s.,  $S$ , is the  
 733 source term playing the role of a constant ionization  
 734 rate in order to maintain the density. Then,  $S L_{\parallel}$  can  
 735 be written as a constant saturation current density,  
 736  $2\alpha n_0 c_s$ . Finally  $n(y)$  can be computed from follow-  
 737 ing equation by matrix method: (see appendix B):

$$\Delta_{\perp} n - \frac{n}{\Lambda^2} = -\frac{J_{\text{RF}}}{e D_{i\perp} L_{\parallel}} - \frac{n_0}{\Lambda^2}. \quad (21)$$

738 with  $\Lambda^2$  still equals to  $L_{\parallel} D_{i\perp} / 2\alpha c_s$ , the typical dif-  
 739 fusion length squared.

### 740 C. Code results and discussions

741 The free parameters in our model are temperature  
 742 (which is assumed constant)  $T_e$ , “bulk density”  $n_0$ ,  
 743 transverse diffusion  $D_{i\perp}$ , transverse mobility  $\mu_{i\perp}$   
 744 through the  $\beta$  coefficient, the parallel scale  $L_{\parallel}$ , the  
 745 perpendicular scale  $L_{\perp}$  (linked to  $\theta$ ), the RF am-  
 746 plitude  $V_{\text{RF}}$  and the wheelbase  $w$  (i.e. the size of  
 747 the transverse gradients). Those seven parameters  
 748 are adjusted by error/test method to get a density  
 749 structure close to the experimental measurements.  
 750 However, there are numerous ways to fit the mea-  
 751 surements due to the seven degrees of freedom of-  
 752 fered by our model. Therefore, the exploitation of  
 753 the code output can only be done qualitatively.

754 The code is used to fit the experimental density  
 755 profiles  $n(y)$  at  $z = 15$  cm and  $x = 0$ , at 1.5 Pa and  
 756 94 mT (best magnetized conditions). Only the poten-  
 757 tial structure at  $\theta = 0^\circ$  is presented here, because  
 758 the potential shape is only enlarged when increas-  
 759 ing the tilting angle. As pointed out in a precedent  
 760 work<sup>47</sup>, the space potential evolution can be decom-  
 761 posed into 3 terms,  $\phi = \bar{\phi} + \phi_t + \tilde{\phi}$  (see Fig.13(a)),  
 762 a stationary term  $\bar{\phi}$ , a transient term  $\phi_t$  which tends  
 763 to 0 as  $t \rightarrow \infty$ , and an oscillatory term  $\tilde{\phi}$ . This  
 764 time evolution is the signature of the capacitive be-  
 765 haviour of the biased flux tube exchanging RF dis-  
 766 placement current with the bulk plasma. As the RF  
 767 potential increases, electrons follows the trend due  
 768 to their high mobility. Whereas ions inertia causes  
 769 the slow decay where the RF potential decreases.

770 For an aligned electrode ( $\theta = 0^\circ$ ), a total perpen-  
 771 dicular capacitance of  $C_{\perp} = 334$  pF is found. This  
 772 gives a transverse sheath capacitance by unit sur-  
 773 face of  $3.71$  pF/cm<sup>2</sup> for  $n_0 = 10^{16}$  m<sup>-3</sup>, which is  
 774 in good agreement with those calculated by Chen<sup>60</sup>.  
 775 The input parameters for  $L_{\perp}$  is 20 mm, which takes  
 776 into account the thickness of the antenna (10 mm),  
 777 plus a ion Larmor radius extension in the direction  
 778 perpendicular to  $\mathbf{B}$  at the electrode surface ( $\rho_{ci} \approx 5$   
 779 mm at  $T_e = 5$  eV), which coincides to Chodura’s<sup>8</sup>  
 780 magnetic pre-sheath. The parallel scale  $L_{\parallel}$  is 50  
 781 mm, which is not the length antenna – wall, but half  
 782 of the ion mean free path instead ( $v_{iN} = 88$  kHz in

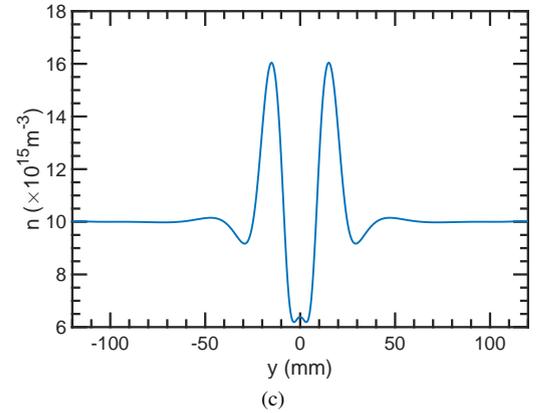
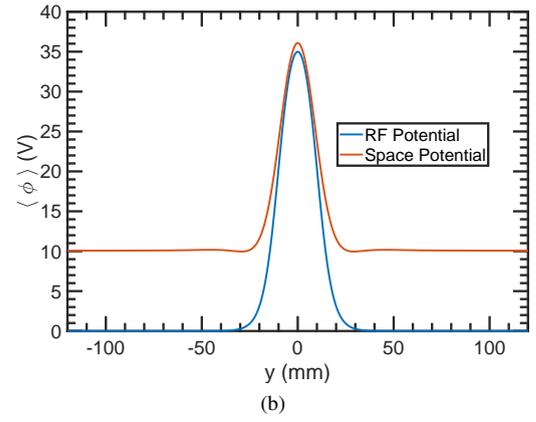
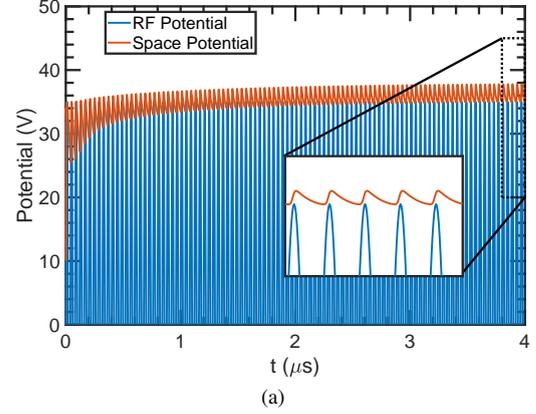


FIG. 13. Code result for the space potential and the density profile in front of the electrode align with magnetic field lines (i.e.  $\theta = 0^\circ$ ). (a) Time evolution of the center of the structure,  $\phi(y=0, t)$ , (b) Averaged space potential over the last five periods from Fig.13(a),  $\langle \phi \rangle_t(y)$  and (c) Resulting density profile  $n(y)$ .

783 our conditions). The input diffusion coefficient is  
 784 acceptable ( $5 \times 10^{-2}$  m<sup>2</sup>/s) and the mobility coef-  
 785 ficient is  $\mu_{i\perp} = 0.36$  m<sup>2</sup>/Vs, thus the parallel mo-  
 786 bility is  $\mu_{i\parallel}^{\text{code}} = \mu_{i\perp} (1 + (\omega_{ci}/v_{iN})^2) = 268$  m<sup>2</sup>/Vs.  
 787 This can be compared with the theoretical mobili-  
 788 ty  $\mu = e/Mv_{iN} \approx 272$  m<sup>2</sup>/Vs. Therefore, the coef-  
 789 ficient provided as input parameters at  $\theta = 0^\circ$  are

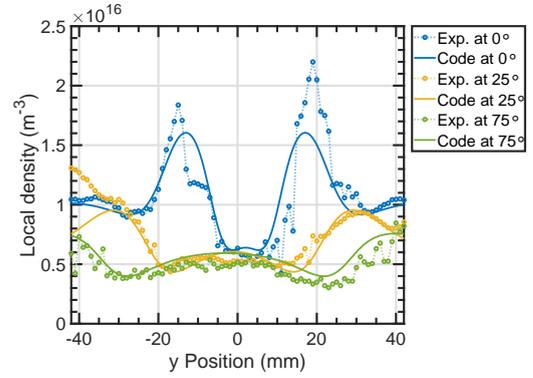
790 acceptable in a physical point of view.

791 The space potential given by the code is a 2-by-2  
792 dimensional matrix for space and time. To compute  
793 the RF transverse current density (using Eq.(19)),  
794 the spacial potential structure is averaged over the  
795 last five RF periods of the simulation. This poten-  
796 tial profile, figure 13(b) is in good agreement with  
797 those obtained experimentally at 15 cm from the  
798 electrode (see figure 5(h)). Then, the density profile  
799 can be calculated in turn by solving equation (20)  
800 numerically. The result is plotted in figure 13(c).  
801 This typical “bunny ears” shape is recovered in the  
802 measurements and is due to transverse currents den-  
803 sities driven by the RF potential structure (ions are  
804 accelerated at the edge of the tube due to the trans-  
805 verse  $\mathbf{E}$  field), bringing density on the outer edge of  
806 the flux tube, and pumping out the inside of the flux  
807 tube.

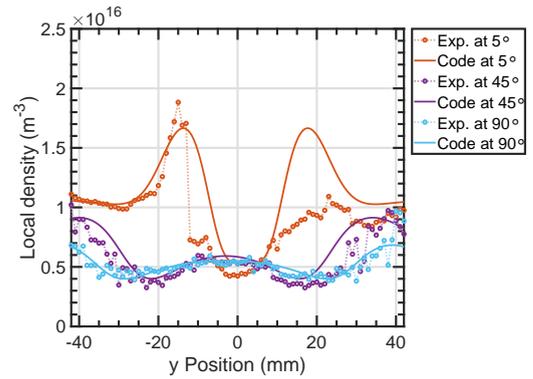
808 Finally, figure 14(a) and 14(b) compares the den-  
809 sity profiles obtained numerically with those ob-  
810 tained by the experiment. Note that for  $\theta = 5^\circ$   
811 and  $25^\circ$ , the negative  $y$  portion of the electrode is  
812 much closer to the measuring point than the posi-  
813 tive  $y$  portion: this geometrical aspect coupled with  
814 transverse diffusion over a longer path is the conse-  
815 quence of the measured asymmetry of the density  
816 profiles at those angles (i.e.  $n(y < 0) > n(y > 0)$ ).  
817 Since our model does not take into account any lon-  
818 gitudinal dynamics, the computed density profile is  
819 perfectly symmetric. Therefore, the higher “ear” is  
820 used as reference for the fitting of the curves

821 The input coefficients at  $0^\circ$  are discussed above.  
822 But as the angle is increased, the coefficients used  
823 are diverging from the physical ones. On the one  
824 hand, diffusion is globally unchanged and stays in  
825 a range between  $0.05$  to  $0.1 \text{ m}^2/\text{s}$ , with a maximum  
826 at  $45^\circ$ . On the other hand, the mobility increases  
827 up to  $\sim 10 \times \mu_{i\parallel}^{\text{theo}}$  at  $45^\circ$  and decreases back to a  
828 smaller value of  $\sim 5 \times \mu_{i\parallel}^{\text{theo}}$  at  $90^\circ$ . However, the  
829 ratio  $\mu_{i\perp}/D_{i\perp}$  increases linearly from  $0$  to  $90^\circ$ . This  
830 growth has a physical meaning though: for small  
831 angles, the lateral surface of the flux tube (which  
832 communicates with the bulk plasma) is smaller,  
833 therefore transverse currents are also smaller. How-  
834 ever, for grazing angles ( $0$  and  $5$  degrees), the input  
835 coefficients are correct in a physical point of view,  
836 and these inclinations matter the most for the fusion  
837 community.

838 The “bunny ears” have a higher amplitude at  
839 small angles. Indeed, the perpendicular currents  
840 are mostly driven by density and potential gradi-  
841 ents, and those gradients are more important in a  
842 narrow flux tube configuration. Just in front of the  
843 electrode, there is a density depletion with respect  
844 to the bulk plasma. Whereas, on the edge of the  
845 flux tube, the potential gradient leads to a density  
846 oversupply, generating these “bunny ears shaped  
847 curves”. Along with this observation, the narrower



(a)



(b)

FIG. 14. Comparison between experiment and code out-  
put for the density profiles. The experimental data are  
taken for 94 mT, 1.5 Pa and 15 cm from antenna at (a)  
0, 25 and 75 degrees – (b) 5, 45 and 90 degrees. The  
gradient length  $w$  is found to be of the order of 1 cm.

848 the flux tube, the more it pumps out particles: the  
849 density inside the tube is much lower than in the  
850 bulk plasma for small angles. Nevertheless, ex-  
851 periment and code are in good agreement, and it  
852 seems obvious that the density profile in a plane  
853 perpendicular to  $\mathbf{B}$ , in front of an RF antenna, is  
854 highly perturbed by transverse currents (which are  
855 enhanced by the RF).

## 856 V. CONCLUSION

857 The behaviour of a magnetized plasma in front  
858 of a RF electrode (or antenna) is one of major in-  
859 terests for the plasma community. A contribution  
860 to this question is provided here, exploiting Lang-  
861 muir probe measurements inside the linear plasma  
862 device ALINE. The probe measurements were per-  
863 formed in several discharge conditions (47 and 94  
864 mT, 1.5 and 3 Pa in He) and 2-D contour plots of  
865 the plasma parameters in the front of a tilted an-  
866 tenna with respect to the magnetic field lines were  
867 established. The main results from the observation

868 of these maps are:

- 869 • The plasma potential varies strongly in the  
870 direction perpendicular to the magnetic field  
871 and remains almost constant over several  
872 centimetres along the magnetic field at pres-  
873 sure of the order of 2 Pa. The plasma poten-  
874 tial is much higher in front of the RF elec-  
875 trode except at low tilting angle (less than  
876  $\approx 20^\circ$ ) for which the plasma biasing due to  
877 sheath rectification is very low.
- 878 • The density maps exhibit a depleted chan-  
879 nel in the central core of the biased flux tube  
880 (connected to the antenna). This depletion  
881 occurs over few centimetres, the typical scale  
882 length of transverse currents. For larger flux  
883 tube (i.e. greater tilting angle), the depletion  
884 appears only at the inner edge while a density  
885 peak grows in the outer edge. In that case the  
886 central region is not disturbed.
- 887 • The floating potential maps seem to be very  
888 dependant on convective electron fluxes and  
889 can be negative even inside the biased flux  
890 tube.

891 To understand the plasma potential amplitude in  
892 front of the antenna as a function of the tilting an-  
893 gle, a model based on current conservation over dif-  
894 ferent surfaces collecting the current all around the  
895 biased flux tube has been developed. At low an-  
896 gle, the active electrode area is almost parallel to  
897 the magnetic field so that classical sheath can not  
898 operate, and then the self-biasing of the connected  
899 plasma is low. At larger angle (over  $\approx 20^\circ$ ) the self-  
900 biasing rises up to reach a fraction of the applied  
901 RF potential. The experiment and the model are  
902 in good agreement, and it was also shown that the  
903 plasma potential increases when the magnetic flux  
904 tube is wider than the typical transverse scale length  
905 of ion currents which deplete the core of the biased  
906 channel.

907 In addition, another model taking into account  
908 the computed electron currents received by the  
909 probe from the measured density and plasma poten-  
910 tial maps (to compute drift current and diamagnetic  
911 currents) allows to recover very nicely the mea-  
912 sured floating potential, proving that this model is  
913 mainly dependant on the electron currents in a plane  
914 perpendicular to the magnetic field.

915 Finally, a numerical fluid model was developed  
916 to understand density maps. Using total current  
917 density conservation and ion flux conservation in-  
918 side a homogeneous flux tube connected to the elec-  
919 trode, the code input parameters (such as mobil-  
920 ity and diffusion coefficients) are in good agree-  
921 ment with the literature for small tilting angles. The  
922 codes outputs allow us to fit the measured density

923 profiles in front of the electrode. Both the numeri-  
924 cal result and the experiment display “bunny ears”  
925 shaped curves for the density profile. The explana-  
926 tion of this shape is done using transverse ion fluxes  
927 (polarization, mobility and diffusion). Inside the  
928 flux channel connected to the probe, the density is  
929 below the bulk plasma density: the transverse cur-  
930 rents deplete the tube, and this depletion is more  
931 important for narrow flux tubes. Whereas, on the  
932 edge of the tube, there is a density oversupply due  
933 to strong potential gradients (i.e. strong  $\mathbf{E}$  field).  
934 This behaviour is enhanced with the RF and lead to  
935 a capacitive behaviour of the RF sheath and of the  
936 space potential structure inside the magnetic chan-  
937 nel.

938 This study provides two main contributions re-  
939 garding the heat fluxes and the triggering of hot  
940 spots on RF antennae.

941 The first is the plasma potential mapping mea-  
942 sured in front of a tilted antenna. These maps have  
943 shown that the RF plasma potential magnetically  
944 connected to the antenna is much lower for small  
945 antennae (or grazing tilt angle with respect to mag-  
946 netic direction), because the connected active area  
947 is simply smaller.

948 The second contribution is the density mapping  
949 and especially the density profiles in the direction  
950 perpendicular to the magnetic field. They show that  
951 the core density, *i.e.* in the central part of the con-  
952 nected flux tube, is strongly depleted compared to  
953 the outer plasma when the flux tube width is smaller  
954 than 1 or 2 cm (which is the order of magnitude  
955 of the ion Larmor radius or the typical RF sheath  
956 width if  $\omega_{pi} > \omega_{ci}$ ). However, for larger channel,  
957 the density is only depleted over the first radial 1 or  
958 2 centimetres at the inner edge of the magnetic flux  
959 tube while the core density is almost undisturbed.

960 This means that a RF antenna smaller than the  
961 typical ion Larmor radius would drain the con-  
962 nected plasma more efficiently. Nevertheless, what-  
963 ever the (active) size of the antenna, it appears that  
964 at the outer edge of the magnetic flux tube there is  
965 an overshoot density structure, which can increase  
966 the heat flux and may induce hot spots, especially  
967 at the edge of the antenna. This has already been  
968 observed on antenna structures in Tokamaks.

969 Finally, concerning the reactive plasma experi-  
970 ments, one can say that the shape of the density  
971 profiles reveals that a homogeneous etching on a  
972 RF biased substrate cannot be achieved unless its  
973 typical width is much larger than a Larmor radius  
974 plus a RF sheath width.

975 The future work will consist in measuring the an-  
976 tenna structure heating with an IR camera to deduce  
977 the heat flux and then compare it to the one calcu-  
978 lated from our potential and density maps.

## 979 ACKNOWLEDGEMENTS

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986 views and opinions expressed herein do not neces-  
987 sarily reflect those of the European Commission.

## 988 DATA AVAILABILITY

989 All datasets underlying the conclusions of the  
990 paper should be available to readers. We encour-  
991 age authors to deposit their datasets in publicly  
992 available repositories (where available and appro-  
993 priate) or present them in the main manuscript.  
994 All research articles must include a data avail-  
995 ability statement informing where the data can be  
996 found. By data we mean the minimal dataset  
997 that would be necessary to interpret, replicate and  
998 build upon the findings reported in the article.  
999 Dataset can be found at [Mendeley Data, DOI :](https://www.mendeley.com/datasets/j7rm7tcrr8/1)  
1000 [10.17632/j7rm7tcrr8.1](https://www.mendeley.com/datasets/j7rm7tcrr8/1).

## 1001 Appendix A: Numeric scheme of solving the 1002 potential structure

In this section is provided the numerical scheme  
for the resolution of the potential profile (see sec-  
tion IV A). The partial differential equation (17)  
needs to be solved numerically. The initial condi-  
tion of  $\varphi(y, t = 0)$  is determined when no perpen-  
dicular currents densities are present, i.e. the r.h.s.  
of Eq.(17) is zero, which gives  $\varphi(y, 0) = \varphi_n$ . Now,  
equation (17) is discretised in time assuming that  
 $\partial_t \Delta \varphi(y, t) = [\Delta \varphi(y, t + \delta t) - \Delta \varphi(y, t)] / \delta t$ , leading  
to the expression:

$$\Delta_{\perp} \varphi_y^{t+\delta t} = [1 - \beta v_{\perp} \delta t] \Delta \varphi_y^t + \frac{v_{\perp} \delta t}{\Lambda^2} \left[ 1 - \frac{e^{\varphi_n - \varphi_y^t}}{2} \left( 1 + e^{\varphi_{RF}(y, t)} \right) \right] \quad (\text{A1})$$

1003 where  $v_{\perp} = \Lambda^2 / C_{\perp} \eta \rho_{ci} = D_{i\perp} / \rho_{ci}^2$  is the charac-  
1004 teristic frequency of the equation. For time step,  $t = 0$ ,  
1005 the r.h.s. of this equation is known, and the spatial  
1006 distribution of  $\varphi$  at time step  $t = \delta t$  can be easily  
1007 found by the mean of finite element method<sup>61</sup>. Let

$$\Delta_{\perp} \varphi_y^{t+\delta t} = \frac{\varphi_y^{t+\delta t} - 2\varphi_y^{t+\delta t} + \varphi_{y-\delta y}^{t+\delta t}}{\delta y^2}. \quad (\text{A2})$$

1008 Thus, equation (A1) can be rewritten into a sim-  
1009 ple vectorial form,

$$\bar{\bar{A}} \cdot \vec{\varphi}^{t+\delta t} = \delta y^2 \mathbf{F}(\vec{\varphi}^t, t, y), \quad (\text{A3})$$

1010 where  $\vec{\varphi}$  is the vector of solutions,  $\bar{\bar{A}}$  is a tridiagonal  
1011 matrix, and  $\mathbf{F}$  the r.h.s. of Eq.(A1), depending  
1012 of the potential structure at previous time step, posi-  
1013 tion, and time. The elements of the matrix are  $A_{i,j} =$   
1014  $\delta_{i-1,j} - 2\delta_{i,j} + \delta_{i,j-1}$  (here  $\delta_{i,j} = 1$  if  $i = j$  and else,  
1015  $\delta_{i,j} = 0$  is the Kronecker symbol). Dirichlet bound-  
1016 ary conditions are used,  $\varphi_{-\delta y}^{t+\delta t} = \varphi_{(N+1)\delta y}^{t+\delta t} = \varphi_n$ . In-  
1017 verting the matrix gives the potential structure at the  
1018 next time step,  $\varphi(y, t + \delta t)$ . This new value of  $\varphi$  is  
1019 in turn injected in the expression of  $\mathbf{F}$  to determine  
1020 the potential structure at  $t + 2\delta t$  by solving Eq.(A3)  
1021 again, and so on.

## 1022 Appendix B: Numeric scheme of solving the 1023 density structure

1024 The numerical scheme for solving the density  
1025 profile (see section IV B) is described in the follow-  
1026 ing:

1027 Equation (21) can also be solved numerically by  
1028 the mean of a finite element difference approach  
1029 and a matrix inversion. Using the same scheme as  
1030 in Eq.(A2), the numerical problem rewrites:

$$\bar{\bar{K}} \cdot \vec{n} = \delta y^2 \mathbf{G}(y) \quad (\text{B1})$$

1031 Where  $\vec{n}$  is the density profile solution,  $\bar{\bar{K}}$  is a tridi-  
1032 agonal matrix, whose elements are  $K_{i,j} = \delta_{i-1,j} -$   
1033  $[2 + \delta y^2 / \Lambda^2] \delta_{i,j} + \delta_{i,j-1}$ , and  $\mathbf{G}$  is the r.h.s. of the  
1034 density equation (21). Again, Dirichlet boundary  
1035 conditions are used,  $n_{-\delta y} = n_{(N+1)\delta y} = n_0$ .

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