Estimation of the number of factors in a multi-factorial Heath-Jarrow-Morton model in electricity markets

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Abstract

In this paper we study the calibration of specific multi-factorial Heath-Jarrow-Morton models to electricity market prices, with a focus on the estimation of the optimal number of factors. We describe a common statistical procedure based on likelihood maximisation and Akaike / Bayesian information criteria, in the case of calibration on futures prices, as well as on both spot and futures prices. We perform a detailed analysis on 6 European markets: Belgium, France, Germany, Italy, Switzerland and UK. The results show a lot of similarities on all the markets considered, especially on the optimal number of factors equal to 5; and on the behaviour of the different factors.

Keywords— Electricity markets, model calibration, model selection, power price model

1 Introduction

Electricity production and furniture have been widely liberalised in a wide set of countries over the last decades. Although their precise organisation varies across places, electricity markets share a common structure linked to the specificities of power: it is not storable and therefore has to be produced exactly when it is consumed. For instance, in Western Europe, the *spot market* takes place everyday and allows to define the amounts of electricity that will be produced (and consumed) during each of the hours in the next day, based on quite accurate forecasts of consumption needs and production capacities. However, as prices are very volatile on the spot market, utilities usually may want to avoid having their full production sold on this market, and can use the *financial futures market* to perform some risk management of their upcoming positions. On the latter, standardised contracts can be exchanged continuously for the next weeks, months, quarters and years or seasons. Grasping the characteristics of the evolution of prices on the futures market is essential to be able to use it efficiently.

We address statistical estimation of a family of models for electricity prices and propose a methodology to select one of those models using information criteria. We consider Heath-Jarrow-Morton (HJM) models, introduced in Heath et al. [15] to represent the dynamics of the forward rates. In that work devoted to the term structure of interest rates, the forward rates processes were led by a sum of N Brownian motions and a drift. Being common to all maturities, this set of stochastic factors was driving the whole forward rate curve. The article of Heath et al. focuses on the use of their model for valuing contingent claims, and some examples are given.

Using such models to represent electricity prices has been done by many authors. Bjerksund et al. [5] defined a 1- and a 3-factor HJM models to represent the dynamics of instantaneous delivery futures in electricity markets. As they acknowledged that the actually traded contracts are flow futures, meaning there is some delivery period, they used an approximation studied by Kemna and Vorst [19]

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to derive a valuation formula for such flow futures contracts. In contrast, Clewlow and Strickland [6] applied HJM models to oil prices: they designed a methodology to search for the best number of factors by doing a Principal Component Analysis (PCA) on their data. Koekebakker and Ollmar [21] followed the same approach on electricity prices, while accounting for the existence of the delivery period. Those last two articles let the volatility functions totally unspecified: the PCA leads to nonparametric volatility functions. As they did, many authors have also looked for the best number of stochastic factors and for the shape of the volatility coefficients: keeping them simple ensures the models can be used operationally for risk management purposes and can allow for simple formulas for prices of derivatives. Lucia and Schwartz [23] designed a 1- and a 2-factor models for the electricity spot price only. Karesen and Husby [18] designed 1- to 4-factor models for electricity prices, accounted for the delivery period, and designed an extended Kalman filter to estimate the models on data from Nordpool market. They suggested to use 2- or 3-factor models, but had no mathematical criterion to argue. Their model incorporates a noise process, which features market imperfections. Manoliu and Tompaidis [24] proposed a sum of two Ornstein-Ühlenbeck mean-reverting processes to represent gas futures prices, which they estimated on Henry Hub price data. Diko et al. [8] studied the risk premia in electricity markets with a 3-factor model that they estimated using a two-step procedure. Kiesel et al. [20] introduced a 2-factor model for electricity prices, which they calibrated on German market to implied volatilities. The same model was studied by Daboussi and Féron [12] where the calibration results show instability of the parameters, depending on the data that are considered. Benth and Koekebakker [1] discussed the use of HJM models for futures contracts in electricity markets. They explained the implications of various modelling choices from a practical viewpoint. Edoli et al. [9] proposed a 2-factor model similar to the one of Kiesel et al. [20]. Their model does not allow for delivery periods, but it can account for more commodities to be correlated. It is applied on oil prices data after many estimation methods are described. Benth et al. [2] and Latini et al. [22] recently proposed HJM-type additive models to jointly represent instantaneous and flow futures. The former article lists conditions so that such models do not allow for arbitrage. It gives examples of simple models suiting their frame. The latter article performs estimation in such an additive model with 2 factors, by minimising the difference between the theoretical and empirical covariations of processes. Féron et al. [11] performed the efficient estimation of a 2-factor model with stochastic volatility, working with a market model representing the dynamics of futures contracts with a delivery duration of one month.

In the context of interest rates, Bhar and Chiarella [3] estimated HJM models on Australian interest rates data. In their model, the volatility is a function of the level of the process. Bhar et al. [4] performed maximum likelihood estimation of a 1-factor model on American short term interest rates data. Heitmann and Trautmann [16] discussed estimation of HJM models on German bond data by performing a PCA, and then by using nonlinear regression to estimate parametrically four specific models. Jeffrey et al. [17] worked on HJM models for interest rates dynamics where the volatility is an unspecified function of the rate level. They acknowledged that there are less Brownian motions than yield curves to be represented in their model, which implies stochastic singularity: some deterministic relationships between yield curves hold using the model, although they do not hold on real-world data.

In order to be able to select a relevant model, one of the main stakes is to represent the volatility structure of prices. Among all possible models, we have to find an equilibrium between a good quality of representation and a simple and parsimonious form. We are focusing on models in which the dynamics of prices is driven by a sum of correlated Brownian motions, with deterministic volatility coefficients which decrease exponentially as time to delivery increases. This class of models is very well known and used in practice for its tractability to deal with option pricing and hedging purposes. It encompasses the set of seminal commodity models [26, 13, 23], which are used for pricing derivatives, for example, in [7] and more recently in [10]. Our aim is not to find the best price model, but rather to find differences and similarities on different electricity markets by means of a quite simple class of HJM models. We focus our study on the number of needed Brownian motions as a function of the market and the data used in order to be parsimonious while reaching a good quality of representation. To this end, we compute the classical Akaike information Criterion (AIC) and Bayesian information criterion (BIC), and we also propose some additional indicators to help the user choose an efficient number of factors.

The rest of the paper is organised as follows. In section 2 we present the model and recall the corresponding dynamics of the prices for spot and futures with delivery periods. The estimation procedures on futures prices as well as on both spot and futures prices are described in Section 3. In Section 4 we precisely describe the data used for the estimation and the markets considered, and we show and analyse the estimation results. Section 5 concludes this work and exposes some perspectives.

2 Model description

In this paper we use the following notations:

- $F_t(T)$ the unitary power futures price quoted at date t for the delivery of one megawatt-hour (hereafter MWh) of electricity at date T;
- $S_t = F_t(t)$ the power spot price;
- $F_t(T,\theta)$ the power futures price quoted at t, delivering 1MWh during all the hours between times T and $T+\theta$, where θ is the length, in hours, of the delivery period.

We consider the classical Heath-Jarrow-Morton model written on the unitary futures price:

$$\frac{dF_t(T)}{F_t(T)} = \sum_{k=1}^{N} e^{-\alpha_k(T-t)}\sigma_k dW_t^k, \tag{1}$$

with $N \geq 1$ the number of stochastic factors, $(W^k)_{k=1,\dots,N}$ a N-dimensional Brownian motion on the filtered space $(\Omega, \mathcal{F}, \mathbf{F} = (\mathcal{F}_t)_{t\geq 0}, \mathbf{P})$, of which components k and k' have correlation $\rho_{k,k'}$, $\sigma_k > 0,\ 1 \leq k \leq N$, and $0 < \alpha_1 < \cdots < \alpha_N$.

By integrating Equation (1) and letting $T = t$, we deduce the expression of the spot price S_t conditionally to $\mathcal{F}_{t_0}$ for $t_0 \leq t$:

$$\begin{aligned} S_t = F_{t_0}(t)\exp\Bigg\{ &-\frac{1}{2}\sum_{k=1}^{N}\sum_{k'=1}^{N}\rho_{k,k'}\sigma_k\sigma_{k'}\frac{1-e^{-(\alpha_k+\alpha_{k'})(t-t_0)}}{\alpha_k+\alpha_{k'}} \\ &+\sum_{k=1}^{N}\int_{t_0}^{t}\sigma_k e^{-\alpha_k(t-s)}dW_s^k\Bigg\}. \end{aligned} \tag{2}$$

Applying Itô formula, we get the dynamics of the spot price, namely

$$\frac{dS_t}{S_t} = \left(\frac{F'_{t_0}(t)}{F_{t_0}(t)} + \frac{1}{2}\sum_{k=1}^{n}\sum_{k'=1}^{N}\sigma_k\sigma_{k'}\rho_{k,k'}\left(1-e^{-(\alpha_k+\alpha_{k'})(t-t_0)}\right)\right)dt + \sum_{k=1}^{N}dZ_t^k,$$

where $dZ_t^k = -\alpha_k Z_t^k dt + \sigma_k dW_t^k$. It is worth emphasising that the dynamics of the spot price can be written as being led by a sum of Ornstein-Ühlenbeck dynamics.

Concerning the futures prices, as in [12], we can consider the non-arbitrage (discrete) relationship between futures products prices and unitary futures prices in the form

$$F_t(T,\theta) = \frac{h}{\theta}\sum_{i=0}^{\theta/h-1}F_t(T+ih)$$

with h a duration equal to 1 hour, from which we can deduce, with (1), the dynamics of the futures product prices:

$$\begin{aligned} \frac{dF_t(T,\theta)}{F_t(T,\theta)} &= \frac{1}{F_t(T,\theta)}\frac{h}{\theta}\sum_{i=0}^{\theta/h-1}F_t(T+ih)\sum_{k=1}^{N}e^{-\alpha_k(T+ih-t)}\sigma_k dW_t^k \\ &= \sum_{k=1}^{N}e^{-\alpha_k(T-t)}\sigma_k\left(\frac{h}{\theta}\sum_{i=0}^{\theta/h-1}\frac{F_t(T+ih)}{F_t(T,\theta)}e^{-\alpha_k ih}\right)dW_t^k. \end{aligned} \tag{3}$$

3 Estimation

In this section we describe the estimation methodology based on the maximum likelihood principle and how we compute the classical AIC and BIC in order to study the optimal number of factors. In the case where only futures products are used to calibrate the model, the likelihood can be written under a specific approximation as in [12]. Also, as already described in [14], we will introduce a Gaussian error model to face the stochastic singularity (see [17]) when the number of model factors is lower than the number of observed products.
In the case where spot prices are used as well as futures prices, the likelihood can be written by means of a Kalman filter.

In the following we consider $n+1$ quotation dates $t_0, ..., t_n$ and we use the notation $\Delta_{t_i} = t_i - t_{i-1}$ and $\Delta_i^n X = X_{t_i} - X_{t_{i-1}}$ for any process X.

3.1 Likelihood of futures prices

We consider the approximation, as in [12], that the shaping factors $\frac{F_t(T+ih)}{F_t(T,\theta)}$ are all equal to 1. Therefore, the dynamics (3) can be written as

$$\frac{dF_t(T,\theta)}{F_t(T,\theta)} = \sum_{k=1}^{N} e^{-\alpha_k(T-t)}\sigma_k \psi_h(\alpha_k,\theta) dW_t^k,$$

with $\psi_h(\alpha,\theta) = \frac{h}{\theta}\frac{1-e^{-\alpha\theta}}{1-e^{-\alpha h}}$.

By denoting $X_t^{T,\theta} = \log(F_t(T,\theta))$ and applying Itô's lemma we get

$$\begin{aligned}\Delta_i^n X^{T,\theta} =& \sum_{k=1}^{N} \int_{t_{i-1}}^{t_i} \psi(\alpha_k,\theta) e^{-\alpha_k(T-t)}\sigma_k dW_t^k \\ &- \frac{1}{2}\sum_{k=1}^{N}\sum_{k'=1}^{N}\int_{t_{i-1}}^{t_i} e^{-(\alpha_k+\alpha_{k'})(T-t)}\rho_{k,k'}\sigma_k\sigma_{k'}\psi_h(\alpha_k,\theta)\psi_h(\alpha_{k'},\theta)dt.\end{aligned} \tag{4}$$

At each time t_i we suppose observing L_i prices of futures products[1] of which prices are $F_t(T_\ell,\theta_\ell)$, for $\ell = 1, \dots, L_i$. When $L_i > N$, the model presents a stochastic singularity (already studied in [17] and in [14] in the case of a two-factor model applied to electricity prices). In order to face this problem we introduce a model error term and assume observing noised returns:

$$\Delta_i^n Y^{T_\ell,\theta_\ell} = \Delta_i^n X^{T_\ell,\theta_\ell} + \varepsilon_i^{T_\ell,\theta_\ell},$$

where $\varepsilon_i^{T_\ell,\theta_\ell}$ are identically distributed according to a Gaussian distribution $\mathcal{N}(0,v^2)$, where v^2 is unknown. Moreover, for all $i = 1, ..., n$, the variables $\varepsilon_i^{T_\ell,\theta_\ell}$ and $\varepsilon_i^{T_{\ell'},\theta_{\ell'}}$ are independent for $1 \le \ell < \ell' \le L_i$, as well as the variables $\varepsilon_i^{T_\ell,\theta_\ell}$ and $\varepsilon_j^{T_{\ell'},\theta_{\ell'}}$ for $1 \le i < j \le n$ and $1 \le \ell \le L_i, 1 \le \ell' \le L_j$.

Therefore, the vector $\mathbf{\Delta_i^n Y} = \left(\Delta_i^n Y^{T_1,\theta_1} \cdots \Delta_i^n Y^{T_{L_i},\theta_{L_i}}\right)'$ is Gaussian $\mathcal{N}(\mathbf{M_i},\mathbf{\Sigma_i})$ with $\mathbf{M_i} = (M_i^\ell)_{1\le\ell\le L_i}$, $\mathbf{\Sigma_i} = (\Sigma_i^{\ell\ell'})_{1\le\ell,\ell'\le L_i}$ and:

$$\begin{aligned}M_i^\ell &= \mathbf{E}\left(\Delta_i^n Y^{T_\ell,\theta_\ell}\right) \\ &= -\frac{1}{2}\sum_{k=1}^{N}\sum_{k'=1}^{N}\rho_{k,k'}\sigma_k\sigma_{k'}\psi_h(\alpha_k,\theta_\ell)\psi_h(\alpha_{k'},\theta_\ell)e^{-(\alpha_k+\alpha_{k'})(T_\ell-t_i)}\frac{1-e^{-(\alpha_k+\alpha_{k'})\Delta_{t_i}}}{\alpha_k+\alpha_{k'}}\end{aligned} \tag{5}$$

[1] In practice the number of observed products may vary, even after removing the redundant products (e.g. a quarter when all the corresponding monthly products are observed), see Section 4.1.

and

$$\begin{aligned}\Sigma_i^{\ell\ell'} &= \mathbf{C}\text{ov}\left(\Delta_i^n Y^{T_\ell,\theta_\ell}, \Delta_i^n Y^{T_{\ell'},\theta_{\ell'}}\right) \\ &= v^2 \mathbf{1}_{\ell=\ell'} \\ &+ \sum_{k=1}^{N}\sum_{k'=1}^{N} \rho_{k,k'}\sigma_k\sigma_{k'}\psi_h(\alpha_k,\theta_\ell)\psi_h(\alpha_{k'},\theta_{\ell'})e^{-\alpha_k(T_\ell-t_i)}e^{-\alpha_{k'}(T_{\ell'}-t_i)}\frac{1-e^{-(\alpha_k+\alpha_{k'})\Delta_{t_i}}}{\alpha_k+\alpha_{k'}},\end{aligned}$$

where $\mathbf{1}_{\ell=\ell'} = 1$ if $\ell = \ell'$ and 0 otherwise. We can then deduce the opposite log-likelihood $\mathcal{L}_n$ of all the observed futures returns:

$$\mathcal{L}_n = \sum_{i=1}^{n} \frac{L_i}{2}\log(2\pi) + \frac{1}{2}\log(\det \mathbf{\Sigma_i}) + \frac{1}{2}\left(\mathbf{\Delta_i^n Y} - \mathbf{M_i}\right)' \mathbf{\Sigma_i}^{-1}\left(\mathbf{\Delta_i^n Y} - \mathbf{M_i}\right).$$

3.2 Likelihood of spot prices and futures products

In this section the objective is to write the multi-factorial model in a discrete formulation with a state-space model. The state variables are denoted Z^k, $1 \leq k \leq N$ and all of them follow an Ornstein-Ühlenbeck process:

$$dZ_t^k = -\alpha_k Z_t^k dt + \sigma_k dW_t^k. \tag{6}$$

Moreover, all initial values $Z_0^1, ..., Z_0^N$ are chosen to be zero. We can then rewrite Equation (4) in terms of a space equation:

$$\begin{aligned}\Delta_i^n X^{T,\theta} &= \sum_{k=1}^{N} \psi_h(\alpha_k,\theta)e^{-\alpha_k(T-t_i)}\left(Z_{t_i}^k - Z_{t_{i-1}}^k e^{-\alpha_k\Delta_{t_i}}\right) \\ &- \frac{1}{2}\sum_{k=1}^{N}\sum_{k'=1}^{N}\rho_{k,k'}\sigma_k\sigma_{k'}\psi_h(\alpha_k,\theta)\psi_h(\alpha_{k'},\theta)e^{-(\alpha_k+\alpha_{k'})(T-t_i)}\frac{1-e^{-(\alpha_k+\alpha_{k'})\Delta_{t_i}}}{\alpha_k+\alpha_{k'}}.\end{aligned} \tag{7}$$

Using the spot price expression (2) and denoting $\widetilde{S}_t = \frac{S_t}{F_{t_0}(t)}$ the seasonality adjusted spot price we get:

$$\begin{aligned}\Delta_i^n X &= \log \widetilde{S}_{t_i} - \log \widetilde{S}_{t_{i-1}} \\ &= \sum_{k=1}^{N}\left(Z_{t_i}^k - Z_{t_{i-1}}^k\right) \\ &- \frac{1}{2}\sum_{k=1}^{N}\sum_{k'=1}^{N}\rho_{k,k'}\sigma_k\sigma_{k'}e^{-(\alpha_k+\alpha_{k'})(t_{i-1}-t_0)}\frac{1-e^{-(\alpha_k+\alpha_{k'})\Delta_{t_i}}}{\alpha_k+\alpha_{k'}}.\end{aligned} \tag{8}$$

For the same reasons as in the previous section, we consider a model (or measurement) error in the observed spot prices. We assume observing $\Delta_i^n Y$ defined as

$$\Delta_i^n Y = \Delta_i^n X + \varepsilon_i,$$

where the random variable ε_i is distributed according to a Gaussian distribution $\mathcal{N}(0, v^2)$ and is independent of all random variables $\varepsilon_j^{T_\ell,\theta_\ell}$, $j = 1, ..., n$, $\ell = 1, ..., L_j$ introduced in Section 3. We must notice that $\Delta_i^n X$ cannot be considered as a price "return" because the underlying is the spot price, which stands for a different delivery period each day. We propose to consider this element in the calibration process in order to work with differences in price logarithms for all observed (futures and spot) prices. Therefore, the spot "return" is considered as an additional term in relation to the previous section.

As a conclusion, the multi-factor model can be written in a state-space model formulation: let us denote

$$\Delta_i^n \mathbf{Y} = \begin{pmatrix} \Delta_i^n Y^{T_1,\theta_1} \\ \vdots \\ \Delta_i^n Y^{T_{L_i},\theta_{L_i}} \\ \Delta_i^n Y \end{pmatrix}, \qquad \mathbf{Z}_i = \begin{pmatrix} Z_{t_i}^1 \\ Z_{t_{i-1}}^1 \\ \vdots \\ Z_{t_i}^N \\ Z_{t_{i-1}}^N \end{pmatrix}, \tag{9}$$

the multi-factor model can be written as follows:

$$\Delta_i^n \mathbf{Y} = \mathbf{M}_i + \mathbf{F}_i \mathbf{Z}_i + \varepsilon_i$$
$$\mathbf{Z}_i = \mathbf{A}_i \mathbf{Z}_{i-1} + \eta_i$$

with elements $\mathbf{M}_i$, $\mathbf{F}_i$, $\mathbf{A}_i$ and the covariance matrices of ε_i and η_i given in Appendix A.

3.3 Criteria: AIC and BIC

The number of degrees of freedom (to account for in the AIC and BIC) is a function of the number N of factors in the model. Given a fixed N, the parameters are decomposed as follows: $2N$ degrees of freedom corresponding to the α_i's and the σ_i's parameters, 1 degree of freedom corresponding to parameter v, and $\frac{N(N+1)}{2}$ degrees of freedom corresponding to the correlation matrix Σ. In total the number of degrees of freedom is then $(N+1)(N+2)/2$. Therefore, in the case of the factorial models described above, the AIC and BIC are given by:

$$AIC = (N+1)(N+2) + 2\mathcal{L}_n$$
$$BIC = \frac{1}{2}(N+1)(N+2)\log n + 2\mathcal{L}_n$$

with $\mathcal{L}_n$ denoting, as in the previous section, the opposite log-likelihood and n the number of quotation dates.

4 Estimation results

4.1 Data and preprocessing description

In this section, we describe the datasets that we used and we explain how we preprocessed them before running our estimation procedures. We also detail the process of seasonal adjustement of spot prices.

4.1.1 Description of the data

We have used data of prices in several European electricity markets, namely Belgium, France, Germany, Italy, Switzerland and the UK. Those prices are available on the websites `https://eex.com` (for the futures prices) and `https://epexspot.com` (for the spot prices). We collected the closing futures prices every business day, from 2017-01-01 to 2018-12-31, for various contracts, delivering 1 MWh of electricity over standardised periods. Those periods can be:

- the nearest (or 2^{nd} nearest, or 3^{rd} nearest...) week (from Monday to Sunday) that has not begun yet. The underlying contracts are named 1 week-ahead (hereafter 1WAH), 2WAH, 3WAH, ... ;
- the nearest months that have not begun yet, corresponding to month-ahead contracts (hereafter MAH);
- the nearest quarters (January–March, April–June, July–September, October–December) that have not begun yet, corresponding to quarter-ahead (QAH) contracts;
- the nearest calendar years that have not begun yet, featuring year-ahead (YAH) contracts, or the nearest season (Winter of Summer : SAH) contracts for the UK.

For each of the previous time spans, a given number of contracts are traded. For every market we collected the following futures contracts: 1 to 4WAH, 1 to 6MAH, 1 to 4QAH and 1 to 2YAH (or 1 to 4SAH for the UK market). This leads to 16 contracts (18 for the UK) considered for the estimation. We must notice that some data are missing, but the maximisation of the likelihood (both directly as in Subsection 3.1 or with the Kalman filter as in Subsection 3.2) can easily deal with a set of missing data. Indeed, in both cases one only has to consider a varying size vector of available prices at each date. Concerning spot prices, 24 prices are issued every day, related to deliveries of 1 MWh over each of the 24 hours of the day after. We computed the average of those 24 prices each day, featuring the price of the delivery of 1 MWh over the following day.

4.1.2 Preprocessing of data

Having collected the same number of prices each day, we faced some redundancy as it happens that

- three monthly contracts exactly cover a quarter contract;
- four quarterly contracts exactly cover a calendar contract.

When we face one of those cases during two consecutive days, we remove the product with longest delivery period (respectively, in the two previous cases, the quarter and the calendar products) on each of those days. By doing so, we can then compute returns of the prices while using the greatest amount of information while avoiding redundancy. Also, we computed the returns at dates of changing products, caring for the specific change. For example, at a date of a month change, e.g. 2017-02-01, we compute the returns between the 2-MAH and the 1MAH, both corresponding to the same futures contract, e.g. March 2017. By doing so, we optimise the quantity of information available in the data, but we do not have the same number L_i of returns each day. As explained earlier, this is acknowledged and easily dealt with in the computation of the log-likelihood.

4.1.3 Seasonal adjustment of spot prices

In the estimation procedure using only futures prices, returns are computed as described in Subsection 3.1. But when spot prices are used, it is necessary to use Kalman filtering as explained in Subsection 3.2, and then we have to remove the seasonality from spot prices. To do so, we assume that the daily spot price S_t at time t is given by $S_t = \widetilde{S}_t F_{t_0}(t)$, where $\widetilde{S}_t$ is the residual that is modelled in Subsection 3.2 and $F_{t_0}(t)$ is the seasonality term, which we represent with dummy variables as

$$F_{t_0}(t) = y_{year(t)} m_{month(t)} d_{weekday(t)},$$

where y_{yr} is a coefficient associated to the year yr, m_{mh} is associated to the month $mh \in \{1, ..., 12\}$, h_{wd} is associated to the weekday $wd \in \{1, ..., 7\}$. Moreover, $year(t)$ refers to the year to which t belongs, $month(t)$ is the number of its month, $weekday(t)$ is the number of its day (Monday being day 1 and Sunday being day 7). Furthermore, we assume that

$$\frac{1}{12}\sum_{j=1}^{12} m_j = 1 \text{ and } \frac{1}{7}\sum_{j=1}^{7} d_j = 1.$$

All the coefficients are estimated with a Least Square procedure under the constraints described above. The spot returns are then computed on the residual $\widetilde{S}_t$.

4.2 Results

For each market, we run the likelihood maximisation on futures prices and on both futures and spot prices, as described respectively in Subsections 3.1 and 3.2, considering from 2 to 10 factors, and we compute the AIC and BIC. The estimation results are given in Appendix B. The graphs show the computed AIC and BIC as a function of the number of factors and the data used (with the notation “F” meaning only futures prices are used, and “F+S” meaning both spot and futures prices are used). Also, in order to simplify the analysis, the estimated parameters are given for 2 to 6 factors only.

As a global remark on all the markets considered in this paper, we can observe that the BIC allows us to discriminate and find the optimal number of factors, contrary to the AIC. In Table 1, first row, we show the optimal number of factors according to the BIC. According to the BIC, the obtained optimal number of factors is equal to 5 for all the markets considered, except in Switzerland where it is equal to 6 when using spot and futures prices. In the second row of Table 1 we show the smaller number N of factors where a correlation close to 1 or -1 appears in the estimated correlation matrix. Except for the German market, the obtained values are smaller than the BIC-optimal N and highlight the fact that only 3 or 4 factors are sufficient to represent the random behaviour of the prices. The additional factors only allow to adjust the volatility function in order to compensate the difficulty of the factorial model (with only exponential volatility functions) to represent the observed volatility, which may incorporate non-monotonic behaviours.

One, quite surprising, result with the BIC-optimal N is that considering spot prices in the estimation does not increase the optimal number of factors, except in Switzerland. However, some other information leads us to see a more intuitive effect of considering spot prices in the estimation. Indeed, the last row of Table 1 shows that considering the spot prices in the estimation adds a random factor in the model. This is also emphasised when looking at the values of the estimated parameters, especially for small N. Indeed, when we compare the estimated parameters with $N-1$ factors on futures prices, and the ones with N factors on spot and futures prices (for $N=3$ in the German market, see Table 8, and at least for $N=4$ for all the other markets) we can see that the first $N-1$ factors are very similar, and that when the spot prices are used, the additional N^{th} factor has a very high value of α_N which is characteristic of the spot prices.

	Be		Fra		Ger		Ita		Sw		UK	
	F	F+S	F	F+S	F	F+S	F	F+S	F	F+S	F	F+S
BIC-optimal N	5	5	5	5	5	5	5	5	5	6	5	5
smaller N, $\max\lvert\rho_{kk'}\rvert \geq 0.99$	3	4	4	5	4	-	3	4	3	4	4	5

Table 1: Optimal number N of factors according to the BIC (first row) and smaller N where there is a correlation greater than 0.99 between two factors.

On all the considered markets (at least for $N \geq 3$), the first factor is a long-term factor with α_1 very close to 0, and σ_1 is approximately equal to the volatility of the long-term contracts 1YAH and 2YAH. This seems to confirm the form of classical models like the ones described, for example, in Schwartz and Smith [25] and more recently in Kiesel et al. [20] and Féron et al. [11], where the first factor is considered with $\alpha_1 = 0$. Also, we can observe a relative stability in the calibration of the first two factors, while the remaining factors show parameter instability when N varies. When we focus our analysis on $N=5$ (i.e. the BIC-optimal number of factors for all the markets), we can observe that the first factor generally presents a weak correlation with the other factors, and that the last three factors are highly correlated, which can explain the instability in the values of mean-reverting and volatility parameters. One exception can be observed in the German market, where the correlations are relatively high for all the factors in the estimation on futures prices, whereas the first factors is more decorrelated from the other ones in the estimation on spot and futures prices. On the contrary, in the Italian market, the considering or missing the spot prices does not significantly change the parameter values.

We may compare our results to the ones available in the literature. In Kiesel et al. [20] the authors perform a calibration on German option prices on futures contracts, they get a set of parameters, presented in their Table 2, that we can compare to ours with $N=2$ (Futures-only line in Tables 8 and 9). While they choose σ_1 to be equal to 0, we estimate it to be very close to 0 as well. Also, they choose the correlation between the two factors to be zero, while our estimation results leads to a relatively weak correlation (0.13). There is a difference of 20% between their estimated value of σ_1 and ours. Their estimated value of α_2 is approximately 24 times higher than ours, and their value for σ_2 is almost 4 times higher than ours. The main differences may come from the fact that we considered short term (weak-ahead contracts) in our estimation, also their dataset was collected approximately 13 years before ours, and specially their calibration was done using implied volatilities on European options (i.e. the authors calibrate the model in the risk-neutral probability) whereas we use maximum likelihood (to calibrate the model in the historical probability).

In Féron and Daboussi [12] the calibration of the same two-factor model is done on marginal volatilities of futures contracts and on spot prices in the French and UK markets. Once again the authors considered $\alpha_1 = 0$, which is confirmed by our estimation results on the futures prices, but is contradicted for the calibration on spot and futures prices (when we consider only 2 factors). They also considered a null correlation between the two factors, which is contradicted by our estimation results, especially when considering spot prices where the estimated correlations are -0.38 and -0.44 respectively in the

French and the UK markets. When we compare with our estimation result with $N = 5$, the estimated parameters in [11] and in this paper are quite similar when considering only futures prices, especially for the mean-reverting values.

It is also possible to draw a comparison between Diko et al. [8] and us. Diko et al. estimate a 3-factor model on French, German and Dutch data. Their estimation setting is somewhat similar to ours, as they use Kalman filtering as well. Yet they use only very short-term contracts, namely day-ahead ones. We compare their parameters, presented in their Table 3, to the ones we got for $N = 3$ for France (Futures+Spot line in Tables 5 and 6) and Germany (Futures+Spot line in Tables 8 and 9). Although values of coefficients α and σ are different, when comparing both coefficients sets, they seldom differ from more than 33% and the orders of magnitude of the coefficients are preserved, meaning for instance that the ratios $\frac{\alpha_2}{\alpha_1}$ are close for each country. We can make similar comments for $\frac{\alpha_3}{\alpha_2}$ of for the volatility coefficients.

5 Conclusion

In this paper, we studied the number of factors to be considered in the specific class of factorial HJM models, widely used in practice and in the literature of pricing and risk management in the power markets. We focused the study on 6 power markets in Europe and applied the classical methods of likelihood maximisation to estimate the parameters, from which we declined the AIC and BIC for selecting the optimal number of factors. We also proposed, in the analysis, some other information which can be helpful to study the needed number of factors and their characteristics. We then formulate some observations that are common to all the considered markets. The main result is that the optimal number, according to the classical BIC, is around 5, for all the markets considered and when the estimation is done on futures prices as well as on both spot and futures prices. This number is significantly lower than the number of prices considered in the calibration (between 16 and 19 contracts, depending on the market and the consideration or not of the spot prices). The optimal number of factors can also be discussed, especially when observing the characteristics of the last factors (with the highest values of the mean-reverting parameter). Indeed the last factors present high (even quasi perfect) correlation, and seem to be only useful to compensate the difficulties of the model (with only exponential volatility functions) to fit the observed volatilities. The number of significant “random factors” is smaller than 5, and considering the spot prices in the estimation leads to one additional random factor to account for. Therefore, in the case of this specific class of factorial HJM models, an acceptable number of factors to take into account may be 3 or 4 when considering only futures prices, and with one additional factor when considering spot prices. The first factor can be considered as a constant volatility function, with a zero-value for the mean-reverting parameter. Moreover, one must assume non-zero correlations between the different factors.

Obviously the considered model class is restrictive, although widely used. The objective of future work is to consider other forms of volatility functions in order to better fit the observed data. Also, one future objective is to consider an HJM factorial model jointly on several markets, and study, by this way, the optimal number of factors to represent several markets prices. We expect, because of interconnections and market price convergence, to get a smaller optimal number of needed factors than the sum of optimal numbers computed independently to each market.

References

[1] Fred Espen Benth and Steen Koekebakker. Stochastic modeling of financial electricity contracts. *Energy Economics*, 30(3):1116–1157, 2008.

[2] Fred Espen Benth, Marco Piccirilli, and Tiziano Vargiolu. Mean-reverting additive energy forward curves in a Heath–Jarrow–Morton framework. *Mathematics and Financial Economics*, 13(4):543–577, 2019.

[3] Ramaprasad Bhar and Carl Chiarella. The estimation of the Heath-Jarrow-Morton model by use of Kalman filtering techniques. In *Computational Approaches to Economic Problems*, pages 113–126. Springer, 1997.

[4] Ramaprasad Bhar, Carl Chiarella, and Thuy-Duong Tô. A maximum likelihood approach to estimation of a class of the Heath-Jarrow-Morton model. Technical report, School of Finance and Economics, Sydney, 2002.

[5] Petter Bjerksund, Heine Rasmussen, and Gunnar Stensland. Valuation and risk management in the Norwegian electricity market. In Endre Bjørndal, Mette Bjørndal, Panos M. Pardalos, and Mikael Rönnqvist, editors, *Energy, Natural Ressources and Environmental Economics*, pages 167–185. Springer, 2000.

[6] Les Clewlow and Chris Strickland. Energy derivatives: Pricing and risk management. *Lacima Group*, 2000.

[7] M. Dahlgren. A continuous time model to price commodity-based swing options. *Review of Derivatives Research*, 8:27–47, 06 2005.

[8] Pavel Diko, Steve Lawford, and Valerie Limpens. Risk premia in electricity forward prices. *Studies in Nonlinear Dynamics & Econometrics*, 10(3), 2006.

[9] Enrico Edoli, Davide Tasinato, and Tiziano Vargiolu. Calibration of a multifactor model for the forward markets of several commodities. *Optimization*, 62(11):1553–1574, 2013.

[10] Emanuele Fabbiani, Andrea Marziali, and Giuseppe De Nicolao. Fast calibration of two-factor models for energy option pricing. Papers 1809.03941, arXiv.org, September 2018.

[11] Olivier Féron, Pierre Gruet, and Marc Hoffmann. Efficient volatility estimation in a two-factor model. *To appear in Scandinavian Journal of Statistics*, 2019. https://onlinelibrary.wiley.com/doi/abs/10.1111/sjos.12431.

[12] Olivier Féron and Elias Daboussi. Calibration of electricity price models. In René Aïd, Michael Ludkovski, and Ronnie Sircar, editors, *Fields Institute Communications*, volume 74, pages 183–210, 01 2015.

[13] Rajna Gibson and Eduardo S Schwartz. Stochastic convenience yield and the pricing of oil contingent claims. *The Journal of Finance*, 45(3):959–976, 1990.

[14] Pierre Gruet. *Some problems of statistics and optimal control for stochastic processes in the field of electricity markets prices modeling.* PhD thesis, Université Paris Diderot, 2015.

[15] David Heath, Robert Jarrow, and Andrew Morton. Bond pricing and the term structure of interest rates: A new methodology for contingent claims valuation. *Econometrica*, 60(1):77–105, 1992.

[16] Frank Heitmann and Siegfried Trautmann. *Gaussian Multi-factor Interest Rate Models: Theory, Estimation and Implications for Option Pricing.* Lehrstuhl f. Finanzwirtschaft, Univ. Mainz, 1995.

[17] Andrew Jeffrey, Dennis Kristensen, Oliver Linton, Thong Nguyen, and Peter CB Phillips. Nonparametric estimation of a multifactor Heath-Jarrow-Morton model: An integrated approach. *Journal of Financial Econometrics*, 2(2):251–289, 2004.

[18] KF Karesen and Egil Husby. A joint state-space model for electricity spot and futures prices. Technical report, Norsk Regnesentral, 2000.

[19] Angelien GZ Kemna and Antonius CF Vorst. A pricing method for options based on average asset values. *Journal of Banking & Finance*, 14(1):113–129, 1990.

[20] Rüdiger Kiesel, Gero Schindlmayr, and Reik H. Börger. A two-factor model for the electricity forward market. *Quantitative Finance*, 9(3):279–287, 2009.

[21] Steen Koekebakker and Fridthjof Ollmar. Forward curve dynamics in the Nordic electricity market. *Managerial Finance*, 31(6):73–94, 2005.

[22] Luca Latini, Marco Piccirilli, and Tiziano Vargiolu. Mean-reverting no-arbitrage additive models for forward curves in energy markets. *Energy Economics*, 79:157–170, 2019.

[23] Julio J Lucia and Eduardo S Schwartz. Electricity prices and power derivatives: Evidence from the nordic power exchange. *Review of derivatives research*, 5(1):5–50, 2002.

[24] Mihaela Manoliu and Stathis Tompaidis. Energy futures prices: term structure models with Kalman filter estimation. *Applied mathematical finance*, 9(1):21–43, 2002.

[25] Eduardo Schwartz and James E Smith. Short-term variations and long-term dynamics in commodity prices. *Management Science*, 46(7):893–911, 2000.

[26] Eduardo S Schwartz. The stochastic behavior of commodity prices: Implications for valuation and hedging. *The journal of finance*, 52(3):923–973, 1997.

A Kalman filter for the likelihood of spot and futures prices

Let us use the notation (9) and rewrite the state-space formulation described in Section 3.2:

$$\Delta_i^n \mathbf{Y} = \mathbf{M}_i + \mathbf{F}_i \mathbf{Z}_i + \varepsilon_i$$
$$\mathbf{Z}_i = \mathbf{A}_i \mathbf{Z}_{i-1} + \eta_i$$

with $\varepsilon_i \sim \mathcal{N}(0, \mathbf{Q}_{\varepsilon_i})$ and $\varepsilon_i \sim \mathcal{N}(0, \mathbf{Q}_{\eta_i})$. The objective of this appendix is to precisely describe all the elements of the state-space model.

A.1 Elements of the space equation

$\Delta_i^n \mathbf{Y}$ is a vector of size $L_i + 1$, corresponding to the number of observed futures returns at date t_i and the spot price "return". The first L_i components correspond to the futures prices returns and the last component corresponds to the spot. The mean vector $\mathbf{M}_i$ then has its L_i first components defined in Equation (5) and its last component defined, accordingly, by the last term in Equation (7).
Concerning matrix $\mathbf{F}_i$, we propose to express it into two different linear forms $\mathbf{F}_i = ((\mathbf{F}_i^f)', (\mathbf{F}_i^s)')'$ corresponding to the futures products and the spot, respectively.
Using Relation (7) we have

$$\mathbf{F}_i^f = G_i^f H_i^f$$

with G_i^f a $(L_i \times N)$ matrix:

$$G_i^f = \begin{bmatrix} \psi(\alpha_1, \theta_1) e^{-\alpha_1 (T_1 - t_i)} & \dots & \psi(\alpha_N, \theta_1) e^{-\alpha_N (T_1 - t_i)} \\ \vdots & \dots & \vdots \\ \psi(\alpha_1, \theta_{L_i}) e^{-\alpha_1 (T_{L_i} - t_i)} & \dots & \psi(\alpha_N, \theta_{L_i}) e^{-\alpha_N (T_{L_i} - t_i)} \end{bmatrix}$$

and H_i^f a $(N \times 2N)$ matrix:

$$H_i^f = \begin{bmatrix} 1 & -e^{-\alpha_1 \Delta t_i} & 0 & \dots & \dots & \dots & 0 \\ 0 & 0 & 1 & -e^{-\alpha_2 \Delta t_i} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 & \dots & \dots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & 0 & 0 \\ 0 & \dots & \dots & \dots & 0 & 1 & -e^{-\alpha_N \Delta t_i} \end{bmatrix}.$$

The matrix H_i^f may not depend on time if the time step Δt_i is constant. However, G_i^f depends on time because of the maturity terms $T_\ell - t_i$.
Concerning the part $\mathbf{F}_i^s$ dedicated to the spot prices, we can use the same decomposition using Relation (8):

$$\mathbf{F}_i^s = G_i^s H_i^s$$

with $G_i^s = \mathbf{1}'_N$ the transpose of a N-dimensional vector composed of ones, and H_i^s a $(N \times 2N)$ matrix:

$$H_i^s = \begin{bmatrix} 1 & -1 & 0 & \dots & \dots & \dots & 0 \\ 0 & 0 & 1 & -1 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 & \dots & \dots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & 0 & 0 \\ 0 & \dots & \dots & \dots & 0 & 1 & -1 \end{bmatrix}.$$

In the calibration tests, we assume that the variance of the model errors is identical for all obeserved products, i.e.

$$\mathbf{Q}_{\varepsilon_i} = v^2 \mathbf{I}_i$$

with $\mathbf{I}_i$ the $(L_i + 1) \times (L_i + 1)$ identity matrix.

A.2 Elements of the state equation

Using the solution of the Ornstein-Ühlenbeck processes on the factors Z^k defined in (6) we get

$$\mathbf{A}_i = \begin{bmatrix} e^{-\alpha_1 \Delta t_i} & 0 & \dots & \dots & \dots & \dots & 0 \\ 1 & 0 & \ddots & \dots & \dots & \dots & 0 \\ \vdots & \vdots & e^{-\alpha_2 \Delta t_i} & 0 & \dots & \dots & \vdots \\ \vdots & \vdots & 1 & 0 & \dots & \dots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \dots & \vdots \\ 0 & \dots & \dots & \dots & 0 & e^{-\alpha_N \Delta t_i} & 0 \\ 0 & \dots & \dots & \dots & \dots & 1 & 0 \end{bmatrix}.$$

If the time step Δt_i is constant, then the matrix $\mathbf{A}_i = \mathbf{A}$ is also constant.
The $(2N \times 2N)$ covariance matrix $\mathbf{Q}_{\eta_i}$ is also deduced from the dynamics of the Ornstein-Ühlenbeck processes. For $1 \leq k, k' \leq N$,

$$\mathbf{Q}_{\eta_i}^{2k-1,2k'-1} = \rho_{k,k'} \sigma_k \sigma_{k'} \frac{1 - e^{-(\alpha_k + \alpha_{k'}) \Delta t_i}}{\alpha_k + \alpha_{k'}},$$

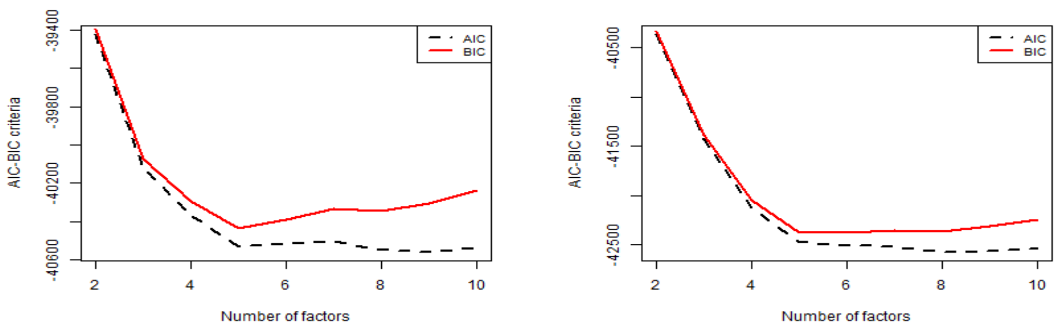
$$\mathbf{Q}_{\eta_i}^{2k-1,2k'} = \mathbf{Q}_{\eta_i}^{2k,2k'-1} = \mathbf{Q}_{\eta_i}^{2k,2k'} = 0.$$

B Estimation results

B.1 Belgian Power market data, 2017-2018

N	Data	f. 1	f. 2	f. 3	f. 4	f. 5	f. 6	f. 7	f. 8	f. 9	f. 10
2	S+F	14.07	399.49								
2	F	6.48	42.82								
3	S+F	6.68	42.33	256.47							
3	F	0.39	25.69	27.00							
4	S+F	0.42	26.34	27.43	262.24						
4	F	0.40	28.11	51.34	53.11						
5	S+F	0.43	23.48	63.15	91.47	100.14					
5	F	0.41	38.89	51.35	75.90	80.96					
6	S+F	0.40	29.37	51.97	53.77	122.94	127.90				
6	F	0.41	41.42	54.14	70.28	70.50	77.44				
7	S+F	0.39	32.90	45.43	48.09	48.09	125.44	130.18			
7	F	0.40	39.52	55.15	68.14	70.99	71.44	82.01			
8	S+F	0.34	25.06	51.24	76.40	83.56	114.47	129.12	129.66		
8	F	0.19	19.05	20.74	56.56	60.88	68.01	68.78	89.42		
9	S+F	0.35	24.20	56.81	83.18	83.45	95.95	110.59	112.26	128.05	
9	F	0.16	7.39	43.96	52.72	60.86	69.24	69.70	76.10	83.33	
10	S+F	0.37	26.64	57.73	80.02	80.04	99.91	108.33	109.79	110.18	124.28
10	F	0.16	7.21	44.94	52.82	58.15	61.21	65.83	75.70	79.88	83.85

Table 2: Estimated values of α_i (yr^{-1}) in Belgian Market data 2017-2018. N is the number of factors, "Data" indicates if the estimation is proceeded on spot and futures (S+F) or only futures (F) prices.



Estimation on futures prices

Estimation on spot and futures prices

Figure 1: AIC-BIC on Belgian Power market data

N	Data	f. 1	f. 2	f. 3	f. 4	f. 5	f. 6	f. 7	f. 8	f. 9	f. 10
2	S+F	1.20	4.26								
2	F	0.81	2.77								
3	S+F	0.84	2.75	4.92							
3	F	0.20	42.51	44.18							
4	S+F	0.21	53.10	54.79	4.91						
4	F	0.21	10.59	301.83	295.88						
5	S+F	0.21	4.27	55.86	262.63	213.18					
5	F	0.21	63.63	203.97	918.09	791.13					
6	S+F	0.21	12.79	361.83	358.17	188.11	182.96				
6	F	0.21	100.01	419.30	899.62	365.49	736.55				
7	S+F	0.20	38.75	358.02	170.24	180.70	219.19	211.53			
7	F	0.21	66.11	389.17	744.34	297.68	319.19	309.40			
8	S+F	0.20	7.35	54.73	100.53	118.34	377.89	356.42	605.27		
8	F	0.18	54.74	68.05	260.48	92.96	194.32	174.98	175.83		
9	S+F	0.20	6.06	80.82	220.29	142.93	147.39	199.08	207.97	268.67	
9	F	0.17	0.81	191.47	508.38	172.02	269.92	247.37	237.65	418.93	
10	S+F	0.20	8.37	114.48	222.47	145.96	141.62	184.61	175.41	138.08	318.63
10	F	0.17	0.78	250.10	640.00	254.85	322.33	342.88	263.80	104.11	406.76

Table 3: Estimated values of σ_i ($\text{yr}^{-1/2}$) in Belgian Market data 2017-2018. N is the number of factors, "Data" indicates if the estimation is proceeded on spot and futures (S+F) or only futures (F) prices.

N	Estimation on futures prices	estimation on spot and futures prices
2	$\begin{bmatrix} 1.00 & -0.31 \\ -0.31 & 1.00 \end{bmatrix}$	$\begin{bmatrix} 1.00 & -0.44 \\ -0.44 & 1.00 \end{bmatrix}$
3	$\begin{bmatrix} 1.00 & -0.03 & 0.04 \\ -0.03 & 1.00 & -1.00 \\ 0.04 & -1.00 & 1.00 \end{bmatrix}$	$\begin{bmatrix} 1.00 & -0.35 & 0.07 \\ -0.35 & 1.00 & -0.71 \\ 0.07 & -0.71 & 1.00 \end{bmatrix}$
4	$\begin{bmatrix} 1.00 & -0.10 & 0.15 & -0.15 \\ -0.10 & 1.00 & -0.93 & 0.93 \\ 0.15 & -0.93 & 1.00 & -1.00 \\ -0.15 & 0.93 & -1.00 & 1.00 \end{bmatrix}$	$\begin{bmatrix} 1.00 & -0.02 & 0.03 & -0.10 \\ -0.02 & 1.00 & -1.00 & 0.49 \\ 0.03 & -1.00 & 1.00 & -0.50 \\ -0.10 & 0.49 & -0.50 & 1.00 \end{bmatrix}$
5	$\begin{bmatrix} 1.00 & -0.05 & 0.06 & -0.05 & 0.05 \\ -0.05 & 1.00 & -0.98 & 0.91 & -0.90 \\ 0.06 & -0.98 & 1.00 & -0.97 & 0.96 \\ -0.05 & 0.91 & -0.97 & 1.00 & -1.00 \\ 0.05 & -0.90 & 0.96 & -1.00 & 1.00 \end{bmatrix}$	$\begin{bmatrix} 1.00 & -0.09 & 0.17 & -0.17 & 0.17 \\ -0.09 & 1.00 & -0.85 & 0.76 & -0.74 \\ 0.17 & -0.85 & 1.00 & -0.98 & 0.96 \\ -0.17 & 0.76 & -0.98 & 1.00 & -1.00 \\ 0.17 & -0.74 & 0.96 & -1.00 & 1.00 \end{bmatrix}$
6	$\begin{bmatrix} 1.00 & -0.04 & 0.04 & 0.05 & -0.22 & 0.03 \\ -0.04 & 1.00 & -0.99 & 0.83 & 0.66 & -0.93 \\ 0.04 & -0.99 & 1.00 & -0.89 & -0.62 & 0.98 \\ 0.05 & 0.83 & -0.89 & 1.00 & 0.22 & -0.95 \\ -0.22 & 0.66 & -0.62 & 0.22 & 1.00 & -0.50 \\ 0.03 & -0.93 & 0.98 & -0.95 & -0.50 & 1.00 \end{bmatrix}$	$\begin{bmatrix} 1.00 & 0.02 & -0.04 & 0.05 & -0.20 & 0.19 \\ 0.02 & 1.00 & -0.92 & 0.91 & 0.34 & -0.38 \\ -0.04 & -0.92 & 1.00 & -1.00 & -0.23 & 0.29 \\ 0.05 & 0.91 & -1.00 & 1.00 & 0.20 & -0.26 \\ -0.20 & 0.34 & -0.23 & 0.20 & 1.00 & -1.00 \\ 0.19 & -0.38 & 0.29 & -0.26 & -1.00 & 1.00 \end{bmatrix}$

Table 4: Estimation of correlation matrices in Belgian Power market data, on only futures (left) and spot and futures prices (right).

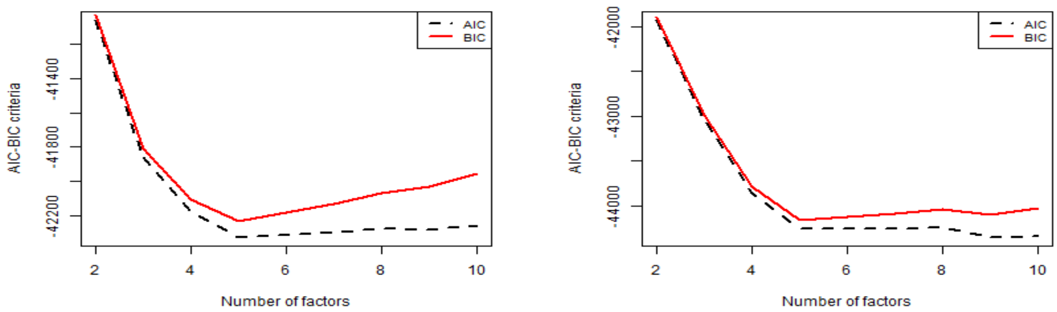
B.2 French Power market data, 2017-2018

N	Data	f. 1	f. 2	f. 3	f. 4	f. 5	f. 6	f. 7	f. 8	f. 9	f. 10
2	S+F	13.37	446.29								
2	F	0.60	27.08								
3	S+F	0.61	25.62	385.49							
3	F	0.08	16.31	59.05							
4	S+F	0.14	21.13	77.31	217.60						
4	F	0.06	9.56	64.21	66.12						
5	S+F	0.05	8.81	67.33	111.77	115.19					
5	F	0.06	10.24	92.31	104.11	113.69					
6	S+F	0.07	9.52	74.68	97.00	99.12	113.17				
6	F	0.05	10.90	85.83	107.38	108.92	111.46				
7	S+F	0.06	9.52	70.44	74.30	116.53	116.55	123.01			
7	F	0.05	10.39	91.48	104.08	104.30	110.53	115.88			
8	S+F	0.07	9.09	72.55	75.13	75.49	114.09	114.09	124.42		
8	F	0.05	10.44	89.04	100.61	105.18	111.48	119.78	125.43		
9	S+F	0.06	10.60	79.51	110.36	111.03	140.16	140.18	158.29	161.08	
9	F	0.05	11.05	97.14	110.36	113.18	141.52	141.70	147.44	205.06	
10	S+F	0.06	10.13	81.84	104.70	111.95	126.69	148.95	153.13	153.13	175.15
10	F	0.00	10.57	96.08	109.47	109.50	150.93	151.55	151.59	160.08	215.62

Table 5: Estimated values of α_i (yr^{-1}) in French Market data 2017-2018. N is the number of factors, "Data" indicates if the estimation is proceeded on spot and futures (S+F) or only futures (F) prices.

N	Data	f. 1	f. 2	f. 3	f. 4	f. 5	f. 6	f. 7	f. 8	f. 9	f. 10
2	S+F	1.10	4.37								
2	F	0.24	1.65								
3	S+F	0.24	1.54	4.62							
3	F	0.17	1.23	3.16							
4	S+F	0.18	1.61	4.86	6.37						
4	F	0.17	0.77	163.51	168.45						
5	S+F	0.17	0.69	19.37	429.53	412.92					
5	F	0.17	0.84	402.83	1088.44	702.49					
6	S+F	0.17	0.76	51.19	140.30	52.75	109.46				
6	F	0.17	0.91	162.90	1231.64	567.02	1491.16				
7	S+F	0.17	0.77	114.47	123.69	120.41	78.52	159.38			
7	F	0.17	0.85	356.41	833.43	372.83	431.07	342.82			
8	S+F	0.17	0.72	218.17	133.27	155.90	96.15	68.79	107.32		
8	F	0.17	0.86	256.00	443.63	210.69	180.08	141.03	79.70		
9	S+F	0.17	0.89	75.18	186.04	190.45	268.94	342.15	342.18	650.07	
9	F	0.17	0.94	430.29	687.47	333.50	318.86	254.12	150.13	236.95	
10	S+F	0.17	0.84	91.31	214.93	134.72	150.45	155.38	123.34	78.75	230.77
10	F	0.16	0.89	347.01	494.53	239.97	255.99	201.25	109.76	64.25	215.09

Table 6: Estimated values of σ_i ($\mathrm{yr}^{-1/2}$) in French Market data 2017-2018. N is the number of factors, "Data" indicates if the estimation is proceeded on spot and futures (S+F) or only futures (F) prices.



Estimation on futures prices — Estimation on spot and futures prices

Figure 2: AIC-BIC on French Power market data

N	Estimation on futures prices	estimation on spot and futures prices
2	$\begin{bmatrix} 1.00 & 0.17 \\ 0.17 & 1.00 \end{bmatrix}$	$\begin{bmatrix} 1.00 & -0.38 \\ -0.38 & 1.00 \end{bmatrix}$
3	$\begin{bmatrix} 1.00 & 0.20 & -0.01 \\ 0.20 & 1.00 & -0.37 \\ -0.01 & -0.37 & 1.00 \end{bmatrix}$	$\begin{bmatrix} 1.00 & 0.16 & -0.11 \\ 0.16 & 1.00 & -0.52 \\ -0.11 & -0.52 & 1.00 \end{bmatrix}$
4	$\begin{bmatrix} 1.00 & -0.02 & 0.21 & -0.20 \\ -0.02 & 1.00 & -0.32 & 0.32 \\ 0.21 & -0.32 & 1.00 & -1.00 \\ -0.20 & 0.32 & -1.00 & 1.00 \end{bmatrix}$	$\begin{bmatrix} 1.00 & 0.23 & -0.08 & -0.03 \\ 0.23 & 1.00 & -0.47 & 0.12 \\ -0.08 & -0.47 & 1.00 & -0.81 \\ -0.03 & 0.12 & -0.81 & 1.00 \end{bmatrix}$
5	$\begin{bmatrix} 1.00 & 0.01 & 0.12 & -0.10 & 0.09 \\ 0.01 & 1.00 & -0.42 & 0.43 & -0.44 \\ 0.12 & -0.42 & 1.00 & -1.00 & 0.99 \\ -0.10 & 0.43 & -1.00 & 1.00 & -1.00 \\ 0.09 & -0.44 & 0.99 & -1.00 & 1.00 \end{bmatrix}$	$\begin{bmatrix} 1.00 & 0.01 & 0.12 & -0.09 & 0.09 \\ 0.01 & 1.00 & -0.14 & 0.15 & -0.15 \\ 0.12 & -0.14 & 1.00 & -0.94 & 0.94 \\ -0.09 & 0.15 & -0.94 & 1.00 & -1.00 \\ 0.09 & -0.15 & 0.94 & -1.00 & 1.00 \end{bmatrix}$
6	$\begin{bmatrix} 1.00 & 0.06 & 0.03 & 0.01 & 0.01 & -0.02 \\ 0.06 & 1.00 & -0.51 & 0.56 & 0.26 & -0.51 \\ 0.03 & -0.51 & 1.00 & -0.95 & -0.82 & 1.00 \\ 0.01 & 0.56 & -0.95 & 1.00 & 0.61 & -0.96 \\ 0.01 & 0.26 & -0.82 & 0.61 & 1.00 & -0.80 \\ -0.02 & -0.51 & 1.00 & -0.96 & -0.80 & 1.00 \end{bmatrix}$	$\begin{bmatrix} 1.00 & -0.00 & 0.19 & -0.15 & -0.16 & 0.17 \\ -0.00 & 1.00 & -0.28 & 0.11 & 0.63 & -0.33 \\ 0.19 & -0.28 & 1.00 & -0.91 & -0.55 & 0.98 \\ -0.15 & 0.11 & -0.91 & 1.00 & 0.17 & -0.95 \\ -0.16 & 0.63 & -0.55 & 0.17 & 1.00 & -0.46 \\ 0.17 & -0.33 & 0.98 & -0.95 & -0.46 & 1.00 \end{bmatrix}$

Table 7: Estimation of correlation matrices in French Power market data, on only futures (left) and spot and futures prices (right).

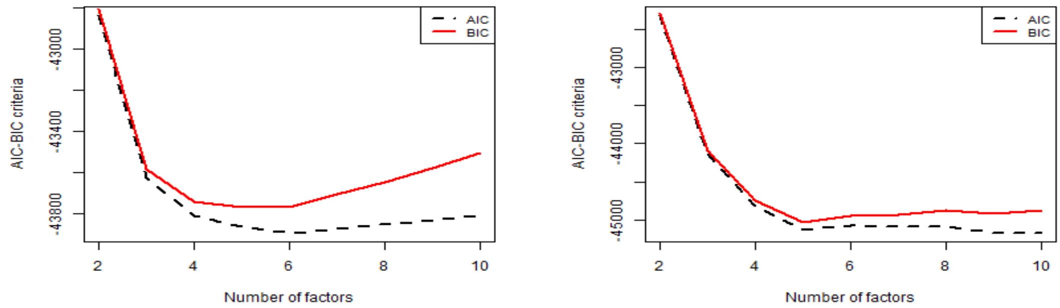
B.3 German Power market data, 2017-2018

N	Data	f. 1	f. 2	f. 3	f. 4	f. 5	f. 6	f. 7	f. 8	f. 9	f. 10
2	S+F	8.68	846.80								
2	F	0.00	33.23								
3	S+F	0.00	31.72	712.80							
3	F	0.00	34.58	66.46							
4	S+F	0.00	38.02	108.36	266.33						
4	F	0.00	16.62	56.41	68.23						
5	S+F	0.00	27.56	44.33	84.93	240.48					
5	F	0.00	21.18	30.50	44.35	49.00					
6	S+F	0.00	9.28	86.75	99.42	115.69	184.67				
6	F	0.00	18.52	76.91	88.68	116.14	161.51				
7	S+F	0.00	25.24	59.05	92.80	125.62	152.48	224.05			
7	F	0.00	17.31	74.39	83.42	118.48	172.61	249.74			
8	S+F	0.00	21.51	50.71	89.55	107.33	153.81	203.07	266.38		
8	F	0.00	16.88	71.38	79.72	113.56	169.94	268.84	372.44		
9	S+F	0.00	21.28	61.21	81.18	81.21	207.43	215.45	239.66	386.79	
9	F	0.00	21.63	61.47	82.91	139.31	227.11	344.01	452.34	555.15	
10	S+F	0.00	20.77	61.61	100.32	106.24	173.94	173.95	177.25	184.89	303.46
10	F	0.00	21.63	61.47	82.91	139.31	227.11	344.01	452.34	555.15	655.88

Table 8: Estimated values of α_i (yr^{-1}) in German Market data 2017-2018. N is the number of factors, "Data" indicates if the estimation is proceeded on spot and futures (S+F) or only futures (F) prices.

N	Data	f. 1	f. 2	f. 3	f. 4	f. 5	f. 6	f. 7	f. 8	f. 9	f. 10
2	S+F	0.61	9.18								
2	F	0.18	1.32								
3	S+F	0.18	1.25	8.98							
3	F	0.18	2.77	5.09							
4	S+F	0.19	2.71	10.12	12.28						
4	F	0.18	1.00	19.66	23.04						
5	S+F	0.18	4.04	10.62	13.46	11.59					
5	F	0.19	6.73	28.78	122.52	103.35					
6	S+F	0.19	0.41	46.25	61.59	28.32	24.67				
6	F	0.18	1.12	66.54	86.04	26.35	45.06				
7	S+F	0.19	2.05	13.07	26.34	20.23	9.28	11.73			
7	F	0.18	1.00	59.83	69.88	17.81	21.63	17.20			
8	S+F	0.18	1.62	8.15	12.48	5.66	4.16	5.84	6.16		
8	F	0.18	0.96	50.29	57.38	16.84	16.77	13.12	9.19		
9	S+F	0.18	1.60	22.85	23.21	12.43	14.18	16.45	6.83	23.97	
9	F	0.18	1.66	21.22	30.93	10.39	16.34	11.55	7.70	5.15	
10	S+F	0.18	1.46	15.71	29.35	18.48	15.66	22.51	19.64	9.30	29.09
10	F	0.18	1.66	21.22	30.93	10.39	16.34	11.55	7.70	5.15	3.41

Table 9: Estimated values of σ_i ($\text{yr}^{-1/2}$) in German Market data 2017-2018. N is the number of factors, "Data" indicates if the estimation is proceeded on spot and futures (S+F) or only futures (F) prices.



Estimation on futures prices — Estimation on spot and futures prices

Figure 3: AIC-BIC on German Power market data

N	Estimation on futures prices	estimation on spot and futures prices
2	$\begin{bmatrix} 1.00 & 0.13 \\ 0.13 & 1.00 \end{bmatrix}$	$\begin{bmatrix} 1.00 & -0.14 \\ -0.14 & 1.00 \end{bmatrix}$
3	$\begin{bmatrix} 1.00 & 0.19 & -0.14 \\ 0.19 & 1.00 & -0.86 \\ -0.14 & -0.86 & 1.00 \end{bmatrix}$	$\begin{bmatrix} 1.00 & 0.12 & -0.01 \\ 0.12 & 1.00 & -0.33 \\ -0.01 & -0.33 & 1.00 \end{bmatrix}$
4	$\begin{bmatrix} 1.00 & -0.15 & 0.30 & -0.30 \\ -0.15 & 1.00 & -0.62 & 0.57 \\ 0.30 & -0.62 & 1.00 & -0.99 \\ -0.30 & 0.57 & -0.99 & 1.00 \end{bmatrix}$	$\begin{bmatrix} 1.00 & 0.50 & -0.59 & 0.49 \\ 0.50 & 1.00 & -0.82 & 0.58 \\ -0.59 & -0.82 & 1.00 & -0.86 \\ 0.49 & 0.58 & -0.86 & 1.00 \end{bmatrix}$
5	$\begin{bmatrix} 1.00 & -0.58 & 0.50 & -0.30 & 0.26 \\ -0.58 & 1.00 & -0.90 & 0.71 & -0.66 \\ 0.50 & -0.90 & 1.00 & -0.94 & 0.91 \\ -0.30 & 0.71 & -0.94 & 1.00 & -1.00 \\ 0.26 & -0.66 & 0.91 & -1.00 & 1.00 \end{bmatrix}$	$\begin{bmatrix} 1.00 & -0.13 & 0.21 & -0.24 & 0.15 \\ -0.13 & 1.00 & -0.92 & 0.75 & -0.53 \\ 0.21 & -0.92 & 1.00 & -0.92 & 0.67 \\ -0.24 & 0.75 & -0.92 & 1.00 & -0.84 \\ 0.15 & -0.53 & 0.67 & -0.84 & 1.00 \end{bmatrix}$
6	$\begin{bmatrix} 1.00 & -0.12 & 0.38 & -0.43 & 0.29 & 0.16 \\ -0.12 & 1.00 & -0.60 & 0.54 & 0.53 & -0.62 \\ 0.38 & -0.60 & 1.00 & -0.98 & -0.55 & 0.97 \\ -0.43 & 0.54 & -0.98 & 1.00 & 0.39 & -0.94 \\ 0.29 & 0.53 & -0.55 & 0.39 & 1.00 & -0.65 \\ 0.16 & -0.62 & 0.97 & -0.94 & -0.65 & 1.00 \end{bmatrix}$	$\begin{bmatrix} 1.00 & -0.10 & 0.36 & -0.53 & 0.59 & -0.02 \\ -0.10 & 1.00 & 0.27 & -0.24 & -0.11 & 0.20 \\ 0.36 & 0.27 & 1.00 & -0.93 & -0.28 & 0.90 \\ -0.53 & -0.24 & -0.93 & 1.00 & -0.08 & -0.79 \\ 0.59 & -0.11 & -0.28 & -0.08 & 1.00 & -0.46 \\ -0.02 & 0.20 & 0.90 & -0.79 & -0.46 & 1.00 \end{bmatrix}$

Table 10: Estimation of correlation matrices in German Power market data, on only futures (left) and spot and futures prices (right).

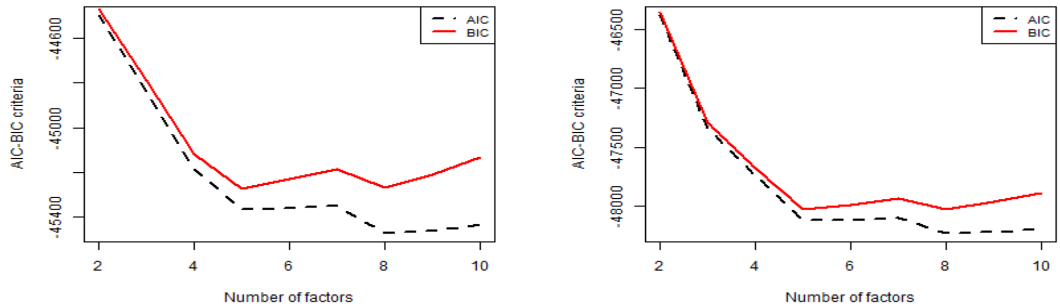
B.4 Italian Power market data, 2017-2018

N	Data	f. 1	f. 2	f. 3	f. 4	f. 5	f. 6	f. 7	f. 8	f. 9	f. 10
2	S+F	4.95	437.78								
2	F	0.23	20.11								
3	S+F	0.24	18.87	373.41							
3	F	0.23	55.99	58.48							
4	S+F	0.23	55.39	59.28	205.18						
4	F	0.11	36.33	44.82	47.56						
5	S+F	0.10	45.11	50.23	72.26	84.41					
5	F	0.10	35.95	60.98	75.55	86.66					
6	S+F	0.12	42.88	50.23	53.06	67.76	89.53				
6	F	0.11	46.91	56.96	68.16	68.18	78.01				
7	S+F	0.12	39.76	54.16	54.17	68.76	68.84	89.35			
7	F	0.10	45.13	55.64	68.45	68.53	77.77	88.43			
8	S+F	0.10	42.88	58.50	65.92	73.24	73.25	100.87	136.25		
8	F	0.10	50.95	63.30	79.13	83.31	118.32	120.01	160.54		
9	S+F	0.11	43.13	55.22	70.82	86.73	87.69	102.59	112.82	132.09	
9	F	0.10	47.94	66.08	89.62	97.19	108.39	119.59	119.63	145.57	
10	S+F	0.11	44.00	54.80	70.90	83.37	92.54	92.55	96.19	115.18	137.19
10	F	0.10	44.90	66.99	89.67	100.25	116.71	116.99	117.53	139.48	157.31

Table 11: Estimated values of α_i (yr^{-1}) in Italian Market data 2017-2018. N is the number of factors, "Data" indicates if the estimation is proceeded on spot and futures (S+F) or only futures (F) prices.

N	Data	f. 1	f. 2	f. 3	f. 4	f. 5	f. 6	f. 7	f. 8	f. 9	f. 10
2	S+F	0.40	2.36								
2	F	0.15	0.71								
3	S+F	0.15	0.66	2.45							
3	F	0.15	60.45	62.69							
4	S+F	0.15	39.50	42.11	4.04						
4	F	0.14	52.05	298.87	250.98						
5	S+F	0.14	145.10	240.70	236.74	142.01					
5	F	0.14	17.34	235.70	566.44	359.57					
6	S+F	0.14	81.77	188.59	150.19	173.61	67.45				
6	F	0.14	194.06	786.40	726.11	498.12	597.41				
7	S+F	0.14	32.21	75.58	93.16	92.72	89.19	73.03			
7	F	0.14	133.54	500.32	318.00	486.42	321.93	136.57			
8	S+F	0.14	55.77	299.96	108.77	261.92	211.64	291.26	91.25		
8	F	0.14	178.71	640.32	314.59	666.30	583.64	275.13	392.96		
9	S+F	0.14	73.68	265.03	309.34	190.87	121.40	164.39	166.84	148.54	
9	F	0.14	84.61	478.11	1091.16	710.84	1075.43	685.36	291.53	697.14	
10	S+F	0.14	91.83	292.59	326.47	152.87	73.93	120.12	125.33	112.76	98.73
10	F	0.14	50.35	391.72	900.90	448.21	799.32	499.77	255.85	268.38	370.86

Table 12: Estimated values of σ_i ($\text{yr}^{-1/2}$) in Italian Market data 2017-2018. N is the number of factors, "Data" indicates if the estimation is proceeded on spot and futures (S+F) or only futures (F) prices.



Estimation on futures prices — Estimation on spot and futures prices

Figure 4: AIC-BIC on Italian Power market data

N	Estimation on futures prices	estimation on spot and futures prices
2	$\begin{bmatrix} 1.00 & 0.13 \\ 0.13 & 1.00 \end{bmatrix}$	$\begin{bmatrix} 1.00 & -0.26 \\ -0.26 & 1.00 \end{bmatrix}$
3	$\begin{bmatrix} 1.00 & 0.21 & -0.21 \\ 0.21 & 1.00 & -1.00 \\ -0.21 & -1.00 & 1.00 \end{bmatrix}$	$\begin{bmatrix} 1.00 & 0.11 & -0.08 \\ 0.11 & 1.00 & -0.44 \\ -0.08 & -0.44 & 1.00 \end{bmatrix}$
4	$\begin{bmatrix} 1.00 & -0.00 & 0.02 & -0.03 \\ -0.00 & 1.00 & -0.99 & 0.99 \\ 0.02 & -0.99 & 1.00 & -1.00 \\ -0.03 & 0.99 & -1.00 & 1.00 \end{bmatrix}$	$\begin{bmatrix} 1.00 & 0.22 & -0.22 & 0.11 \\ 0.22 & 1.00 & -1.00 & 0.75 \\ -0.22 & -1.00 & 1.00 & -0.76 \\ 0.11 & 0.75 & -0.76 & 1.00 \end{bmatrix}$
5	$\begin{bmatrix} 1.00 & 0.06 & -0.03 & 0.03 & -0.04 \\ 0.06 & 1.00 & -0.98 & 0.96 & -0.94 \\ -0.03 & -0.98 & 1.00 & -1.00 & 0.99 \\ 0.03 & 0.96 & -1.00 & 1.00 & -1.00 \\ -0.04 & -0.94 & 0.99 & -1.00 & 1.00 \end{bmatrix}$	$\begin{bmatrix} 1.00 & 0.20 & -0.19 & 0.17 & -0.16 \\ 0.20 & 1.00 & -1.00 & 0.98 & -0.97 \\ -0.19 & -1.00 & 1.00 & -0.99 & 0.98 \\ 0.17 & 0.98 & -0.99 & 1.00 & -1.00 \\ -0.16 & -0.97 & 0.98 & -1.00 & 1.00 \end{bmatrix}$
6	$\begin{bmatrix} 1.00 & 0.13 & -0.13 & 0.31 & -0.12 & -0.15 \\ 0.13 & 1.00 & -1.00 & 0.98 & 0.91 & -0.98 \\ -0.13 & -1.00 & 1.00 & -0.98 & -0.93 & 0.99 \\ 0.31 & 0.98 & -0.98 & 1.00 & 0.85 & -0.97 \\ -0.12 & 0.91 & -0.93 & 0.85 & 1.00 & -0.95 \\ -0.15 & -0.98 & 0.99 & -0.97 & -0.95 & 1.00 \end{bmatrix}$	$\begin{bmatrix} 1.00 & 0.24 & -0.73 & 0.92 & -0.15 & 0.07 \\ 0.24 & 1.00 & -0.84 & -0.13 & 0.90 & -0.91 \\ -0.73 & -0.84 & 1.00 & -0.43 & -0.55 & 0.59 \\ 0.92 & -0.13 & -0.43 & 1.00 & -0.51 & 0.45 \\ -0.15 & 0.90 & -0.55 & -0.51 & 1.00 & -0.99 \\ 0.07 & -0.91 & 0.59 & 0.45 & -0.99 & 1.00 \end{bmatrix}$

Table 13: Estimation of correlation matrices in Italian Power market data, on only futures (left) and spot and futures prices (right).

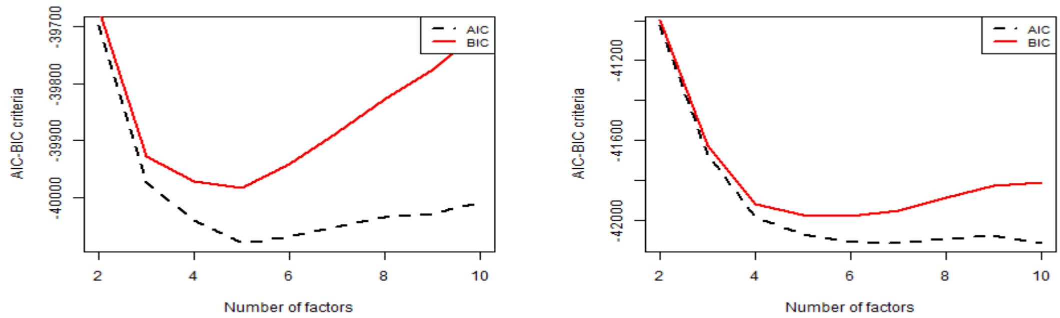
B.5 Swiss Power market data, 2017-2018

N	Data	f. 1	f. 2	f. 3	f. 4	f. 5	f. 6	f. 7	f. 8	f. 9	f. 10
2	S+F	6.56	581.44								
2	F	0.12	16.51								
3	S+F	0.14	15.57	541.17							
3	F	0.04	38.29	39.82							
4	S+F	0.06	39.30	41.81	345.60						
4	F	0.07	29.26	48.49	52.33						
5	S+F	0.09	34.34	46.69	59.06	232.12					
5	F	0.07	24.70	71.25	96.08	100.35					
6	S+F	0.08	17.23	70.49	72.59	166.85	174.06				
6	F	0.06	17.96	84.57	97.93	103.43	104.51				
7	S+F	0.07	16.15	79.72	96.14	145.11	152.19	166.53			
7	F	0.05	16.98	82.96	92.60	94.18	94.99	119.67			
8	S+F	0.05	14.89	80.60	95.56	149.06	152.82	168.58	177.70		
8	F	0.05	17.52	83.08	97.32	98.06	105.10	111.26	111.61		
9	S+F	0.06	14.67	79.22	97.79	145.05	152.55	186.90	198.60	228.18	
9	F	0.06	17.43	98.80	115.63	117.71	138.75	141.56	154.29	179.12	
10	S+F	0.06	16.47	86.58	97.49	185.05	268.60	278.69	278.74	287.99	357.33
10	F	0.05	15.77	101.17	117.09	117.39	147.10	147.21	151.44	171.04	184.88

Table 14: Estimated values of α_i (yr^{-1}) in Swiss Market data 2017-2018. N is the number of factors, "Data" indicates if the estimation is proceeded on spot and futures (S+F) or only futures (F) prices.

N	Data	f. 1	f. 2	f. 3	f. 4	f. 5	f. 6	f. 7	f. 8	f. 9	f. 10
2	S+F	0.60	4.80								
2	F	0.17	0.85								
3	S+F	0.18	0.80	4.84							
3	F	0.16	51.25	52.31							
4	S+F	0.17	33.78	34.95	4.74						
4	F	0.17	7.49	86.84	82.70						
5	S+F	0.17	14.92	44.07	33.34	6.58					
5	F	0.17	2.79	88.87	804.38	726.08					
6	S+F	0.17	1.22	293.04	309.22	222.69	209.51				
6	F	0.17	1.31	274.21	1040.39	271.57	532.36				
7	S+F	0.17	1.10	100.86	193.77	166.16	167.72	236.95			
7	F	0.17	1.18	231.69	609.42	133.13	201.91	112.02			
8	S+F	0.17	0.96	99.23	174.17	87.10	123.10	79.17	174.48		
8	F	0.17	1.25	192.85	604.05	126.35	209.07	130.58	85.38		
9	S+F	0.17	0.93	65.44	122.18	57.96	54.17	58.27	45.41	71.03	
9	F	0.17	1.23	478.50	1347.50	312.25	608.07	488.61	294.99	494.15	
10	S+F	0.17	1.10	155.13	218.51	208.96	94.23	92.54	92.94	76.41	172.55
10	F	0.17	1.03	507.16	1291.24	360.37	671.36	546.83	310.45	334.85	316.65

Table 15: Estimated values of σ_i ($\text{yr}^{-1/2}$) in Swiss Market data 2017-2018. N is the number of factors, "Data" indicates if the estimation is proceeded on spot and futures (S+F) or only futures (F) prices.



Estimation on futures prices

Estimation on spot and futures prices

Figure 5: AIC-BIC on Swiss Power market data

N	Estimation on futures prices	estimation on spot and futures prices
2	$\begin{bmatrix} 1.00 & 0.12 \\ 0.12 & 1.00 \end{bmatrix}$	$\begin{bmatrix} 1.00 & -0.21 \\ -0.21 & 1.00 \end{bmatrix}$
3	$\begin{bmatrix} 1.00 & 0.25 & -0.25 \\ 0.25 & 1.00 & -1.00 \\ -0.25 & -1.00 & 1.00 \end{bmatrix}$	$\begin{bmatrix} 1.00 & 0.10 & -0.02 \\ 0.10 & 1.00 & -0.33 \\ -0.02 & -0.33 & 1.00 \end{bmatrix}$
4	$\begin{bmatrix} 1.00 & -0.16 & 0.28 & -0.29 \\ -0.16 & 1.00 & -0.91 & 0.88 \\ 0.28 & -0.91 & 1.00 & -1.00 \\ -0.29 & 0.88 & -1.00 & 1.00 \end{bmatrix}$	$\begin{bmatrix} 1.00 & 0.24 & -0.25 & 0.06 \\ 0.24 & 1.00 & -1.00 & 0.40 \\ -0.25 & -1.00 & 1.00 & -0.42 \\ 0.06 & 0.40 & -0.42 & 1.00 \end{bmatrix}$
5	$\begin{bmatrix} 1.00 & -0.06 & 0.22 & -0.23 & 0.22 \\ -0.06 & 1.00 & -0.75 & 0.69 & -0.68 \\ 0.22 & -0.75 & 1.00 & -0.99 & 0.99 \\ -0.23 & 0.69 & -0.99 & 1.00 & -1.00 \\ 0.22 & -0.68 & 0.99 & -1.00 & 1.00 \end{bmatrix}$	$\begin{bmatrix} 1.00 & -0.12 & 0.20 & -0.24 & 0.19 \\ -0.12 & 1.00 & -0.96 & 0.90 & -0.54 \\ 0.20 & -0.96 & 1.00 & -0.98 & 0.66 \\ -0.24 & 0.90 & -0.98 & 1.00 & -0.74 \\ 0.19 & -0.54 & 0.66 & -0.74 & 1.00 \end{bmatrix}$
6	$\begin{bmatrix} 1.00 & -0.05 & 0.29 & -0.29 & 0.33 & 0.25 \\ -0.05 & 1.00 & -0.48 & 0.46 & -0.59 & -0.36 \\ 0.29 & -0.48 & 1.00 & -1.00 & 0.91 & 0.99 \\ -0.29 & 0.46 & -1.00 & 1.00 & -0.93 & -0.99 \\ 0.33 & -0.59 & 0.91 & -0.93 & 1.00 & 0.85 \\ 0.25 & -0.36 & 0.99 & -0.99 & 0.85 & 1.00 \end{bmatrix}$	$\begin{bmatrix} 1.00 & -0.15 & 0.46 & -0.47 & 0.45 & -0.44 \\ -0.15 & 1.00 & -0.43 & 0.42 & -0.16 & 0.15 \\ 0.46 & -0.43 & 1.00 & -1.00 & 0.79 & -0.76 \\ -0.47 & 0.42 & -1.00 & 1.00 & -0.80 & 0.77 \\ 0.45 & -0.16 & 0.79 & -0.80 & 1.00 & -1.00 \\ -0.44 & 0.15 & -0.76 & 0.77 & -1.00 & 1.00 \end{bmatrix}$

Table 16: Estimation of correlation matrices in Swiss Power market data, on only futures (left) and spot and futures prices (right).

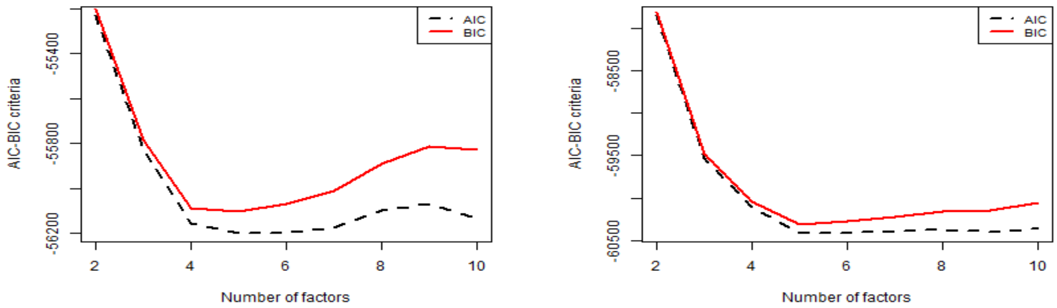
B.6 UK market data, 2017-2018

N	Data	f. 1	f. 2	f. 3	f. 4	f. 5	f. 6	f. 7	f. 8	f. 9	f. 10
2	S+F	1.09	259.64								
2	F	0.06	12.01								
3	S+F	0.06	11.76	158.98							
3	F	0.02	16.24	47.11							
4	S+F	0.03	16.13	71.34	112.44						
4	F	0.00	11.01	59.83	76.30						
5	S+F	0.01	12.73	66.60	75.87	83.33					
5	F	0.00	11.30	67.10	103.18	207.87					
6	S+F	0.00	10.26	63.89	77.07	95.39	97.01				
6	F	0.00	13.00	64.29	104.32	140.94	196.37				
7	S+F	0.00	11.35	59.62	78.03	83.89	96.49	98.50			
7	F	0.00	12.47	65.45	105.02	127.01	197.51	278.36			
8	S+F	0.00	10.90	58.65	78.71	83.38	96.16	101.83	104.99		
8	F	0.00	11.16	51.52	75.59	145.18	297.41	371.34	377.45		
9	S+F	0.00	12.17	59.43	81.52	115.82	140.40	175.62	218.28	279.02	
9	F	0.00	9.94	49.45	85.59	149.80	296.43	369.25	375.32	410.60	
10	S+F	0.00	11.98	50.59	72.48	115.66	168.44	235.28	307.50	385.46	473.14
10	F	0.00	11.56	69.05	93.97	128.47	199.49	358.54	374.38	465.39	475.19

Table 17: Estimated values of α_i (yr^{-1}) in UK Market data 2017-2018. N is the number of factors, "Data" indicates if the estimation is proceeded on spot and futures (S+F) or only futures (F) prices.

N	Data	f. 1	f. 2	f. 3	f. 4	f. 5	f. 6	f. 7	f. 8	f. 9	f. 10
2	S+F	0.26	0.74								
2	F	0.15	0.46								
3	S+F	0.14	0.45	0.87							
3	F	0.14	0.84	1.42							
4	S+F	0.14	0.75	3.27	3.08						
4	F	0.13	0.57	11.69	14.73						
5	S+F	0.14	0.73	98.81	236.38	138.43					
5	F	0.13	0.58	11.99	28.32	42.33					
6	S+F	0.13	0.54	31.50	55.84	34.86	58.35				
6	F	0.13	0.71	13.37	46.72	38.14	23.53				
7	S+F	0.13	0.62	19.23	39.24	27.44	16.74	18.17			
7	F	0.13	0.67	12.63	41.66	19.01	24.41	13.53			
8	S+F	0.13	0.58	15.24	26.74	20.33	12.56	9.07	9.64		
8	F	0.13	0.59	5.86	7.73	2.36	1.53	1.32	1.49		
9	S+F	0.13	0.68	15.19	28.40	7.53	10.03	4.82	2.60	4.53	
9	F	0.13	0.51	3.83	5.83	2.40	1.20	1.07	1.28	1.16	
10	S+F	0.13	0.68	8.68	13.82	4.05	4.12	2.51	1.29	1.38	2.16
10	F	0.13	0.60	22.10	50.42	28.36	19.78	6.15	6.42	8.17	8.43

Table 18: Estimated values of σ_i ($\text{yr}^{-1/2}$) in UK Market data 2017-2018. N is the number of factors, "Data" indicates if the estimation is proceeded on spot and futures (S+F) or only futures (F) prices.



Estimation on futures prices — Estimation on spot and futures prices

Figure 6: AIC-BIC on UK market data

N	Estimation on futures prices	estimation on spot and futures prices
2	$\begin{bmatrix} 1.00 & 0.19 \\ 0.19 & 1.00 \end{bmatrix}$	$\begin{bmatrix} 1.00 & -0.44 \\ -0.44 & 1.00 \end{bmatrix}$
3	$\begin{bmatrix} 1.00 & 0.38 & -0.38 \\ 0.38 & 1.00 & -0.79 \\ -0.38 & -0.79 & 1.00 \end{bmatrix}$	$\begin{bmatrix} 1.00 & 0.21 & -0.32 \\ 0.21 & 1.00 & -0.65 \\ -0.32 & -0.65 & 1.00 \end{bmatrix}$
4	$\begin{bmatrix} 1.00 & 0.35 & -0.09 & 0.06 \\ 0.35 & 1.00 & -0.47 & 0.41 \\ -0.09 & -0.47 & 1.00 & -0.99 \\ 0.06 & 0.41 & -0.99 & 1.00 \end{bmatrix}$	$\begin{bmatrix} 1.00 & 0.36 & -0.30 & 0.18 \\ 0.36 & 1.00 & -0.65 & 0.45 \\ -0.30 & -0.65 & 1.00 & -0.95 \\ 0.18 & 0.45 & -0.95 & 1.00 \end{bmatrix}$
5	$\begin{bmatrix} 1.00 & 0.40 & -0.22 & 0.23 & -0.33 \\ 0.40 & 1.00 & -0.45 & 0.38 & -0.32 \\ -0.22 & -0.45 & 1.00 & -0.98 & 0.89 \\ 0.23 & 0.38 & -0.98 & 1.00 & -0.95 \\ -0.33 & -0.32 & 0.89 & -0.95 & 1.00 \end{bmatrix}$	$\begin{bmatrix} 1.00 & 0.21 & 0.13 & -0.14 & 0.14 \\ 0.21 & 1.00 & -0.61 & 0.59 & -0.58 \\ 0.13 & -0.61 & 1.00 & -1.00 & 1.00 \\ -0.14 & 0.59 & -1.00 & 1.00 & -1.00 \\ 0.14 & -0.58 & 1.00 & -1.00 & 1.00 \end{bmatrix}$
6	$\begin{bmatrix} 1.00 & 0.38 & -0.15 & 0.12 & -0.03 & -0.34 \\ 0.38 & 1.00 & -0.56 & 0.38 & -0.04 & -0.92 \\ -0.15 & -0.56 & 1.00 & -0.97 & 0.83 & 0.64 \\ 0.12 & 0.38 & -0.97 & 1.00 & -0.94 & -0.54 \\ -0.03 & -0.04 & 0.83 & -0.94 & 1.00 & 0.25 \\ -0.34 & -0.92 & 0.64 & -0.54 & 0.25 & 1.00 \end{bmatrix}$	$\begin{bmatrix} 1.00 & 0.31 & 0.01 & -0.02 & -0.13 & 0.08 \\ 0.31 & 1.00 & -0.44 & 0.42 & 0.15 & -0.26 \\ 0.01 & -0.44 & 1.00 & -1.00 & -0.94 & 0.98 \\ -0.02 & 0.42 & -1.00 & 1.00 & 0.93 & -0.98 \\ -0.13 & 0.15 & -0.94 & 0.93 & 1.00 & -0.98 \\ 0.08 & -0.26 & 0.98 & -0.98 & -0.98 & 1.00 \end{bmatrix}$

Table 19: Estimation of correlation matrices in UK market data, on only futures (left) and spot and futures prices (right).