Valid attacks in Argumentation Frameworks with Recursive Attacks (IRIT/RR–2019–02–FR)

Claudette Cayrol, Jorge Fandinno, Luis Fariñas del Cerro, Marie-Christine Lagasquie-Schiex

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(long version)

Claudette Cayrol,
Jorge Fandinno*,
Luis Fariñas del Cerro,
M-Christine Lagasquie-Schiex

Université de Toulouse, IRIT,
118 route de Narbonne, 31062 Toulouse, France
{ccayrol, jorge.fandinno, luis, lagasq}@irit.fr

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Abstract

The purpose of this work is to study a generalisation of Dung's abstract argumentation frameworks that allows representing *recursive attacks*, that is, a class of attacks whose targets are other attacks. We do this by developing a theory of argumentation where the classic role of *attacks* in defeating arguments is replaced by a subset of them, which is "extension-dependent" and which, intuitively, represents a set of "valid attacks" with respect to the extension. The studied theory displays a conservative generalisation of Dung's semantics (complete, preferred, stable and grounded) and also of its principles (conflict-freeness, acceptability and admissibility). Furthermore, despite its conceptual differences, we are also able to show that our theory agrees with the AFRA interpretation of recursive attacks for the complete, preferred, stable and grounded semantics and with a recent flattening method.

Contents

1 Introduction

Argumentation has become an essential paradigm for Knowledge Representation and, especially, for reasoning from contradictory information [1, 13] and for formalizing the exchange of arguments between agents in, *e.g.*, negotiation [2]. Formal abstract frameworks have greatly eased the modelling and study of argumentation. For instance, a Dung's argumentation framework (AF) [13] consists of a collection of arguments interacting with each other through an attack relation, enabling to determine "acceptable" sets of arguments called *extensions*.

A natural generalisation of Dung's argumentation frameworks consists in allowing higher-order attacks (also called recursive attacks in literature) that target other attacks. Here is an example in the legal field, borrowed from [3].

Example 1. *The lawyer says that the defendant did not have intention to kill the victim (Argument b). The prosecutor says that the defendant threw a sharp knife towards the victim (Argument a). So, there is an attack from a to b. And the intention to kill should be inferred. Then the lawyer says that the defendant was in a habit of throwing the knife at his wife's foot once drunk. This latter argument (Argument c) is better considered attacking the attack from a to b, than argument a itself. Now the prosecutor's argumentation seems no longer sufficient for proving the intention to kill. This example is represented as a recursive framework in Figure 1.* □

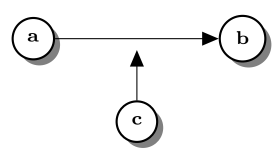

Figure 1: An acyclic recursive framework

Another example, borrowed from [4, 11], will be taken as a running example.

Example 2. *Suppose Bob is making decisions about his Christmas holidays, and is willing to buy cheap last minute offers. He knows there are deals for travelling to Gstaad (g) or Cuba (c). Suppose that Bob has a preference for skiing (p) and knows that Gstaad is a renowned ski resort. However, the weather service reports that it has not snowed in Gstaad (n). So it might not be possible to ski there. Suppose finally that Bob is informed that the ski resort in Gstaad has a good amount of artificial snow, that makes it anyway possible to ski there (a). The different attacks are represented on Figure 2.* □

The idea of encompassing attacks to attacks in abstract argumentation frameworks has been first considered in [5] in the context of an extended framework handling argument strengths and their propagation. Then, a semantics for *recursive frameworks* has been introduced in [18], motivated by the fact that attacks to attacks come from preferences between conflicting arguments.

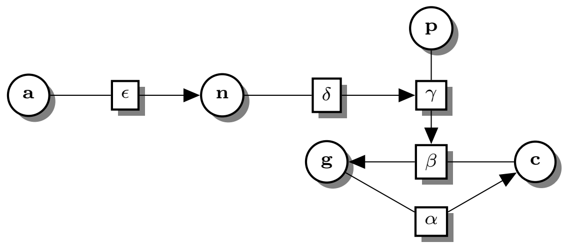

Figure 2: Bob's dilemma: arguments are in circle and attacks in square.

More recently, recursive frameworks have been studied in [4] under the name of AFRA (Argumentation Framework with Recursive Attacks). This version describes abstract argumentation frameworks in which the interactions can be either attacks between arguments or attacks from an argument to another attack. A translation of an AFRA into an AF is defined by the addition of some new arguments and the attacks they produce or they receive. Note that AFRA have been extended in order to handle recursive support interactions together with recursive attacks [11, 12]. However, when supports are removed, these approaches go back to AFRA.

Similar works have proposed to handle recursive frameworks through the definition of a Meta-Argumentation Framework (MAF). The idea goes back to [17]. Recent work [8] uses the addition of meta-arguments that enable to encode the notions of "grounded attack" and "valid attack" (the notion of grounded attack is about the source of the attack and the notion of valid attack is about the link between the source and the target of the attack, *i.e.* the role of the interaction itself). A common point of these approaches (AFRA, MAF) for taking into account higher-order attacks is the fact that they somehow change the role that attacks play in Dung's frameworks.

Example 3. *Consider the argumentation framework corresponding to Figure 3.*

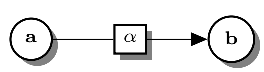

Figure 3: A simple Dung's framework

According to Dung's theory, the framework depicted in Figure 3 has three conflict-free sets: $\varnothing$, $\{a\}$ *and* $\{b\}$. *The set* $\{a, b\}$ *is not conflict-free. However,* $\{a, b\}$ *may become conflict-free if we consider that the attack* α *is not "valid", and so has no impact. This is the case, for instance, in AFRA which translates this argumentation framework into a new AF by converting* α *into a new argument as in Figure 4. In this new framework, it is easy to observe that* $\{a, b\}$ *is considered conflict-free in AFRA because there is no attack between* a *and* b. *In some sense, the connection between an attack and its source has been lost.*

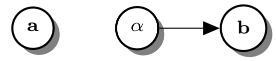

Figure 4: AF framework for AFRA of Figure 3

As another example of this behaviour, the set $\{\alpha, b\}$ *is not AFRA-conflict-free despite the fact that the source of* α*, the argument* a*, is not in the set.* □

In this paper, we study an alternative semantics for argumentation frameworks with recursive attacks based on the following intuitive principles:

P1 The role played in Dung's argumentation frameworks by attacks in defeating arguments is now played by a subset of these attacks, which is "extension-dependent" and represents the "valid attacks" with respect to that extension.

P2 It is a conservative generalisation of Dung's framework for the definitions of conflict-free, admissible, complete, preferred, stable and grounded extensions.

For instance, in the proposed semantics, the conflict-free extensions of the framework of Figure 3 are precisely Dung's conflict-free extensions: $\varnothing$, $\{a\}$ and $\{b\}$. Besides, as we will see later, the attack α is valid with respect to all three extensions because it is not the target of any attack. It is worth noting that, despite its conceptual difference with respect to AFRA, we are able to prove a one-to-one correspondence between our complete, preferred, stable and grounded extensions and the corresponding AFRA extensions, in which the set of "acceptable" arguments are the same. This offers an alternative view for the semantics of recursive attacks that we believe to be closer to Dung's intuitive understanding.

The paper is organized as follows: the necessary background is recalled in Section 2; semantics for recursive frameworks are defined in Section 3; then Sections 4 to 6 present the comparison with existing works (AFRA in Section 4, MAF in Section 5 and Dung's frameworks in Section 6); the notion of inhibited attacks is discussed in Section 7; finally, we conclude in Section 8. Proofs of some additional results can be found in the Appendix.

2 Background

In this section we review the necessary background for Dung's argumentation frameworks.

Definition 1 (D-framework). *A* Dung's abstract argumentation framework (D-framework *for short) is a pair* $\mathbf{AF} = \langle \mathbf{A},\mathbf{R} \rangle$ *where* $\mathbf{A}$ *is a set of arguments and* $\mathbf{R} \subseteq \mathbf{A} \times \mathbf{A}$ *is a relation representing attacks over arguments.* □

For instance, Figure 3 represents the D-framework $\mathbf{AF}_3 = \langle \mathbf{A}_3, \mathbf{R}_3 \rangle$ with the set of arguments $\mathbf{A}_3 = \{a, b\}$ and the attack relation $\mathbf{R}_3 = \{(a, b)\}$.

Definition 2 (Defeat, acceptability). *Given some D-framework* $\mathbf{AF} = \langle \mathbf{A},\mathbf{R} \rangle$ *and some set of arguments* $S \subseteq \mathbf{A}$*, an argument* $a \in \mathbf{A}$ *is said to be*[1]

i) defeated w.r.t. S in $\mathbf{AF}$ *iff* $\exists b \in S$ *such that* $(b, a) \in \mathbf{R}$*, and*
ii) acceptable w.r.t. S in $\mathbf{AF}$ *iff for every argument* $b \in \mathbf{A}$ *with* $(b, a) \in \mathbf{R}$*, there is* $c \in S$ *such that* $(c, b) \in \mathbf{R}$*.* □

To obtain shorter definitions we will also use the following notations:

$$Def(S) \stackrel{\text{def}}{=} \{\ a \in \mathbf{A} \ \big|\ \exists b \in S \text{ s.t. } (b, a) \in \mathbf{R}\ \}$$
$$Acc(S) \stackrel{\text{def}}{=} \{\ a \in \mathbf{A} \ \big|\ \forall b \in \mathbf{A},\ (b, a) \in \mathbf{R} \text{ implies } b \in Def(S)\ \}$$

respectively denote the set of all defeated and acceptable arguments w.r.t. S.

Definition 3 (Semantics). *Given a D-framework* $\mathbf{AF} = \langle \mathbf{A}, \mathbf{R} \rangle$*, a set of arguments* $S \subseteq \mathbf{A}$ *is said to be*

i) conflict-free *iff* $S \cap Def(S) = \varnothing$*,*
ii) naive *iff it is* $\subseteq$*-maximal*[2] *conflict-free,*
iii) admissible *iff it is conflict-free and* $S \subseteq Acc(S)$*,*
iv) complete *iff it is conflict-free and* $S = Acc(S)$*,*
v) preferred *iff it is* $\subseteq$*-maximal admissible,*
vi) grounded *iff it is* $\subseteq$*-minimal complete,*
vii) stable *iff it is conflict-free and* $S \cup Def(S) = \mathbf{A}$*.* □

Theorem 1 ([13]). *Given a D-framework* $\mathbf{AF} = \langle \mathbf{A}, \mathbf{R} \rangle$*, the following assertions hold:*

i) every complete set is also admissible,
ii) every preferred set is also complete, and
iii) every stable set is also preferred. □

For instance, in Example 3, the argument a is accepted w.r.t. any set S because there is no argument $x \in \mathbf{A}$ such that $(x, a) \in \mathbf{R}$. Furthermore, b is defeated and non-acceptable w.r.t. the set $\{a\}$. Then, it is easy to check that $\{a\}$ is stable (and, thus, conflict-free, admissible, complete and preferred). The empty set $\varnothing$ is admissible, but not complete; and the set $\{b\}$ is conflict-free, but not admissible.

[1] "iff" (resp "w.r.t.") stands for "if and only if" (resp. "with respect to").
[2] With $\subseteq$ denoting the standard set inclusion relation.

3 Semantics for recursive attacks

Definition 4 (Recursive Argumentation Framework (RAF))**.** *A* recursive argumentation framework $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *is a quadruple where* $\mathbf{A}$ *and* $\mathbf{K}$ *are (possibly infinite) disjunct sets respectively representing arguments and attack names, and where* $\mathbf{s} : \mathbf{K} \longrightarrow \mathbf{A}$ *and* $\mathbf{t} : \mathbf{K} \longrightarrow \mathbf{A} \cup \mathbf{K}$ *are functions respectively mapping each attack to its source and its target.* □

For instance, the argumentation framework of Example 3 corresponds to $\mathbf{RAF}_3 = \langle \mathbf{A}_3, \mathbf{K}_3, \mathbf{s}_3, \mathbf{t}_3 \rangle$ where $\mathbf{A}_3 = \{a, b\}$, $\mathbf{K}_3 = \{\alpha\}$, $\mathbf{s}_3(\alpha) = a$ and $\mathbf{t}_3(\alpha) = b$. In general, given any D-framework $\mathbf{AF} = \langle \mathbf{A}, \mathbf{R} \rangle$, we may obtain its corresponding recursive argumentation framework $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ by defining a set of attack names $\mathbf{K} = \{ \alpha_{(a,b)} \mid (a, b) \in \mathbf{R} \}$. Functions $\mathbf{s}$ and $\mathbf{t}$ are straightforwardly defined by mapping each attack $(a, b) \in \mathbf{R}$ as follows: $\mathbf{s}(\alpha_{(a,b)}) = a$ and $\mathbf{t}(\alpha_{(a,b)}) = b$.

It is worth noting that our definition allows the existence of several attacks between the same arguments. Though this does not make any difference for frameworks without recursive attacks, for recursive ones, it allows representing attacks between the same arguments that are valid in different contexts. For instance, in the example of Figure 5, there are two attacks between a and b,

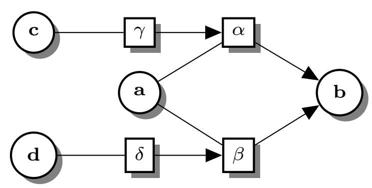

Figure 5: A recursive framework representing attacks in different contexts

namely α and β, which represent different contexts as they are attacked by different arguments.

Definition 5 (Structure)**.** *A pair* $\mathfrak{A} = \langle S, \Gamma \rangle$ *is said to be a* structure *of some* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *iff it satisfies:* $S \subseteq \mathbf{A}$ *and* $\Gamma \subseteq \mathbf{K}$*.* □

Intuitively, the set S represents the set of "acceptable" arguments w.r.t. the structure $\mathfrak{A}$, while Γ represents the set of "valid attacks" w.r.t. $\mathfrak{A}$. Any attack[3] $\alpha \in \overline{\Gamma}$ is understood as non-valid and, in this sense, it cannot defeat the argument or attack that it is targeting.

For the rest of this section we assume that all definitions and results are relative to some given framework $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$. We extend now the definition of defeated arguments (Definition 2) using the set Γ instead of the attack relation $\mathbf{R}$: given a structure of the form $\mathfrak{A} = \langle S, \Gamma \rangle$, we define:

$$Def(\mathfrak{A}) \stackrel{\text{def}}{=} \{ a \in \mathbf{A} \mid \exists \alpha \in \Gamma,\ \mathbf{s}(\alpha) \in S \text{ and } \mathbf{t}(\alpha) = a \} \tag{1}$$

[3] By $\overline{\Gamma} \stackrel{\text{def}}{=} \mathbf{K} \setminus \Gamma$ we denote the set complement of Γ.

In other words, an argument $a \in \mathbf{A}$ is defeated w.r.t. $\mathfrak{A}$ iff there is a "valid attack" w.r.t. $\mathfrak{A}$ that targets a and whose source is "acceptable" w.r.t. $\mathfrak{A}$. It is interesting to observe that we may define the *attack relation* associated with some structure $\mathfrak{A} = \langle S, \Gamma \rangle$ as follows:

$$\mathbf{R}_{\mathfrak{A}} \stackrel{\text{def}}{=} \{ \ (\mathbf{s}(\alpha), \mathbf{t}(\alpha)) \ \big| \ \alpha \in \Gamma \ \} \tag{2}$$

and that, using this relation, we can rewrite (1) as:

$$\mathit{Def}(\mathfrak{A}) \ \stackrel{\text{def}}{=} \ \{ \ a \in \mathbf{A} \ \big| \ \exists b \in S \text{ s.t. } (b, a) \in \mathbf{R}_{\mathfrak{A}} \ \} \tag{3}$$

Now, it is easy to see that our definition can be obtained from Dung's definition of defeat (Definition 2) just by replacing the attack relation $\mathbf{R}$ by the attack relation $\mathbf{R}_{\mathfrak{A}}$ associated with the structure $\mathfrak{A}$, or in other words, by replacing the set of all attacks in the argumentation framework by the set of the "valid attacks" w.r.t. the structure $\mathfrak{A}$, as stated in **P1**. Analogously, by

$$\mathit{Inh}(\mathfrak{A}) \ \stackrel{\text{def}}{=} \ \{ \ \alpha \in \mathbf{K} \ \big| \ \exists b \in S \text{ s.t. } (b, \alpha) \in \mathbf{R}_{\mathfrak{A}} \ \} \tag{4}$$

we denote a set of attacks that, intuitively, represents the "inhibited attacks"[4] w.r.t. $\mathfrak{A}$.

We are now ready to extend the definition of acceptable argument w.r.t. a set (Definition 2):

Definition 6 (Acceptability in a RAF)**.** *An element $x \in (\mathbf{A} \cup \mathbf{K})$ is said to be* acceptable *w.r.t. some structure $\mathfrak{A}$ iff every attack $\alpha \in \mathbf{K}$ with $\mathbf{t}(\alpha) = x$ satisfies either (i) $\mathbf{s}(\alpha) \in \mathit{Def}(\mathfrak{A})$ or (ii) $\alpha \in \mathit{Inh}(\mathfrak{A})$.* □

By $\mathit{Acc}(\mathfrak{A})$, we denote the set containing all acceptable arguments and attacks w.r.t. $\mathfrak{A}$. We also define the following order relations that will help us defining preferred structures: for any pair of structures $\mathfrak{A} = \langle S, \Gamma \rangle$ and $\mathfrak{A}' = \langle S', \Gamma' \rangle$, we write $\mathfrak{A} \sqsubseteq \mathfrak{A}'$ iff $(S \cup \Gamma) \subseteq (S' \cup \Gamma')$ and we write $\mathfrak{A} \sqsubseteq_{ar} \mathfrak{A}'$ iff $S \subseteq S'$. As usual, we say that a structure $\mathfrak{A}$ is $\sqsubseteq$-maximal (resp. $\sqsubseteq_{ar}$-maximal) iff every $\mathfrak{A}'$ that satisfies $\mathfrak{A} \sqsubseteq \mathfrak{A}'$ (resp. $\mathfrak{A} \sqsubseteq_{ar} \mathfrak{A}'$) also satisfies $\mathfrak{A}' \sqsubseteq \mathfrak{A}$ (resp. $\mathfrak{A}' \sqsubseteq_{ar} \mathfrak{A}$).

Definition 7 (Semantics in a RAF)**.** *A structure $\mathfrak{A} = \langle S, \Gamma \rangle$ is said to be:*

i) conflict-free *iff $S \cap \mathit{Def}(\mathfrak{A}) = \varnothing$ and $\Gamma \cap \mathit{Inh}(\mathfrak{A}) = \varnothing$,*
ii) naive *iff it is a $\sqsubseteq$-maximal conflict-free structure,*
iii) admissible *iff it is conflict-free and $(S \cup \Gamma) \subseteq \mathit{Acc}(\mathfrak{A})$,*
iv) complete *iff it is conflict-free and $\mathit{Acc}(\mathfrak{A}) = (S \cup \Gamma)$,*
v) preferred *iff it is a $\sqsubseteq$-maximal admissible structure,*
vi) grounded *iff it is a $\sqsubseteq$-minimal complete structure,*
vii) arg-preferred *iff it is a $\sqsubseteq_{ar}$-maximal preferred structure,*
viii) stable[5] *iff $S = \overline{\mathit{Def}(\mathfrak{A})}$ and $\Gamma = \overline{\mathit{Inh}(\mathfrak{A})}$.* □

[4] We will deepen about the intuition of inhibited attacks in Section 7.

[5] By $\overline{\mathit{Def}(\mathfrak{A})} \stackrel{\text{def}}{=} \mathbf{A} \setminus \mathit{Def}(\mathfrak{A})$ we denote the non-defeated arguments. Similarly, by $\overline{\mathit{Inh}(\mathfrak{A})} \stackrel{\text{def}}{=} \mathbf{K} \setminus \mathit{Inh}(\mathfrak{A})$ we denote the non-inhibited attacks. Note also that $S = \overline{\mathit{Def}(\mathfrak{A})}$ and $\Gamma = \overline{\mathit{Inh}(\mathfrak{A})}$ already implies conflict-freeness.

Example 1 (cont'd) Let **RAF** be the recursive argumentation framework corresponding to Figure 6 (Figure 6 is Figure 1 completed with the attack names).

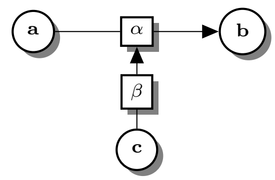

Figure 6: An acyclic recursive framework

It is easy to check that this framework has a unique complete, preferred and stable structure $\mathfrak{A} = \langle\{a, b, c\}, \{\beta\}\rangle$. Furthermore, there are nine admissible structures that are not complete: $\langle\{a, c\}, \{\beta\}\rangle$, $\langle\{b, c\}, \{\beta\}\rangle$, $\langle\{a\}, \{\beta\}\rangle$, $\langle\{c\}, \{\beta\}\rangle$, $\langle\emptyset, \{\beta\}\rangle$, $\langle\{a, c\}, \emptyset\rangle$, $\langle\{a\}, \emptyset\rangle$, $\langle\{c\}, \emptyset\rangle$ and $\langle\emptyset, \emptyset\rangle$. There are also other conflict-free structures that are not admissible:

- $\langle\emptyset, \{\alpha, \beta\}\rangle$, $\langle\emptyset, \{\alpha\}\rangle$,
- $\langle\{a\}, \{\alpha, \beta\}\rangle$, $\langle\{a\}, \{\alpha\}\rangle$,
- $\langle\{b\}, \{\alpha, \beta\}\rangle$, $\langle\{b\}, \{\beta\}\rangle$, $\langle\{b\}, \{\alpha\}\rangle$, $\langle\{b\}, \emptyset\rangle$.
- $\langle\{c\}, \{\alpha\}\rangle$,
- $\langle\{a, b\}, \{\beta\}\rangle$, $\langle\{a, b\}, \emptyset\rangle$,
- $\langle\{a, c\}, \{\alpha\}\rangle$,
- $\langle\{b, c\}, \{\alpha\}\rangle$, $\langle\{b, c\}, \emptyset\rangle$ and
- $\langle\{a, b, c\}, \emptyset\rangle$.

□

It is worth to mention that preferred and arg-preferred structures do not necessarily coincide, since there exist preferred structures that do not contain a maximal set of arguments as shown by the following example:

Example 4. *Let* **RAF** *be the recursive argumentation framework corresponding to the graph depicted in Figure 7.*
Both $\mathfrak{A} = \langle\{a, b\}, \{\beta\}\rangle$ *and* $\mathfrak{A}' = \langle\{a\}, \{\alpha, \beta\}\rangle$ *are preferred structures of* **RAF**,

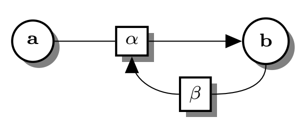

Figure 7: A RAF in which preferred and arg-preferred structures do not coincide

but only the former contains a maximal set of arguments and thus $\mathfrak{A}$ *is the unique arg-preferred structure.* □

It is worth to note that functions mapping structures to sets of defeated arguments, inhibited attacks and acceptable elements (arguments and attacks) are monotonic in the following sense:

Observation 1. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some recursive framework and* $\mathfrak{A}, \mathfrak{A}'$ *be two structures such that* $\mathfrak{A} \sqsubseteq \mathfrak{A}'$. *Then, it follows that* $\mathit{Def}(\mathfrak{A}) \subseteq \mathit{Def}(\mathfrak{A}')$ *and* $\mathit{Inh}(\mathfrak{A}) \subseteq \mathit{Inh}(\mathfrak{A}')$. □

Observation 2. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some recursive framework and* $\mathfrak{A}, \mathfrak{A}'$ *be two structures such that* $\mathfrak{A} \sqsubseteq \mathfrak{A}'$. *Then, it follows that* $\mathit{Acc}(\mathfrak{A}) \subseteq \mathit{Acc}(\mathfrak{A}')$.

Proof. Let $x \in \mathit{Acc}(\mathfrak{A})$. Pick any $\alpha \in \mathbf{K}$ with $\mathbf{t}(\alpha) = x$. Since $x \in \mathit{Acc}(\mathfrak{A})$, it follows that either $\mathbf{s}(\alpha) \in \mathit{Def}(\mathfrak{A})$ or $\alpha \in \mathit{Inh}(\mathfrak{A})$ holds. Furthermore, from Observation 1, we have $\mathit{Def}(\mathfrak{A}) \subseteq \mathit{Def}(\mathfrak{A}')$ and $\mathit{Inh}(\mathfrak{A}) \subseteq \mathit{Inh}(\mathfrak{A}')$. In its turn, this implies that $x \in \mathit{Acc}(\mathfrak{A}')$. □

Using Observation 2, we are able to show now that, as in Dung's argumentation theory, there is also a kind of Fundamental Lemma for argumentation frameworks with recursive attacks. For the sake of compactness, we will adopt the following notations: Given a structure $\mathfrak{A} = \langle S, \Gamma \rangle$ and a set $T \subseteq (\mathbf{A} \cup \mathbf{K})$ containing arguments and attacks, by $\mathfrak{A} \cup T \stackrel{\text{def}}{=} \langle S \cup (T \cap \mathbf{A}), \Gamma \cup (T \cap \mathbf{K}) \rangle$ we denote the result of extending $\mathfrak{A}$ with the elements in T.

Lemma 3.1 (Fundamental Lemma)**.** *Let* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be an admissible structure and* $x, y \in \mathit{Acc}(\mathfrak{A})$ *be any pair of acceptable elements. Then, the following assertions hold:*

i) $\mathfrak{A}' = \mathfrak{A} \cup \{x\}$ *is an admissible structure, and*

ii) $y \in \mathit{Acc}(\mathfrak{A}')$.

Proof. First, note that it can be checked that $\mathfrak{A}' = \langle S', \Gamma' \rangle$ is conflict-free (see Lemma A.4 in the Appendix for more details). Furthermore, since $\mathfrak{A}$ is admissible and $x \in \mathit{Acc}(\mathfrak{A})$, we have that $(S \cup \Gamma \cup \{x\}) \subseteq \mathit{Acc}(\mathfrak{A})$. Moreover, since $\mathfrak{A} \sqsubseteq \mathfrak{A}'$, we also have that $\mathit{Acc}(\mathfrak{A}) \subseteq \mathit{Acc}(\mathfrak{A}')$ (Observation 2) and, thus, we obtain

$$(S' \cup \Gamma') \;=\; (S \cup \Gamma \cup \{x\}) \;\subseteq\; \mathit{Acc}(\mathfrak{A}) \;\subseteq\; \mathit{Acc}(\mathfrak{A}')$$

Consequently, $\mathfrak{A}'$ is admissible and $y \in \mathit{Acc}(\mathfrak{A}')$. □

Moreover, admissible structures form a complete partial order with preferred structures as maximal elements:

Proposition 1. *The set of all admissible structures forms a complete partial order with respect to* $\sqsubseteq$. *Furthermore, for every admissible structure* $\mathfrak{A}$*, there exists an (arg-)preferred structure* $\mathfrak{A}'$ *such that* $\mathfrak{A} \sqsubseteq \mathfrak{A}'$.

Proof. First note that $\langle \varnothing, \varnothing \rangle$ is always admissible and that $\langle \varnothing, \varnothing \rangle \sqsubseteq \mathfrak{A}$ for any structure $\mathfrak{A}$. Furthermore, for every chain $\mathfrak{A}_0 \sqsubseteq \mathfrak{A}_1 \sqsubseteq \ldots$ with $\mathfrak{A}_i = \langle S_i, \Gamma_i \rangle$, it follows that $\mathfrak{A}_i \sqsubseteq \mathfrak{A}$ where $\mathfrak{A} = \langle S, \Gamma \rangle$ with $S = \bigcup_{0 \leq i} S_i$ and $\Gamma = \bigcup_{0 \leq i} \Gamma_i$. It can be checked that $\mathfrak{A}$ is conflict-free (Lemma A.6). Let us show now that $\mathfrak{A}$ is admissible, that is, that every element in $\mathfrak{A}$ is acceptable. Pick $x \in (\Gamma \cup S)$ and any attack $\beta \in \mathbf{K}$ with $\mathbf{t}(\beta) = x$. Then, $x \in (\Gamma_i \cup S_i)$ for some $0 \leq i$. Since $\mathfrak{A}_i$ is admissible, this implies that $x \in \mathit{Acc}(\mathfrak{A}_i)$ and, thus, there is $\gamma \in \Gamma_i \subseteq \Gamma$ such that $\mathbf{s}(\gamma) \in S_i \subseteq S$ and $\mathbf{t}(\gamma) = \beta$. Hence, $x \in \mathit{Acc}(\mathfrak{A})$ and, thus, $\mathfrak{A}$ is admissible.

To show that, for every admissible structure $\mathfrak{A}$, there is some preferred structure $\mathfrak{A}'$ such that $\mathfrak{A} \sqsubseteq \mathfrak{A}'$, suppose, for the sake of contradiction, that there is some admissible structure $\mathfrak{A}$ such that no preferred structure $\mathfrak{A}'$ with $\mathfrak{A} \sqsubseteq \mathfrak{A}'$ exists. Then, there must be some infinite chain $\mathfrak{A} \sqsubseteq \mathfrak{A}_1 \sqsubseteq \mathfrak{A}_2 \sqsubseteq \dots$. However, as shown above, it follows that there is some $\mathfrak{A}'$ such that $\mathfrak{A}_i \sqsubseteq \mathfrak{A}'$ for all $\mathfrak{A}_i$ and, thus, $\mathfrak{A}'$ is a preferred structure. □

Let us now introduce the definition of the characteristic function:

Definition 8 (Characteristic Function)**.** *The* characteristic function $F_{\mathbf{RAF}}$ *of a recursive argumentation framework* **RAF** *is a function over structures satisfying:* $F_{\mathbf{RAF}}(\mathfrak{A}) = \langle Acc(\mathfrak{A}) \cap \mathbf{A},\ Acc(\mathfrak{A}) \cap \mathbf{K} \rangle$. □

Proposition 2. *The characteristic function is $\sqsubseteq$-monotonic. Therefore,*

i) the set of fixpoints of $F_{\mathbf{RAF}}$ is a complete lattice,

ii) there exists a least fixpoint of $F_{\mathbf{RAF}}$,

iii) if $\mathfrak{A}$ is an admissible structure, then there is a unique $\sqsubseteq$-minimal complete structure $\mathfrak{A}'$ such that $\mathfrak{A} \sqsubseteq \mathfrak{A}'$.

Proof. Let $\mathfrak{A} \sqsubseteq \mathfrak{A}'$ be two structures. Then, from Observation 2, it follows that $Acc(\mathfrak{A}) \subseteq Acc(\mathfrak{A}')$ and, thus, that $F_{\mathbf{RAF}}(\mathfrak{A}) \sqsubseteq F_{\mathbf{RAF}}(\mathfrak{A}')$. In other words $F_{\mathbf{RAF}}$ is $\sqsubseteq$-monotonic. Then, since the set of structures forms a complete lattice, for Knaster-Tarski theorem, it follows that the set of fixpoints of $F_{\mathbf{RAF}}$ is also a complete lattice. Conditions ii) and iii) are direct consequences of this fact. □

Proposition 3. *There is always a unique grounded structure which coincides with the least fixpoint of the characteristic function.*

Proof. Note that complete structures coincide with the fixpoints of the characteristic function. Then, this is a direct consequence of Proposition 2. □

The following result shows that the usual relation between extensions also holds for structures.

Theorem 2. *The following assertions hold:*

i) every complete structure is also admissible,

ii) the grounded structure is complete,

iii) every preferred structure is also complete,

iv) every stable structure is also preferred, and

v) every stable structure is also naive.

Proof. Note that i) and ii) follow directly by definition.

iii) By definition, every preferred structure $\mathfrak{A} = \langle S, \Gamma \rangle$ is also admissible. Hence, to show that $\mathfrak{A}$ is complete, it is enough to prove that $Acc(\mathfrak{A}) \subseteq (S \cup \Gamma)$. Pick any $x \in Acc(\mathfrak{A})$. Then, from the Fundamental Lemma, it follows that $\mathfrak{A}' = (\mathfrak{A} \cup \{x\})$ is also admissible and that $\mathfrak{A} \sqsubseteq \mathfrak{A}'$. Furthermore, since $\mathfrak{A}$ is preferred, it follows that $\mathfrak{A}$ is a $\sqsubseteq$-maximal admissible structure and, thus, $\mathfrak{A} \sqsubseteq \mathfrak{A}'$ implies that $\mathfrak{A} = \mathfrak{A}'$. Hence, $x \in (S \cup \Gamma)$ holds and, thus, it follows that $Acc(\mathfrak{A}) \subseteq (S \cup \Gamma)$ and that $\mathfrak{A}$ is complete.

iv) Assume that $\mathfrak{A}$ is a stable structure. We have to prove that $\mathfrak{A}$ is also a $\sqsubseteq$-maximal admissible structure.

We first prove that $\mathfrak{A}$ is admissible. By definition, $\mathfrak{A}$ is conflict-free and satisfies $S = \overline{\mathit{Def}(\mathfrak{A})}$ and $\Gamma = \overline{\mathit{Inh}(\mathfrak{A})}$. Pick $x \in (\Gamma \cup S)$ and any attack $\beta \in \mathbf{K}$ with $\mathbf{t}(\beta) = x$. As $\mathfrak{A}$ is conflict-free, either $\beta \notin \Gamma$ or $\mathbf{s}(\beta) \notin S$. Hence, either $\beta \in \mathit{Inh}(\mathfrak{A})$ or $\mathbf{s}(\beta) \in \mathit{Def}(\mathfrak{A})$. Thus, it follows that $x \in \mathit{Acc}(\mathfrak{A})$ and that $\mathfrak{A}$ is admissible.

Now assume $\mathfrak{A}' = \langle S', \Gamma' \rangle$ to be some admissible structure such that $\mathfrak{A} \sqsubseteq \mathfrak{A}'$. Since $\mathfrak{A}'$ is admissible and thus, conflict-free, it follows from Lemma A.5 that $(S' \cup \Gamma') \subseteq \mathit{Acc}(\mathfrak{A}') \subseteq \overline{\mathit{Def}(\mathfrak{A}')} \cup \overline{\mathit{Inh}(\mathfrak{A}')}$. Furthermore, from Observation 1, and $\mathfrak{A} \sqsubseteq \mathfrak{A}'$, it follows that

$$(\mathit{Def}(\mathfrak{A}) \cup \mathit{Inh}(\mathfrak{A})) \quad \subseteq \quad (\mathit{Def}(\mathfrak{A}') \cup \mathit{Inh}(\mathfrak{A}'))$$

and thus, $\overline{\mathit{Def}(\mathfrak{A}')} \cup \overline{\mathit{Inh}(\mathfrak{A}')} \subseteq \overline{\mathit{Def}(\mathfrak{A})} \cup \overline{\mathit{Inh}(\mathfrak{A})}$. Hence we have $(S' \cup \Gamma') \subseteq \overline{\mathit{Def}(\mathfrak{A})} \cup \overline{\mathit{Inh}(\mathfrak{A})}$. Furthermore, since $\mathfrak{A}$ is stable, it holds that $\overline{\mathit{Def}(\mathfrak{A})} \cup \overline{\mathit{Inh}(\mathfrak{A})} \subseteq (S \cup \Gamma)$ and thus, that $(S' \cup \Gamma') \subseteq (S \cup \Gamma)$ Recall that $\mathfrak{A} \sqsubseteq \mathfrak{A}'$ implies $(S \cup \Gamma) \subseteq (S' \cup \Gamma')$ and thus, $\mathfrak{A} = \mathfrak{A}'$. That is, $\mathfrak{A}$ is a $\sqsubseteq$-maximal admissible structure and, consequently, $\mathfrak{A}$ is a preferred one.

v) Since $\mathfrak{A}$ is a stable structure, by definition, it follows that $\mathfrak{A}$ is conflict-free. Furthermore, since every argument $a \notin S$ belongs to $\mathit{Def}(\mathfrak{A})$ (resp. every attack $\alpha \notin \Gamma$ belongs to $\mathit{Inh}(\mathfrak{A})$), it is easy to see that $\mathfrak{A}$ is also a $\sqsubseteq$-maximal conflict-free structure. □

Example 5. *As a further example, consider the framework* **RAF** *corresponding to Figure 8. This framework has a unique complete and (arg-)preferred struc-*

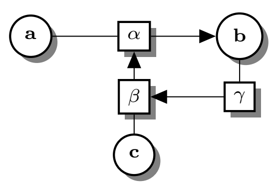

Figure 8: A cyclic recursive framework

ture $\mathfrak{A} = \langle \{a, c\}, \{\gamma\} \rangle$, but no stable one. Note that α and β are neither acceptable nor inhibited w.r.t. $\mathfrak{A}$: β is not inhibited because b is not in the structure, so α is not acceptable. α is not inhibited because β is not in the structure. And β is not acceptable because b is not defeated (as α is not in the structure). □

Example 2 (cont'd) Consider the framework RAF represented in Figure 2. This framework has a unique complete, preferred, grounded and stable structure: $\mathfrak{A}_0 = \langle \{a, g, p\}, \{\alpha, \epsilon, \gamma, \delta\} \rangle$. Among the 63 admissible structures, we also find $\mathfrak{A}_1 = \langle \{a\}, \{\epsilon\} \rangle$, $\mathfrak{A}_2 = \langle \{a\}, \{\epsilon, \delta\} \rangle$, and $\mathfrak{A}_3 = \langle \{a\}, \{\alpha, \epsilon, \gamma, \delta\} \rangle$. □

4 Relation with AFRA

In this section, we establish correspondences between our semantics for recursive frameworks and the semantics for AFRA. In [4] a recursive framework is turned into a Dung's framework by adding new arguments and attacks using the following notion of defeat:

Definition 9 (Defeat in AFRA). *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$. *An attack* $\alpha \in \mathbf{K}$ *is said to* directly defeat $x \in \mathbf{A} \cup \mathbf{K}$ *iff* $\mathbf{t}(\alpha) = x$. *It is said to* indirectly defeat $\beta \in \mathbf{K}$ *iff* α *directly defeats* $\mathbf{s}(\beta)$. *Then,* α *is said to* defeat $x \in \mathbf{A} \cup \mathbf{K}$ *iff* α *directly defeats* x *or* α *indirectly defeats* x. □

For instance, in Example 5, it is easy to see that α directly defeats b and indirectly defeats γ. Hence, α defeats both b and γ. Attacks β and γ directly defeat α and β, respectively. It has been shown in [4] that AFRA extensions can be characterized as the extensions of a Dung's framework whose new set of arguments contains both arguments and attacks and whose new attack relation is the defeat relation of Definition 9. In this sense, under AFRA, the argumen-

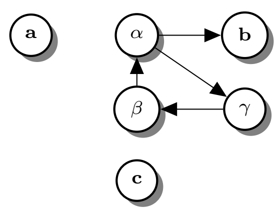

Figure 9: AF framework for AFRA framework of Ex. 5

tation framework of Example 5 is turned into the one in Figure 9 and it can be checked that it has a unique complete and preferred extension $\{a, c\}$ and no stable one. We recall next the formal definitions of AFRA from [4]:

Definition 10 (AFRA-acceptability). *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *and* $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$. *Then, an element* $x \in (\mathbf{A} \cup \mathbf{K})$ *is said to be* AFRA-acceptable *w.r.t.* $\mathcal{E}$ *iff for every* $\alpha \in \mathbf{K}$ *that defeats* x, *there is* $\beta \in \mathcal{E}$ *that defeats* α. □

Definition 11 (AFRA-extensions). *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *and a set* $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$, $\mathcal{E}$ *is said to be:*

i) AFRA-conflict-free *iff* $\nexists \alpha, x \in \mathcal{E}$ *s.t.* α *defeats* x,

ii) AFRA-admissible *iff* $\mathcal{E}$ *is AFRA-conflict-free and each element of* $\mathcal{E}$ *is AFRA-acceptable w.r.t.* $\mathcal{E}$,

iii) AFRA-complete *iff it is AFRA-admissible and every* $x \in (\mathbf{A} \cup \mathbf{K})$ *which is AFRA-acceptable w.r.t.* $\mathcal{E}$ *belongs to* $\mathcal{E}$,

iv) AFRA-preferred *iff it is a* $\subseteq$*-maximal AFRA-admissible extension,*

v) AFRA-grounded *iff it is a* $\subseteq$*-minimal AFRA-complete extension,*

vi) AFRA-stable *iff it is AFRA-conflict-free and, for every* $x \in (\mathbf{A} \cup \mathbf{K})$, $x \notin \mathcal{E}$ *implies that* x *is defeated by some* $\alpha \in \mathcal{E}$. □

As illustrated by Example 3, AFRA does not preserve Dung's notion of conflict-freeness.

Observation 3. *AFRA is not a conservative generalisation of Dung's approach.* ◻

In order to characterize the relation between our approach and AFRA, we will need the following notation. Given some structure $\mathfrak{A} = \langle S, \Gamma \rangle$, by

$$\mathtt{Afra}(\mathfrak{A}) \quad \stackrel{\text{def}}{=} \quad S \cup \{ \ \alpha \in \Gamma \mid \mathbf{s}(\alpha) \in S \ \}$$

we denote its corresponding AFRA-extension.

Note that the AFRA-extension corresponding to a given structure only contains the attacks of the structure whose source belongs to the structure. The other attacks of the structure do not appear in the AFRA-extension. Intuitively, this selection is motivated by the fact that, in an AFRA-extension, any attack directly carries a conflict against its target, even if its source is not accepted, something which we avoid in our framework.

Proposition 4. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be a recursive framework and* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be some conflict-free structure. Then,* $\mathtt{Afra}(\mathfrak{A})$ *is AFRA-conflict-free.*

Proof. Consider a conflict-free structure $\mathfrak{A} = \langle S, \Gamma \rangle$. Suppose that $\mathtt{Afra}(\mathfrak{A})$ is is not an AFRA-conflict-free extension. Then, there are $\alpha, x \in \mathtt{Afra}(\mathfrak{A})$ s.t. α defeats x and, thus, $\alpha \in \Gamma'$ with $\Gamma' = \{ \ \alpha \in \Gamma \mid \mathbf{s}(\alpha) \in S \ \}$. That is, $\alpha \in \Gamma$ and $\mathbf{s}(\alpha) \in S$.

If $x \in S$, then we have that $\mathbf{t}(\alpha) = x \in \mathit{Def}(\mathfrak{A})$ which is a contradiction with the fact that $S \cap (\mathit{Def}(\mathfrak{A})) \neq \varnothing$ holds because $\mathfrak{A}$ is conflict-free.

Otherwise, $x \notin S$ implies $x \in \mathbf{K}$. Then, $x \in \mathtt{Afra}(\mathfrak{A})$ implies that $x \in \Gamma'$ and, thus, that $x \in \Gamma$ and $\mathbf{s}(x) \in S$. Furthermore, α defeats x implies that either $\mathbf{t}(\alpha) = x$ or $\mathbf{t}(\alpha) = \mathbf{s}(x)$ holds. The former, $\mathbf{t}(\alpha) = x$, implies that $x \in \mathit{Inh}(\mathfrak{A})$ which is a contradiction with the fact that $(\Gamma \cap \mathit{Inh}(\mathfrak{A})) \neq \varnothing$ follows from $\mathfrak{A}$ being conflict-free. The latter, $\mathbf{t}(\alpha) = \mathbf{s}(x)$, is a contradiction with the fact that $\mathbf{s}(x) \in S$ and the fact that $(S \cap \mathit{Def}(\mathfrak{A})) \neq \varnothing$ follows from $\mathfrak{A}$ being conflict-free. ◻

For the converse of Proposition 4, we need to introduce some extra notation: Given some set $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$, by $S_{\mathcal{E}} \stackrel{\text{def}}{=} (\mathcal{E} \cap \mathbf{A})$ and $\tilde{\Gamma}_{\mathcal{E}} \stackrel{\text{def}}{=} (\mathcal{E} \cap \mathbf{K})$, we respectively denote the set of arguments and attacks of $\mathcal{E}$. Then, we define $\tilde{\mathfrak{A}}_{\mathcal{E}} = \langle S_{\mathcal{E}}, \tilde{\Gamma}_{\mathcal{E}} \rangle$ and we denote by

$$\Gamma_{\mathcal{E}} \ \stackrel{\text{def}}{=} \ \tilde{\Gamma}_{\mathcal{E}} \cup \{ \ \alpha \in (\mathit{Acc}(\tilde{\mathfrak{A}}_{\mathcal{E}}) \cap \mathbf{K}) \mid \mathbf{s}(\alpha) \notin \mathcal{E} \ \} \tag{5}$$

the set of "non-inhibited attacks" w.r.t. $\mathcal{E}$.

Finally, we can define the structure corresponding to some AFRA-extension $\mathcal{E}$ as $\mathfrak{A}_{\mathcal{E}} \stackrel{\text{def}}{=} \langle S_{\mathcal{E}}, \Gamma_{\mathcal{E}} \rangle$. Intuitively, this is due to the fact that, in AFRA, an attack is inhibited whenever its source is defeated. Hence, we need to add to the structure all those attacks whose only reason for being defeated according to AFRA is because of the attacks towards their source.

Proposition 5. *Given some recursive framework* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *and an AFRA-conflict-free set* $\mathcal{E} \subseteq \mathbf{A} \cup \mathbf{K}$*, it follows that* $\mathfrak{A}_{\mathcal{E}}$ *is a conflict-free structure.* □

Proof. Let $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$ be an AFRA-conflict-free set and pick $a \in Def(\mathfrak{A}_{\mathcal{E}})$. Then, there is some $\alpha \in \mathtt{Afra}(\mathfrak{A}_{\mathcal{E}})$ such that α defeats a (Lemma A.7). Furthermore, since $\mathcal{E}$ is AFRA-conflict-free, $\alpha \in (\mathcal{E} \cap \mathbf{K})$ ($\alpha \in \mathtt{Afra}(\mathfrak{A}_{\mathcal{E}})$ means that $\alpha \in \Gamma_{\mathcal{E}}$ and $s(\alpha) \in S_{\mathcal{E}}$; so by definition of $\Gamma_{\mathcal{E}}$ it follows that $\alpha \in \tilde{\Gamma}_{\mathcal{E}}$) implies that $a \notin \mathcal{E}$ and, by definition, this implies that $a \notin S_{\mathcal{E}}$. Then, since a is an arbitrary argument of $Def(\mathfrak{A}_{\mathcal{E}})$ it follows that $(S_{\mathcal{E}} \cap Def(\mathfrak{A}_{\mathcal{E}})) = \varnothing$ (and also that $(S_{\mathcal{E}} \cap Def(\tilde{\mathfrak{A}}_{\varepsilon})) = \varnothing$).

Similarly, pick any $\alpha \in Inh(\tilde{\mathfrak{A}}_{\mathcal{E}})$. Then, there exists some attack $\beta \in \tilde{\Gamma}_{\mathcal{E}}$ such that $\mathbf{s}(\beta) \in S_{\mathcal{E}}$ and $\mathbf{t}(\beta) = \alpha$. As above, this implies that β (directly) defeats α and, since $\beta \in \Gamma_{\mathcal{E}}$ and $\mathbf{s}(\beta) \in S_{\mathcal{E}} \subseteq \mathcal{E}$, it follows that $\beta \in (\mathcal{E} \cap \mathbf{K})$. Moreover, since $\mathcal{E}$ is an AFRA-conflict-free extension, this implies that $\alpha \notin \mathcal{E}$, so $\alpha \notin \tilde{\Gamma}_{\mathcal{E}}$. Hence, we obtain $(\tilde{\Gamma}_{\mathcal{E}} \cap Inh(\tilde{\mathfrak{A}}_{\mathcal{E}})) = \varnothing$, which together with $(S_{\mathcal{E}} \cap Def(\tilde{\mathfrak{A}}_{\mathcal{E}})) = \varnothing$, implies that $\tilde{\mathfrak{A}}_{\mathcal{E}}$ is conflict-free. Moreover, $\alpha \in Inh(\tilde{\mathfrak{A}}_{\mathcal{E}})$ implies that $\alpha \notin \Gamma_{\mathcal{E}}$. Hence, we also obtain $(\Gamma_{\mathcal{E}} \cap Inh(\tilde{\mathfrak{A}}_{\mathcal{E}})) = \varnothing$.

Finally, to see that $\mathfrak{A}_{\mathcal{E}}$ is also conflict-free, it remains to be proven that $(\Gamma_{\mathcal{E}} \cap Inh(\mathfrak{A}_{\mathcal{E}})) = \varnothing$ holds. First suppose, for the sake of contradiction, that $(\tilde{\Gamma}_{\mathcal{E}} \cap Inh(\mathfrak{A}_{\mathcal{E}})) \neq \varnothing$. Since $(\tilde{\Gamma}_{\mathcal{E}} \cap Inh(\tilde{\mathfrak{A}}_{\mathcal{E}})) \neq \varnothing$, there is $\alpha \in \Gamma_{\mathcal{E}} \setminus \tilde{\Gamma}_{\mathcal{E}}$ such that $\mathbf{s}(\alpha) \in S_{\mathcal{E}}$ and $\mathbf{t}(\alpha) \in \tilde{\Gamma}_{\mathcal{E}}$. However, $\alpha \in \Gamma_{\mathcal{E}} \setminus \tilde{\Gamma}_{\mathcal{E}}$ implies $\mathbf{s}(\alpha) \notin S_{\mathcal{E}}$ which is a contradiction. Hence, $(S_{\mathcal{E}} \cap Def(\mathfrak{A}_{\mathcal{E}})) = \varnothing$ holds. Suppose now that $(\Gamma_{\mathcal{E}} \cap Inh(\mathfrak{A}_{\mathcal{E}})) \neq \varnothing$. Then, there is $\alpha \in \Gamma_{\mathcal{E}} \setminus \tilde{\Gamma}_{\mathcal{E}}$ and $\beta \in \Gamma_{\mathcal{E}}$ such that $\mathbf{t}(\beta) = \alpha$ and $\mathbf{s}(\beta) \in \Gamma_{\mathcal{E}}$. Note that $\alpha \in \Gamma_{\mathcal{E}} \setminus \tilde{\Gamma}_{\mathcal{E}}$ implies that $\alpha \in Acc(\tilde{\mathfrak{A}}_{\mathcal{E}})$ and, thus, there is $\gamma \in \tilde{\Gamma}_{\mathcal{E}}$ such that $\mathbf{t}(\gamma) \in \{\beta, \mathbf{s}(\beta)\}$ and $\mathbf{s}(\beta) \in S_{\mathcal{E}}$. If $\mathbf{t}(\gamma) = \beta$, then $\beta \in Inh(\tilde{\mathfrak{A}}_{\mathcal{E}})$ and, from $(\Gamma_{\mathcal{E}} \cap Inh(\tilde{\mathfrak{A}}_{\mathcal{E}})) = \varnothing$, it follows that $\beta \notin \tilde{\Gamma}_{\mathcal{E}}$ and, thus, $\beta \in Acc(\tilde{\mathfrak{A}}_{\mathcal{E}})$. Then, then there must be $\delta \in \Gamma_{\mathcal{E}}$ such that $\mathbf{t}(\delta) = \gamma$ and $\mathbf{s}(\delta) \in S_{\mathcal{E}}$. This implies that $\beta \in (\Gamma_{\mathcal{E}} \cap Inh(\tilde{\mathfrak{A}}_{\mathcal{E}})) = \varnothing$ which is a contradiction. Hence, it must be that $\mathbf{t}(\gamma) = \mathbf{s}(\beta)$. But this implies that $\mathbf{s}(\beta) \in (S_{\mathcal{E}} \cap Def(\tilde{\mathfrak{A}}_{\mathcal{E}}))$ which is also a contradiction. Consequently, $\mathfrak{A}_{\mathcal{E}}$ is also conflict-free. □

We can also extend the result of Proposition 4 to the admissible semantics as follows:

Proposition 6. *Let* **RAF** *be some recursive framework and let* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be an admissible structure. Then,* $\mathtt{Afra}(\mathfrak{A})$ *is AFRA-admissible.* □

Proof. By definition, $\mathfrak{A}$ being admissible implies that it is conflict-free and, from Proposition 4, that $\mathtt{Afra}(\mathfrak{A})$ is AFRA-conflict-free. In addition, since $\mathfrak{A}$ is admissible, then $(S \cup \Gamma) \subseteq Acc(\mathfrak{A})$ and it follows that every argument $a \in (\mathtt{Afra}(\mathfrak{A}) \cap \mathbf{A})$ is AFRA-acceptable w.r.t. $\mathtt{Afra}(\mathfrak{A})$ (Lemma A.8). Furthermore, by construction, every attack $\alpha \in (\mathtt{Afra}(\mathfrak{A}) \cap \mathbf{K})$ satisfies that $\mathbf{s}(\alpha) \in S$. Hence, both α and $\mathbf{s}(\alpha)$ are acceptable w.r.t. $\mathfrak{A}$ and, thus, it follows that α is AFRA-acceptable w.r.t. $\mathtt{Afra}(\mathfrak{A})$ (Lemma A.9). Consequently, if $\mathfrak{A}$ is admissible, it implies that $\mathtt{Afra}(\mathfrak{A})$ is AFRA-admissible. □

On the other hand, the result of Proposition 5 does not hold for the admissible semantics. For instance, considering the argumentation framework of

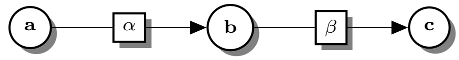

Figure 10: A Dung's framework with two attacks

Figure 10, the set $\mathcal{E} = \{\alpha, c\}$ is AFRA-admissible, but the corresponding structure $\mathfrak{A}_{\mathcal{E}} = \langle \{c\}, \{\alpha, \beta\} \rangle$ is not an admissible structure (because a is not in the structure). This discrepancy follows from the fact that, in AFRA, α defeats β despite of the absence of a while in our approach attacks whose source is not accepted cannot defeat other arguments or attacks. This difference disappears if we consider what we call *closed* sets. We say that $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$ is *closed* iff every attack $\alpha \in (\mathcal{E} \cap \mathbf{K})$ satisfies $\mathbf{s}(\alpha) \in \mathcal{E}$. Then, we have the following result:

Lemma 4.1. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some recursive framework and* $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$ *be some closed AFRA-conflict-free extension. Then, every AFRA-acceptable element* x *w.r.t.* $\mathcal{E}$ *satisfies* $x \in \mathit{Acc}(\mathfrak{A}_{\mathcal{E}})$. □

Proof. Let $x \in (\mathbf{A} \cup \mathbf{K})$ be an AFRA-acceptable element w.r.t. $\mathcal{E}$ and pick any attack $\alpha \in \mathbf{K}$ such that $\mathbf{t}(\alpha) = x$. Since x is AFRA-acceptable w.r.t. $\mathcal{E}$, there is $\beta \in \mathcal{E}$ such that β defeats α. Note that $\beta \in \mathcal{E}$ implies that $\beta \in \Gamma_{\mathcal{E}}$ and that β defeats α implies that either $\mathbf{t}(\beta) = \alpha$ or $\mathbf{t}(\beta) = \mathbf{s}(\alpha)$ holds.

Furthermore, as $\mathcal{E}$ is assumed to be closed, $\beta \in \mathcal{E}$ implies that $\mathbf{s}(\beta) \in \mathcal{E}$. Then, this implies $\mathbf{s}(\beta) \in S_{\mathcal{E}}$ and, thus, that $\mathbf{t}(\beta) \in (\mathit{Inh}(\mathfrak{A}_{\mathcal{E}}) \cup \mathit{Def}(\mathfrak{A}_{\mathcal{E}}))$ holds.

Hence, the fact that either $\mathbf{t}(\beta) = \alpha$ or $\mathbf{t}(\beta) = \mathbf{s}(\alpha)$ holds implies that either $\alpha \in \mathit{Inh}(\mathfrak{A}_{\mathcal{E}})$ or $\mathbf{s}(\alpha) \in \mathit{Def}(\mathfrak{A}_{\mathcal{E}})$ must also hold and thus, $x \in \mathit{Acc}(\mathfrak{A}_{\mathcal{E}})$. □

Lemma 4.2. *Given a recursive framework* **RAF** *and a AFRA-admissible closed set* $\mathcal{E} \subseteq \mathbf{A} \cup \mathbf{K}$*, it follows that* $\tilde{\mathfrak{A}}_{\mathcal{E}}$ *is admissible.* □

Proof. First note that, since $\mathcal{E}$ is an AFRA-admissible extension, it is also AFRA-conflict-free and, from Proposition 5, it follows that $\mathfrak{A}_{\mathcal{E}}$ is a conflict-free structure. Furthermore, it is easy to see that $\tilde{\mathfrak{A}}_{\mathcal{E}} \sqsubseteq \mathfrak{A}_{\mathcal{E}}$ and, thus, $\tilde{\mathfrak{A}}_{\mathcal{E}}$ is also conflict-free. In addition, since $\mathcal{E}$ is AFRA-admissible, then every element belonging to $\mathcal{E}$ is AFRA-acceptable w.r.t. $\mathcal{E}$. Then, from Lemma 4.1 this implies $(S_{\mathcal{E}} \cup \tilde{\Gamma}_{\mathcal{E}}) = \mathcal{E} \subseteq \mathit{Acc}(\mathfrak{A}_{\mathcal{E}})$. Thus, $\tilde{\mathfrak{A}}_{\mathcal{E}}$ is admissible. □

Proposition 7. *Given some recursive framework* **RAF** *and an AFRA-admissible closed set* $\mathcal{E} \subseteq \mathbf{A} \cup \mathbf{K}$*, it follows that* $\mathfrak{A}_{\mathcal{E}}$ *is admissible.* □

Proof. From Lemma 4.2, it follows that $\tilde{\mathfrak{A}}_{\mathcal{E}}$ is admissible. Then, from the Fundamental Lemma, it directly follows that $\mathfrak{A}_{\mathcal{E}}$ is also admissible. □

An interesting observation is that all AFRA-complete extensions are closed sets.[6] Then, building on this observation plus Propositions 4, 5, 6 and 7, we

[6]See Lemma A.16 in the Appendix.

can prove the two following propositions:[7]

Proposition 8. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *and a structure* $\mathfrak{A} = \langle S, \Gamma \rangle$*, the following assertions hold:*

i) if $\mathfrak{A}$ *is complete, then* $\mathtt{Afra}(\mathfrak{A})$ *is AFRA-complete,*
ii) if $\mathfrak{A}$ *is grounded, then* $\mathtt{Afra}(\mathfrak{A})$ *is AFRA-grounded,*
iii) if $\mathfrak{A}$ *is preferred, then* $\mathtt{Afra}(\mathfrak{A})$ *is AFRA-preferred,*
iv) if $\mathfrak{A}$ *is stable, then* $\mathtt{Afra}(\mathfrak{A})$ *is AFRA-stable.* □

Example 2 (cont'd) For the framework represented in Figure 2, there is a unique AFRA-complete (preferred, stable) extension: $\mathcal{E} = \{a, g, p, \alpha, \epsilon, \gamma\}$. Recall that $\mathfrak{A}_0 = \langle \{a, g, p\}, \{\alpha, \epsilon, \gamma, \delta\} \rangle$ and, thus, we have $\mathcal{E} = \mathtt{Afra}(\mathfrak{A}_0)$, but $\delta \notin \mathcal{E}$. Indeed, no AFRA-admissible extension contains δ. Analogously, we have $\mathtt{Afra}(\mathfrak{A}_1) = \mathtt{Afra}(\mathfrak{A}_2) = \mathtt{Afra}(\mathfrak{A}_3) = \{a, \epsilon\}$. Moreover, among the AFRA-admissible extensions, we find $\{a, g, \epsilon, \gamma\}$ which is not closed. The associated structure $\mathfrak{A}_4 = \langle \{a, g\}, \{\epsilon, \gamma\} \rangle$ is not an admissible structure. □

The converse of Proposition 8 also holds:

Proposition 9. *Given a* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *and a set* $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$*, the following assertions hold:*

i) if $\mathcal{E}$ *is AFRA-complete, then* $\mathfrak{A}_{\mathcal{E}}$ *is a complete structure,*
ii) if $\mathcal{E}$ *is AFRA-grounded, then* $\mathfrak{A}_{\mathcal{E}}$ *is a grounded structure,*
iii) if $\mathcal{E}$ *is AFRA-preferred, then* $\mathfrak{A}_{\mathcal{E}}$ *is a preferred structure,*
iv) if $\mathcal{E}$ *is AFRA-stable, then* $\mathfrak{A}_{\mathcal{E}}$ *is a stable structure.* □

Interestingly, for the complete semantics, the following property holds:

Proposition 10. *The following assertions hold:*

i) if $\mathcal{E}$ *is AFRA-complete (or just a closed AFRA-conflict-free extension), then it follows that* $\mathtt{Afra}(\mathfrak{A}_{\mathcal{E}}) = \mathcal{E}$*, and*
ii) if $\mathfrak{A}$ *is a complete structure, then* $\mathfrak{A}_{\mathtt{Afra}(\mathfrak{A})} = \mathfrak{A}$. □

Proof. First, note that by definition $S_{\mathcal{E}} = (\mathcal{E} \cap \mathbf{A})$ and, therefore, it holds that $(\mathtt{Afra}(\mathfrak{A}_{\mathcal{E}}) \cap \mathbf{A}) = (\mathcal{E} \cap \mathbf{A})$. Furthermore, $\alpha \in (\mathtt{Afra}(\mathfrak{A}_{\mathcal{E}}) \cap \mathbf{K})$ satisfies that $\alpha \in \Gamma_{\mathcal{E}}$ and $\mathbf{s}(\alpha) \in S_{\mathcal{E}}$. Moreover, note that $\alpha \in \Gamma_{\mathcal{E}}$ implies that either $\alpha \in (\mathcal{E} \cap \mathbf{K}) \subseteq \mathcal{E}$ or $\mathbf{s}(\alpha) \notin \mathcal{E}$. However, the latter is a contradiction with the facts that $\mathbf{s}(\alpha) \in S_{\mathcal{E}}$ and $S_{\mathcal{E}} = (\mathcal{E} \cap \mathbf{A}) \subseteq \mathcal{E}$. Hence, it follows that $(\mathtt{Afra}(\mathfrak{A}_{\mathcal{E}}) \cap \mathbf{K}) \subseteq (\mathcal{E} \cap \mathbf{K})$ holds. This plus $(\mathtt{Afra}(\mathfrak{A}_{\mathcal{E}}) \cap \mathbf{A}) = (\mathcal{E} \cap \mathbf{A})$ imply $(\mathtt{Afra}(\mathfrak{A}_{\mathcal{E}}) \subseteq \mathcal{E})$. Assume now that $\mathcal{E}$ is closed (it is the case if $\mathcal{E}$ is complete, due to Lemma A.16) and pick $\alpha \in (\mathcal{E} \cap \mathbf{K})$. By definition, it follows that $\alpha \in \Gamma_{\mathcal{E}}$. Furthermore, since $\alpha \in \mathcal{E}$ and $\mathcal{E}$ is closed, it follows that $\mathbf{s}(\alpha) \in \mathcal{E}$ and, thus, that $\mathbf{s}(\alpha) \in S_{\mathcal{E}}$. Consequently, it follows that $\alpha \in \mathtt{Afra}(\mathfrak{A}_{\mathcal{E}})$ and that $\mathtt{Afra}(\mathfrak{A}_{\mathcal{E}}) = \mathcal{E}$ holds.

For ii), let $\mathfrak{A}$ be of the form $\mathfrak{A} = \langle S, \Gamma \rangle$ and note that $S_{\mathtt{Afra}(\mathfrak{A})} = S$ is easy to check. Then to show that $\Gamma \subseteq \Gamma_{\mathtt{Afra}(\mathfrak{A})}$, pick any attack $\alpha \in \Gamma$. If $\mathbf{s}(\alpha) \in$

[7] The proofs can be found in the Appendix.

S, then $\alpha \in (\texttt{Afra}(\mathfrak{A}) \cap \mathbf{K})$ and thus, $\alpha \in \Gamma_{\texttt{Afra}(\mathfrak{A})}$. Otherwise, $\mathbf{s}(\alpha) \notin S$ and $\alpha \in (Acc(\mathfrak{A}) \cap \mathbf{K})$ because $\mathfrak{A} = \langle S, \Gamma \rangle$ is admissible. Hence, $\alpha \in \Gamma_{\texttt{Afra}(\mathfrak{A})}$ (Lemma A.10) and $\Gamma \subseteq \Gamma_{\texttt{Afra}(\mathfrak{A})}$ follows. The other way around. Let Γ' denote $\texttt{Afra}(\mathfrak{A}) \cap \mathbf{K}$, and $\mathfrak{A}'$ denote the structure $\langle S, \Gamma' \rangle$. $\alpha \in \Gamma_{\texttt{Afra}(\mathfrak{A})}$ implies that either $\alpha \in \Gamma' \subseteq \Gamma$ or both $\mathbf{s}(\alpha) \notin \texttt{Afra}(\mathfrak{A})$ and $\alpha \in Acc(\mathfrak{A}')$. Furthermore, the latter plus $S_{\texttt{Afra}(\mathfrak{A})} = S$ imply that $\alpha \in Acc(\mathfrak{A}')$. Note that this plus $\mathfrak{A}' \sqsubseteq \mathfrak{A}$ imply $\alpha \in Acc(\mathfrak{A})$ (Observation 2) and, since $\mathfrak{A}$ is complete, this implies that $\alpha \in \Gamma$. Therefore, $\Gamma_{\texttt{Afra}(\mathfrak{A})} = \Gamma$ and $\mathfrak{A}_{\texttt{Afra}(\mathfrak{A})} = \mathfrak{A}$ holds. □

Taking together Propositions 8, 9 and 10, we can prove the following one-to-one correspondence:

Theorem 3. *For each semantics $\sigma \in \{$complete, stable, preferred, grounded$\}$: The function* $\texttt{Afra}(\cdot)$ *is a one-to-one correspondence between the sets of all σ-structures and the set of all AFRA-σ-extensions.* □

Proof. First, note that for every complete (resp. preferred, stable, grounded) structure $\mathfrak{A}$, we have that $\texttt{Afra}(\mathfrak{A})$ is an AFRA-complete (resp. preferred, stable, grounded) extension (Proposition 8). To see that $\texttt{Afra}(\cdot)$ is injective take two complete structures $\mathfrak{A}$ and $\mathfrak{A}'$ such that $\texttt{Afra}(\mathfrak{A}) = \texttt{Afra}(\mathfrak{A}')$. Then, it is obvious that $\mathfrak{A}_{\texttt{Afra}(\mathfrak{A})} = \mathfrak{A}_{\texttt{Afra}(\mathfrak{A}')}$ and, from Proposition 10, we obtain that $\mathfrak{A} = \mathfrak{A}'$. That is, that $\texttt{Afra}(\cdot)$ is injective. Finally, note that for any AFRA-complete (resp. preferred, stable, grounded) extension $\mathcal{E}$, we have that $\mathfrak{A}_{\mathcal{E}}$ is a complete (resp. preferred, stable, grounded) structure (Proposition 9), and that $\texttt{Afra}(\mathfrak{A}_{\mathcal{E}}) = \mathcal{E}$, so $\texttt{Afra}(\cdot)$ is surjective. □

Note that given the one-to-one correspondence between preferred structures and AFRA-preferred extensions, there are AFRA-preferred extensions that do not correspond to arg-preferred ones and thus, they do not contain a maximal set of arguments. For instance, $\{a, b, \beta\}$ and $\{a, \alpha\}$ are both AFRA-preferred extensions in Example 4, but only the former contains a maximal set of arguments. Note also that for conflict-freeness and admissibility, the correspondence is not necessarily one-to-one. For instance, both $\mathfrak{A} = \langle \{a, c\}, \{\alpha\} \rangle$ and $\mathfrak{A}' = \langle \{a, c\}, \{\alpha, \beta\} \rangle$ are admissible structures of the framework of Figure 10 and both of them correspond to the same AFRA-admissible set $\texttt{Afra}(\mathfrak{A}) = \texttt{Afra}(\mathfrak{A}') = \{a, c, \alpha\}$. Recall that β is acceptable w.r.t. $\mathfrak{A}'$ because it is not attacked. However, it is not AFRA-acceptable w.r.t. $\{a, c, \alpha, \beta\}$ because, in AFRA, α defeats β and α is not itself defeated (in fact, $\{a, c, \alpha, \beta\}$ is not even AFRA-conflict-free).

An interesting consequence of Theorem 3 and Proposition 12 in [4] is that complexity for RAFs' semantics does not increase w.r.t. Dung's frameworks. From the results in [14] we obtain the following corollary:

Proposition 11. *The problem of credulous acceptance in a RAF w.r.t. the complete, the preferred or the stable semantics (whether there exists some preferred or stable structure containing some argument) is* NP*-complete. The problem of sceptical acceptance in a RAF w.r.t. the preferred (resp. stable) semantics is* Π_2^{P}*-complete (resp.* coNP*-complete).* □

5 Relation with MAF

In this section, we establish correspondences between our semantics for recursive frameworks and the semantics for MAF.

The idea of turning recursive frameworks into Dung's frameworks by adding meta-arguments goes back to [17], in the general case where an attack can be attacked, and an attack itself can also attack an argument. The translation approach proposed in [17] adopts the view that the target of an attack from a to b is "jointly attacked" by the attack itself and the source a. Then, a joint attack is implemented through the addition of three meta-arguments. Another approach has been proposed in [8], where an interaction can be attacked by an argument, but is not allowed to attack an argument or another interaction. In [8] a recursive framework is turned into a Dung's framework using the addition of meta-arguments that enable to encode the notions of "grounded" attack and "valid" attack. The idea is that there are two ways for weakening a higher-order attack: either by attacking it, or by weakening its source (as in a D-framework). The first way impacts the "validity" of the attack, whereas the second way impacts the "groundness" of the attack. Groundness is taken into account through a kind of supporting link between an attack and its source, which is encoded by adding a meta-argument. The following example illustrates these notions.

Example 6. *Consider the RAF depicted in Figure 11. According to [8], Fig-*

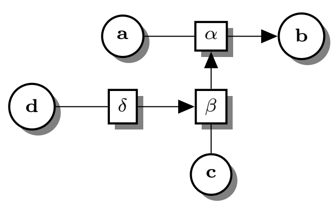

Figure 11: A RAF with two higher-order attacks

ure 12 depicts the MAF associated with this RAF, with attacks becoming new arguments in the MAF: For instance, the attack from a to b becomes a new argument α in the MAF.

In addition the meta-argument $N_{a\alpha}$ encodes the link between α and its source a, ensuring that α can be accepted only if its source a is accepted. □

In the case when the source of an attack is restricted to be an argument (as in a RAF), there is a correspondence between the encodings proposed by [8] and [17]. So, in this section, we have chosen to focus on the approach of [8], since it is more suited for establishing correspondences with RAF semantics; indeed, an attack is turned into an attack name, as in a RAF, and attack names may appear in MAF extensions as arguments.

As a MAF is a D-framework, Definition 3 can be applied for defining semantics of the MAF. For instance, the preferred extension of the MAF pictured in

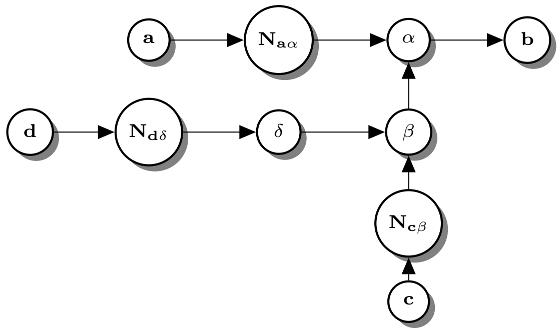

Figure 12: MAF for RAF of Figure 11 (following [8])

Figure 12 is $\{a, c, d, \alpha, \delta\}$.

Given a RAF and its associated MAF, correspondences can be established between RAF-structures and MAF-extensions, for various semantics. Formally, the MAF associated with a given RAF is defined as follows:

Definition 12 (From [8]). *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be a recursive framework. The associated MAF of this RAF is* $\mathbf{MAF} = \langle \mathbf{A}', \mathbf{R}' \rangle$ *with:*

- $\mathbf{A}' = \mathbf{A} \cup \mathbf{K} \cup \{N_{\mathbf{s}(\alpha)\alpha} | \alpha \in \mathbf{K}\}$ *(this last subset will be denoted by* $\mathbf{N}$*),*
- $\mathbf{R}' = \{(\mathbf{s}(\alpha), N_{\mathbf{s}(\alpha)\alpha}) | \alpha \in \mathbf{K}\} \cup \{(N_{\mathbf{s}(\alpha)\alpha}, \alpha) | \alpha \in \mathbf{K}\} \cup \{(\alpha, \mathbf{t}(\alpha)) | \alpha \in \mathbf{K}\}$. □

The following results directly follow from Definition 12:

Observation 4. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *and its associated* $\mathbf{MAF} = \langle \mathbf{A}', \mathbf{R}' \rangle$.

i) There are only three types of attacks in $\mathbf{R}'$*: either from* $\mathbf{A}$ *to* $\mathbf{N}$*, or from* $\mathbf{N}$ *to* $\mathbf{K}$*, or from* $\mathbf{K}$ *to* $\mathbf{A} \cup \mathbf{K}$.

ii) For all $N_{a\alpha} \in \mathbf{N}$*,* $N_{a\alpha}$ *is involved in only two attacks belonging to* $\mathbf{R}'$*:* $(a, N_{a\alpha})$ *and* $(N_{a\alpha}, \alpha)$.

iii) For all $\alpha \in \mathbf{K}$*,* $\mathbf{s}(\alpha)$ *is the only attacker of* $N_{\mathbf{s}(\alpha)\alpha}$ *and so the only defender of* α *against* $N_{\mathbf{s}(\alpha)\alpha}$.

iv) For all $a \in \mathbf{A}$*,* a *is unattacked in* $\mathbf{RAF}$ *iff* a *is unattacked in* $\mathbf{MAF}$.

v) For all $\alpha \in \mathbf{K}$*,* α *is always attacked in* $\mathbf{MAF}$. □

A MAF-extension can be associated with a given structure, and conversely a RAF-structure can be associated with a given MAF-extension.

Notation 1 (From RAF-structures to MAF-extensions). *Given a* $\mathbf{RAF}$ *structure* $\mathfrak{A} = \langle S, \Gamma \rangle$*:*

- $\mathcal{E}'_{\mathfrak{A}}$ *denotes the set* $S \cup \{\alpha \in \Gamma$ *s.t.* $\mathbf{s}(\alpha) \in S\}$.

- $\mathtt{Maf}(\mathfrak{A})$ *denotes the set* $\mathcal{E}'_{\mathfrak{A}} \cup \{N_{\mathbf{s}(\alpha)\alpha}$ *s.t.* $\mathbf{s}(\alpha) \notin S$ *and* $\mathbf{s}(\alpha) \in Def(\mathfrak{A})\}$.

In other words, $\mathcal{E}'_{\mathfrak{A}}$ is made of the arguments of S, and the attacks of Γ whose source belongs to S. $\mathtt{Maf}(\mathfrak{A})$ is obtained from $\mathcal{E}'_{\mathfrak{A}}$ by adding the elements $N_{\mathbf{s}(\alpha)\alpha}$ of $\mathbf{N}$ such that $\alpha \notin \mathcal{E}'_{\mathfrak{A}}$ and $N_{\mathbf{s}(\alpha)\alpha}$ is defended by $\mathcal{E}'_{\mathfrak{A}}$.[8] The following properties directly result from Observation 4:

Observation 5. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *and its associated* $\mathbf{MAF} = \langle \mathbf{A}', \mathbf{R}' \rangle$*. Let* $a \in \mathbf{A}$ *and* $\alpha \in \mathbf{K}$*,*

i) If α *is acceptable w.r.t.* $\mathtt{Maf}(\mathfrak{A})$ *in* $\mathbf{MAF}$*, then* $\mathbf{s}(\alpha) \in (\mathtt{Maf}(\mathfrak{A}) \cap \mathbf{A}) = S$*.*

ii) $a \in Def(\mathfrak{A})$ *iff* $\mathtt{Maf}(\mathfrak{A})$ *defeats* a *in* $\mathbf{MAF}$*.*

iii) $\alpha \in Inh(\mathfrak{A})$ *iff* $\mathtt{Maf}(\mathfrak{A}) \cap \mathbf{K}$ *defeats* α *in* $\mathbf{MAF}$*.*

Notation 2 (From MAF-extensions to RAF-structures)**.** *Let* $\langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be a RAF and its associated* $\mathbf{MAF} = \langle \mathbf{A}', \mathbf{R}' \rangle$*. Given some set* $\mathcal{E} \subseteq \mathbf{A}'$*:*

- $\mathcal{E}_a$ *(resp.* $\mathcal{E}_k$*,* $\mathcal{E}_n$*) denotes the set* $\mathcal{E} \cap \mathbf{A}$ *(resp.* $\mathcal{E} \cap \mathbf{K}$*,* $\mathcal{E} \cap \mathbf{N}$*).*
- $\Gamma_{\mathcal{E}}$ *denotes the set* $\mathcal{E}_k \cup \{\alpha \in (\mathbf{K} \setminus \mathcal{E}_k)$ *s.t.* $\alpha \in Acc(\mathfrak{A}'_{\mathcal{E}})$ *and* $\mathbf{s}(\alpha) \notin \mathcal{E}_a\}$*, where* $\mathfrak{A}'_{\mathcal{E}}$ *denotes the structure* $\langle \mathcal{E}_a, \mathcal{E}_k \rangle$*.*
- $\mathfrak{A}_{\mathcal{E}}$ *denotes the structure* $\langle S_{\mathcal{E}}, \Gamma_{\mathcal{E}} \rangle$ *where* $S_{\mathcal{E}} = \mathcal{E}_a$*.*

In other words, $\Gamma_{\mathcal{E}}$ contains the attacks that belong to $\mathcal{E}$, and also the attacks that do not belong to $\mathcal{E}$ but are acceptable w.r.t. $\mathfrak{A}'_{\mathcal{E}}$, even if they are not acceptable w.r.t. $\mathcal{E}$, because of their source. Intuitively, this is due to the fact that an attack α cannot be acceptable w.r.t. $\mathcal{E}$ if $\mathbf{s}(\alpha) \notin \mathcal{E}$, whereas this is not a problem for structure-based semantics.

Example 6 (cont'd) There is only one complete (resp. preferred, grounded, stable) structure in this RAF: $\mathfrak{A} = \langle \{a, c, d\}, \{\alpha, \delta\} \rangle$. We have $\mathcal{E}'_{\mathfrak{A}} = \mathtt{Maf}(\mathfrak{A}) = \{a, c, d, \alpha, \delta\}$. Conversely, consider the set $\mathcal{E} = \{a, c, d, \alpha, \delta\}$ in the MAF. Then, $\mathfrak{A}'_{\mathcal{E}} = \mathfrak{A}_{\mathcal{E}} = \langle \{a, c, d\}, \{\alpha, \delta\} \rangle$. □

Example 7. *Consider the RAF depicted in Figure 10. The associated MAF is represented in Figure 13. Then, for RAF-structure* $\mathfrak{A} = \langle \{a, c\}, \{\alpha, \beta\} \rangle$*, we have*

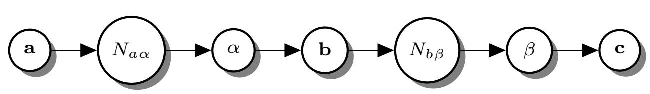

Figure 13: MAF for RAF of Figure 10

that β *belongs to* Γ*, but its source* b *does not belong to* S*. So,* $\mathcal{E}'_{\mathfrak{A}} = \{a, c, \alpha\}$ *and* $\mathtt{Maf}(\mathfrak{A}) = \{a, c, \alpha, N_{b\beta}\}$*. For the set* $\mathcal{E} = \{a, \alpha, N_{b\beta}\}$ *in the MAF, we have* $\mathfrak{A}'_{\mathcal{E}} = \langle \{a\}, \{\alpha\} \rangle$ *and that* β *is acceptable w.r.t.* $\mathfrak{A}'_{\mathcal{E}}$*; so* $\Gamma_{\mathcal{E}} = \{\alpha, \beta\}$ *and* $\mathfrak{A}_{\mathcal{E}} = \langle \{a\}, \{\alpha, \beta\} \rangle$*.* □

[8] This last condition is mandatory for proving the formal links between RAF and MAF.

As illustrated by the above example, there is a correspondence between the complete (stable, preferred, grounded) RAF-structures and the corresponding MAF-extensions, according to the following result:

Proposition 12. *Given* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *and its associated* $\mathbf{MAF} = \langle \mathbf{A}', \mathbf{R}' \rangle$*. For each semantics* $\sigma \in \{$*complete, stable, preferred, grounded*$\}$*:*

i) For each σ*-structure* $\mathfrak{A}$ *of* $\mathbf{RAF}$*,* $\mathtt{Maf}(\mathfrak{A})$ *is a* σ*-extension of* $\mathbf{MAF}$*.*

ii) For each σ*-extension* $\mathcal{E}$ *of* $\mathbf{MAF}$*,* $\mathfrak{A}_{\mathcal{E}}$ *is a* σ*-structure of* $\mathbf{RAF}$*.*

The above result can be proved[9] in a similar way as for Propositions 8 and 9, but using the following results about conflict-freeness and acceptability requirements for MAF.

Proposition 13. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *and its associated* $\mathbf{MAF} = \langle \mathbf{A}', \mathbf{R}' \rangle$*.*

i) Let $\mathfrak{A}$ *be a conflict-free structure in* $\mathbf{RAF}$*. Then, both* $\mathcal{E}'_{\mathfrak{A}}$ *and* $\mathtt{Maf}(\mathfrak{A})$ *are conflict-free in* $\langle \mathbf{A}', \mathbf{R}' \rangle$*.*

ii) Let $\mathcal{E}$ *be a conflict-free subset in* $\langle \mathbf{A}', \mathbf{R}' \rangle$*. Then, both* $\mathfrak{A}'_{\mathcal{E}}$ *and* $\mathfrak{A}_{\mathcal{E}}$ *are conflict-free structures in* $\mathbf{RAF}$*.*

Proof.
For i), let $\mathfrak{A} = \langle S, \Gamma \rangle$ be a conflict-free structure in $\mathbf{RAF}$.

1. Assume that $\mathcal{E}'_{\mathfrak{A}} = S \cup \{\alpha \in \Gamma \text{ s.t. } \mathbf{s}(\alpha) \in S\}$ is not conflict-free in $\langle \mathbf{A}', \mathbf{R}' \rangle$. Due to Observation 4, the only possible conflict comes from an attack of the form $(\alpha, \mathbf{t}(\alpha))$ with $\alpha \in (\mathcal{E}'_{\mathfrak{A}} \cap \Gamma)$ and $\mathbf{t}(\alpha) \in \mathcal{E}'_{\mathfrak{A}}$ (*i.e.* $\mathbf{t}(\alpha) \in S \subseteq \mathbf{A}$ or $\mathbf{t}(\alpha) \in (\mathcal{E}'_{\mathfrak{A}} \cap \Gamma) \subseteq \mathbf{K}$). Moreover, for $\alpha \in (\mathcal{E}'_{\mathfrak{A}} \cap \Gamma)$ we have $\mathbf{s}(\alpha) \in S$; so, $\mathbf{t}(\alpha)$ is either defeated (if $\mathbf{t}(\alpha) \in \mathbf{A}$) or inhibited (if $\mathbf{t}(\alpha) \in \mathbf{K}$) w.r.t. $\mathfrak{A}$. And so there is a contradiction with $\mathfrak{A}$ being conflict-free.

2. Assume that $\mathtt{Maf}(\mathfrak{A}) = \mathcal{E}'_{\mathfrak{A}} \cup \{N_{\mathbf{s}(\alpha)\alpha} \text{ s.t. } \mathbf{s}(\alpha) \notin S \text{ and } \mathbf{s}(\alpha) \in Def(\mathfrak{A})\}$ is not conflict-free. From the first part of the proof, the only possible conflict comes from an attack from S to $(\mathtt{Maf}(\mathfrak{A}) \cap \mathbf{N})$ or from $(\mathtt{Maf}(\mathfrak{A}) \cap \mathbf{N})$ to $(\mathtt{Maf}(\mathfrak{A}) \cap \Gamma)$.
   In the first case, there is an attack of the form $(\mathbf{s}(\alpha), N_{\mathbf{s}(\alpha)\alpha})$ with $\mathbf{s}(\alpha) \in S$ and $N_{\mathbf{s}(\alpha)\alpha} \in (\mathtt{Maf}(\mathfrak{A}) \cap \mathbf{N})$. However, for $N_{\mathbf{s}(\alpha)\alpha} \in (\mathtt{Maf}(\mathfrak{A}) \cap \mathbf{N})$, we have $\mathbf{s}(\alpha) \notin S$. So there is a contradiction about $\mathbf{s}(\alpha)$.
   In the second case, there is an attack of the form $(N_{\mathbf{s}(\alpha)\alpha}, \alpha)$ with $N_{\mathbf{s}(\alpha)\alpha} \in (\mathtt{Maf}(\mathfrak{A}) \cap \mathbf{N})$ and $\alpha \in (\mathtt{Maf}(\mathfrak{A}) \cap \Gamma)$. However, for $\alpha \in (\mathtt{Maf}(\mathfrak{A}) \cap \Gamma)$, we have $\mathbf{s}(\alpha) \in S$ so there is a contradiction with $N_{\mathbf{s}(\alpha)\alpha} \in (\mathtt{Maf}(\mathfrak{A}) \cap \mathbf{N})$.

For ii), let $\mathcal{E}$ be a conflict-free subset in $\langle \mathbf{A}', \mathbf{R}' \rangle$.

[9]The formal proof can be found in the Appendix.

1. Assume that $\mathfrak{A}'_{\mathcal{E}} = \langle \mathcal{E}_a, \mathcal{E}_k \rangle$ is not a conflict-free structure in **RAF**. So either $\mathcal{E}_a \cap Def(\mathfrak{A}'_{\mathcal{E}}) \neq \varnothing$ (Case 1) or $\mathcal{E}_k \cap Inh(\mathfrak{A}'_{\mathcal{E}}) \neq \varnothing$ (Case 2).
   In Case 1, there are $a \in \mathcal{E}_a, \beta \in \mathcal{E}_k$ s.t. $\mathbf{s}(\beta) \in \mathcal{E}_a$ and $\mathbf{t}(\beta) = a$. So, due to Definition 12, we have $(\beta, a) \in \mathbf{R}'$. That is in contradiction with $\mathcal{E}$ being conflict-free in the MAF.
   In Case 2, there are $\alpha \in \mathcal{E}_k, \beta \in \mathcal{E}_k$ s.t. $\mathbf{s}(\beta) \in \mathcal{E}_a$ and $\mathbf{t}(\beta) = \alpha$. So, due to Definition 12, we have $(\beta, \alpha) \in \mathbf{R}'$. That is in contradiction with $\mathcal{E}$ being conflict-free in the MAF.

2. Assume that $\mathfrak{A}_{\mathcal{E}} = \langle \mathcal{E}_a, \Gamma_{\mathcal{E}} \rangle$ is not a conflict-free structure in **RAF**. So either $\mathcal{E}_a \cap Def(\mathfrak{A}_{\mathcal{E}}) \neq \varnothing$ (Case 1) or $\Gamma_{\mathcal{E}} \cap Inh(\mathfrak{A}_{\mathcal{E}}) \neq \varnothing$ (Case 2).
   Let us recall that $\Gamma_{\mathcal{E}} = \mathcal{E}_k \cup \{\alpha \notin \mathcal{E}_k \text{ s.t. } \mathbf{s}(\alpha) \notin \mathcal{E}_a \text{ and } \alpha \in Acc(\mathfrak{A}'_{\mathcal{E}})\}$.
   In Case 1, there are $a \in \mathcal{E}_a, \beta \in \Gamma_{\mathcal{E}}$ s.t. $\mathbf{s}(\beta) \in \mathcal{E}_a$ and $\mathbf{t}(\beta) = a$. Due to the definition of $\Gamma_{\mathcal{E}}$, as $\mathbf{s}(\beta) \in \mathcal{E}_a$, we have $\beta \in \mathcal{E}_k$. So we are back to the first part of the proof (Case 1) and we get a contradiction with $\mathcal{E}$ being conflict-free in the MAF.
   In Case 2, there are $\alpha \in \Gamma_{\mathcal{E}}, \beta \in \Gamma_{\mathcal{E}}$ s.t. $\mathbf{s}(\beta) \in \mathcal{E}_a$ and $\mathbf{t}(\beta) = \alpha$. Due to the definition of $\Gamma_{\mathcal{E}}$, as $\mathbf{s}(\beta) \in \mathcal{E}_a$, we have $\beta \in \mathcal{E}_k$. Due to the first part of the proof (Case 2), we cannot have $\alpha \in \mathcal{E}_k$. So we have $\mathbf{s}(\alpha) \notin \mathcal{E}_a$ and $\alpha \in Acc(\mathfrak{A}'_{\mathcal{E}})$. From the second condition, it follows that $\beta \in Inh(\mathfrak{A}'_{\mathcal{E}})$ or $\mathbf{s}(\beta) \in Def(\mathfrak{A}'_{\mathcal{E}})$. However, $\mathbf{s}(\beta) \in \mathcal{E}_a$, and $\beta \in \mathcal{E}_k$. Moreover $\mathfrak{A}'_{\mathcal{E}} = \langle \mathcal{E}_a, \mathcal{E}_k \rangle$ is a conflict-free structure, due to the first part of the proof. So we obtain a contradiction. □

Proposition 14. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *and its associated* $\mathbf{MAF} = \langle \mathbf{A}', \mathbf{R}' \rangle$*. Let* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be a conflict-free structure in* **RAF***. Let* $a \in \mathbf{A}$ *and* $\alpha \in \mathbf{K}$*.*

i) If a is acceptable w.r.t. $\mathfrak{A}$ in **RAF***, then a is acceptable w.r.t.* $\mathtt{Maf}(\mathfrak{A})$ *in* $\langle \mathbf{A}', \mathbf{R}' \rangle$*.*

ii) If α is acceptable w.r.t. $\mathfrak{A}$ in **RAF***, and $\mathbf{s}(\alpha) \in S$, then α is acceptable w.r.t.* $\mathtt{Maf}(\mathfrak{A})$ *in* $\langle \mathbf{A}', \mathbf{R}' \rangle$*.*

Proof.
For i), let a be acceptable w.r.t. $\mathfrak{A}$ in **RAF**. We have to prove that a is acceptable w.r.t. $\mathtt{Maf}(\mathfrak{A})$ in $\langle \mathbf{A}', \mathbf{R}' \rangle$.
If a is not attacked in **MAF**, it is trivially acceptable w.r.t. $\mathtt{Maf}(\mathfrak{A})$. So, let us assume that $a \in \mathbf{A}$ is attacked in **MAF**. Due to Observation 4, there is $\alpha \in \mathbf{K}$ with $(\alpha, a) \in \mathbf{R}'$, or in other words $\alpha \in \mathbf{K}$ with $\mathbf{t}(\alpha) = a$. So there are also in $\mathbf{R}'$ the attacks $(\mathbf{s}(\alpha), N_{\mathbf{s}(\alpha)\alpha})$ and $(N_{\mathbf{s}(\alpha)\alpha}, \alpha)$.
As a is acceptable w.r.t. $\mathfrak{A}$ in **RAF**, either $\alpha \in Inh(\mathfrak{A})$, or $\mathbf{s}(\alpha) \in Def(\mathfrak{A})$. It means that there is $\beta \in \Gamma$ s.t. $\mathbf{s}(\beta) \in S$ and $\mathbf{t}(\beta) \in \{\alpha, \mathbf{s}(\alpha)\}$. So $\beta \in (\mathtt{Maf}(\mathfrak{A}) \cap \Gamma)$.
If $\mathbf{t}(\beta) = \alpha$ then $(\beta, \alpha) \in \mathbf{R}'$ and we have that $\mathtt{Maf}(\mathfrak{A})$ attacks α.
If $\mathbf{t}(\beta) = \mathbf{s}(\alpha)$ then $\mathbf{s}(\alpha) \in Def(\mathfrak{A})$ and, since $\mathfrak{A}$ is assumed to be conflict-free, we have $\mathbf{s}(\alpha) \notin S$; so we can prove that $N_{\mathbf{s}(\alpha)\alpha} \in (\mathtt{Maf}(\mathfrak{A}) \cap \mathbf{N})$. And then $\mathtt{Maf}(\mathfrak{A})$ attacks α.

For *ii*), let α be acceptable w.r.t. $\mathfrak{A}$ in **RAF**, with $\mathbf{s}(\alpha) \in S$. We have to prove that α is acceptable w.r.t. $\mathtt{Maf}(\mathfrak{A})$ in $\langle \mathbf{A}', \mathbf{R}' \rangle$.
$\alpha \in \mathbf{K}$ is always attacked in $\langle \mathbf{A}', \mathbf{R}' \rangle$. Due to Observation 4, we have to consider two kinds of attack, namely an attack of the form $(N_{\mathbf{s}(\alpha)\alpha}, \alpha)$ and an attack of the form (γ, α) with $\gamma \in \mathbf{K}$.
In the first case, as it is assumed that $\mathbf{s}(\alpha) \in S$, we have $\mathbf{s}(\alpha) \in \mathtt{Maf}(\mathfrak{A})$. As we also have the attack $(\mathbf{s}(\alpha), N_{\mathbf{s}(\alpha)\alpha})$ in $\mathbf{R}'$, we conclude that $\mathtt{Maf}(\mathfrak{A})$ attacks $N_{\mathbf{s}(\alpha)\alpha}$.
In the second case, as α is acceptable w.r.t. $\mathfrak{A}$ in **RAF**, $\gamma \in Inh(\mathfrak{A})$, or $\mathbf{s}(\gamma) \in Def(\mathfrak{A})$. It means that there is $\beta \in \Gamma$ s.t. $\mathbf{s}(\beta) \in S$ and $\mathbf{t}(\beta) \in \{\gamma, \mathbf{s}(\gamma)\}$. So $\beta \in (\mathtt{Maf}(\mathfrak{A}) \cap \Gamma)$. As done in the first part of the proof, we prove that $\mathtt{Maf}(\mathfrak{A})$ attacks γ. □

Note that the second result of the above proposition does not hold if we drop the condition $\mathbf{s}(\alpha) \in S$, as shown on the following example:

Example 7 (cont'd) Consider the conflict-free structure $\mathfrak{A} = \langle \{a, c\}, \{\alpha\} \rangle$. We have $\mathtt{Maf}(\mathfrak{A}) = \{a, c, \alpha, N_{b\beta}\}$. β is acceptable w.r.t. $\mathfrak{A}$ since it is not attacked in the RAF. However, β is not acceptable w.r.t. $\mathtt{Maf}(\mathfrak{A})$ in the associated MAF, since b does not belong to $\mathtt{Maf}(\mathfrak{A})$.

Note also that Proposition 14 does not hold if we replace $\mathtt{Maf}(\mathfrak{A})$ by $\mathcal{E}'_{\mathfrak{A}}$.

Example 7 (cont'd) Let $\mathfrak{A} = \langle \{a, c\}, \{\alpha\} \rangle$. We have $\mathcal{E}'_{\mathfrak{A}} = \{a, c, \alpha\}$. c is acceptable w.r.t. $\mathfrak{A}$. However, c is not acceptable w.r.t. $\mathcal{E}'_{\mathfrak{A}}$ in the associated MAF.

Moreover, Example 7 illustrates one difference about acceptability between MAF and RAF:

Example 7 (cont'd) Consider the set $\mathcal{E} = \{a, \alpha, N_{b\beta}\}$ in the MAF. $\mathfrak{A}'_{\mathcal{E}} = \langle \{a\}, \{\alpha\} \rangle$. β is acceptable w.r.t. $\mathfrak{A}'_{\mathcal{E}}$, whereas it is not acceptable w.r.t. $\mathcal{E}$ since $b \notin \mathcal{E}$.

As in the case of AFRA, we also have a one-to-one correspondence for complete, grounded, preferred and stable semantics:

Theorem 4. *For each semantics $\sigma \in \{complete, stable, preferred, grounded\}$: The function $\mathtt{Maf}(\cdot)$ is a one-to-one correspondence between the sets of all σ-structures and the set of all σ-extensions.* □

The proof of Theorem 4 is also similar to the proof of Theorem 3 for AFRA, but using the following result:

Proposition 15. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *and its associated* $\mathbf{MAF} = \langle \mathbf{A}', \mathbf{R}' \rangle$. *The following assertions hold:*

i) If $\mathfrak{A}$ is a complete structure of **RAF**, *then* $\mathfrak{A}_{\mathtt{Maf}(\mathfrak{A})} = \mathfrak{A}$.

ii) If $\mathcal{E}$ is a complete extension of **MAF**, *then* $\mathtt{Maf}(\mathfrak{A}_{\mathcal{E}}) = \mathcal{E}$. □

6 Conservative generalisation

As mentioned in the introduction, our theory aims to be a conservative generalisation of Dung's theory (**P2**). Indeed, given the one-to-one correspondence between complete, preferred, grounded and stable structures and their corresponding AFRA-extensions and between the latter and Dung's extensions [4] in the case of non-recursive frameworks, it immediately follows that there exists a one-to-one correspondence between complete, preferred, grounded and stable structures and their corresponding Dung's extensions.

On the other hand, this is not the case when we consider only conflict-freeness or admissibility. As mentioned in the introduction, $\{a, b\}$ is an AFRA-conflict-free extension of the non-recursive argumentation framework of Example 3. From Proposition 5, this implies that the corresponding structure $\langle\{a, b\}, \varnothing\rangle$, is a conflict-free structure.

It is worth to note that, in Dung's argumentation frameworks, every attack is considered as "valid" in the sense that it may affect its target. The following definition strengthens the notion of structure by adding a kind of reinstatement principle on attacks, that forces every attack that cannot be defeated to be "valid".

Definition 13 (D-structure)**.** *A* d-structure $\mathfrak{A} = \langle S, \Gamma\rangle$ *is a structure that satisfies* $(Acc(\mathfrak{A}) \cap \mathbf{K}) \subseteq \Gamma$. □

Definition 14 (Semantics with D-structures)**.** *A* conflict-free *(resp.* naive, admissible, complete, preferred, grounded, stable*)* d-structure *is a conflict-free (resp. naive, admissible, complete, preferred, grounded, stable) structure which is also a d-structure.* □

As a direct consequence of Definition 7, we have:

Observation 6. *Every complete structure is also a d-structure.* □

Observation 6 plus Theorem 3 immediately imply the existence of a one-to-one correspondence between complete (resp. grounded, preferred or stable) d-structures and their corresponding AFRA and Dung's extensions. In order to establish a correspondence between conflict-free (resp. admissible) d-structures and their corresponding Dung's extensions, we need to define what it means for a set of arguments to be an extension of some recursive framework.

Definition 15 (Argument extensions)**.** *Given* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t}\rangle$*. Let* $S \subseteq \mathbf{A}$ *be a set of arguments.* S *is* conflict-free *(resp. naive,* admissible, complete, preferred, grounded, stable*) w.r.t.* $\mathbf{RAF}$ *iff there is some* $\Gamma \subseteq \mathbf{K}$ *such that* $\mathfrak{A} = \langle S, \Gamma\rangle$ *is a conflict-free (resp. naive, admissible, complete, preferred, grounded, stable) d-structure of* $\mathbf{RAF}$*.* □

Definition 15 allows us to talk about sets of arguments instead of structures. Before formalising the fact that Definition 15 characterizes a conservative generalisation of Dung's argumentation framework, we define the attack relation associated with some framework in a similar way to the attack relation associated with some structure: $\mathbf{R_{RAF}} \stackrel{\text{def}}{=} \{ (\mathbf{s}(\alpha), \mathbf{t}(\alpha)) \mid \alpha \in \mathbf{K} \}$. Note that, since every structure $\mathfrak{A} = \langle S, \Gamma \rangle$ satisfies $\Gamma \subseteq \mathbf{K}$, it clearly follows that $\mathbf{R}_{\mathfrak{A}} \subseteq \mathbf{R_{RAF}}$. We also precise what we mean by non-recursive framework:

Definition 16 (Non-recursive framework)**.** *A framework* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *is said to be* non-recursive *iff* $\mathbf{R_{RAF}} \subseteq \mathbf{A} \times \mathbf{A}$*.* ◻

That is, non-recursive frameworks are those in which no attack targets another attack. Given a non-recursive framework **RAF**, it is easy to observe that $\mathbf{AF} = \langle \mathbf{A}, \mathbf{R_{RAF}} \rangle$ is a D-framework (Definition 1). In this sense, by $\mathbf{RAF}^D \stackrel{\text{def}}{=} \langle \mathbf{A}, \mathbf{R_{RAF}} \rangle$, we denote the D-framework associated with some **RAF**.

Observation 7. *Every d-structure* $\mathfrak{A} = \langle S, \Gamma \rangle$ *of any non-recursive framework satisfies* $\Gamma = \mathbf{K}$*.* ◻

Proof. Pick any $\alpha \in \mathbf{K}$. Since **RAF** is non-recursive, there is no $\beta \in \mathbf{K}$ s.t. $\mathbf{t}(\beta) = \alpha$ and thus $\alpha \in Acc(\mathfrak{A})$. As $\mathfrak{A}$ is a d-structure, $\alpha \in \Gamma$. ◻

Proposition 16. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some non-recursive framework and* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be some d-structure. Then, any argument* $a \in \mathbf{A}$ *satisfies:* $a \in Def(\mathfrak{A})$ *iff it is defeated w.r.t.* S *in* $\mathbf{RAF}^D$ *(Definition 2).* ◻

Proof. Recall that, by definition, it follows that $\mathbf{RAF}^D = \langle \mathbf{A}, \mathbf{R_{RAF}} \rangle$. Then, from Observation 7, it follows that $\mathbf{R}_{\mathfrak{A}} = \mathbf{R_{RAF}}$ for every d-structure $\mathfrak{A}$. Then, the result follows by observing that the definition of $Def(\mathfrak{A})$ (Equation (1)) is obtained from the defeated definition (Definition 2) by just replacing relation $\mathbf{R_{RAF}}$ by $\mathbf{R}_{\mathfrak{A}}$. ◻

Proposition 17. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some non-recursive framework and* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be some conflict-free d-structure. Then, any argument* $a \in \mathbf{A}$ *satisfies:* $a \in Acc(\mathfrak{A})$ *iff* a *is acceptable w.r.t.* S *in* $\mathbf{RAF}^D$ *(Definition 2).* ◻

Proof. First note that, since **RAF** is non-recursive, it follows that $Inh(\mathfrak{A}) = \varnothing$ and, thus, that $\overline{Inh(\mathfrak{A})} = \mathbf{K}$ holds. Furthermore, from Observation 7, it also follows that $\Gamma = \mathbf{K}$. Hence, we may rewrite the definition of acceptability as follows:
$a \in \mathbf{A}$ is acceptable with respect to some d-structure $\mathfrak{A}$ in **RAF**
iff every $\alpha \in \mathbf{K}$ with $\mathbf{t}(\alpha) = a$ satisfies $\mathbf{s}(\alpha) \in Def(\mathfrak{A})$
iff for every $b \in \mathbf{A}$, $(b, a) \in \mathbf{R}_{\mathfrak{A}}$ implies $b \in Def(\mathfrak{A})$
iff for every $b \in \mathbf{A}$, $(b, a) \in \mathbf{R_{RAF}}$ implies $b \in Def(\mathfrak{A})$
iff a is acceptable w.r.t. S in $\mathbf{RAF}^D$ (Definition 2). ◻

Theorem 5. *For each semantics* $\sigma \in \{$*conflict-free, naive, admissible, complete, preferred, grounded, stable*$\}$*: A set of arguments* $S \subseteq \mathbf{A}$ *is a* σ*-extension w.r.t. some non-recursive* **RAF** *(Definition 15) iff it is a* σ*-extension w.r.t.* $\mathbf{RAF}^D$ *(Definition 3).* ◻

Proof. First note that due to Observation 7, a set S is a σ-extension of some non-recursive **RAF** iff $\mathfrak{A} = \langle S, \mathbf{K} \rangle$ is a σ-structure. Then,

1. Conflict-free: $\mathfrak{A} = \langle S, \mathbf{K} \rangle$ is a conflict-free structure in **RAF**
iff $S \cap Def(\mathfrak{A}) = \varnothing$ and $\mathbf{K} \cap Inh(\mathfrak{A}) = \varnothing$
iff $S \cap Def(\mathfrak{A}) = \varnothing$ (note that $Inh(\mathfrak{A}) = \varnothing$)
iff $S \cap Def(S) = \varnothing$ (Proposition 16)
iff S is a conflict-free extension of $\mathbf{RAF}^D$.

2. Naive: $\mathfrak{A} = \langle S, \mathbf{K} \rangle$ is a naive structure in **RAF**
iff $\mathfrak{A}$ is a $\sqsubseteq$-maximal conflict-free structure
iff $\mathfrak{A}$ is conflict-free and there is not conflict-free $\mathfrak{A}' = \langle S', \Gamma' \rangle$ s.t. $S' \supset S$
iff S is a conflict-free and there is not conflict-free S' s.t. $S' \supset S$
iff S is a $\subseteq$-maximal conflict-free extension
iff S is a naive extension of $\mathbf{RAF}^D$.

3. Admissible: $\mathfrak{A} = \langle S, \mathbf{K} \rangle$ is an admissible structure in **RAF**
iff $\mathfrak{A}$ is conflict-free and $(S \cup \mathbf{K}) \subseteq Acc(\mathfrak{A})$
iff S is conflict-free and $(S \cup \mathbf{K}) \subseteq Acc(\mathfrak{A})$
iff S is conflict-free and $S \subseteq Acc(\mathfrak{A})$ (since no attack is attacked)
iff S is conflict-free and $S \subseteq Acc(S)$ (Proposition 17)
iff S is an admissible extension of $\mathbf{RAF}^D$.

4. Complete: $\mathfrak{A} = \langle S, \mathbf{K} \rangle$ is a complete structure in **RAF**
iff $\mathfrak{A}$ is admissible and $Acc(\mathfrak{A}) \subseteq (S \cup \mathbf{K})$
iff S is admissible and $Acc(\mathfrak{A}) \subseteq (S \cup \mathbf{K})$
iff S is admissible and $(Acc(\mathfrak{A}) \cap \mathbf{A}) \subseteq S$
iff S is admissible and $Acc(S) \subseteq S$ (Proposition 17)
iff S is a complete extension of $\mathbf{RAF}^D$.

5. Preferred: $\mathfrak{A} = \langle S, \mathbf{K} \rangle$ is a preferred structure in **RAF**
iff $\mathfrak{A}$ is admissible and $\nexists \mathfrak{A}' = \langle S', \Gamma' \rangle$ s.t. admissible and $(S \cup \mathbf{K}) \subset (S' \cup \Gamma')$
iff $\mathfrak{A}$ is admissible and $\nexists \mathfrak{A}' = \langle S', \mathbf{K} \rangle$ s.t. admissible and $S \subset S'$
iff S is admissible and $\nexists \mathfrak{A}' = \langle S', \mathbf{K} \rangle$ s.t. admissible and $S \subset S'$
iff S is admissible and $\nexists S'$ admissible s.t. $S \subset S'$
iff S is a preferred extension of $\mathbf{RAF}^D$.

6. Grounded: $\mathfrak{A} = \langle S, \mathbf{K} \rangle$ is a grounded structure in **RAF**
iff $\mathfrak{A}$ is a $\sqsubseteq$-minimal complete structure
iff $\mathfrak{A}$ is complete and there is not complete structure $\mathfrak{A}' = \langle S', \Gamma' \rangle$ s.t. $S' \subset S$
iff S is complete and there is not complete extension S' s.t. $S' \subset S$
iff S is a $\subseteq$-minimal complete extension
iff S is a grounded extension of $\mathbf{RAF}^D$.

7. Stable: $\mathfrak{A} = \langle S, \mathbf{K} \rangle$ is a stable structure in **RAF**
iff $\mathfrak{A}$ is conflict-free, $S = \overline{Def(\mathfrak{A})}$ and $\mathbf{K} = \overline{Inh(\mathfrak{A})}$
iff S is conflict-free, $S = \overline{Def(\mathfrak{A})}$ and $\mathbf{K} = \overline{Inh(\mathfrak{A})}$
iff S is conflict-free and $S = \overline{Def(\mathfrak{A})}$ (no attack is attacked)
iff S is conflict-free and $S = \overline{Def(S)}$ (Proposition 16)
iff S is a stable extension of $\mathbf{RAF}^D$.

$\square$

Due to Observation 6, it follows directly that:

Corollary 1. *For each semantics* $\sigma \in \{complete, preferred, grounded, stable\}$*:* $\mathfrak{A} = \langle S, \mathbf{K} \rangle$ *is a* σ*-structure w.r.t. a non-recursive* **RAF** *(Definition 7) iff* S *is* σ*-extension w.r.t.* $\mathbf{RAF}^D$ *(Definition 3).* □

For the naive semantics, note that we cannot take simply naive structures. For instance, the simple Dung's framework of Figure 3 has two naive extensions, $\{a\}$ and $\{b\}$ and five naive structures, $\mathfrak{A}_1 = \langle \{a\}, \varnothing \rangle$, $\mathfrak{A}_2 = \langle \{a\}, \{\alpha\} \rangle$, $\mathfrak{A}_3 = \langle \{b\}, \varnothing \rangle$, $\mathfrak{A}_4 = \langle \{b\}, \{\alpha\} \rangle$ and $\mathfrak{A}_5 = \langle \{a, b\}, \varnothing \rangle$. It is easy to see that structures $\mathfrak{A}_1$ and $\mathfrak{A}_2$ correspond to the naive extension $\{a\}$ while structures $\mathfrak{A}_3$ and $\mathfrak{A}_4$ correspond to the naive extension $\{b\}$. However, structure $\mathfrak{A}_5$ corresponds to the set $\{a, b\}$ which is not conflict-free. Note also that $\mathfrak{A}_5$ (and also $\mathfrak{A}_1$ and $\mathfrak{A}_3$) are not d-structures because α is acceptable (it is not attacked) and does not belong to the structures. Hence, the notion of naive d-structure provides an alternative semantics for the naive semantics, which is a conservative extension of the naive semantics for Dung's frameworks. Similarly, the notion of d-structure provides alternative semantics for the principles of conflict-freeness and admissibility.

Example 1 (cont'd) Among the conflict-free structures that are not admissible, only five are conflict-free d-structures: $\langle \varnothing, \{\alpha, \beta\} \rangle$, $\langle \{a\}, \{\alpha, \beta\} \rangle$, $\langle \{b\}, \{\alpha, \beta\} \rangle$, $\langle \{a, b\}, \{\beta\} \rangle$, $\langle \{b\}, \{\beta\} \rangle$. Similarly, among the admissible structures that are not complete, only five are admissible d-structures: $\langle \{a, c\}, \{\beta\} \rangle$, $\langle \{b, c\}, \{\beta\} \rangle$, $\langle \{a\}, \{\beta\} \rangle$, $\langle \{c\}, \{\beta\} \rangle$ and $\langle \varnothing, \{\beta\} \rangle$. Furthermore, among these we have only three naive d-structures: $\langle \{a, b\}, \{\beta\} \rangle$, $\langle \{a, c\}, \{\beta\} \rangle$ and $\langle \{b, c\}, \{\beta\} \rangle$. □

Example 2 (cont'd) There are admissible structures w.r.t. the framework represented in Figure 2 that are not d-structures: for instance $\mathfrak{A}_1 = \langle \{a\}, \{\epsilon\} \rangle$ and $\mathfrak{A}_2 = \langle \{a\}, \{\epsilon, \delta\} \rangle$. Indeed, each d-structure must contain the attacks that are not targeted by any other attack, that is, $\{\epsilon, \alpha, \delta\}$. Moreover each d-structure containing a must also contain γ. □

As mentioned above, RAF naive d-structures provide a generalisation for the naive semantics for recursive frameworks, while taking $\subseteq$-maximal AFRA-conflict-free sets does not provide a conservative generalisation of the naive semantics, as illustrated by the following example:

Example 8. *The framework corresponding to Figure 10 has two naive extensions:* $\{a, c\}$ *and* $\{b\}$*. Accordingly, in RAF, we have two naive d-structures:* $\mathfrak{A}_0 = \langle \{a, c\}, \{\alpha, \beta\} \rangle$ *and* $\mathfrak{A}_1 = \langle \{b\}, \{\alpha, \beta\} \rangle$*. However, there is no* $\subseteq$*-maximal AFRA-conflict-free set whose only acceptable argument is* b*: the sets* $\{b, \alpha\}$ *and* $\{b, \alpha, \beta\}$ *are not AFRA-conflict-free because* α *defeats* b *while the sets* $\{b\}$ *and* $\{b, \beta\}$ *are not* $\subseteq$*-maximal because* $\{a, b, \beta\}$ *is AFRA-conflict-free.* □

7 Inhibited attacks

In this section, the intuition behind the concept of inhibited attacks is deepened and precisely defined. Indeed, we may expect that attacks that are inhibited do not have any effect on their targets, that is, we may remove them without modifying the condition of the structure.

Example 9. *Let* **RAF** *be the recursive argumentation framework of Figure 6 and* $\mathfrak{A} = \langle \{a, b, c\}, \{\beta\} \rangle$ *its unique complete structure. It is easy to check that* α *is inhibited w.r.t.* $\mathfrak{A}$ *because* c *and* β *belong to the structure and* α *is the target of* β*. According to the above intuition, we may expect that this would imply that there is a "somehow" corresponding structure* $\mathfrak{A}'$ *which is complete w.r.t. some* $\mathbf{RAF}'$ *obtained by removing* α*. Note that, in this case, removing* α *also implies removing* β *because there cannot be attacks without target. In fact, the resulting* $\mathbf{RAF}'$ *is a recursive framework with arguments* $\{a, b, c\}$ *and no attack. It is easy to check that* $\mathfrak{A}' = \langle \{a, b, c\}, \varnothing \rangle$ *is complete (also preferred and stable) w.r.t* $\mathbf{RAF}'$ *and that it shares with* $\mathfrak{A}$ *the set of "acceptable" arguments.* □

Let us now formalise this intuition:

Definition 17. *Given some recursive framework* **RAF** *and two different attacks* β, α*, we define:* $\beta \prec \alpha$ *iff there is some chain of attacks* $\delta_1, \delta_2, \ldots \delta_n$ *such that* $\delta_1 = \beta$*,* $\delta_n = \alpha$ *and* $\mathbf{t}(\delta_i) = \delta_{i+1}$ *for* $1 \leq i < n$*.* □

For instance, in the argumentation framework of Figure 6, we have that $\beta \prec \alpha$. On the other hand, neither $\alpha \prec \beta$, nor $\beta \prec \alpha$ hold for the argumentation framework of Figure 10. Note that $\prec$ is the empty relation for any non-recursive framework. As usual, by $\preceq$ we denote the reflexive closure of $\prec$. Given an attack α and a set of attacks Γ, by $\Gamma^{-\alpha} \stackrel{\text{def}}{=} \Gamma \setminus \{ \beta \in \mathbf{K} \mid \beta \preceq \alpha \}$ we denote the set of attacks obtained by removing the attack α from Γ. Furthermore, by $\mathbf{RAF}^{-\alpha} = \langle \mathbf{A}, \mathbf{K}^{-\alpha}, \mathbf{s}^{-\alpha}, \mathbf{t}^{-\alpha} \rangle$, with $\mathbf{s}^{-\alpha}$ and $\mathbf{t}^{-\alpha}$ the restrictions of $\mathbf{s}$ and $\mathbf{t}$ to $\mathbf{K}^{-\alpha}$, we denote the framework obtained by removing the attack α from $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$. Similarly, by $\mathfrak{A}^{-\alpha} = \langle S, \Gamma^{-\alpha} \rangle$ we denote the structure obtained by removing the attack α from the structure $\mathfrak{A} = \langle S, \Gamma \rangle$.

Example 9 (cont'd) Let **RAF** be the recursive argumentation framework of Figure 6. Then $\mathbf{RAF}^{-\alpha} = \langle \mathbf{A}, \varnothing, \mathbf{s}^{-\alpha}, \mathbf{t}^{-\alpha} \rangle$ with $\mathbf{A} = \{a, b, c\}$ because $\beta \prec \alpha$ implies that $\beta \notin \mathbf{K}^{-\alpha}$. Furthermore, if $\mathfrak{A} = \langle \{a, b, c\}, \{\beta\} \rangle$, then $\mathfrak{A}^{-\alpha} = \langle \{a, b, c\}, \varnothing \rangle$ which is a stable structure of $\mathbf{RAF}^{-\alpha}$. □

Proposition 19 below formalises the intuitions presented in the previous example.

Proposition 18. *Let* **RAF** *be some recursive framework,* $\mathfrak{A}$ *be some conflict-free (resp. admissible) structure and* $\alpha \in \mathit{Inh}(\mathfrak{A})$ *be some inhibited attack w.r.t.* $\mathfrak{A}$*. Then,* $\mathfrak{A}^{-\alpha}$ *is a conflict-free (resp. admissible) structure of* $\mathbf{RAF}^{-\alpha}$*.*

Proof. Let $\mathfrak{A} = \langle S, \Gamma \rangle$ and $\mathfrak{A}^{-\alpha} = \langle S, \Gamma^{-\alpha} \rangle$; and pick some argument $a \in S$. Then, if $\mathfrak{A}$ is conflict-free, it follows that[10] $a \notin \mathit{Def}(\mathbf{RAF}, \mathfrak{A})$. Suppose, for the

[10] $\mathit{Def}(\mathbf{RAF}, \mathfrak{A})$ denotes the defeated arguments with respect to structure $\mathfrak{A}$ and argumentation framework **RAF**.

sake of contradiction, that $a \in \mathit{Def}(\mathbf{RAF}^{-\alpha}, \mathfrak{A})$. Then, there is some attack $\beta \in \Gamma^{-\alpha}$ such that $\mathbf{t}(\beta) = a$ and $\mathbf{s}(\beta) \in S$. Furthermore, $\beta \in \Gamma^{-\alpha}$ plus $\Gamma^{-\alpha} \subseteq \Gamma$ imply $\beta \in \Gamma$ which, in its turn, implies that $a \in \mathit{Def}(\mathbf{RAF}, \mathfrak{A})$, which is a contradiction. Hence, $a \notin \mathit{Def}(\mathbf{RAF}^{-\alpha}, \mathfrak{A})$. Similarly, for any $\beta \in \Gamma^{-\alpha} \subseteq \Gamma$ we have that $\beta \notin \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})$ and, thus, $\beta \notin \mathit{Inh}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$ (see Lemma A.41 in the Appendix for more details). As a result, we have that $\mathfrak{A}^{-\alpha}$ is conflict-free w.r.t. $\mathbf{RAF}^{-\alpha}$.

Assume now that $\mathfrak{A}$ is admissible w.r.t. $\mathbf{RAF}$. Then, it is conflict-free and we have just seen that this implies that $\mathfrak{A}^{-\alpha}$ is conflict-free w.r.t. $\mathbf{RAF}^{-\alpha}$. Furthermore, since $\mathfrak{A}$ is admissible, it follows that $(S \cup \Gamma) \subseteq \mathit{Acc}(\mathbf{RAF}, \mathfrak{A})$. Suppose, for the sake of contradiction, that there is some $x \in \mathit{Acc}(\mathbf{RAF}, \mathfrak{A})$ such that $x \notin \mathit{Acc}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$. Hence, there is $\beta \in \Gamma^{-\alpha} \subseteq \Gamma$ such that $\mathbf{t}(\beta) = x$ and $\mathbf{s}(\beta) \notin \mathit{Def}(\mathbf{RAF}, \mathfrak{A}) \setminus \mathit{Def}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$. Then, there is $\gamma \in \Gamma \setminus \Gamma^{-\alpha}$ such that $\mathbf{t}(\gamma) = \mathbf{s}(\beta)$. However, this implies that $\gamma = \alpha$ and, since α is inhibited, we have that $\alpha \notin \Gamma$, which is a contradiction. Hence, $x \in \mathit{Acc}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$ follows and thus, we get that $\mathfrak{A}^{-\alpha}$ is admissible w.r.t. $\mathbf{RAF}^{-\alpha}$. ◻

In a similar way, this result can be extended to all other semantics (see the Appendix for a detailed proof).

Proposition 19. *Let* $\mathbf{RAF}$ *be some recursive framework and let* $\sigma \in \{naive, complete, preferred, grounded, stable\}$ *be a semantics. If* $\mathfrak{A}$ *is a* σ*-structure of* $\mathbf{RAF}$ *and* $\alpha \in \mathit{Inh}(\mathfrak{A})$ *is some inhibited attack w.r.t.* $\mathfrak{A}$*, then,* $\mathfrak{A}^{-\alpha}$ *is a* σ*-structure of* $\mathbf{RAF}^{-\alpha}$*.* ◻

8 Conclusion and future works

In this work we have extended Dung's abstract argumentation framework with recursive attacks. One of the essential characteristics of this extension is its conservative nature with respect to Dung's approach (when d-structures are considered). The other one is that semantics are given with respect to the notion of "valid attacks" which play a role analogous to attacks in Dung's frameworks. In contrast with meta-argumentation approaches, we propose a theory where valid attacks remain explicit, and distinct from arguments, within the notion of structure. Despite these differences, we proved a one-to-one correspondence with AFRA and MAF extensions in the case of the complete, preferred , grounded and stable semantics, while retaining a one-to-one correspondence with Dung's frameworks in the case of conflict-free and admissible extensions.

For a better understanding of the RAF framework, future work should include the study of other semantics (stage, semi-stable, and ideal). Moreover, we are interested in enriching the translation proposed by [6, 7, 16, 19] from Dung's framework into propositional logic and ASP, in order to capture RAF. Another line of further research will be to extend our approach by taking into account bipolar interactions [11, 20] (case when arguments and attacks may be attacked or supported). First works have been done in that direction (see [9] for a particular case of support – the evidential one – and [10] for another case of support – the necessary one).

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A Proofs

A.1 Proofs of Section 3

Lemma A.1. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathfrak{A}$ *be a conflict-free structure. Then, it follows that* $Acc(\mathfrak{A}) \cap Def(\mathfrak{A}) = \varnothing$ *and* $Acc(\mathfrak{A}) \cap Inh(\mathfrak{A}) = \varnothing$

Proof. Assume that $a \in (Acc(\mathfrak{A}) \cap Def(\mathfrak{A}))$. Then, there is $\alpha \in \Gamma$ with $s(\alpha) \in S$ and $t(\alpha) = a$. Since $a \in Acc(\mathfrak{A})$, it follows that either $s(\alpha) \in Def(\mathfrak{A})$ or $\alpha \in Inh(\mathfrak{A})$ holds. Both situations are impossible since $\mathfrak{A}$ is conflict-free, meaning that $S \cap Def(\mathfrak{A}) = \varnothing$ and $\Gamma \cap Inh(\mathfrak{A}) = \varnothing$.

The same reasoning holds for $\beta \in (Acc(\mathfrak{A}) \cap Inh(\mathfrak{A}))$ replacing a by β. □

Lemma A.2. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be some an admissible structure. Then, any acceptable argument* $a \in (Acc(\mathfrak{A}) \cap \mathbf{A})$ *satisfies that* $\mathfrak{A}' = \langle S \cup \{a\}, \Gamma \rangle$ *is conflict-free.*

Proof. Let $S' = (S \cup \{a\})$ and suppose, for the sake of contradiction, that $\mathfrak{A}'$ is not conflict-free, that is, that either $(S' \cap Def(\mathfrak{A}')) \neq \varnothing$ or $(\Gamma \cap Inh(\mathfrak{A}')) \neq \varnothing$ holds.

a) In the first case, there is $a' \in (S' \cap Def(\mathfrak{A}'))$. So there is $\alpha \in \Gamma$ such that $\mathbf{t}(\alpha) = a'$ and $\mathbf{s}(\alpha) \in S'$. Either $a' = a$ or $a' \in S$.

   Assume first that $a' = a$. Since $a \in Acc(\mathfrak{A})$, it follows that either $\alpha \in Inh(\mathfrak{A})$ or $s(\alpha) \in Def(\mathfrak{A})$ holds. However, we know that $\mathfrak{A}$ is conflict-free, so it is impossible that $\alpha \in (\Gamma \cap Inh(\mathfrak{A}))$. So, it must be the case that $s(\alpha) \in Def(\mathfrak{A})$ holds and, thus, that $s(\alpha) \notin S$ (also because $\mathfrak{A}$ is conflict-free). Hence, $s(\alpha) = a$. Due to Lemma A.1, it is impossible to have $a \in (Acc(\mathfrak{A}) \cap Def(\mathfrak{A}))$ and, thus, $s(\alpha) \in Def(\mathfrak{A})$ plus $s(\alpha) = a$ imply that $a \notin Acc(\mathfrak{A})$. This is a contradiction with the fact $a \in Acc(\mathfrak{A})$.

   Assume now that $a' \neq a$ and, thus, that $a' \in S$. Since $(S \cap Def(\mathfrak{A})) = \varnothing$, we have that $a' \notin Def(\mathfrak{A})$. Hence, $s(\alpha) \notin S$ and $s(\alpha) = a$ hold. Since $\mathfrak{A}$ is admissible and $a' \in S$, it follows that $a' \in Acc(\mathfrak{A})$. Furthermore, since $\mathbf{t}(\alpha) = a'$, it also follows that either $\alpha \in Inh(\mathfrak{A})$ or $s(\alpha) \in Def(\mathfrak{A})$. The former is in contradiction with the fact that $\mathfrak{A}$ is admissible (and thus conflict-free). Furthermore, from Lemma A.1 and the fact that $\mathbf{s}(\alpha) = a$, the latter implies that $a \notin Acc(\mathfrak{A})$ which is a contradiction, too.

b) If $(\Gamma \cap Inh(\mathfrak{A}')) \neq \varnothing$, then there is some attack $\beta \in \Gamma$ with $\beta \in Inh(\mathfrak{A}')$ and thus, there is also some $\alpha \in \Gamma$ such that $\mathbf{t}(\alpha) = \beta$ and $\mathbf{s}(\alpha) \in S'$. Since $\mathfrak{A}$ is conflict-free, $\beta \notin Inh(\mathfrak{A})$ which implies that $\mathbf{s}(\alpha) \notin S$ and thus, $\mathbf{s}(\alpha) = a$ holds. Since $\mathfrak{A}$ is admissible and $\beta \in \Gamma$, it follows that $\beta \in Acc(\mathfrak{A})$. Furthermore, since $\mathbf{t}(\alpha) = \beta$, it must be that either $\alpha \in Inh(\mathfrak{A})$ or $s(\alpha) \in Def(\mathfrak{A})$ holds. The former is in contradiction with the fact that $\alpha \in \Gamma$ and the latter implies that $a \notin Acc(\mathfrak{A})$ which is in contradiction with the hypothesis.

Consequently, $\mathfrak{A}'$ is conflict-free. □

Lemma A.3. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be some admissible structure. Then, any attack* $\alpha \in (Acc(\mathfrak{A}) \cap \mathbf{K})$ *satisfies that* $\mathfrak{A}' = \langle S, \Gamma \cup \{\alpha\} \rangle$ *is conflict-free.*

Proof. Let $\Gamma' = (\Gamma \cup \{\alpha\})$ and suppose, for the sake of contradiction, that $\mathfrak{A}'$ is not conflict free, that is, either $(S \cap Def(\mathfrak{A}')) \neq \varnothing$ or $(\Gamma' \cap Inh(\mathfrak{A}')) \neq \varnothing$.

1. In the first case, there is $a \in S$ with $a \in Def(\mathfrak{A}')$. So there is $\beta \in \Gamma'$ such that $\mathbf{t}(\beta) = a$ and $\mathbf{s}(\beta) \in S$. Since $\mathfrak{A}$ is conflict-free, $a \notin Def(\mathfrak{A})$ and thus, $\beta \notin \Gamma$ and $\beta = \alpha$ follow. Then, $\beta \in Acc(\mathfrak{A})$. Furthermore, since $\mathfrak{A}$ is admissible and $a \in S$, we have that $a \in Acc(\mathfrak{A})$ and thus, that either $\beta \in Inh(\mathfrak{A})$ or $s(\beta) \in Def(\mathfrak{A})$ holds. From Lemma A.1, the former is in contradiction with the fact that $\beta \in Acc(\mathfrak{A})$ and, since $\mathfrak{A}$ is conflict-free, the latter is in contradiction with the fact that $\mathbf{s}(\beta) \in S$.

2. If $(\Gamma' \cap Inh(\mathfrak{A}')) \neq \varnothing$, there is some attack $\alpha' \in \Gamma'$ with $\alpha' \in Inh(\mathfrak{A}')$. So there is $\beta \in \Gamma'$ such that $\mathbf{t}(\beta) = \alpha'$ and $\mathbf{s}(\beta) \in S$. Furthermore, either $\alpha' = \alpha$ or $\alpha' \in \Gamma$.

   Assume first that $\alpha' = \alpha$. Since $\alpha \in Acc(\mathfrak{A})$, it follows that either $\beta \in Inh(\mathfrak{A})$ or $s(\beta) \in Def(\mathfrak{A})$. However, we know that $\mathfrak{A}$ is conflict-free, so it is impossible that $s(\beta) \in (S \cap Def(\mathfrak{A}))$. So we must have $\beta \in Inh(\mathfrak{A})$ and thus, that $\beta \notin \Gamma$ (also because $\mathfrak{A}$ is conflict-free). Hence $\beta = \alpha$. Due to Lemma A.1, it is impossible to have $\beta \in (Acc(\mathfrak{A}) \cap Inh(\mathfrak{A}))$ and thus, $\alpha \notin Acc(\mathfrak{A})$. This is in contradiction with the hypothesis on α.

   Assume now that $\alpha' \neq \alpha$ and thus that $\alpha' \in \Gamma$. Since $\mathfrak{A}$ is conflict-free, it follows that $(\Gamma \cap Inh(\mathfrak{A})) = \varnothing$ and thus, that $\alpha' \notin Inh(\mathfrak{A})$. So $\beta \notin \Gamma$, that is, $\beta = \alpha$ and, thus, that $\beta \in Acc(\mathfrak{A})$. Furthermore, since $\mathfrak{A}$ is admissible and $\alpha' \in \Gamma$, we have that $\alpha' \in Acc(\mathfrak{A})$. As $\mathbf{t}(\beta) = \alpha'$, either $\beta \in Inh(\mathfrak{A})$ or $s(\beta) \in Def(\mathfrak{A})$. From Lemma A.1, the former is in contradiction with the fact that $\beta \in Acc(\mathfrak{A})$ and, since $\mathfrak{A}$ is conflict-free, the latter is in contradiction with the fact that $\mathbf{s}(\beta) \in S$.

Consequently, $\mathfrak{A}'$ is conflict-free. ◻

Lemma A.4. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be some an admissible structure. Then, any element* $x \in Acc(\mathfrak{A})$ *satisfies that* $\mathfrak{A}' = \mathfrak{A} \cup \{x\}$ *is conflict-free.*

Proof. If $x \in \mathbf{A}$, the result follows directly from Lemma A.2. Otherwise, $x \in \mathbf{K}$, and the result follows from Lemma A.3. ◻

Lemma A.5. *Any conflict-free structure* $\mathfrak{A} = \langle S, \Gamma \rangle$ *satisfies:* $Acc(\mathfrak{A}) \subseteq (\overline{Def(\mathfrak{A})} \cup \overline{Inh(\mathfrak{A})})$. ◻

Proof. It follows directly from Lemma A.1 and the definitions of $\overline{Def(\mathfrak{A})}$ and $\overline{Inh(\mathfrak{A})}$. ◻

Lemma A.6. *Let* $\mathbf{RAF} = \langle \mathbf{A},\mathbf{K},\mathbf{s},\mathbf{t}\rangle$ *be some non-recursive framework and* $\mathfrak{A}_0 \sqsubseteq \mathfrak{A}_1 \sqsubseteq \dots$ *be some sequence of conflict-free structures such that* $\mathfrak{A}_i = \langle S_i, \Gamma_i\rangle$. *Let us also define* $\mathfrak{A} = \langle \bigcup_{0\leq i} S_i, \bigcup_{0\leq i} \Gamma_i\rangle$. *Then,* $\mathfrak{A}$ *is conflict-free.* □

Proof. Suppose, for the sake of contradiction, that $\mathfrak{A}$ is not conflict-free. Then, either $(S\cap Def(\mathfrak{A})) \neq \varnothing$ or $(\Gamma\cap Inh(\mathfrak{A})) \neq \varnothing$ (with $S = \bigcup_{0\leq i} S_i$ and $\Gamma = \bigcup_{0\leq i} \Gamma_i$). Pick any argument $x \in (S \cap Def(\mathfrak{A}))$ (resp. attack $x \in \Gamma \cap Inh(\mathfrak{A})$)). Then, $x \in Def(\mathfrak{A})$ (resp. $x \in Inh(\mathfrak{A})$) implies that there is $\alpha \in \Gamma$ such that $\mathbf{t}(\alpha) = x$ and $\mathbf{s}(\alpha) \in S$. Hence, there is $0 \leq i$ such that $\alpha \in \Gamma_i$ and $0 \leq j$ such that $\mathbf{s}(\alpha) \in S_j$. Let $k = \max\{i, j\}$. Then, $\alpha \in \Gamma_k$ and $\mathbf{s}(\alpha) \in S_k$ which means that $x \in Def(\mathfrak{A}_k)$ (resp. $x \in Inh(\mathfrak{A}_k)$). Moreover, there is $0 \leq l$ such that $x \in S_l$ (resp. $x \in \Gamma_l$). Let $m = \max\{k, l\}$. Then, $x \in S_m$ (resp. $x \in \Gamma_m$), and from Observation 1, we have that $Def(\mathfrak{A}_k) \subseteq Def(\mathfrak{A}_m)$ (resp. $Inh(\mathfrak{A}_k) \subseteq Inh(\mathfrak{A}_m)$). That is in contradiction with the fact that $\mathfrak{A}_m$ is conflict-free.

Hence, $\mathfrak{A}$ must be conflict-free. □

A.2 Proofs of Section 4

Lemma A.7. *Let* $\mathbf{RAF} = \langle \mathbf{A},\mathbf{K},\mathbf{s},\mathbf{t}\rangle$ *be a framework and* $\mathfrak{A}$ *be a structure. Then,* $x \in (Def(\mathfrak{A}) \cup Inh(\mathfrak{A}))$ *implies that there is some* $\alpha \in \mathtt{Afra}(\mathfrak{A})$ *such that* α *defeats* x. □

Proof. Since $x \in (Def(\mathfrak{A}) \cup Inh(\mathfrak{A}))$, there is $\alpha \in \Gamma$ s.t. $\mathbf{t}(\alpha) = x$ and $\mathbf{s}(\alpha) \in S$. Note that $\alpha \in \Gamma$ and $\mathbf{s}(\alpha) \in \Gamma$ imply that $\alpha \in \mathtt{Afra}(\mathfrak{A})$ and that $\mathbf{t}(\alpha) = x$ implies that α defeats x. □

Lemma A.8. *Let* $\mathbf{RAF} = \langle \mathbf{A},\mathbf{K},\mathbf{s},\mathbf{t}\rangle$ *be some framework and* $\mathfrak{A}$ *be some structure. Then, every* $a \in (Acc(\mathfrak{A}) \cap \mathbf{A})$ *is AFRA-acceptable w.r.t.* $\mathtt{Afra}(\mathfrak{A})$. □

Proof. Pick any attack $\alpha \in \mathbf{K}$ such that α defeats a. Then, $\mathbf{t}(\alpha) = a$ and, since $a \in Acc(\mathfrak{A})$ it follows that either $\alpha \in Inh(\mathfrak{A})$ or $\mathbf{s}(\alpha) \in Def(\mathfrak{A})$.

If $\alpha \in Inh(\mathfrak{A})$, then there is $\beta \in \Gamma$ such that $\mathbf{s}(\beta) \in S$ and $\mathbf{t}(\beta) = \alpha$. Note that $\beta \in \Gamma$ plus $\mathbf{s}(\beta) \in S$ imply $\beta \in \mathtt{Afra}(\mathfrak{A})$ and that $\mathbf{t}(\beta) = \alpha$ implies that β defeats α. Hence, the fact that a is AFRA-acceptable w.r.t. $\mathcal{E}$ follows.

Otherwise, $\mathbf{s}(\alpha) \in Def(\mathfrak{A})$, and, there is $\beta \in \Gamma$ such that $\mathbf{s}(\beta) \in S$ and $\mathbf{t}(\beta) = \mathbf{s}(\alpha)$. As above, $\beta \in \Gamma$ plus $\mathbf{s}(\beta) \in S$ imply $\beta \in \mathtt{Afra}(\mathfrak{A})$, and $\mathbf{t}(\beta) = \mathbf{s}(\alpha)$ implies that β defeats α. Hence, the fact that a is AFRA-acceptable w.r.t. $\mathcal{E}$ follows.

In consequence, it holds that a is AFRA-acceptable w.r.t. $\mathcal{E}$. □

Lemma A.9. *Let* $\mathbf{RAF} = \langle \mathbf{A},\mathbf{K},\mathbf{s},\mathbf{t}\rangle$ *be some framework and* $\mathfrak{A} = \langle S, \Gamma\rangle$ *be some structure. Then, every* $\alpha \in (Acc(\mathfrak{A}) \cap \mathbf{K})$ *that satisfies* $\mathbf{s}(\alpha) \in Acc(\mathfrak{A})$*, is also AFRA-acceptable w.r.t.* $\mathtt{Afra}(\mathfrak{A})$. □

Proof. Pick any attack $\beta \in \mathbf{K}$ such that β defeats α. Then, either $\mathbf{t}(\beta) = \alpha$ or $\mathbf{t}(\beta) = \mathbf{s}(\alpha)$.

If the latter, then Lemma A.8 plus $\mathbf{s}(\alpha) \in \mathit{Acc}(\mathfrak{A})$ imply that $\mathbf{s}(\alpha)$ is AFRA-acceptable w.r.t. $\mathtt{Afra}(\mathfrak{A})$ and, thus, that there is some $\gamma \in \mathtt{Afra}(\mathfrak{A})$ that defeats β.

If the former, $\alpha \in \mathit{Acc}(\mathfrak{A})$ implies that either $\beta \in \mathit{Inh}(\mathfrak{A})$ or $\mathbf{s}(\beta) \in \mathit{Def}(\mathfrak{A})$. Assume $\beta \in \mathit{Inh}(\mathfrak{A})$. Then, there is $\gamma \in \Gamma$ such that $\mathbf{s}(\gamma) \in S$ and $\mathbf{t}(\gamma) = \beta$. Note that $\gamma \in \Gamma$ plus $\mathbf{s}(\gamma) \in S$ imply $\gamma \in \mathtt{Afra}(\mathfrak{A})$ and that $\mathbf{t}(\gamma) = \beta$ implies that γ defeats β.

Otherwise, $\mathbf{s}(\beta) \in \mathit{Def}(\mathfrak{A})$ and there is $\gamma \in \Gamma$ such that $\mathbf{s}(\gamma) \in S$ and $\mathbf{t}(\gamma) = \mathbf{s}(\beta)$. As above, $\gamma \in \Gamma$ plus $\mathbf{s}(\gamma) \in S$ imply $\gamma \in \mathtt{Afra}(\mathfrak{A})$ and $\mathbf{t}(\gamma) = \mathbf{s}(\beta)$ implies that γ defeats β.

Hence, for any attack $\beta \in \mathbf{K}$ that defeats α, there is some attack $\gamma \in \mathtt{Afra}(\mathfrak{A})$ that defeats β. That is, the fact that α is AFRA-acceptable w.r.t. $\mathtt{Afra}(\mathfrak{A})$ follows. □

Lemma A.10. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathfrak{A}$ *be some structure. Then,* $\alpha \in (\mathit{Acc}(\mathfrak{A}) \cap \mathbf{K})$ *and* $\mathbf{s}(\alpha) \notin S$ *imply* $\alpha \in \Gamma_{\mathtt{Afra}(\mathfrak{A})}$. □

Proof. Let $\mathfrak{A}' = \langle S, \Gamma_{\mathtt{Afra}(\mathfrak{A})} \rangle$ and let us show that $\alpha \in \mathit{Acc}(\mathfrak{A}')$. Pick any $\beta \in \mathbf{K}$ such that $\mathbf{t}(\beta) = \alpha$. Since $\alpha \in \mathit{Acc}(\mathfrak{A})$, it follows that either $\beta \in \mathit{Inh}(\mathfrak{A})$ or $\mathbf{s}(\beta) \in \mathit{Def}(\mathfrak{A})$. If the former, there is $\gamma \in \Gamma$ such that $\mathbf{t}(\gamma) = \beta$ and $\mathbf{s}(\gamma) \in S$. Note that $\gamma \in \Gamma$ plus $\mathbf{s}(\gamma) \in S$ imply that $\gamma \in \mathtt{Afra}(\mathfrak{A})$ and, thus, that $\gamma \in \Gamma_{\mathtt{Afra}(\mathfrak{A})}$ and that $\beta \in \mathit{Inh}(\mathfrak{A}')$. Similarly, $\mathbf{s}(\beta) \in \mathit{Def}(\mathfrak{A})$ implies that there is $\gamma \in \Gamma$ such that $\mathbf{t}(\gamma) = \mathbf{s}(\beta)$ and $\mathbf{s}(\gamma) \in S$ and, thus, $\mathbf{s}(\beta) \in \mathit{Def}(\mathfrak{A}')$. Hence, any $\beta \in \mathbf{K}$ with $\mathbf{t}(\beta) = \alpha$ satisfies either $\beta \in \mathit{Inh}(\mathfrak{A}')$ or $\mathbf{s}(\beta) \in \mathit{Def}(\mathfrak{A}')$. That is, $\alpha \in \mathit{Acc}(\mathfrak{A}')$. Hence, to show that $\alpha \in \Gamma_{\mathtt{Afra}(\mathfrak{A})}$ it is enough to prove that $\mathbf{s}(\alpha) \notin \mathtt{Afra}(\mathfrak{A})$ which directly follows from the fact that $\mathbf{s}(\alpha) \notin S$. □

Lemma A.11. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be an framework and* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be some admissible structure. Then,* $S = S_{\mathtt{Afra}(\mathfrak{A})}$ *and* $\Gamma \subseteq \Gamma_{\mathtt{Afra}(\mathfrak{A})}$ *hold.* □

Proof. Note that, by definition, it follows that $S_{\mathtt{Afra}(\mathfrak{A})} = (\mathtt{Afra}(\mathfrak{A}) \cap \mathbf{A}) = S$.

Then, to show that $\Gamma \subseteq \Gamma_{\mathtt{Afra}(\mathfrak{A})}$ holds, pick any attack $\alpha \in \Gamma$. If $\mathbf{s}(\alpha) \in S$, then $\alpha \in (\mathtt{Afra}(\mathfrak{A}) \cap \mathbf{K})$ and thus, $\alpha \in \Gamma_{\mathtt{Afra}(\mathfrak{A})}$. Otherwise, $\mathbf{s}(\alpha) \notin S$ and $\alpha \in (\mathit{Acc}(\mathfrak{A}) \cap \mathbf{K})$ as $\mathfrak{A} = \langle S, \Gamma \rangle$ is admissible. So, from Lemma A.10, it follows that $\alpha \in \Gamma_{\mathtt{Afra}(\mathfrak{A})}$. □

Lemma A.12. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathfrak{A}$ *be a complete structure. Then, it follows that* $\mathit{Acc}(\mathfrak{A}) \subseteq (S_{\mathtt{Afra}(\mathfrak{A})} \cup \Gamma_{\mathtt{Afra}(\mathfrak{A})})$. □

Proof. Let $\mathfrak{A} = \langle S, \Gamma \rangle$. Note that, since $\mathfrak{A}$ is complete, it follows that $\mathit{Acc}(\mathfrak{A}) \subseteq (S \cup \Gamma)$ and, thus, the result follows directly from Lemma A.11. □

Lemma A.13. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework,* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be a complete structure and* x *be some AFRA-acceptable element w.r.t.* $\mathtt{Afra}(\mathfrak{A})$. *Then,* $x \in (S \cup \Gamma)$ *holds.* □

Proof. From Lemma 4.1, the hypothesis implies that $x \in Acc(\mathfrak{A}_{\texttt{Afra}(\mathfrak{A})})$. Note that, since $\mathfrak{A}$ is complete, Proposition 10 implies that $\mathfrak{A}_{\texttt{Afra}(\mathfrak{A})} = \mathfrak{A}$ and, thus, that $x \in Acc(\mathfrak{A})$ and that $x \in (S \cup \Gamma)$ (recall that $\mathfrak{A}$ is complete). □

Lemma A.14. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be a complete structure. Then,* $\texttt{Afra}(\mathfrak{A})$ *is AFRA-complete.* □

Proof. Since $\mathfrak{A}$ is a complete structure, it is admissible and, in addition, it satisfies $(S \cup \Gamma) = Acc(\mathfrak{A})$. From Proposition 6 the former implies that $\texttt{Afra}(\mathfrak{A})$ is AFRA-admissible. Hence, to show that $\texttt{Afra}(\mathfrak{A})$ is AFRA-complete, it is enough to prove that every acceptable element x w.r.t. $\texttt{Afra}(\mathfrak{A})$ belongs to $\texttt{Afra}(\mathfrak{A})$.

Pick any AFRA-acceptable element $x \in (\mathbf{A} \cup \mathbf{K})$ w.r.t. $\texttt{Afra}(\mathfrak{A})$. From Lemma A.13, this implies that $x \in (S \cup \Gamma)$. Note that, by construction, we have that $S \subseteq \texttt{Afra}(\mathfrak{A})$. Furthermore, if $x \in \Gamma$, then Lemma 1 in [4] plus the fact that x is AFRA-acceptable w.r.t. $\texttt{Afra}(\mathfrak{A})$, imply that $\mathbf{s}(x)$ is AFRA-acceptable w.r.t. $\texttt{Afra}(\mathfrak{A})$ and, from Lemma A.13 again, this implies that $\mathbf{s}(x) \in S$. By definition, $x \in \Gamma$ plus $\mathbf{s}(x) \in S$ imply $x \in \texttt{Afra}(\mathfrak{A})$ and, thus, that $\Gamma \subseteq \texttt{Afra}(\mathfrak{A})$. Therefore, we have that every AFRA-acceptable element x w.r.t. $\texttt{Afra}(\mathfrak{A})$ belongs to $\texttt{Afra}(\mathfrak{A})$ and, thus, that $\texttt{Afra}(\mathfrak{A})$ is an AFRA-complete extension. □

Lemma A.15. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$ *be a closed set. Then, it follows that* $\texttt{Afra}(\mathfrak{A}_{\mathcal{E}}) = \mathcal{E}$. □

Proof. Note that, by definition, it follows that

$$(\texttt{Afra}(\mathfrak{A}_{\mathcal{E}}) \cap \mathbf{A}) \;=\; S_{\mathcal{E}} \;=\; (\mathcal{E} \cap \mathbf{A})$$

It remains to be shown that $(\texttt{Afra}(\mathfrak{A}_{\mathcal{E}}) \cap \mathbf{K}) = (\mathcal{E} \cap \mathbf{K})$. By definition, it follows that $\Gamma_{\mathcal{E}} \supseteq (\mathcal{E} \cap \mathbf{K})$. Therefore, $\alpha \in \mathcal{E}$ implies that $\alpha \in \Gamma_{\mathcal{E}}$ and, since $\mathcal{E}$ is closed, this implies that $\mathbf{s}(\alpha) \in (\mathcal{E} \cap \mathbf{A}) = S_{\mathcal{E}}$ and, thus, that $\alpha \in (\texttt{Afra}(\mathfrak{A}_{\mathcal{E}}) \cap \mathbf{K})$. In its turn, this implies $(\texttt{Afra}(\mathfrak{A}_{\mathcal{E}}) \cap \mathbf{K}) \supseteq (\mathcal{E} \cap \mathbf{K})$.

On the other hand, every $\alpha \in (\texttt{Afra}(\mathfrak{A}_{\mathcal{E}}) \cap \mathbf{K})$ satisfies $\alpha \in \Gamma_{\mathcal{E}}$ and $\mathbf{s}(\alpha) \in S_{\mathcal{E}} \subseteq \mathcal{E}$. Together, these two facts imply $\alpha \in (\mathcal{E} \cap \mathbf{K})$. Hence, $(\texttt{Afra}(\mathfrak{A}_{\mathcal{E}}) \cap \mathbf{K}) = (\mathcal{E} \cap \mathbf{K})$ holds and, thus, $\texttt{Afra}(\mathfrak{A}_{\mathcal{E}}) = \mathcal{E}$ follows. □

Lemma A.16. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathcal{E}$ *be some AFRA-complete extension. Then,* $\mathcal{E}$ *is closed.* □

Proof. Pick any attack $\alpha \in (\mathcal{E} \cap \mathbf{K})$. Then, since $\mathcal{E}$ is AFRA-complete, this implies that α is AFRA-acceptable w.r.t. $\mathcal{E}$ and, from Lemma 1 in [4], this implies that $\mathbf{s}(\alpha)$ is also AFRA-acceptable w.r.t. $\mathcal{E}$. This plus the fact that $\mathcal{E}$ is AFRA-complete imply that $\mathbf{s}(\alpha) \in \mathcal{E}$. □

Lemma A.17. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$ *be a AFRA-complete extension. Then,* $\mathfrak{A}_{\mathcal{E}}$ *is a complete structure.* □

Proof. By definition, every AFRA-complete extension is also AFRA-admissible. Furthermore, from Lemma A.16, every AFRA-complete extension is also closed, thus, Proposition 7 implies that $\mathfrak{A}_{\mathcal{E}}$ is an admissible structure. Then, to show that $\mathfrak{A}_{\mathcal{E}}$ is a complete structure, it is enough to prove the following inclusion: $Acc(\mathfrak{A}_{\mathcal{E}}) \subseteq (S_{\mathcal{E}} \cup \Gamma_{\mathcal{E}})$. Let us recall that, from Lemma A.12, it follows that

$$Acc(\mathfrak{A}_{\mathcal{E}}) \quad \subseteq \quad (S_{\texttt{Afra}(\mathfrak{A}_{\mathcal{E}})} \cup \Gamma_{\texttt{Afra}(\mathfrak{A}_{\mathcal{E}})})$$

and that, from Lemma A.15, it follows $\texttt{Afra}(\mathfrak{A}_{\mathcal{E}}) = \mathcal{E}$. As a result, $Acc(\mathfrak{A}_{\mathcal{E}}) \subseteq (S_{\mathcal{E}} \cup \Gamma_{\mathcal{E}})$ holds and, thus, $\mathfrak{A}_{\mathcal{E}}$ is complete. □

Lemma A.18. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathcal{E} \subseteq \mathcal{E}' \subseteq (\mathbf{A} \cup \mathbf{K})$ *be two AFRA-complete extensions. Then,* $\mathfrak{A}_{\mathcal{E}} \sqsubseteq \mathfrak{A}_{\mathcal{E}'}$. □

Proof. First, note that

$$S_{\mathcal{E}} \ = \ (\mathcal{E} \cap \mathbf{A}) \ \subseteq \ (\mathcal{E}' \cap \mathbf{A}) \ = \ S_{\mathcal{E}'}$$

Let $S = S_{\mathcal{E}}$, $S' = S_{\mathcal{E}'}$, $\Gamma = (\mathcal{E} \cap \mathbf{K})$ and $\Gamma' = (\mathcal{E}' \cap \mathbf{K})$. Let also $\mathfrak{A} = \langle S, \Gamma \rangle$ and $\mathfrak{A}' = \langle S', \Gamma' \rangle$. Then,

$$\begin{array}{lcl} \Gamma_{\mathcal{E}} & = & (\mathcal{E} \cap \mathbf{K}) \cup \{ \ \alpha \in Acc(\mathfrak{A}) \ \big| \ \mathbf{s}(\alpha) \notin S \ \} \\ \Gamma_{\mathcal{E}'} & = & (\mathcal{E}' \cap \mathbf{K}) \cup \{ \ \alpha \in Acc(\mathfrak{A}') \ \big| \ \mathbf{s}(\alpha) \notin S' \ \} \end{array}$$

Hence, to show $\Gamma_{\mathcal{E}} \subseteq \Gamma_{\mathcal{E}'}$, it is enough to prove

$$\{ \ \alpha \in Acc(\mathfrak{A}) \ \big| \ \mathbf{s}(\alpha) \notin S \ \} \subseteq \mathcal{E}' \cup \{ \ \alpha \in Acc(\mathfrak{A}') \ \big| \ \mathbf{s}(\alpha) \notin S' \ \}$$

Furthermore, from Observation 2 and the fact that $\mathfrak{A} \sqsubseteq \mathfrak{A}'$, it follows that $Acc(\mathfrak{A}) \subseteq Acc(\mathfrak{A}')$ and, thus, it is enough to show that every $\alpha \in Acc(\mathfrak{A})$ satisfies that: $\mathbf{s}(\alpha) \notin S$ implies that either $\mathbf{s}(\alpha) \notin S'$ or $\alpha \in \mathcal{E}'$.

Suppose, for the sake of contradiction, that there is some $\alpha \in (Acc(\mathfrak{A}) \setminus \mathcal{E}')$ that satisfies $\mathbf{s}(\alpha) \notin S$ and $\mathbf{s}(\alpha) \in S'$. Since by hypothesis $\mathcal{E}'$ is AFRA-complete, $\alpha \notin \mathcal{E}'$ implies that α is not AFRA-acceptable w.r.t. $\mathcal{E}'$ and, thus, there is some $\beta \in \mathbf{K}$ that defeats α and is not defeated by any $\gamma \in \mathcal{E}'$. That is, either $\mathbf{t}(\beta) = \alpha$ or $\mathbf{t}(\beta) = \mathbf{s}(\alpha)$. If the latter, then β also defeats $\mathbf{s}(\alpha)$. But, then $\mathbf{s}(\alpha) \in S'$ implies $\mathbf{s}(\alpha) \in \mathcal{E}'$ which, in its turn, implies that $\mathbf{s}(\alpha)$ is AFRA-acceptable w.r.t. $\mathcal{E}'$ and, thus, that β is defeated by some $\gamma \in \mathcal{E}'$ which is a contradiction with the above. Hence, it must be that $\mathbf{s}(\beta) = \alpha$ and, thus, that $\alpha \notin Acc(\mathfrak{A})$ which is a contradiction with the assumption. Hence, $\Gamma \subseteq \Gamma'$ holds. □

Lemma A.19. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathcal{E} \subset \mathcal{E}' \subseteq (\mathbf{A} \cup \mathbf{K})$ *be two AFRA-complete extensions. Then,* $\mathfrak{A}_{\mathcal{E}} \sqsubset \mathfrak{A}_{\mathcal{E}'}$. □

Proof. First note that from Lemma A.19, we have $\mathfrak{A}_{\mathcal{E}} \sqsubseteq \mathfrak{A}_{\mathcal{E}'}$. Suppose, for the sake of contradiciton that $\mathfrak{A}_{\mathcal{E}} = \mathfrak{A}_{\mathcal{E}'}$. Then, $S \subset S'$ implies $\mathfrak{A} \sqsubset \mathfrak{A}_{\mathcal{E}'}$, which is a contradiction with the assumption. Hence, it must be that $S = S'$ holds.

Furthermore, since $\mathcal{E} \subset \mathcal{E}'$, there is some element $x \in (\mathcal{E}' \backslash \mathcal{E})$ and, since $S = S'$, it follows that $x \in \mathbf{K}$. From Lemma A.16 and the fact that $\mathcal{E}'$ is AFRA-complete, it follows that $\mathcal{E}'$ is closed and, thus, $x \in \mathcal{E}'$ implies that $\mathbf{s}(x) \in \mathcal{E}'$. This implies $\mathbf{s}(x) \in S$ and, since $S = S'$, that $\mathbf{s}(x) \in S$ and $\mathbf{s}(x) \in \mathcal{E}$. This plus $x \notin \mathcal{E}$ imply that $x \notin \Gamma$ and, thus, that $\Gamma \subset \Gamma'$ and $\mathfrak{A} \sqsubset \mathfrak{A}'$ hold. □

Lemma A.20. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be a preferred structure. Then,* $\mathtt{Afra}(\mathfrak{A})$ *is AFRA-preferred.* □

Proof. Since $\mathfrak{A}$ is a preferred structure, it is admissible and, in addition, there is no admissible structure $\mathfrak{A}$ such that $\mathfrak{A} \sqsubseteq \mathfrak{A}'$. From Proposition 6, the former implies that $\mathtt{Afra}(\mathfrak{A})$ is AFRA-admissible. Hence, to show that $\mathtt{Afra}(\mathfrak{A})$ is AFRA-preferred, it is enough to prove that there does not exist any AFRA-admissible extension $\mathcal{E}$ such that $\mathtt{Afra}(\mathfrak{A}) \subset \mathcal{E}$.

Suppose, for the sake of contradiction, that there exists any AFRA-admissible extension $\mathcal{E}$ such that $\mathtt{Afra}(\mathfrak{A}) \subset \mathcal{E}$. Since $\mathcal{E}$ is AFRA-admissible, from Theorem 2 in [4], there is some AFRA-preferred extension $\mathcal{E}'$ such that $\mathtt{Afra}(\mathfrak{A}) \subset \mathcal{E} \subseteq \mathcal{E}'$. Furthermore, from Lemma 4 in [4], it follows that $\mathcal{E}'$ is also AFRA-complete and, thus, from Lemma A.17, that $\mathfrak{A}_{\mathcal{E}'}$ is a complete structure. Furthermore, since $\mathfrak{A}$ is a preferred structure, from Theorem 2, it follows that $\mathfrak{A}$ is also complete and thus, from Lemma A.14, that $\mathtt{Afra}(\mathfrak{A})$ is AFRA-complete. From Lemma A.19 and the fact that both $\mathtt{Afra}(\mathfrak{A})$ and $\mathcal{E}'$ are complete, $\mathtt{Afra}(\mathfrak{A}) \subset \mathcal{E}$ implies that $\mathfrak{A}_{\mathtt{Afra}(\mathfrak{A})} \sqsubset \mathfrak{A}_{\mathcal{E}'}$ Moreover, since $\mathfrak{A}$ is complete, from Proposition 10, it follows that $\mathfrak{A}_{\mathtt{Afra}(\mathfrak{A})} = \mathfrak{A}$ and, thus, that $\mathfrak{A} \sqsubset \mathfrak{A}_{\mathcal{E}'}$. This is a contradiction with the assumption that $\mathfrak{A}$ is a preferred structure. Hence, $\mathtt{Afra}(\mathfrak{A})$ is an AFRA-preferred set. □

Lemma A.21. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be a stable structure. Then,* $\mathtt{Afra}(\mathfrak{A})$ *is AFRA-stable.* □

Proof. Since $\mathfrak{A}$ is a stable structure, it is conflict-free and, in addition, it satisfies $S = \overline{\mathit{Def}(\mathfrak{A})}$ and $\Gamma = \overline{\mathit{Inh}(\mathfrak{A})}$. From Proposition 4, the former implies that $\mathtt{Afra}(\mathfrak{A})$ is AFRA-conflict-free. Hence, to show that $\mathtt{Afra}(\mathfrak{A})$ is AFRA-stable, it is enough to prove that, for every $x \in ((\mathbf{A} \cup \mathbf{K}) \setminus \mathtt{Afra}(\mathfrak{A}))$, there is $\alpha \in \mathtt{Afra}(\mathfrak{A})$ such that α defeats x.

First, note that $x \in ((\mathbf{A} \cup \mathbf{K}) \setminus \mathtt{Afra}(\mathfrak{A}))$ implies that either $x \notin (S \cup \Gamma)$ or $x \in \Gamma$ but $\mathbf{s}(x) \notin S$. Since $\mathfrak{A}$ is stable, the former implies that $x \in (\mathit{Def}(\mathfrak{A}) \cup \mathit{Inh}(\mathfrak{A}))$ and, from Lemma A.7, this implies that there is some $\alpha \in \mathtt{Afra}(\mathfrak{A})$ such that α defeats x. On the other hand, the latter implies that $\mathbf{s}(x) \notin S$ and, thus, that $\mathbf{s}(x) \in \mathit{Def}(\mathfrak{A})$. From Lemma A.7, this implies that there is some $\alpha \in \mathtt{Afra}(\mathfrak{A})$ such that α defeats $\mathbf{s}(x)$ and, thus, that defeats x. □

Proof of Proposition 8. Conditions i), iii), and iv) follow directly from Lemmas A.14, A.20 and A.21, respectively. For ii), note that $\mathfrak{A}$ being grounded implies that it is also complete and, thus, that $\mathtt{Afra}(\mathfrak{A})$ is AFRA-complete. Suppose, for the sake of contradiction, that $\mathtt{Afra}(\mathfrak{A})$ is not AFRA-grounded and, thus, that there is some set $\mathcal{E}$ which is AFRA-complete and satisfies $\mathcal{E} \subset \mathtt{Afra}(\mathfrak{A})$. Then, from Lemmas A.17 and A.19, it respectively follows that $\mathfrak{A}_{\mathcal{E}}$ is a complete structure and that $\mathfrak{A}_{\mathcal{E}} \sqsubset \mathfrak{A}_{\mathtt{Afra}(\mathfrak{A})}$. Note that, from Proposition 10, we have that $\mathfrak{A}_{\mathtt{Afra}(\mathfrak{A})} = \mathfrak{A}$ and, thus, we get $\mathfrak{A}_{\mathcal{E}} \sqsubset \mathfrak{A}$ which is a contradiction with the fact that $\mathfrak{A}$ is a grounded structure. Hence, $\mathtt{Afra}(\mathfrak{A})$ is AFRA-grounded. □

Lemma A.22. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$ *be a closed set. Then, it follows that* $x \in (\overline{Def(\mathfrak{A}_{\mathcal{E}})} \cup \overline{Inh(\mathfrak{A}_{\mathcal{E}})})$ *implies that there is no* $\alpha \in \mathcal{E}$ *such that* α *directly defeats* x. □

Proof. Suppose, for the sake of contradiction, that there is $\alpha \in \mathcal{E}$ such that α directly defeats x. If α directly defeats x, then $\mathbf{t}(\alpha) = x$. Furthermore, since $\mathcal{E}$ is closed, $\alpha \in \mathcal{E}$ implies that $\mathbf{s}(\alpha) \in \mathcal{E}$ and, thus, that $\alpha \in \Gamma_{\mathcal{E}}$ and that $\mathbf{s}(\alpha) \in S_{\mathcal{E}}$. This implies that $x \in (Def(\mathfrak{A}_{\mathcal{E}}) \cup Inh(\mathfrak{A}_{\mathcal{E}}))$ which is a contradiction with the assumption. □

Lemma A.23. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$ *be a closed set. Then,* $a \in \overline{Def(\mathfrak{A}_{\mathcal{E}})}$ *implies that there is no* $\alpha \in \mathcal{E}$ *such that* α *defeats* a. □

Proof. $a \in \overline{Def(\mathfrak{A}_{\mathcal{E}})}$ implies that $a \in \mathbf{A}$ and, thus, α defeats a only if α directly defeats a. Then, the result follows directly from Lemma A.22. □

Lemma A.24. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$ *be a closed set. Then,* $\alpha \in \overline{Inh(\mathfrak{A}_{\mathcal{E}})}$ *and* $\mathbf{s}(\alpha) \in \overline{Def(\mathfrak{A}_{\mathcal{E}})}$ *imply that there is no* $\beta \in \mathcal{E}$ *such that* β *defeats* α. □

Proof. From Lemma A.22, it follows that there is is no $\beta \in \mathcal{E}$ such that β directly defeats α. Suppose, for the sake of contradiction, that there is $\beta \in \mathcal{E}$ such that β indirectly defeats α. This implies that $\mathbf{t}(\beta) = \mathbf{s}(\alpha)$. Furthermore, since $\mathcal{E}$ is closed, it follows that $\beta \in \mathcal{E}$ implies that $\mathbf{s}(\beta) \in \mathcal{E}$ and, thus, that $\beta \in \Gamma_{\mathcal{E}}$ and that $\mathbf{s}(\beta) \in S_{\mathcal{E}}$. This implies that $\mathbf{s}(\alpha) \in Def(\mathfrak{A}_{\mathcal{E}})$ which is a contradiction with the assumption. □

Lemma A.25. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$ *be a closed AFRA-conflict-free set. Then,* $x \in (\overline{Def(\mathfrak{A}_{\mathcal{E}})} \cup \overline{Inh(\mathfrak{A}_{\mathcal{E}})})$ *implies that there is no* $\alpha \in \mathcal{E}$ *such that* α *defeats* x. □

Proof. Pick any $x \in (\overline{Def(\mathfrak{A})} \cup \overline{Inh(\mathfrak{A}_{\mathcal{E}})})$. If $x \in \mathbf{A}$, from Lemma A.23, it follows that there is no $\alpha \in \mathcal{E}$ such that α defeats x. Otherwise, $x \in \mathbf{K}$ and $x \in \overline{Inh(\mathfrak{A}_{\mathcal{E}})}$. Since $\mathcal{E}$ is closed, it follows that $\mathbf{s}(x) \in \mathcal{E}$ and $\mathbf{s}(x) \in S_{\mathcal{E}}$. Furthermore, since $\mathcal{E}$ is conflict-free, Proposition 5 implies that $\mathfrak{A}_{\mathcal{E}}$ is conflict-free. Then, $\mathbf{s}(x) \in S_{\mathcal{E}}$ implies $\mathbf{s}(x) \in \overline{Def(\mathfrak{A}_{\mathcal{E}})}$. From Lemma A.24, this plus $x \in \overline{Inh(\mathfrak{A}_{\mathcal{E}})}$ imply that there is no $\alpha \in \mathcal{E}$ such that α defeats x. □

Lemma A.26. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework. Then, every AFRA-stable extension is closed.* □

Proof. Note that every AFRA-stable extension is also AFRA-complete (Lemmas 4 and 5 in [4]) and, thus, Lemma A.16 implies that every AFRA-stable extension is also closed. □

Lemma A.27. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$ *be a AFRA-stable extension. Then, it follows that* $(\overline{Def(\mathfrak{A}_\mathcal{E})} \cup \overline{Inh(\mathfrak{A}_\mathcal{E})}) \subseteq \mathcal{E}$. □

Proof. By definition every AFRA-stable extension is AFRA-conflict-free. Furthermore, from Lemma A.26, every AFRA-stable extension is closed. Then, Lemma A.25 implies that, for every $x \in (\overline{Def(\mathfrak{A})} \cup \overline{Inh(\mathfrak{A}_\mathcal{E})})$, there is no $\alpha \in \mathcal{E}$ such that α defeats x. Then, since $\mathcal{E}$ is AFRA-stable, this implies that $x \in \mathcal{E}$ and, consequently, that $(\overline{Def(\mathfrak{A})} \cup \overline{Inh(\mathfrak{A}_\mathcal{E})}) \subseteq \mathcal{E}$ holds. □

Lemma A.28. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$ *be a AFRA-stable extension. Then,* $\mathfrak{A}_\mathcal{E}$ *is a stable structure.* □

Proof. Since by definition every AFRA-stable extension is AFRA-conflict-free, it follows that $\mathfrak{A}_\mathcal{E}$ is a conflict-free structure (Proposition 5). Then, to show that $\mathfrak{A}_\mathcal{E}$ is stable, it is enough to prove $S_\mathcal{E} = \overline{Def(\mathfrak{A}_\mathcal{E})}$ and $\Gamma_\mathcal{E} = \overline{Inh(\mathfrak{A}_\mathcal{E})}$. Note that, since $\mathfrak{A}_\mathcal{E}$ is conflict-free, it follows that $S \subseteq \overline{Def(\mathfrak{A}_\mathcal{E})}$ and $\Gamma \subseteq \overline{Inh(\mathfrak{A}_\mathcal{E})}$ hold. Furthermore, from Lemma A.27, it follows that

$$(\overline{Def(\mathfrak{A}_\mathcal{E})} \cup \overline{Inh(\mathfrak{A}_\mathcal{E})}) \quad \subseteq \quad \mathcal{E} \quad \subseteq \quad (S_\mathcal{E} \cup \Gamma_\mathcal{E})$$

and, thus, that $S = \overline{Def(\mathfrak{A}_\mathcal{E})}$ and $\Gamma = \overline{Inh(\mathfrak{A}_\mathcal{E})}$ hold. Consequently, $\mathfrak{A}_\mathcal{E}$ is a stable structure. □

Lemma A.29. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework. Every AFRA-preferred extension is closed.* □

Proof. Note that every AFRA-preferred extension is also AFRA-complete (see Lemma 4 in [4]) and, thus, Lemma A.16 implies that every AFRA-preferred extension is also closed. □

Lemma A.30. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathfrak{A} \sqsubseteq \mathfrak{A}'$ *be two structures. Then,* $\mathtt{Afra}(\mathfrak{A}) \subseteq \mathtt{Afra}(\mathfrak{A}')$. □

Proof. Let $\mathfrak{A} = \langle S, \Gamma \rangle$ and $\mathfrak{A}' = \langle S', \Gamma' \rangle$. Then,

$$(\mathtt{Afra}(\mathfrak{A}) \cap \mathbf{A}) \;=\; S \;\subseteq\; S' \;=\; (\mathtt{Afra}(\mathfrak{A}') \cap \mathbf{A})$$

Furthermore,

$$\begin{array}{rcl} (\mathtt{Afra}(\mathfrak{A}) \cap \mathbf{K}) & = & \{\, \alpha \in \Gamma \;\big|\; \mathbf{s}(\alpha) \in S \,\} \\ (\mathtt{Afra}(\mathfrak{A}) \cap \mathbf{K}) & = & \{\, \alpha \in \Gamma' \;\big|\; \mathbf{s}(\alpha) \in S' \,\} \end{array}$$

and, thus, $(\mathtt{Afra}(\mathfrak{A}) \cap \mathbf{K}) \subseteq (\mathtt{Afra}(\mathfrak{A}) \cap \mathbf{K})$. These two facts together imply $\mathtt{Afra}(\mathfrak{A}) \subseteq \mathtt{Afra}(\mathfrak{A}')$. □

Lemma A.31. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathfrak{A} \sqsubset \mathfrak{A}'$ *be two complete structures. Then,* $\mathtt{Afra}(\mathfrak{A}) \subset \mathtt{Afra}(\mathfrak{A}')$. □

Proof. First note that, from Lemma A.31, we have that $\mathtt{Afra}(\mathfrak{A}) \subset \mathtt{Afra}(\mathfrak{A}')$. Suppose, for the sake of contradiction that $\mathtt{Afra}(\mathfrak{A}) = \mathtt{Afra}(\mathfrak{A}')$. Then, it is obvious that $\mathfrak{A}_{\mathtt{Afra}(\mathfrak{A})} = \mathfrak{A}_{\mathtt{Afra}(\mathfrak{A}')}$. However, from Proposition 10, this implies that $\mathfrak{A} = \mathfrak{A}'$ which is a contradiction with the assumption. Hence, $\mathtt{Afra}(\mathfrak{A}) \subset \mathtt{Afra}(\mathfrak{A}')$ holds. □

Lemma A.32. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$ *be a AFRA-preferred extension. Then,* $\mathfrak{A}_{\mathcal{E}}$ *is a preferred structure.* □

Proof. By definition every AFRA-preferred extension is AFRA-admissible. Furthermore, from Lemma A.29, it follows that AFRA-preferred extensions are closed. Then, from Proposition 7, it follows that $\mathfrak{A}_{\mathcal{E}}$ is an admissible structure. Suppose, for the sake of contradiction, that $\mathfrak{A}_{\mathcal{E}}$ is not preferred and, thus, that there is some admissible structure $\mathfrak{A}'$ such that $\mathfrak{A}_{\mathcal{E}} \sqsubset \mathfrak{A}'$. Then, from Proposition 1, there is some preferred structure $\mathfrak{A}''$ such that $\mathfrak{A}_{\mathcal{E}} \sqsubset \mathfrak{A}' \sqsubseteq \mathfrak{A}''$ and, from Theorem 2, we have that $\mathfrak{A}''$ is complete. Then, from Proposition 7, it follows that $\mathtt{Afra}(\mathfrak{A}'')$ is admissible and, from Lemma A.31 and Proposition 10, that $\mathcal{E} = \mathtt{Afra}(\mathfrak{A}_{\mathcal{E}}) \subset \mathtt{Afra}(\mathfrak{A}'')$. This is a contradiction with the fact that $\mathcal{E}$ is AFRA-preferred. Hence, $\mathfrak{A}_{\mathcal{E}}$ must be a preferred structure. □

Proof of Proposition 9. Conditions i), iii), and iv) directly follow from Lemmas A.17, A.32 and A.28, respectively. For ii) note that $\mathcal{E}$ being AFRA-grounded implies that it is also AFRA-complete and, thus, that $\mathfrak{A}_{\mathcal{E}}$ is a complete structure. Suppose, for the sake of contradiction, that $\mathfrak{A}_{\mathcal{E}}$ is not AFRA-grounded and, thus, that there is some complete structure $\mathfrak{A}$ which satisfies $\mathfrak{A} \sqsubset \mathfrak{A}_{\mathcal{E}}$. Then, from Proposition 8 and Lemma A.31, it respectively follows that $\mathtt{Afra}(\mathfrak{A})$ is a AFRA-complete and that $\mathtt{Afra}(\mathfrak{A}) \subset \mathtt{Afra}(\mathfrak{A}_{\mathcal{E}})$. Note that, from Proposition 10, we have that $\mathtt{Afra}(\mathfrak{A}_{\mathcal{E}}) = \mathcal{E}$ and, thus, we get $\mathtt{Afra}(\mathfrak{A}) \subset \mathcal{E}$ which is a contradiction with the fact that $\mathcal{E}$ is a AFRA-grounded. Hence, $\mathfrak{A}_{\mathcal{E}}$ must be a grounded structure. □

Proof of Proposition 11. Let $a \in \mathbf{A}$ be some argument. Then, from Theorem 3 and Proposition 12 in [4], it follows that some structure $\mathfrak{A} = \langle S, \Gamma \rangle$ is preferred (resp. stable) w.r.t. **RAF**

> iff $\mathtt{Afra}(\mathfrak{A})$ is a preferred (resp. stable) extension w.r.t. **RAF**
> iff $\mathtt{Afra}(\mathfrak{A})$ is a preferred (resp. stable) extension w.r.t. $\mathbf{RAF}_{AF}$ with $\mathbf{RAF}_{AF}$ is the corresponding Dung framework of **RAF** as given by Def. 19 in [4].

Hence, a is credulous accepted w.r.t. **RAF** and the preferred (resp. stable) semantics

iff there is some preferred (resp. stable) structure $\mathfrak{A} = \langle S, \Gamma \rangle$ of **RAF** such that $a \in S$
iff there is some preferred (resp. stable) extension $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$ of **RAF** such that $a \in \mathcal{E}$
iff there is some preferred (resp. stable) extension $\mathcal{E} \subseteq (\mathbf{A} \cup \mathbf{K})$ of $\mathbf{RAF}_{AF}$ such that $a \in \mathcal{E}$.
iff a is credulous accepted w.r.t. $\mathbf{RAF}_{AF}$ and the preferred (resp. stable) semantics.

Then, since credulous acceptance for Dung's frameworks w.r.t. the preferred and the stable semantics is NP-complete [15] and $\mathbf{RAF}_{AF}$ can be computed in polynomial time, it follows that credulous acceptance for RAFs is in NP. Hardness, follows from the fact that every Dung's framework is also a RAF and that, from Theorem 5, the preferred (resp. stable) semantics for RAFs are conservative generalisations.

Analogously, since sceptical acceptance for Dung's frameworks w.r.t. the preferred (resp. stable) semantics is coNP-complete (resp. Π_2^{P}-complete) [15], it follows that sceptical acceptance for RAFs w.r.t. the preferred (resp. stable) semantics is coNP-complete (resp. Π_2^{P}-complete).

Finally, for the complete semantics, note that from Theorem 2, every preferred structure is also a complete structure and, thus, if an argument is credulous accepted w.r.t. the preferred semantics, it is also credulous accepted w.r.t. the complete semantics. Furthermore, every complete $\mathfrak{A}$ structure is admissible and, from Proposition 1, this implies that there is a preferred structure $\mathfrak{A}'$ such that $\mathfrak{A} \sqsubseteq \mathfrak{A}'$. This implies that, if an argument is credulous accepted w.r.t. the complete semantics, it is also credulous accepted w.r.t. the preferred semantics. □

A.3 Proofs of Section 5

Lemma A.33. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *and its associated* $\mathbf{MAF} = \langle \mathbf{A}', \mathbf{R}' \rangle$*. Let* $\mathcal{E}$ *be an admissible extension of* $\langle \mathbf{A}', \mathbf{R}' \rangle$*.*

1. *If* $\mathcal{E} \cap \mathbf{K}$ *contains* α*, then* $\mathcal{E}$ *contains* $\mathbf{s}(\alpha)$*.*
2. *If* $\mathcal{E}$ *contains* $N_{\mathbf{s}(\alpha)\alpha}$ *then* $\mathcal{E} \cap \mathbf{K}$ *attacks* $\mathbf{s}(\alpha)$*.*
3. *Moreover, if* $\mathcal{E}$ *is complete the equivalence holds:* $\mathcal{E}$ *contains* $N_{\mathbf{s}(\alpha)\alpha}$ *if and only if* $\mathcal{E} \cap \mathbf{K}$ *attacks* $\mathbf{s}(\alpha)$*.*

Proof. Let $N_{\mathbf{s}(\alpha)\alpha} \in \mathbf{N}$. Due to the definition of $\mathbf{R}'$ (see Definition 12), the only attack to $N_{\mathbf{s}(\alpha)\alpha}$ is the attack $(\mathbf{s}(\alpha), N_{\mathbf{s}(\alpha)\alpha})$.

1. As $\mathcal{E}$ is admissible, α is acceptable w.r.t. $\mathcal{E}$. As $\mathbf{s}(\alpha)$ is the only defender of α against $N_{\mathbf{s}(\alpha)\alpha}$, $\mathbf{s}(\alpha)$ must belong to $\mathcal{E}$.
2. If $\mathcal{E}$ is admissible and contains $N_{\mathbf{s}(\alpha)\alpha}$, $N_{\mathbf{s}(\alpha)\alpha}$ is acceptable w.r.t. $\mathcal{E}$ so $\mathcal{E}$ must attack $\mathbf{s}(\alpha)$. Due to the definition of $\mathbf{R}'$ again (see Definition 12), this attack comes from $\mathcal{E} \cap \mathbf{K}$.

3. If $\mathcal{E}$ is complete and $\mathcal{E} \cap \mathbf{K}$ attacks $\mathbf{s}(\alpha)$ then $N_{\mathbf{s}(\alpha)\alpha}$ is acceptable w.r.t. $\mathcal{E}$. So $N_{\mathbf{s}(\alpha)\alpha}$ must belong to $\mathcal{E}$.

$\square$

Lemma A.34. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *and its associated* $\mathbf{MAF} = \langle \mathbf{A}', \mathbf{R}' \rangle$*. Let* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be a structure. Let* $\mathfrak{A}'$ *be the structure* $\langle S, \mathtt{Maf}(\mathfrak{A}) \cap \mathbf{K} \rangle$*. It holds that* $Acc(\mathfrak{A}) = Acc(\mathfrak{A}')$*.*

Proof. By definition $\mathtt{Maf}(\mathfrak{A}) \cap \mathbf{K} = \{\alpha \in \Gamma \text{ s.t. } \mathbf{s}(\alpha) \in S\})$. Let $x \in \mathbf{A} \cup \mathbf{K}$. By definition, $x \in Acc(\mathfrak{A})$ if and only if for each attack $\beta \in \mathbf{K}$ such that $\mathbf{t}(\beta) = x$, there exists $\gamma \in \Gamma$ with $\mathbf{s}(\gamma) \in S$ and $\mathbf{t}(\gamma) \in \{\beta, \mathbf{s}(\beta)\}$. Obviously, $\gamma \in \Gamma$ with $\mathbf{s}(\gamma) \in S$ is equivalent to $\gamma \in \mathtt{Maf}(\mathfrak{A}) \cap \mathbf{K}$ with $\mathbf{s}(\gamma) \in S$. So $x \in Acc(\mathfrak{A})$ if and only if $x \in Acc(\mathfrak{A}')$. $\square$

Lemma A.35. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *and its associated* $\mathbf{MAF} = \langle \mathbf{A}', \mathbf{R}' \rangle$*. Let* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be a structure,* $\mathtt{Maf}(\mathfrak{A})$ *be the associated MAF-extension, and* $\mathfrak{A}_{\mathtt{Maf}(\mathfrak{A})} = \langle S_{\mathtt{Maf}(\mathfrak{A})}, \Gamma_{\mathtt{Maf}(\mathfrak{A})} \rangle$ *be the structure associated with the extension* $\mathtt{Maf}(\mathfrak{A})$*. The following assertions hold:*

1. $S_{\mathtt{Maf}(\mathfrak{A})} = S$
2. *If* $\mathfrak{A}$ *is an admissible structure, then* $\Gamma \subseteq \Gamma_{\mathtt{Maf}(\mathfrak{A})}$
3. *If* $(Acc(\mathfrak{A}) \cap \mathbf{K}) \subseteq \Gamma$*, then* $\Gamma_{\mathtt{Maf}(\mathfrak{A})} \subseteq \Gamma$
4. *If* $\mathfrak{A}$ *is a complete structure, then* $\Gamma = \Gamma_{\mathtt{Maf}(\mathfrak{A})}$

Proof.

1. By definition of $\mathfrak{A}_{\mathtt{Maf}(\mathfrak{A})}$ we have $S_{\mathtt{Maf}(\mathfrak{A})} = \mathtt{Maf}(\mathfrak{A}) \cap \mathbf{A}$. By definition of $\mathtt{Maf}(\mathfrak{A})$, we have $\mathtt{Maf}(\mathfrak{A}) \cap \mathbf{A} = S$.

2. By definition, $\Gamma_{\mathtt{Maf}(\mathfrak{A})} = (\mathtt{Maf}(\mathfrak{A}) \cap \mathbf{K}) \cup \{\alpha \notin (\mathtt{Maf}(\mathfrak{A}) \cap \mathbf{K}) \text{ s.t. } \alpha \in Acc((\mathtt{Maf}(\mathfrak{A}) \cap \mathbf{A}), (\mathtt{Maf}(\mathfrak{A}) \cap \mathbf{K})) \text{ and } \mathbf{s}(\alpha) \notin (\mathtt{Maf}(\mathfrak{A}) \cap \mathbf{A})\}$. Note that by definition, $(\mathtt{Maf}(\mathfrak{A}) \cap \mathbf{A}) = S$ and $(\mathtt{Maf}(\mathfrak{A}) \cap \mathbf{K}) = \{\gamma \in \Gamma \text{ with } \mathbf{s}(\gamma) \in S\}$. And from Lemma A.34, we have $Acc((\mathtt{Maf}(\mathfrak{A}) \cap \mathbf{A}), (\mathtt{Maf}(\mathfrak{A}) \cap \mathbf{K})) = Acc(\mathfrak{A})$.
   Let $\alpha \in \Gamma$. If $\mathbf{s}(\alpha) \in S$, $\alpha \in (\mathtt{Maf}(\mathfrak{A}) \cap \mathbf{K})$ so $\alpha \in \Gamma_{\mathtt{Maf}(\mathfrak{A})}$. If $\mathbf{s}(\alpha) \notin S$, as $\mathfrak{A}$ is admissible, we have $\alpha \in Acc(\mathfrak{A})$, so α belongs to the second part of $\Gamma_{\mathtt{Maf}(\mathfrak{A})}$.

3. Assume that $(Acc(\mathfrak{A}) \cap \mathbf{K}) \subseteq \Gamma$. Let $\alpha \in \Gamma_{\mathtt{Maf}(\mathfrak{A})}$. If $\alpha \in (\mathtt{Maf}(\mathfrak{A}) \cap \mathbf{K}) \subseteq \Gamma$, we have $\alpha \in \Gamma$. If $\alpha \notin (\mathtt{Maf}(\mathfrak{A}) \cap \mathbf{K})$, by definition of $\Gamma_{\mathtt{Maf}(\mathfrak{A})}$, we have $\alpha \in Acc(\mathfrak{A})$. Due to the assumption, it follows that $\alpha \in \Gamma$.

4. The proof follows directly from the above items of this lemma.

$\square$

Lemma A.36. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *and its associated* $\mathbf{MAF} = \langle \mathbf{A}', \mathbf{R}' \rangle$*. Let* $\mathcal{E}$ *be an extension of* $\mathbf{MAF}$*. Let* $\mathfrak{A}_{\mathcal{E}}$ *be its associated structure and* $\mathtt{Maf}(\mathfrak{A}_{\mathcal{E}})$ *be the MAF-extension associated with* $\mathfrak{A}_{\mathcal{E}}$*. The following assertions hold:*

1. $\mathtt{Maf}(\mathfrak{A}_{\mathcal{E}}) \cap \mathbf{A} = \mathcal{E} \cap \mathbf{A}$
2. $\mathtt{Maf}(\mathfrak{A}_{\mathcal{E}}) \cap \mathbf{K} \subseteq \mathcal{E} \cap \mathbf{K}$
3. *If* $\mathcal{E}$ *is admissible, then* $\mathtt{Maf}(\mathfrak{A}_{\mathcal{E}}) \cap \mathbf{K} = \mathcal{E} \cap \mathbf{K}$
4. *If* $\mathcal{E}$ *is admissible, then* $\mathcal{E} \cap \mathbf{N} \subseteq \mathtt{Maf}(\mathfrak{A}_{\mathcal{E}}) \cap \mathbf{N}$
5. *If* $\mathcal{E}$ *is complete then* $\mathtt{Maf}(\mathfrak{A}_{\mathcal{E}}) \cap \mathbf{N} = \mathcal{E} \cap \mathbf{N}$

Proof. By definition, $\mathfrak{A}_{\mathcal{E}} = \langle S_{\mathcal{E}}, \Gamma_{\mathcal{E}} \rangle$ with $S_{\mathcal{E}} = \mathcal{E}_a = \mathcal{E} \cap \mathbf{A}$, $\mathcal{E}_k = \mathcal{E} \cap \mathbf{K}$, and $\Gamma_{\mathcal{E}} = \mathcal{E}_k \cup \{\alpha \notin \mathcal{E}_k \text{ s.t. } \alpha \in Acc(\mathcal{E}_a, \mathcal{E}_k) \text{ and } \mathbf{s}(\alpha) \notin \mathcal{E}_a\}$.
Then, $\mathtt{Maf}(\mathfrak{A}_{\mathcal{E}})$ is such that $\mathtt{Maf}(\mathfrak{A}_{\mathcal{E}}) \cap \mathbf{A} = S_{\mathcal{E}}$, $\mathtt{Maf}(\mathfrak{A}_{\mathcal{E}}) \cap \mathbf{K} = \{\alpha \in \Gamma_{\mathcal{E}}$ such that $\mathbf{s}(\alpha) \in S_{\mathcal{E}}\}$, and $\mathtt{Maf}(\mathfrak{A}_{\mathcal{E}}) \cap \mathbf{N} = \{N_{\mathbf{s}(\alpha)\alpha} \text{ s.t. } \mathbf{s}(\alpha) \notin S_{\mathcal{E}} \text{ and } \mathbf{s}(\alpha) \in Def(\mathfrak{A}_{\mathcal{E}})\}$.

1. Obviously, $\mathtt{Maf}(\mathfrak{A}_{\mathcal{E}}) \cap \mathbf{A} = \mathcal{E} \cap \mathbf{A}$.
2. By definition of $\Gamma_{\mathcal{E}}$, if $\alpha \in \Gamma_{\mathcal{E}}$ with $\mathbf{s}(\alpha) \in S_{\mathcal{E}} = \mathcal{E}_a$ then $\alpha \in \mathcal{E}_k$. So, $\mathtt{Maf}(\mathfrak{A}_{\mathcal{E}}) \cap \mathbf{K} \subseteq \mathcal{E}_k$.
3. Assume that $\mathcal{E}$ is admissible. Let $\alpha \in \mathcal{E}_k$. From Lemma A.33, we have $\mathbf{s}(\alpha) \in \mathcal{E}_a$. As $\mathcal{E}_k \subseteq \Gamma_{\mathcal{E}}$ it follows that $\alpha \in \Gamma_{\mathcal{E}}$ and $\mathbf{s}(\alpha) \in \mathcal{E}_a$. So $\alpha \in \mathtt{Maf}(\mathfrak{A}_{\mathcal{E}}) \cap \mathbf{K}$. From the above item we conclude that $\mathtt{Maf}(\mathfrak{A}_{\mathcal{E}}) \cap \mathbf{K} = \mathcal{E}_k$.
4. Assume that $\mathcal{E}$ is admissible. Let $N_{\mathbf{s}(\alpha)\alpha} \in \mathcal{E} \cap \mathbf{N}$. From Lemma A.33, we know that $\mathcal{E}_k$ attacks $\mathbf{s}(\alpha)$. So there exists $\beta \in \mathcal{E}_k$ that attacks $\mathbf{s}(\alpha)$. From Lemma A.33 again, we have $\mathbf{s}(\beta) \in \mathcal{E}$. So, $\mathbf{s}(\alpha)$ is attacked by $\beta \in \mathcal{E}_k \subseteq \Gamma_{\mathcal{E}}$ with $\mathbf{s}(\beta) \in \mathcal{E}_a$. That means that $\mathbf{s}(\alpha) \in Def(\mathfrak{A}_{\mathcal{E}})$. Moreover, as $\mathcal{E}$ is conflict-free, $\mathbf{s}(\alpha) \notin \mathcal{E}_a$. It follows that $N_{\mathbf{s}(\alpha)\alpha} \in \mathtt{Maf}(\mathfrak{A}_{\mathcal{E}}) \cap \mathbf{N}$.
5. Assume that $\mathcal{E}$ is complete. Let $N_{\mathbf{s}(\alpha)\alpha} \in \mathtt{Maf}(\mathfrak{A}_{\mathcal{E}}) \cap \mathbf{N}$. By definition, $\mathbf{s}(\alpha) \notin S_{\mathcal{E}} = \mathcal{E}_a$ and $\mathbf{s}(\alpha) \in Def(S_{\mathcal{E}}, \Gamma_{\mathcal{E}})$. So there exists $\beta \in \Gamma_{\mathcal{E}}$ with $\mathbf{s}(\beta) \in \mathcal{E}_a$ such that β attacks $\mathbf{s}(\alpha)$. By definition of $\Gamma_{\mathcal{E}}$ it follows that $\beta \in \mathcal{E}_k$. Then from Lemma A.33, we conclude that $\mathcal{E}$ contains $N_{\mathbf{s}(\alpha)\alpha}$. Then from the above item of this lemma, as E is also admissible, we conclude that $\mathtt{Maf}(\mathfrak{A}_{\mathcal{E}}) \cap \mathbf{N} = \mathcal{E} \cap \mathbf{N}$.

□

Lemma A.37. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *and its associated* $\mathbf{MAF} = \langle \mathbf{A}', \mathbf{R}' \rangle$*. Let* $\mathcal{E}$ *and* $\mathcal{E}'$ *be two complete extensions of* $\mathbf{MAF}$ *such that* $\mathcal{E} \subseteq \mathcal{E}'$ *then* $\mathfrak{A}_{\mathcal{E}} \subseteq \mathfrak{A}_{\mathcal{E}'}$*.*

Proof. By definition, $\mathfrak{A}_{\mathcal{E}} = \langle S_{\mathcal{E}}, \Gamma_{\mathcal{E}} \rangle$ with $S_{\mathcal{E}} = \mathcal{E}_a = \mathcal{E} \cap \mathbf{A}$, $\mathcal{E}_k = \mathcal{E} \cap \mathbf{K}$, and $\Gamma_{\mathcal{E}} = \mathcal{E}_k \cup \{\alpha \notin \mathcal{E}_k \text{ s.t. } \alpha \in Acc(\mathcal{E}_a, \mathcal{E}_k) \text{ and } \mathbf{s}(\alpha) \notin \mathcal{E}_a\}$.
First, we have $\mathcal{E}_a \subseteq \mathcal{E}'_a$ and $\mathcal{E}_k \subseteq \mathcal{E}'_k$. Then, due to Lemma A.33, as $\mathcal{E}$ (resp. $\mathcal{E}'$) is admissible, $\mathbf{s}(\alpha) \notin \mathcal{E}_a$ (resp. $\mathbf{s}(\alpha) \notin \mathcal{E}'_a$) implies that $\alpha \notin \mathcal{E}_k$ (resp. $\alpha \notin \mathcal{E}'_k$). So it is enough to prove that: $\{\alpha \in Acc(\mathcal{E}_a, \mathcal{E}_k) \text{ such that } \mathbf{s}(\alpha) \notin \mathcal{E}_a\} \subseteq$

$\mathcal{E}'_k \cup \{\alpha \in Acc(\mathcal{E}'_a, \mathcal{E}'_k)$ such that $\mathbf{s}(\alpha) \notin \mathcal{E}'_a\}$.
Furthermore, we have $Acc(\mathcal{E}_a, \mathcal{E}_k) \subseteq Acc(\mathcal{E}'_a, \mathcal{E}'_k)$ so it is enough to show that: For each $\alpha \in Acc(\mathcal{E}_a, \mathcal{E}_k)$ such that $\mathbf{s}(\alpha) \notin \mathcal{E}_a$, either $\alpha \in \mathcal{E}'_k$ or $\mathbf{s}(\alpha) \notin \mathcal{E}'_a$.
Assume that the contrary holds. So there is $\alpha \in Acc(\mathcal{E}_a, \mathcal{E}_k)$ such that $\alpha \notin \mathcal{E}'_k$, $\mathbf{s}(\alpha) \notin \mathcal{E}_a$ and $\mathbf{s}(\alpha) \in \mathcal{E}'_a$. As $\mathcal{E}'$ is a complete extension of **MAF**, α is not acceptable w.r.t. $\mathcal{E}'$. Moreover, as $\mathbf{s}(\alpha) \in \mathcal{E}'_a$, α is defended by $\mathcal{E}'$ against its attacker $N_{\mathbf{s}(\alpha)\alpha}$. So there must exist another attacker of α, say β, such that $\mathcal{E}'$ does not attack β. That implies that $N_{\mathbf{s}(\beta)\beta} \notin \mathcal{E}'$, and from Lemma A.33 that $\mathcal{E}'$ does not attack $\mathbf{s}(\beta)$. So $\mathcal{E}'$ attacks neither β, nor $\mathbf{s}(\beta)$; this fact will be denoted by (*).
Moreover, as $\alpha \in Acc(\mathcal{E}_a, \mathcal{E}_k)$, either $\beta \in Inh(\mathcal{E}_a, \mathcal{E}_k)$ (Case 1), or $\mathbf{s}(\beta) \in Def(\mathcal{E}_a, \mathcal{E}_k)$ (Case 2). It follows that there is $\gamma \in \mathcal{E}_k \subseteq \mathcal{E}'_k$ with $\mathbf{s}(\gamma) \in \mathcal{E}_a$ such that $\mathbf{t}(\gamma) = \beta$ in Case 1 (resp. $\mathbf{t}(\gamma) = \mathbf{s}(\beta)$ in Case 2). So $\mathcal{E}'_k$ attacks β in Case 1 (resp. $\mathbf{s}(\beta)$ in Case 2), which is in contradiction with the fact (*).

□

Lemma A.38. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *and its associated* $\mathbf{MAF} = \langle \mathbf{A}', \mathbf{R}' \rangle$. *Let* $\mathfrak{A}$ *and* $\mathfrak{A}'$ *be two conflict-free structures of* **RAF** *such that* $\mathfrak{A} \subseteq \mathfrak{A}'$ *then* $\mathtt{Maf}(\mathfrak{A}) \subseteq \mathtt{Maf}(\mathfrak{A}')$.

Proof. By definition, $\mathfrak{A}$ being the structure $\langle S, \Gamma \rangle$, $\mathtt{Maf}(\mathfrak{A}) = S \cup \{\alpha \in \Gamma$ s.t. $\mathbf{s}(\alpha) \in S\} \cup \{N_{\mathbf{s}(\alpha)\alpha}$ s.t. $\mathbf{s}(\alpha) \notin S$ and $\mathbf{s}(\alpha) \in Def(\mathfrak{A})\}$.
First, we have $S \subseteq S'$ and $\{\alpha \in \Gamma$ s.t. $\mathbf{s}(\alpha) \in S\} \subseteq \{\alpha \in \Gamma'$ s.t. $\mathbf{s}(\alpha) \in S'\}$. So it is enough to prove that $(\mathtt{Maf}(\mathfrak{A}) \cap \mathbf{N}) \subseteq (\mathtt{Maf}(\mathfrak{A}') \cap \mathbf{N})$.
Let $x = N_{\mathbf{s}(\alpha)\alpha} \in \mathtt{Maf}(\mathfrak{A}) \cap \mathbf{N}$. As $\mathfrak{A} \subseteq \mathfrak{A}'$, we have $Def(\mathfrak{A}) \subseteq Def(\mathfrak{A}')$. So $\mathbf{s}(\alpha) \in Def(\mathfrak{A}')$. Hence, as $\mathfrak{A}'$ is a conflict-free structure, it is impossible to have $\mathbf{s}(\alpha) \in S'$. So $x = N_{\mathbf{s}(\alpha)\alpha} \in \mathtt{Maf}(\mathfrak{A}') \cap \mathbf{N}$.

□

Proposition 20. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *and its associated* $\mathbf{MAF} = \langle \mathbf{A}', \mathbf{R}' \rangle$.

1. *Let* $\mathfrak{A}$ *be an admissible structure of* **RAF**. *Then,* $\mathtt{Maf}(\mathfrak{A})$ *is an admissible extension of* $\langle \mathbf{A}', \mathbf{R}' \rangle$.
2. *Let* $\mathcal{E}$ *be an admissible extension of* $\langle \mathbf{A}', \mathbf{R}' \rangle$. $\mathfrak{A}'_{\mathcal{E}}$ *and* $\mathfrak{A}_{\mathcal{E}}$ *are admissible structures of* **RAF**.

Proof.

1. Let $\mathfrak{A} = \langle S, \Gamma \rangle$ be an admissible structure of **RAF**.
   From Proposition 13, $\mathtt{Maf}(\mathfrak{A})$ is conflict-free in **MAF**. It remains to prove that $\forall x \in \mathtt{Maf}(\mathfrak{A})$, x is acceptable w.r.t. $\mathtt{Maf}(\mathfrak{A})$. Three cases must be considered for x:
   (a) Let $x \in \mathtt{Maf}(\mathfrak{A}) \cap \mathbf{A}$. So $x \in S$. As $\mathfrak{A}$ is admissible, x is acceptable w.r.t. $\mathfrak{A}$. From Proposition 14, it follows that x is acceptable w.r.t. $\mathtt{Maf}(\mathfrak{A})$ in **MAF**.

(b) Let $x \in \mathtt{Maf}(\mathfrak{A}) \cap \mathbf{K}$. As $\mathfrak{A}$ is admissible, and $x \in \Gamma$, x is acceptable w.r.t. $\mathfrak{A}$. Moreover from the definition of $\mathtt{Maf}(\mathfrak{A})$ we have $\mathbf{s}(x) \in S$. So Proposition 14 applies and we conclude that x is acceptable w.r.t. $\mathtt{Maf}(\mathfrak{A})$ in **MAF**.

(c) Let $x \in \mathtt{Maf}(\mathfrak{A}) \cap \mathbf{N}$. So x has the form $N_{\mathbf{s}(\alpha)\alpha}$ with $\mathbf{s}(\alpha) \notin S$ and $\mathbf{s}(\alpha) \in Def(\mathfrak{A})$.
The only possible attack to x is from $\mathbf{s}(\alpha)$. As $\mathbf{s}(\alpha) \in Def(\mathfrak{A})$, $\exists \beta \in \Gamma$ s.t. $\mathbf{s}(\beta) \in S$ and $\mathbf{t}(\beta) = \mathbf{s}(\alpha)$. From the definition of $\mathtt{Maf}(\mathfrak{A})$, it follows that $\beta \in \mathtt{Maf}(\mathfrak{A})$ and then that $\mathtt{Maf}(\mathfrak{A})$ attacks $\mathbf{s}(\alpha)$. So $N_{\mathbf{s}(\alpha)\alpha}$ is acceptable w.r.t. $\mathtt{Maf}(\mathfrak{A})$ in **MAF**.

Hence we have proved that $\mathtt{Maf}(\mathfrak{A})$ is an admissible extension of $\langle \mathbf{A}', \mathbf{R}' \rangle$.

2. Let $\mathcal{E}$ be an admissible extension of $\langle \mathbf{A}', \mathbf{R}' \rangle$.

(a) From Proposition 13, $\mathfrak{A}'_{\mathcal{E}}$ is a conflict-free structure in **RAF**.
It remains to prove that $\forall x \in (\mathcal{E}_a \cup \mathcal{E}_k)$, x is acceptable w.r.t. $\mathfrak{A}'_{\mathcal{E}}$.
If x is unattacked in **RAF**, then it is obviously acceptable w.r.t. $\mathfrak{A}'_{\mathcal{E}}$.
Otherwise two cases must be considered for x:

i. Let $x \in \mathcal{E}_a$. Assume that x is attacked by $\alpha \in \mathbf{K}$. We have to prove that either $\alpha \in Inh(\mathfrak{A}'_{\mathcal{E}})$ or $\mathbf{s}(\alpha) \in Def(\mathfrak{A}'_{\mathcal{E}})$.
The attack α is encoded in the MAF with the three following attacks in $\mathbf{R}'$: $(\mathbf{s}(\alpha), N_{\mathbf{s}(\alpha)\alpha})$, $(N_{\mathbf{s}(\alpha)\alpha}, \alpha)$ and (α, x).
As $\mathcal{E}$ is assumed to be admissible, $\mathcal{E}$ attacks α. So either $N_{\mathbf{s}(\alpha)\alpha} \in \mathcal{E}$ (Case 1), or $\mathcal{E}_k$ attacks α (Case 2). Moreover, due to Lemma A.33, Case 1 also implies that $\mathcal{E}_k$ attacks $\mathbf{s}(\alpha)$.
So there exists $\beta \in \mathcal{E}_k$ s.t. β attacks $\mathbf{s}(\alpha)$ in Case 1 (resp. β attacks α in Case 2). As $\mathcal{E}$ is admissible, $\mathcal{E}$ defends β against $N_{\mathbf{s}(\beta)\beta}$ (Observation 4). So $\mathbf{s}(\beta) \in \mathcal{E}_a$. That fact with $\beta \in \mathcal{E}_k$ prove that $\mathbf{s}(\alpha) \in Def(\mathfrak{A}'_{\mathcal{E}})$ (resp. $\alpha \in Inh(\mathfrak{A}'_{\mathcal{E}})$).

ii. Let $x \in \mathcal{E}_k$. Assume that x is attacked by $\alpha \in \mathbf{K}$. We have to prove that either $\alpha \in Inh(\mathfrak{A}'_{\mathcal{E}})$ or $\mathbf{s}(\alpha) \in Def(\mathfrak{A}'_{\mathcal{E}})$. We can do exactly the same reasoning as for the first case ($x \in \mathcal{E}_a$).
So we have proved that the structure $\mathfrak{A}'_{\mathcal{E}}$ is admissible.

(b) From Proposition 13, $\mathfrak{A}_{\mathcal{E}}$ is a conflict-free structure in **RAF**.
It remains to prove that $\forall x \in (\mathcal{E}_a \cup \Gamma_{\mathcal{E}})$, x is acceptable w.r.t. $\mathfrak{A}_{\mathcal{E}}$.
We recall that $\Gamma_{\mathcal{E}} = \mathcal{E}_k \cup \{\alpha \notin \mathcal{E}_k \text{ s.t. } \alpha \in Acc(\mathfrak{A}'_{\mathcal{E}})\}$.
From the first part of the proof, $\mathfrak{A}'_{\mathcal{E}}$ is admissible. So $\forall x \in (\mathcal{E}_a \cup \mathcal{E}_k)$, x is acceptable w.r.t. $\mathfrak{A}'_{\mathcal{E}}$ and then w.r.t. $\mathfrak{A}_{\mathcal{E}}$. It remains to consider $x \in \Gamma_{\mathcal{E}} \setminus \mathcal{E}_k$. In that case, due to the definition of $\Gamma_{\mathcal{E}}$, $x \in Acc(\mathfrak{A}'_{\mathcal{E}})$ so x is acceptable w.r.t. $\mathfrak{A}_{\mathcal{E}}$.

So we have proved that the structure $\mathfrak{A}_{\mathcal{E}}$ is admissible.

□

Proof of Proposition 12.

1. Let $\mathfrak{A}$ be a structure of **RAF**.

   (a) (σ = complete) Let $\mathfrak{A} = \langle S, \Gamma \rangle$ be a complete structure of **RAF**. Let us recall that $\texttt{Maf}(\mathfrak{A}) = S \cup \{\alpha \in \Gamma \text{ s.t. } \mathbf{s}(\alpha) \in S\} \cup \{N_{\mathbf{s}(\alpha)\alpha} \text{ s.t. } \mathbf{s}(\alpha) \notin S \text{ and } \mathbf{s}(\alpha) \in \mathit{Def}(\mathfrak{A})\}$.
   By definition, $\mathfrak{A}$ is an admissible structure and satisfies $\mathit{Acc}(\mathfrak{A}) \subseteq S \cup \Gamma$. From Proposition 20, we have that $\texttt{Maf}(\mathfrak{A})$ is an admissible extension of $\langle \mathbf{A}', \mathbf{R}' \rangle$. So it remains to prove that $\forall x \in \mathbf{A}'$, if x is acceptable w.r.t. $\texttt{Maf}(\mathfrak{A})$, then $x \in \texttt{Maf}(\mathfrak{A})$.
   Three cases must be considered for x:

      i. Let $x \in \mathbf{A}$ being acceptable w.r.t. $\texttt{Maf}(\mathfrak{A})$. Assume that $x \notin \texttt{Maf}(\mathfrak{A})$. Then $x \notin S$ and so $x \notin \mathit{Acc}(\mathfrak{A})$, due to the assumption on $\mathfrak{A}$. $x \notin \mathit{Acc}(\mathfrak{A})$ means that there is $\beta \in \mathbf{K}$ with $\mathbf{t}(\beta) = x$ and such that $\beta \notin \mathit{Inh}(\mathfrak{A})$ and $\mathbf{s}(\beta) \notin \mathit{Def}(\mathfrak{A})$; this fact will be denoted by (*). So we have the attack (β, x) in **MAF**.
      As x is acceptable w.r.t. $\texttt{Maf}(\mathfrak{A})$, we know that $\texttt{Maf}(\mathfrak{A})$ attacks β in **MAF**. So, either $N_{\mathbf{s}(\beta)\beta} \in \texttt{Maf}(\mathfrak{A})$ (Case 1), or there exists $\gamma \in (\texttt{Maf}(\mathfrak{A}) \cap \mathbf{K})$ that attacks β (Case 2).
      In Case 1, by definition of $\texttt{Maf}(\mathfrak{A})$, $\mathbf{s}(\beta) \in \mathit{Def}(\mathfrak{A})$, which is in contradiction with the fact (*). In Case 2, by definition of $\texttt{Maf}(\mathfrak{A})$, we have $\gamma \in \Gamma$ and $\mathbf{s}(\gamma) \in S$. So $\beta \in \mathit{Inh}(\mathfrak{A})$, which is in contradiction with the fact (*).
      So we have proved that x must belong to $\texttt{Maf}(\mathfrak{A})$.

      ii. Let $\alpha \in \mathbf{K}$ being acceptable w.r.t. $\texttt{Maf}(\mathfrak{A})$. From Observation 5, it follows that $\mathbf{s}(\alpha) \in S$. Now, assume that $\alpha \notin \texttt{Maf}(\mathfrak{A})$. By definition of $\texttt{Maf}(\mathfrak{A})$, it follows that $\alpha \notin \Gamma$ and so $\alpha \notin \mathit{Acc}(\mathfrak{A})$, due to the assumption on $\mathfrak{A}$.
      The rest of the proof is analogous to the proof of the first item. $\alpha \notin \mathit{Acc}(\mathfrak{A})$ means that there is $\beta \in \mathbf{K}$ with $\mathbf{t}(\beta) = \alpha$ and such that $\beta \notin \mathit{Inh}(\mathfrak{A})$ and $\mathbf{s}(\beta) \notin \mathit{Def}(\mathfrak{A})$; this fact will be denoted by (*). So we have the attack (β, α) in **MAF**.
      As α is acceptable w.r.t. $\texttt{Maf}(\mathfrak{A})$, we know that $\texttt{Maf}(\mathfrak{A})$ attacks β in **MAF**. So, either $N_{\mathbf{s}(\beta)\beta} \in \texttt{Maf}(\mathfrak{A})$ (Case 1), or there exists $\gamma \in (\texttt{Maf}(\mathfrak{A}) \cap \mathbf{K})$ that attacks β (Case 2).
      In Case 1, by definition of $\texttt{Maf}(\mathfrak{A})$, $\mathbf{s}(\beta) \in \mathit{Def}(\mathfrak{A})$, which is in contradiction with the fact (*). In Case 2, by definition of $\texttt{Maf}(\mathfrak{A})$, we have $\gamma \in \Gamma$ and $\mathbf{s}(\gamma) \in S$. So $\beta \in \mathit{Inh}(\mathfrak{A})$, which is in contradiction with the fact (*).
      So we have proved that α must belong to $\texttt{Maf}(\mathfrak{A})$.

      iii. Let $x \in \mathbf{N}$ being acceptable w.r.t. $\texttt{Maf}(\mathfrak{A})$. x has the form $N_{\mathbf{s}(\alpha)\alpha}$. As $\mathbf{s}(\alpha)$ is the only attacker of x, we know that $\texttt{Maf}(\mathfrak{A})$ attacks $\mathbf{s}(\alpha)$ in **MAF**. So there exists $\beta \in \texttt{Maf}(\mathfrak{A}) \cap \mathbf{K}$ with $(\beta, \mathbf{s}(\alpha)) \in \mathbf{R}'$. By definition of $\texttt{Maf}(\mathfrak{A})$, we have $\beta \in \Gamma$ and

$\mathbf{s}(\beta) \in S$. Hence $\mathbf{s}(\alpha) \in Def(\mathfrak{A})$. As $\mathfrak{A}$ is conflict-free, it implies that $\mathbf{s}(\alpha) \notin S$. So we have $N_{\mathbf{s}(\alpha)\alpha} \in \mathtt{Maf}(\mathfrak{A}) \cap \mathbf{N}$.

(b) (σ = stable) Let $\mathfrak{A} = \langle S, \Gamma \rangle$ be a stable structure of **RAF**. By definition, $\mathfrak{A}$ is a conflict-free structure that satisfies: $\mathbf{A} \setminus S \subseteq Def(\mathfrak{A})$ and $\mathbf{K} \setminus \Gamma \subseteq Inh(\mathfrak{A})$. First, from Proposition 13, we have that $\mathtt{Maf}(\mathfrak{A})$ is conflict-free in $\langle \mathbf{A}', \mathbf{R}' \rangle$. Then, we have to prove that $\forall x \in \mathbf{A}' \setminus \mathtt{Maf}(\mathfrak{A})$, x is attacked by $\mathtt{Maf}(\mathfrak{A})$.
Three cases must be considered for x:

i. Let $x \in \mathbf{A} \setminus \mathtt{Maf}(\mathfrak{A})$. Then $x \in \mathbf{A} \setminus S$. By assumption on $\mathfrak{A}$, it follows that $x \in Def(\mathfrak{A})$ and from Observation 5 it follows that $\mathtt{Maf}(\mathfrak{A})$ attacks x.

ii. Let $\alpha \in \mathbf{K} \setminus \mathtt{Maf}(\mathfrak{A})$. Then either $\alpha \notin \Gamma$ (Case 1), or $\alpha \in \Gamma$ and $\mathbf{s}(\alpha) \notin S$ (Case 2).
In Case 1, as $\mathfrak{A}$ is a stable structure, $\alpha \in Inh(\mathfrak{A})$ and from Observation 5 it follows that $\mathtt{Maf}(\mathfrak{A})$ attacks α. In Case 2, as $\mathfrak{A}$ is a stable structure, $\mathbf{s}(\alpha) \in Def(\mathfrak{A})$. Moreover, $\mathbf{s}(\alpha) \notin S$, so $N_{\mathbf{s}(\alpha)\alpha} \in \mathtt{Maf}(\mathfrak{A}) \cap \mathbf{N}$. As $(N_{\mathbf{s}(\alpha)\alpha}, \alpha) \in \mathbf{R}'$, we conclude that $\mathtt{Maf}(\mathfrak{A})$ attacks α.

iii. Let $x \in \mathbf{N} \setminus \mathtt{Maf}(\mathfrak{A})$. x has the form $N_{\mathbf{s}(\alpha)\alpha}$ with $\mathbf{s}(\alpha) \in S$ or $\mathbf{s}(\alpha) \notin Def(\mathfrak{A})$. Note that, as $\mathfrak{A}$ is stable, if $\mathbf{s}(\alpha) \notin Def(\mathfrak{A})$ then $\mathbf{s}(\alpha) \in S$. Then, if $\mathbf{s}(\alpha) \in S$, as $(\mathbf{s}(\alpha), N_{\mathbf{s}(\alpha)\alpha}) \in \mathbf{R}'$ and $S \subseteq \mathtt{Maf}(\mathfrak{A})$, we conclude that $\mathtt{Maf}(\mathfrak{A})$ attacks x.

So we have proved that $\mathtt{Maf}(\mathfrak{A})$ is a stable extension of **MAF**.

(c) (σ = preferred) Let $\mathfrak{A}$ be a preferred structure. By definition, $\mathfrak{A}$ is a $\subseteq$-maximal admissible structure. Moreover $\mathfrak{A}$ is a complete structure.
So, from Proposition 12 (item 1 (a)), $\mathtt{Maf}(\mathfrak{A})$ is a complete extension of **MAF**.
Assume that $\mathtt{Maf}(\mathfrak{A})$ is not a preferred extension of **MAF**. Then there exists $\mathcal{E}'$ an admissible extension of **MAF** that strictly contains $\mathtt{Maf}(\mathfrak{A})$. It can be assumed that $\mathcal{E}'$ is a $\subseteq$-maximal admissible extension of **MAF**. So $\mathcal{E}'$ is preferred and thus complete.
From Lemma A.37, it follows that $\mathfrak{A}_{\mathtt{Maf}(\mathfrak{A})} \subseteq \mathfrak{A}_{\mathcal{E}'}$. From Proposition 15, we have $\mathfrak{A}_{\mathtt{Maf}(\mathfrak{A})} = \mathfrak{A}$. So, $\mathfrak{A} \subseteq \mathfrak{A}_{\mathcal{E}'}$. As $\mathfrak{A}$ is preferred, it follows that $\mathfrak{A} = \mathfrak{A}_{\mathcal{E}'}$.
From Proposition 15 again, $\mathtt{Maf}(\mathfrak{A}_{\mathcal{E}'}) = \mathcal{E}'$ so $\mathtt{Maf}(\mathfrak{A}) = \mathcal{E}'$. That is in contradiction with the assumption that $\mathcal{E}'$ strictly contains $\mathcal{E}_{\mathfrak{A}}$.
Hence, we have proved that $\mathtt{Maf}(\mathfrak{A})$ is a preferred extension of **MAF**.

(d) (σ = grounded) Let $\mathfrak{A} = \langle S, \Gamma \rangle$ be the grounded structure of **RAF**. By definition, $\mathfrak{A}$ is the $\subseteq$-minimal complete structure. From Proposition 12 (item 1 (a)) $\mathtt{Maf}(\mathfrak{A})$ is a complete extension of **MAF**. Assume that there is $\mathcal{E}'$ a complete extension that is strictly included in $\mathtt{Maf}(\mathfrak{A})$. From Lemma A.37, we have $\mathfrak{A}_{\mathcal{E}'} \subseteq \mathfrak{A}_{\mathtt{Maf}(\mathfrak{A})}$. As $\mathfrak{A}_{\mathtt{Maf}(\mathfrak{A})} = \mathfrak{A}$, due to Proposition 15, we have $\mathfrak{A}_{\mathcal{E}'} \subseteq \mathfrak{A}$. Proposition 12 (item 2 (a)), $\mathfrak{A}_{\mathcal{E}'}$ is a complete structure, so by assumption on $\mathfrak{A}$ it follows that $\mathfrak{A}_{\mathcal{E}'} = \mathfrak{A}$. Hence, $\mathtt{Maf}(\mathfrak{A}_{\mathcal{E}'}) = \mathtt{Maf}(\mathfrak{A})$, and from Proposition 15 again, $\mathcal{E}' = \mathtt{Maf}(\mathfrak{A})$. That is in contradiction with the fact that $\mathcal{E}'$ is strictly included in $\mathtt{Maf}(\mathfrak{A})$. So we have proved that $\mathtt{Maf}(\mathfrak{A})$ is a $\subseteq$-minimal complete extension of **MAF**, or in other words the grounded extension of **MAF**.

2. Let $\mathcal{E}$ be a set of $\mathbf{A}'$.

   (a) (σ = complete) Let $\mathcal{E}$ be a complete extension of $\langle \mathbf{A}', \mathbf{R}' \rangle$. Let us recall that $\mathfrak{A}_{\mathcal{E}} = \langle \mathcal{E}_a, \Gamma_{\mathcal{E}} \rangle$, where $\Gamma_{\mathcal{E}} = \mathcal{E}_k \cup \{\alpha \notin \mathcal{E}_k \text{ s.t. } \alpha \in Acc(\mathfrak{A}'_{\mathcal{E}})$ and $\mathbf{s}(\alpha) \notin \mathcal{E}_a\}$, and $\mathfrak{A}'_{\mathcal{E}} = \langle \mathcal{E}_a, \mathcal{E}_k \rangle$.
   By definition, $\mathcal{E}$ is an admissible extension of **MAF** and $\forall x \in \mathbf{A}'$, if x is acceptable w.r.t. $\mathcal{E}$, then $x \in \mathcal{E}$.
   From Proposition 20, we have that $\mathfrak{A}_{\mathcal{E}}$ is an admissible structure. So it remains to prove that $Acc(\mathfrak{A}_{\mathcal{E}}) \subseteq \mathcal{E}_a \cup \Gamma_{\mathcal{E}}$. Two cases must be considered:

      i. Let $a \in \mathbf{A} \cap Acc(\mathfrak{A}_{\mathcal{E}})$. Assume that $a \notin \mathcal{E}_a$. As $\mathcal{E}$ is a complete extension of **MAF**, a is not acceptable w.r.t. $\mathcal{E}$. So there exists an attack (β, a) in $\mathbf{R}'$ such that $\mathcal{E}$ does not attack β. That implies that $N_{\mathbf{s}(\beta)\beta} \notin \mathcal{E}$, and from Lemma A.33 that $\mathcal{E}$ does not attack $\mathbf{s}(\beta)$. So $\mathcal{E}$ attacks neither β, nor $\mathbf{s}(\beta)$; this fact will be denoted by (*).
      Moreover, as $a \in Acc(\mathfrak{A}_{\mathcal{E}})$, either $\beta \in Inh(\mathfrak{A}_{\mathcal{E}})$ (Case 1), or $\mathbf{s}(\beta) \in Def(\mathfrak{A}_{\mathcal{E}})$ (Case 2). It follows that there is $\gamma \in \Gamma_{\mathcal{E}}$ with $\mathbf{s}(\gamma) \in \mathcal{E}_a$ such that $\mathbf{t}(\gamma) = \beta$ in Case 1 (resp. $\mathbf{t}(\gamma) = \mathbf{s}(\beta)$ in Case 2). These conditions on γ and the definition of $\Gamma_{\mathcal{E}}$ imply that γ must belong to $\mathcal{E}_k$. So we have that $\mathcal{E}$ attacks β in Case 1 (resp. $\mathbf{s}(\beta)$ in Case 2). Hence we obtain a contradiction with the fact (*) and consequently we have proved that $a \in \mathcal{E}_a$.

      ii. Let $\alpha \in \mathbf{K} \cap Acc(\mathfrak{A}_{\mathcal{E}})$. Assume that $\alpha \notin \Gamma_{\mathcal{E}}$. It follows that $\alpha \notin \mathcal{E}_k$ and either $\mathbf{s}(\alpha) \in \mathcal{E}_a$ or $\alpha \notin Acc(\mathfrak{A}'_{\mathcal{E}})$. Let us successively consider the two cases.

         A. Assume that $\alpha \notin \mathcal{E}_k$ and $\mathbf{s}(\alpha) \in \mathcal{E}_a$. As $\mathcal{E}$ is a complete extension of **MAF**, α is not acceptable w.r.t. $\mathcal{E}$. As $\mathbf{s}(\alpha) \in \mathcal{E}_a$, α is defended by $\mathcal{E}$ against its attacker $N_{\mathbf{s}(\alpha)\alpha}$. So there must exist another attacker of α, say β, such that $\mathcal{E}$ does not attack β. That implies that $N_{\mathbf{s}(\beta)\beta} \notin \mathcal{E}$, and from Lemma A.33 that $\mathcal{E}$ does not attack $\mathbf{s}(\beta)$. So $\mathcal{E}$ attacks neither β, nor $\mathbf{s}(\beta)$; this fact will be denoted by (*).
         Moreover, as $\alpha \in Acc(\mathfrak{A}_{\mathcal{E}})$, either $\beta \in Inh(\mathfrak{A}_{\mathcal{E}})$ (Case 1), or $\mathbf{s}(\beta) \in Def(\mathfrak{A}_{\mathcal{E}})$ (Case 2). It follows that there is $\gamma \in \Gamma_{\mathcal{E}}$ with $\mathbf{s}(\gamma) \in \mathcal{E}_a$ such that $\mathbf{t}(\gamma) = \beta$ in Case 1 (resp. $\mathbf{t}(\gamma) = \mathbf{s}(\beta)$ in Case 2). These conditions on γ and the definition of $\Gamma_{\mathcal{E}}$ imply that γ must belong to $\mathcal{E}_k$. So we have that $\mathcal{E}$ attacks β in Case 1 (resp. $\mathbf{s}(\beta)$ in Case 2). Hence we obtain a contradiction with the fact (*).

B. It remains to consider the case when $\alpha \notin \mathcal{E}_k$, $\mathbf{s}(\alpha) \notin \mathcal{E}_a$ and $\alpha \notin Acc(\mathfrak{A}'_{\mathcal{E}})$. Let us recall that $\mathfrak{A}'_{\mathcal{E}}$ is the structure $\langle \mathcal{E}_a, \mathcal{E}_k \rangle$. $\alpha \notin Acc(\mathfrak{A}'_{\mathcal{E}})$ implies that α is attacked in **RAF** by $\beta \in \mathbf{K}$ such that $\beta \notin Inh(\mathfrak{A}'_{\mathcal{E}})$ and $\mathbf{s}(\beta) \notin Def(\mathfrak{A}'_{\mathcal{E}})$; this fact will be denoted by (**).
Moreover, as $\alpha \in Acc(\mathfrak{A}_{\mathcal{E}})$, either $\beta \in Inh(\mathfrak{A}_{\mathcal{E}})$ (Case 1), or $\mathbf{s}(\beta) \in Def(\mathfrak{A}_{\mathcal{E}})$ (Case 2). It follows that there is $\gamma \in \Gamma_{\mathcal{E}}$ with $\mathbf{s}(\gamma) \in \mathcal{E}_a$ such that $\mathbf{t}(\gamma) = \beta$ in Case 1 (resp. $\mathbf{t}(\gamma) = \mathbf{s}(\beta)$ in Case 2). These conditions on γ and the definition of $\Gamma_{\mathcal{E}}$ imply that γ must belong to $\mathcal{E}_k$. So we have that $\beta \in Inh(\mathfrak{A}'_{\mathcal{E}})$ in Case 1 (resp. $\mathbf{s}(\beta) \in Def(\mathfrak{A}'_{\mathcal{E}})$ in Case 2). Hence we obtain a contradiction with with the fact (**).

So we have proved that α must belong to $\Gamma_{\mathcal{E}}$.

(b) (σ = stable) Let $\mathcal{E}$ be a stable extension of $\langle \mathbf{A}', \mathbf{R}' \rangle$. First, from Proposition 13, we have that $\mathfrak{A}_{\mathcal{E}}$ is a conflict-free structure in **RAF**. Then, we have to prove that $\mathbf{A} \setminus \mathcal{E}_a \subseteq Def(\mathfrak{A}_{\mathcal{E}})$ and $\mathbf{K} \setminus \Gamma_{\mathcal{E}} \subseteq Inh(\mathfrak{A}_{\mathcal{E}})$.

i. Let $x \in \mathbf{A} \setminus \mathcal{E}_a$. As $\mathcal{E}$ is stable, $\mathcal{E}$ attacks x. As $x \in \mathbf{A}$ all the attackers of x belong to $\mathbf{K}$. So there is $\alpha \in \mathcal{E}_k$ that attacks x. Note that $\alpha \in \Gamma_{\mathcal{E}}$ by definition of $\Gamma_{\mathcal{E}}$.
Moreover, as $\mathcal{E}$ is a stable extension of **MAF**, $\mathcal{E}$ is admissible. As α is attacked by $N_{\mathbf{s}(\alpha)\alpha}$, $\mathcal{E}$ contains the only attacker of $N_{\mathbf{s}(\alpha)\alpha}$, that is $\mathbf{s}(\alpha)$. So $\mathbf{s}(\alpha) \in \mathcal{E}_a$. By definition, $\alpha \in \mathcal{E}_r$ and $\mathbf{s}(\alpha) \in \mathcal{E}_a$ imply that $x \in Def(\mathfrak{A}'_{\mathcal{E}})$. As $\Gamma_{\mathcal{E}}$ contains $\mathcal{E}_k$ it follows that we also have $x \in Def(\mathfrak{A}_{\mathcal{E}})$.

ii. Let $\alpha \in \mathbf{K} \setminus \Gamma_{\mathcal{E}}$. It follows that $\alpha \notin \mathcal{E}_k$ and either $\mathbf{s}(\alpha) \in \mathcal{E}_a$ or $\alpha \notin Acc(\mathfrak{A}'_{\mathcal{E}})$. Let us successively consider the two cases.

A. Assume that $\alpha \notin \mathcal{E}_k$ and $\mathbf{s}(\alpha) \in \mathcal{E}_a$. As $\mathcal{E}$ is stable, $\mathcal{E}$ attacks α. As $\mathcal{E}$ is conflict-free and contains $\mathbf{s}(\alpha)$, it follows that $N_{\mathbf{s}(\alpha)\alpha} \notin \mathcal{E}$. So there exists $\beta \in \mathcal{E}_k$ that attacks α.
Moreover, as $\mathcal{E}$ is a stable extension of **MAF**, $\mathcal{E}$ is admissible. As β is attacked by $N_{\mathbf{s}(\beta)\beta}$, $\mathcal{E}$ contains the only attacker of $N_{\mathbf{s}(\beta)\beta}$, that is $\mathbf{s}(\beta)$. So $\mathbf{s}(\beta) \in \mathcal{E}_a$. By definition, $\beta \in \mathcal{E}_k$ and $\mathbf{s}(\beta) \in \mathcal{E}_a$ imply that $\alpha \in Inh(\mathfrak{A}'_{\mathcal{E}})$. As $\Gamma_{\mathcal{E}}$ contains $\mathcal{E}_k$ it follows that we also have $\alpha \in Inh(\mathfrak{A}_{\mathcal{E}})$.

B. It remains to consider the case when $\alpha \notin \mathcal{E}_k$, $\mathbf{s}(\alpha) \notin \mathcal{E}_a$ and $\alpha \notin Acc(\mathfrak{A}'_{\mathcal{E}})$. Let us recall that $\mathfrak{A}'_{\mathcal{E}}$ is the structure $\langle \mathcal{E}_a, \mathcal{E}_k \rangle$. $\alpha \notin Acc(\mathfrak{A}'_{\mathcal{E}})$ implies that α is attacked in **RAF** by $\beta \in \mathbf{K}$ such that $\beta \notin Inh(\mathfrak{A}'_{\mathcal{E}})$ and $\mathbf{s}(\beta) \notin Def(\mathfrak{A}'_{\mathcal{E}})$; this fact will be denoted by (*).
If $\mathbf{s}(\beta) \notin \mathcal{E}_a$, from the first part of this proof, it follows that $\mathbf{s}(\beta) \in Def(\mathfrak{A}'_{\mathcal{E}})$. That is in contradiction with the fact (*). So we have $\mathbf{s}(\beta) \in \mathcal{E}_a$.
If $\beta \notin \Gamma_{\mathcal{E}}$, as $\mathbf{s}(\beta) \in \mathcal{E}_a$, from the first item of the second part of this proof, it follows that $\beta \in Inh(\mathfrak{A}'_{\mathcal{E}})$. That is in contradiction with the fact (*). So we have $\beta \in \Gamma_{\mathcal{E}}$.

By definition, $\mathbf{s}(\beta) \in \mathcal{E}_a$, $\beta \in \Gamma_{\mathcal{E}}$ and β attacks α imply that $\alpha \in Inh(\mathfrak{A}_{\mathcal{E}})$.

In both cases, we have proved that $\alpha \in Inh(\mathfrak{A}_{\mathcal{E}})$.

(c) (σ = preferred) Let $\mathcal{E}$ be a preferred extension of **MAF**. By definition, $\mathcal{E}$ is a $\subseteq$-maximal admissible extension. Moreover $\mathcal{E}$ is a complete extension. So, from Proposition 12 (item 2 (a)), $\mathfrak{A}_{\mathcal{E}}$ is a complete structure of **RAF**.
Assume that $\mathfrak{A}_{\mathcal{E}}$ is not a preferred structure of **RAF**. Then there exists $\mathfrak{A}'$ an admissible structure that strictly contains $\mathfrak{A}_{\mathcal{E}}$. It can be assumed that $\mathfrak{A}'$ is a $\subseteq$-maximal admissible structure of **RAF**. So $\mathfrak{A}'$ is preferred and thus complete.
From Lemma A.38, it follows that $\mathtt{Maf}(\mathfrak{A}_{\mathcal{E}}) \subseteq \mathtt{Maf}(\mathfrak{A}')$. From Proposition 15, we have $\mathtt{Maf}(\mathfrak{A}_{\mathcal{E}}) = \mathcal{E}$. So, $\mathcal{E} \subseteq \mathtt{Maf}(\mathfrak{A}')$. As $\mathcal{E}$ is preferred, it follows that $\mathcal{E} = \mathtt{Maf}(\mathfrak{A}')$.
From Proposition 15 again, $\mathfrak{A}_{\mathtt{Maf}(\mathfrak{A}')} = \mathfrak{A}'$ so $\mathfrak{A}_{\mathcal{E}} = \mathfrak{A}'$.
That is in contradiction with the assumption that $\mathfrak{A}'$ strictly contains $\mathfrak{A}_{\mathcal{E}}$. Hence, we have proved that $\mathfrak{A}_{\mathcal{E}}$ is a preferred structure.

(d) (σ = grounded) Let $\mathcal{E}$ be the grounded extension of **MAF**. By definition, $\mathcal{E}$ is the $\subseteq$-minimal complete extension. From Proposition 12 (item 2 (a)), $\mathfrak{A}_{\mathcal{E}}$ is a complete structure of **RAF**. Assume that there is $\mathfrak{A}'$ a complete structure that is strictly included in $\mathfrak{A}_{\mathcal{E}}$. From Lemma A.38, we have $\mathtt{Maf}(\mathfrak{A}') \subseteq \mathtt{Maf}(\mathfrak{A}_{\mathcal{E}})$ As $\mathtt{Maf}(\mathfrak{A}_{\mathcal{E}}) = \mathcal{E}$, due to Proposition 15, we have $\mathtt{Maf}(\mathfrak{A}') \subseteq \mathcal{E}$. From Proposition 12 (item 1 (a)), $\mathtt{Maf}(\mathfrak{A}')$ is a complete extension, so by assumption on $\mathcal{E}$ it follows that $\mathtt{Maf}(\mathfrak{A}') = \mathcal{E}$. Hence, $\mathfrak{A}_{\mathtt{Maf}(\mathfrak{A}')} = \mathfrak{A}_{\mathcal{E}}$, and from Proposition 15 again, $\mathfrak{A}' = \mathfrak{A}_{\mathcal{E}}$. That is in contradiction with the fact that $\mathfrak{A}'$ is strictly included in $\mathfrak{A}_{\mathcal{E}}$. So we have proved that $\mathfrak{A}_{\mathcal{E}}$ is a $\subseteq$-minimal complete structure of **RAF**, or in other words is the grounded extension of **RAF**. □

Proof of Proposition 15.

1. The proof follows directly from Lemma A.35.
2. The proof follows directly from Lemma A.36. □

A.4 Proofs of Section 7

Note: By $\downarrow\alpha = \{\ \beta \in \mathbf{K} \mid \beta \preceq \alpha\ \}$ we denote the down set generated by α. Furthermore, for some argumentation framework **RAF** and structure $\mathfrak{A}$, by $Def(\mathbf{RAF}, \mathfrak{A})$ and $Inh(\mathbf{RAF}, \mathfrak{A})$ we respectively denote the defeated arguments and inhibited attacks w.r.t. **RAF** and $\mathfrak{A}$. This allows us to relate defeated arguments (resp. inhibited attacks) w.r.t. different argumentation frameworks.

Lemma A.39. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework and* $\alpha, \beta \in \mathbf{K}$ *be be two attacks and* $x \in (\mathbf{A} \cup \mathbf{K})$ *be some argument or attack. Then,* $\alpha \neq \beta$, $\mathbf{t}(\beta) = x$ *and* $x \notin\downarrow\alpha$ *imply* $\beta \notin\downarrow\alpha$*.*

Proof. Suppose, for the sake of contradiction, that $\beta \in\downarrow\alpha$. Then, since $\alpha \neq \beta$, it follows that there is some chain $\delta_0, \delta_1, \delta_2, \delta_n$ such that $\mathbf{t}(\delta_i) = \delta_{i+1}$ and $\delta_0 = \beta$ and $\delta_n = \alpha$. But $\delta_0 = \beta$ plus $\mathbf{t}(\beta) = x$ imply that $\delta_1 = x$ and, thus, that $x \in\downarrow\alpha$. This is a contradiction with the assumption. Consequently, it must be that $\beta \notin\downarrow\alpha$. □

Lemma A.40. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework,* $\mathfrak{A}$ *be some structure and* $\alpha \in \mathbf{K}$ *be some attack. Then,* $\mathit{Def}(\mathbf{RAF}, \mathfrak{A}) \supseteq \mathit{Def}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$*.* □

Proof. Let $\mathfrak{A} = \langle S, \Gamma \rangle$ and $\mathfrak{A}^{-\alpha} = \langle S, \Gamma^{-\alpha} \rangle$. Pick any $a \in \mathit{Def}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$. Then, there is some $\beta \in \Gamma^{-\alpha}$ such that $\mathbf{t}(\beta) = a$ and $\mathbf{s}(\beta) \in S$. Furthermore, $\beta \in \Gamma^{-\alpha}$ plus $\Gamma^{-\alpha} \subseteq \Gamma$ imply $\beta \in \Gamma$ which, in its turn, implies that $a \in \mathit{Def}(\mathbf{RAF}, \mathfrak{A})$. □

Lemma A.41. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework,* $\mathfrak{A}$ *be some structure and* $\alpha \in \mathbf{K}$ *be some attack. Then,* $\mathit{Inh}(\mathbf{RAF}, \mathfrak{A})^{-\alpha} \supseteq \mathit{Inh}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$*.* □

Proof. Let $\mathfrak{A} = \langle S, \Gamma \rangle$ and $\mathfrak{A}^{-\alpha} = \langle S, \Gamma^{-\alpha} \rangle$. Pick any $\beta \in \mathit{Inh}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$. Then, there is some $\gamma \in \Gamma^{-\alpha}$ such that $\mathbf{t}(\gamma) = \beta$ and $\mathbf{s}(\gamma) \in S$. Furthermore, $\gamma \in \Gamma^{-\alpha}$ plus $\Gamma^{-\alpha} \subseteq \Gamma$ imply $\gamma \in \Gamma$ which, in its turn, implies that $\beta \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})$. Furthermore, $\beta \in \mathit{Inh}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$ implies $\beta \not\leq \alpha$. Hence, $\beta \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})^{-\alpha}$. □

Lemma A.42. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework,* $\mathfrak{A}$ *be some conflict-free structure w.r.t.* $\mathbf{RAF}$ *and* $\alpha \in \mathbf{K}$ *be some attack. Then,* $\mathfrak{A}^{-\alpha}$ *is conflict-free w.r.t.* $\mathbf{RAF}^{-\alpha}$*.* □

Proof. Let $\mathfrak{A} = \langle S, \Gamma \rangle$ and $\mathfrak{A}^{-\alpha} = \langle S, \Gamma^{-\alpha} \rangle$. Pick $a \in S$. Then, since $\mathfrak{A}$ is conflict-free, it follows that $a \notin \mathit{Def}(\mathbf{RAF}, \mathfrak{A})$ and, from Lemma A.40, that $a \notin \mathit{Def}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$. Similarly, $\beta \in \Gamma^{-\alpha}$ implies $\beta \notin \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})$. From Lemma A.41, this implies $\beta \notin \mathit{Inh}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$. Hence, $\mathfrak{A}^{-\alpha}$ is conflict-free w.r.t. $\mathbf{RAF}^{-\alpha}$. □

Lemma A.43. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework,* $\mathfrak{A}$ *be some structure and* $\alpha \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})$ *be some inhibited attack.*
Then, it follows that $\mathit{Acc}(\mathbf{RAF}, \mathfrak{A})^{-\alpha} \supseteq \mathit{Acc}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$*.* □

Proof. Let $\mathfrak{A} = \langle S, \Gamma \rangle$ and $\mathfrak{A}^{-\alpha} = \langle S, \Gamma^{-\alpha} \rangle$. Pick any $x \in \mathit{Acc}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$ and $\gamma \in \mathbf{K}$ such that $\mathbf{t}(\gamma) = x$. Then, $x \in \mathit{Acc}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$ implies $x \in (\mathbf{A} \cup \mathbf{K}^{-\alpha})$ and, thus, that $x \notin\downarrow\alpha$. From Lemma A.39, this plus $\mathbf{t}(\gamma) = x$ imply that either $\gamma = \alpha$ or $\gamma \notin\downarrow\alpha$. On the one hand, by hypothesis we have that $\alpha \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})$ and, thus, the former implies $\gamma \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})$. On the other hand, the latter implies $\gamma \in \mathbf{K}^{-\alpha}$ and, thus, $x \in \mathit{Acc}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$ implies that $\gamma \in \mathit{Inh}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$ or $\mathbf{s}(\gamma) \in \mathit{Def}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$. From Lemma A.41 and A.40, this implies that $\gamma \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})$ or $\mathbf{s}(\gamma) \in \mathit{Def}(\mathbf{RAF}, \mathfrak{A})$. Hence, $x \in \mathit{Acc}(\mathbf{RAF}, \mathfrak{A})$. Finally, we have that $x \notin\downarrow\alpha$ implies $x \in \mathit{Acc}(\mathbf{RAF}, \mathfrak{A})^{-\alpha}$. □

Lemma A.44. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework,* $\mathfrak{A}$ *be some structure and* $\alpha \in (\mathit{Inh}(\mathbf{RAF}, \mathfrak{A}) \setminus \Gamma)$ *be some inhibited attack w.r.t.* $\mathfrak{A}$*. Then, it follows that* $\mathit{Def}(\mathbf{RAF}, \mathfrak{A}) = \mathit{Def}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$*.* □

Proof. Let $\mathfrak{A} = \langle S, \Gamma \rangle$ and $\mathfrak{A}^{-\alpha} = \langle S, \Gamma^{-\alpha} \rangle$. From Lemma A.40, it follows that $\mathit{Def}(\mathbf{RAF}, \mathfrak{A}) \supseteq \mathit{Def}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$. Pick now any argument $a \in \mathit{Def}(\mathbf{RAF}, \mathfrak{A})$. Then, there is some $\beta \in \Gamma$ such that $\mathbf{t}(\beta) = a$ and $\mathbf{s}(\beta) \in S$. Furthermore, since $\alpha \notin \Gamma$, it follows that $\beta \neq \alpha$. Moreover, every $\gamma \in\downarrow\alpha$ satisfies either $\gamma = \alpha$ or $\mathbf{t}(\gamma) \in \mathbf{K}$ and, thus, that $\beta \notin\downarrow\alpha$. This implies that $\beta \in \Gamma^{-\alpha}$ and, thus, that $a \in \mathit{Def}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$ □

Lemma A.45. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework,* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be some structure and* $\alpha \in (\mathit{Inh}(\mathbf{RAF}, \mathfrak{A}) \setminus \Gamma)$ *be some inhibited attack w.r.t.* $\mathfrak{A}$*. Then, it follows that* $\mathit{Inh}(\mathbf{RAF}, \mathfrak{A})^{-\alpha} = \mathit{Inh}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$*.* □

Proof. First note that, from Lemma A.41, it follows that

$$\mathit{Inh}(\mathbf{RAF}, \mathfrak{A})^{-\alpha} \supseteq \mathit{Inh}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$$

Let $\mathfrak{A} = \langle S, \Gamma \rangle$ and $\mathfrak{A}^{-\alpha} = \langle S, \Gamma^{-\alpha} \rangle$. Pick now any $\beta \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})^{-\alpha}$.

Then, there is some $\gamma \in \Gamma$ such that $\mathbf{t}(\gamma) = \beta$ and $\mathbf{s}(\gamma) \in S$. Furthermore, since $\alpha \notin \Gamma$, it follows that $\gamma \neq \alpha$. Moreover, $\beta \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})^{-\alpha}$ implies that $\beta \not\preceq \alpha$. Then, since $\gamma \prec \beta$, that $\gamma \not\preceq \alpha$. Hence, it follows that $\gamma \in \Gamma^{-\alpha}$ and $\beta \in \mathit{Inh}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$. □

Lemma A.46. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework,* $\mathfrak{A}$ *be some structure and* $\alpha \in (\mathit{Inh}(\mathbf{RAF}, \mathfrak{A}) \setminus \Gamma)$ *be some inhibited attack w.r.t.* $\mathfrak{A}$*.*
Then, it follows that $(\mathit{Acc}(\mathbf{RAF}, \mathfrak{A})^{-\alpha}) = \mathit{Acc}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$*.* □

Proof. First note that, from Lemma A.43, it follows that

$$\mathit{Acc}(\mathbf{RAF}, \mathfrak{A})^{-\alpha} \supseteq \mathit{Acc}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$$

Let $\mathfrak{A} = \langle S, \Gamma \rangle$ and $\mathfrak{A}^{-\alpha} = \langle S, \Gamma^{-\alpha} \rangle$. Pick any $\beta \in \mathit{Acc}(\mathbf{RAF}, \mathfrak{A})^{-\alpha}$ and $\gamma \in \mathbf{K}^{-\alpha} \subseteq \mathbf{K}$ such that $\mathbf{t}(\gamma) = \beta$. Since $\beta \in \mathit{Acc}(\mathbf{RAF}, \mathfrak{A})$, it follows that either $\gamma \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})$ or $\mathbf{s}(\gamma) \in \mathit{Def}(\mathbf{RAF}, \mathfrak{A})$. Furthermore, $\gamma \in \mathbf{K}^{-\alpha}$ implies $\gamma \not\preceq \alpha$ and, from Lemma A.45 and A.44, this implies that either $\gamma \in \mathit{Inh}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}')$ or $\mathbf{s}(\gamma) \in \mathit{Def}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}')$. Hence, $\beta \in \mathit{Acc}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}')$ and, thus, $\mathit{Acc}(\mathbf{RAF}, \mathfrak{A})^{-\alpha}) = \mathit{Acc}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$. □

Lemma A.47. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework,* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be some admissible structure w.r.t.* $\mathbf{RAF}$ *and* $\alpha \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})$ *be some inhibited attack. Then, it follows that* $\mathfrak{A}^{-\alpha}$ *is admissible w.r.t.* $\mathbf{RAF}^{-\alpha}$*.* □

Proof. Since $\mathfrak{A}$ is an admissible structure w.r.t. $\mathbf{RAF}$, it is conflict-free and, from Lemma A.42, this implies that $\mathfrak{A}^{-\alpha}$ is conflict-free w.r.t. $\mathbf{RAF}^{-\alpha}$. Furthermore, since $\mathfrak{A}$ is admissible, it follows that $(S \cup \Gamma) \subseteq \mathit{Acc}(\mathbf{RAF}, \mathfrak{A})$. From Lemma A.46, this implies $(S \cup \Gamma^{-\alpha}) \subseteq \mathit{Acc}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$ and, thus, that $\mathfrak{A}^{-\alpha}$ is admissible w.r.t. $\mathbf{RAF}^{-\alpha}$. □

Lemma A.48. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework,* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be some complete structure w.r.t.* $\mathbf{RAF}$ *and* $\alpha \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})$ *be some inhibited attack. Then, it follows that* $\mathfrak{A}^{-\alpha}$ *is complete w.r.t.* $\mathbf{RAF}^{-\alpha}$*.* □

Proof. Since $\mathfrak{A}$ is a complete structure w.r.t. $\mathbf{RAF}$, it follows that it is admissible and, from Lemma A.47, this implies that $\mathfrak{A}^{-\alpha}$ is admissible w.r.t. $\mathbf{RAF}^{-\alpha}$. Furthermore, since $\mathfrak{A}$ is complete, it follows that $(S \cup \Gamma) \supseteq \mathit{Acc}(\mathbf{RAF}, \mathfrak{A})$. From Lemma A.46, this implies $(S \cup \Gamma^{-\alpha}) \supseteq \mathit{Acc}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$ and, thus, that $\mathfrak{A}^{-\alpha}$ is complete w.r.t. $\mathbf{RAF}^{-\alpha}$. □

Lemma A.49. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework,* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be some conflict-free structure w.r.t.* $\mathbf{RAF}$ *and* $\alpha \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})$ *be some inhibited attack. Let* $\mathfrak{A}' = \langle S', \Gamma' \rangle$ *some conflict-free structure w.r.t.* $\mathbf{RAF}^{-\alpha}$ *such that* $\Gamma^{-\alpha} \subseteq \Gamma'$*. Then, the structure* $\mathfrak{A}'' = \langle S', \Gamma \cup \Gamma' \rangle$ *is conflict-free w.r.t.* $\mathbf{RAF}$*.* □

Proof. Let $\Gamma'' = \Gamma \cup \Gamma'$. Pick first $a \in S'$ and any $\beta \in \Gamma''$ such that $\mathbf{t}(\beta) = a$. Then, since $\mathbf{t}(\beta) = a$, it follows that $\beta \not\preceq \alpha$ and, thus, that $\beta \in \mathbf{K}^{-\alpha}$. Hence, $\beta \in \Gamma$ implies $\beta \in \Gamma^{-\alpha} \subseteq \Gamma'$. Then, since $\mathfrak{A}'$ is conflict-free, it follows that $\mathbf{s}(\beta) \notin S'$. This implies that every $\beta \in \Gamma''$ with $\mathbf{t}(\beta) = a$ satisfies $\mathbf{s}(\beta) \notin S'$ and, thus, that $a \notin \mathit{Def}(\mathbf{RAF}, \mathfrak{A}'')$.

Pick now $\gamma, \beta \in \Gamma''$ such that $\mathbf{t}(\beta) = \gamma$. Suppose, for the sake of contradiction, that $\gamma \in (\Gamma'' \backslash \Gamma)$ and $\beta \in (\Gamma'' \backslash \Gamma')$. Then, $\gamma \notin (\Gamma'' \backslash \Gamma)$ implies $\gamma \in \Gamma' \subseteq \mathbf{K}^{-\alpha}$ and, thus, it follows that $\gamma \not\preceq \alpha$ and $\beta \not\preceq \alpha$. On the other hand, $\beta \in (\Gamma'' \setminus \Gamma')$ implies $\beta \in \Gamma$ which, since $\beta \not\preceq \alpha$, implies $\beta \in \Gamma^{-\alpha} \subseteq \Gamma'$. This is a contradiction with the assumption. Similarly, suppose that $\gamma \in (\Gamma'' \setminus \Gamma')$ and $\beta \in (\Gamma'' \setminus \Gamma)$. Then, $\gamma \in (\Gamma'' \setminus \Gamma')$ implies and $\gamma \in \Gamma$. Furthermore, since $\Gamma^{-\alpha} \subseteq \Gamma'$, it follows that $\gamma \notin \Gamma'$ implies $\gamma \notin \Gamma^{-\alpha}$ and, this plus $\gamma \in \Gamma$, imply $\gamma \preceq \alpha$. Since $\mathbf{t}(\beta) = \gamma$, the latter implies that $\beta \preceq \alpha$ holds and, thus, that $\beta \notin \Gamma'$. This is a contradiction with the assumption that $\beta \in (\Gamma'' \setminus \Gamma)$. Hence, either $\gamma, \beta \in \Gamma$ or $\gamma, \beta \in \Gamma'$ must hold. In both cases, the fact that $\mathfrak{A}$ and $\mathfrak{A}'$ are conflict-free imply $\mathbf{s}(\beta) \notin S$. This implies that every $\beta \in \Gamma''$ with $\mathbf{t}(\beta) = \gamma$ satisfies $\mathbf{s}(\beta) \notin S$ and, thus, that $\gamma \notin \mathit{Inh}(\mathbf{RAF}, \mathfrak{A}'')$. Consequently, $\mathfrak{A}''$ is conflict-free. □

Lemma A.50. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework,* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be some naive structure w.r.t.* $\mathbf{RAF}$ *and* $\alpha \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})$ *be some inhibited attack. Then,* $\mathfrak{A}^{-\alpha}$ *is naive w.r.t.* $\mathbf{RAF}^{-\alpha}$*.* □

Proof. Since $\mathfrak{A}$ is a naive structure w.r.t. $\mathbf{RAF}$, it follows that it is conflict-free and, from Proposition 18, this implies that $\mathfrak{A}^{-\alpha}$ is conflict-free w.r.t. $\mathbf{RAF}^{-\alpha}$. Suppose, for the sake of contradiction, that there exists some structure $\mathfrak{A}' = \langle S', \Gamma' \rangle$ which is conflict-free w.r.t. $\mathbf{RAF}^{-\alpha}$ and that satisfies $\mathfrak{A}^{-\alpha} \sqsubset \mathfrak{A}'$. Then, we may assume without loss of generality that $\mathfrak{A}'$ is also naive w.r.t. $\mathbf{RAF}^{-\alpha}$.

Let $\Gamma'' = (\Gamma \cup \Gamma')$ and let $\mathfrak{A}'' = \langle S', \Gamma'' \rangle$ be some structure. Then, from Lemma A.49, it follows that $\mathfrak{A}''$ is conflict-free. Furthermore, note that by construction, $\mathfrak{A}, \mathfrak{A}' \sqsubseteq \mathfrak{A}''$ holds and, thus, the fact that $\mathfrak{A}$ is a naive structure implies $\mathfrak{A} \sqsupseteq \mathfrak{A}''$. On the other hand, $\mathfrak{A}^{-\alpha} \sqsubset \mathfrak{A}'$ implies that there is some element $x \in ((S' \cup \Gamma') \setminus (S^{-\alpha} \cup \Gamma^{-\alpha})$. Moreover, $x \in (S' \cup \Gamma')$ implies that $x \not\preceq \alpha$ and that $x \in (S' \cup \Gamma'')$. Since $\mathfrak{A} \sqsupseteq \mathfrak{A}''$, the latter implies that $x \in (S \cup \Gamma)$ which, together with $x \not\preceq \alpha$, implies that $x \in (S^{-\alpha} \cup \Gamma^{-\alpha})$. This is a contradiction and, consequently, we have that $\mathfrak{A}^{-\alpha}$ must be naive. $\square$

Lemma A.51. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework,* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be some grounded structure w.r.t.* $\mathbf{RAF}$ *and* $\alpha \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})$ *be some inhibited attack. Then,* $\mathfrak{A}^{-\alpha}$ *is grounded w.r.t.* $\mathbf{RAF}^{-\alpha}$*.* $\square$

Proof. Since $\mathfrak{A}$ is a grounded structure w.r.t. $\mathbf{RAF}$, it follows that it is complete and, from Lemma A.48, this implies that $\mathfrak{A}^{-\alpha}$ is complete w.r.t. $\mathbf{RAF}^{-\alpha}$. Suppose, for the sake of contradiction, that there is some structure $\mathfrak{A}' = \langle S', \Gamma' \rangle$ which is complete w.r.t. $\mathbf{RAF}^{-\alpha}$ and that satisfies $\mathfrak{A}^{-\alpha} \sqsupset \mathfrak{A}'$. Then, we also have $\mathfrak{A} \sqsupset \mathfrak{A}'$. So $\mathfrak{A}'$ must not be complete w.r.t. $\mathbf{RAF}$. Obviously, since $\mathfrak{A}$ is conflict-free and $\mathfrak{A} \sqsupseteq \mathfrak{A}'$, we have that $\mathfrak{A}'$ is also conflict-free. Suppose, for the sake of contradiction, that $\mathfrak{A}'$ is not admissible w.r.t. $\mathbf{RAF}$. Then, there is $x \in (S' \cup \Gamma')$ and $\beta \in \mathbf{K}$ such that $\mathbf{t}(\beta) = x$ and there is no $\gamma \in \Gamma \supseteq \Gamma'$ such that $\mathbf{t}(\gamma) \in \{\beta, \mathbf{s}(\beta)\}$ and $\mathbf{s}(\gamma) \in S \supseteq S'$. Since $\mathfrak{A}$ is admissible w.r.t. $\mathbf{RAF}^{-\alpha}$, it must be that $\beta \notin \mathbf{K}^{-\alpha}$ and, thus, that $\beta \preceq \alpha$. However, this is a contradiction with the fact that $\mathfrak{A}^{-\alpha} \sqsupset \mathfrak{A}'$. Hence, $\mathfrak{A}'$ must be admissible. Suppose now that $\mathfrak{A}'$ is not complete w.r.t. $\mathbf{RAF}$. Then, there is some $x \in \mathit{Acc}(\mathbf{RAF}, \mathfrak{A}')$ such that $x \notin (S' \cup \Gamma')$. Hence, for every $\beta \in \mathbf{K}$ such that $\mathbf{t}(\beta) = x$ and there is $\gamma_\beta \in \Gamma'$ such that $\mathbf{t}(\gamma_\beta) \in \{\beta, \mathbf{s}(\beta)\}$ and $\mathbf{s}(\gamma_\beta) \in S'$. On the other hand, since $\mathfrak{A}'$ is complete w.r.t. $\mathbf{RAF}^{-\alpha}$, this means that $x \in \mathit{Acc}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}')$ and, thus, that $\gamma_\beta \preceq \alpha$. This is a contradiction with the fact that $\gamma_\beta \in \Gamma' \subseteq \Gamma^{-\alpha}$. Hence, $\mathfrak{A}'$ is complete w.r.t. $\mathbf{RAF}$ which is a contradiction with the fact that $\mathfrak{A}$ is grounded. Thus, $\mathfrak{A}^{-\alpha}$ must be grounded. $\square$

Lemma A.52. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework,* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be some admissible structure w.r.t.* $\mathbf{RAF}$ *and* $\alpha \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})$ *be some inhibited attack. Let* $\mathfrak{A}' = \langle S', \Gamma' \rangle$ *some admissible structure w.r.t.* $\mathbf{RAF}^{-\alpha}$ *such that* $\mathfrak{A}^{-\alpha} \sqsubseteq \mathfrak{A}'$*. Then, the structure* $\mathfrak{A}'' = \langle S', \Gamma \cup \Gamma' \rangle$ *is admissible w.r.t.* $\mathbf{RAF}$*.* $\square$

Proof. From Lemma A.49, it follows that $\mathfrak{A}''$ is conflict-free. Furthermore, since $\mathfrak{A}$ is admissible, it follows that $(S \cup \Gamma) \subseteq \mathit{Acc}(\mathbf{RAF}, \mathfrak{A})$. Note also that $\mathfrak{A}^{-\alpha} \sqsubseteq \mathfrak{A}'$ and $\alpha \in \mathbf{K}$ implies that $\Gamma = \Gamma^{-\alpha} \subseteq \Gamma'$ and, thus, that $\mathfrak{A} \sqsubseteq \mathfrak{A}''$. Then, from Observation 2, it follows that $(S \cup \Gamma) \subseteq \mathit{Acc}(\mathbf{RAF}, \mathfrak{A}'')$. Pick now any attack $\gamma \in (\Gamma'' \setminus \Gamma)$ and any attack $\beta \in \mathbf{K}$ such that $\mathbf{t}(\beta) = \gamma$. Since $\gamma \in (\Gamma'' \setminus \Gamma)$, it follows that $\gamma \in \Gamma' \subseteq \mathbf{K}^{-\alpha}$ and, thus, that $\gamma \not\preceq \alpha$ and $\beta \not\preceq \alpha$. Hence, $\beta \in \mathbf{K}$ implies $\beta \in \mathbf{K}^{-\alpha}$. Furthermore, since $\mathfrak{A}'$ is admissible w.r.t. $\mathbf{RAF}^{-\alpha}$, this implies that either $\mathbf{s}(\beta) \in \mathit{Def}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$ or $\beta \in \mathit{Inh}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})$. From Lemma A.44 and A.45 respectively, this implies that either

$$\mathbf{s}(\beta) \in \mathit{Def}(\mathbf{RAF}, \mathfrak{A})^{-\alpha} \subseteq \mathit{Def}(\mathbf{RAF}, \mathfrak{A})$$

or

$$\beta \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})^{-\alpha} \subseteq \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})$$

holds.

Furthermore, since $\mathfrak{A} \sqsubseteq \mathfrak{A}''$, Observation 1 implies either $\mathbf{s}(\beta) \in \mathit{Def}(\mathbf{RAF}, \mathfrak{A}'')$

or $\beta \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A}'')$. Hence, every $\gamma \in (\Gamma'' \setminus \Gamma)$ satisfies that $\gamma \in \mathit{Acc}(\mathbf{RAF}, \mathfrak{A}'')$ and, thus, $(S \cup \Gamma'') \subseteq \mathit{Acc}(\mathbf{RAF}, \mathfrak{A}'')$. This means that $\mathfrak{A}''$ is admissible. $\square$

Lemma A.53. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework,* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be some preferred structure w.r.t.* $\mathbf{RAF}$ *and* $\alpha \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})$ *be some inhibited attack. Then,* $\mathfrak{A}^{-\alpha}$ *is preferred w.r.t.* $\mathbf{RAF}^{-\alpha}$. $\square$

Proof. Since $\mathfrak{A}$ is a preferred structure w.r.t. $\mathbf{RAF}$, it follows that it is admissible and, from Lemma A.47, this implies that $\mathfrak{A}^{-\alpha}$ is admissible w.r.t. $\mathbf{RAF}^{-\alpha}$. Suppose, for the sake of contradiction, that there is some structure $\mathfrak{A}' = \langle S', \Gamma' \rangle$ which is admissible w.r.t. $\mathbf{RAF}^{-\alpha}$ and that satisfies $\mathfrak{A}^{-\alpha} \sqsubset \mathfrak{A}'$. Then, from Proposition 3.1, we may assume without loss of generality that $\mathfrak{A}'$ is also preferred w.r.t. $\mathbf{RAF}^{-\alpha}$.

Let $\Gamma'' = (\Gamma \cup \Gamma')$ and let $\mathfrak{A}'' = \langle S', \Gamma'' \rangle$ be some structure. Then, from Lemma A.52, it follows that $\mathfrak{A}''$ is admissible. Furthermore, note that by construction, $\mathfrak{A}, \mathfrak{A}' \sqsubseteq \mathfrak{A}''$ holds and, thus, the fact that $\mathfrak{A}$ is a preferred structure implies $\mathfrak{A} \sqsupseteq \mathfrak{A}''$. On the other hand, $\mathfrak{A}^{-\alpha} \sqsubset \mathfrak{A}'$ implies that there is some element $x \in ((S' \cup \Gamma') \setminus (S^{-\alpha} \cup \Gamma^{-\alpha})$. Moreover, $x \in (S' \cup \Gamma')$ implies that $x \not\preceq \alpha$ and that $x \in (S' \cup \Gamma'')$. Since $\mathfrak{A} \sqsupseteq \mathfrak{A}''$, the latter implies that $x \in (S \cup \Gamma)$ which, together with $x \not\preceq \alpha$, implies that $x \in (S^{-\alpha} \cup \Gamma^{-\alpha})$. This is a contradiction and, consequently, we have that $\mathfrak{A}^{-\alpha}$ must be preferred. $\square$

Lemma A.54. *Let* $\mathbf{RAF} = \langle \mathbf{A}, \mathbf{K}, \mathbf{s}, \mathbf{t} \rangle$ *be some framework,* $\mathfrak{A} = \langle S, \Gamma \rangle$ *be some stable structure w.r.t.* $\mathbf{RAF}$ *and* $\alpha \in \mathit{Inh}(\mathbf{RAF}, \mathfrak{A})$ *be some inhibited attack. Then, it follows that* $\mathfrak{A}^{-\alpha}$ *is stable w.r.t.* $\mathbf{RAF}^{-\alpha}$. $\square$

Proof. Since $\mathfrak{A}$ is a stable structure w.r.t. $\mathbf{RAF}$, it follows that it is conflict-free and, from Lemma A.42, this implies that $\mathfrak{A}^{-\alpha}$ is conflict-free w.r.t. $\mathbf{RAF}^{-\alpha}$. Furthermore, since $\mathfrak{A}$ is stable, it follows that $S = \overline{\mathit{Def}(\mathbf{RAF}, \mathfrak{A})}$ and $\Gamma = \overline{\mathit{Inh}(\mathbf{RAF}, \mathfrak{A})}$. From Lemma A.44 and A.45 respectively, this implies that $S = \overline{\mathit{Def}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})}$ and $\Gamma^{-\alpha} = \overline{\mathit{Inh}(\mathbf{RAF}^{-\alpha}, \mathfrak{A}^{-\alpha})}$. This implies that $\mathfrak{A}^{-\alpha}$ is stable w.r.t. $\mathbf{RAF}^{-\alpha}$. $\square$

Proof of Proposition 19. The fact that $\mathfrak{A}^{-\alpha}$ is complete, naive, grounded, preferred or stable respectively follows from Lemmas A.48, A.50, A.51, A.53 and A.54. $\square$