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Comparison of uncertainty quantification process using statistical and data mining algorithms

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Abstract

Uncertainty quantification has always been an important topic in model reduction and simulation of complex systems. In this aspect, Global Sensitivity Analysis (GSA) methods such as Fourier Amplitude Sensitivity Test (FAST) are well recognized as effective algorithms. Recently, some data-based meta-modeller such as Random Forest (RF) also developed their own variable importance selection solutions for parameters with perturbations. This paper proposes a visual comparison of these two uncertainty quantification methods, using datasets retrieved from vibroacoustic models. Their results have a lot in common and are capable to explain many results. The remarkable agreement between methods under fundamentally different definitions can potentially improve their compatibility in various occasions.

Key words: Global Sensitivity Analysis, Random Forest, FAST, Sound Transmission Loss, Sandwich panel, Composite material

1. Introduction

The trend of increasing complexity of mathematical models in various domains has resulted increasing parametric uncertainties for their inputs. Whereas these uncertainties may often impact directly the output, the need for model uncertainty quantification has been greatly raised. Focusing on analytical expressions, Sensitivity Analysis (SA) is a traditional way to get the uncertainty of output explained by the uncertainties of inputs. Fixing all variables except one to observe its influence on the output, which is exactly the spirit of Local Sensitivity Analysis (LSA), has already been performed by scientists and engineers through thousands of years. Later entering the information age, with the support of computational calculation capability, some more stochastic algorithms and data-based algorithms have been developed and have got apparent advantage towards old LSA methods.

Global Sensitivity Analysis (GSA), namely being distinguished as the opposite conception of Local Sensitivity Analysis, is a category of advanced SA methods. The key idea of GSA is to vary all the inputs together and to study

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simultaneously their sensitivity using the same datasets. This approach makes it possible to estimate the interaction effects among variables [1] and to avoid the curse of dimensionality [2]. Literally, GSA methods can be applied on most kind of mathematical models with quantitative inputs and outputs. Beginning by some applications in chemistry [3], GSA has been proved effective in civil engineering [4], climate change [5], safety measurements [6], and among others. Fourier Amplitude Sensitivity Test (FAST) algorithm, firstly proposed by Cukier et al. [7], is one of the most efficient GSA algorithms. Improved by Saltelli and Bolado [8], FAST has now been perfectly integrated into the sensitivity indices proposed in the paper of Sobol' [9], with a rather brief computational approach [10]. It can calculate Sensitivity Indices (SIs) based on a unique analytical expansion of ANOVA (ANalysis Of VAriance), and its application has been introduced into vibro-acoutstic domain since recent year [11]. In general, GSA methods help to indicate the variables which should get fixed or be paid on attention with priority in case of model optimization and condensation.

Since GSA mainly serves for metamodelling, data mining models themselves are metamodels based on great number of samples. As other metamodels, data mining models can do estimations and predictions (classifiers and regressioners), and are also capable for some extra functions such as clustering. Generally the implementation of these models do not rely on preliminary studies such as GSA: they often regard the uncertainty of inputs as part of the models themselves. For some algorithms such as Random Forest (RF), they can further rank the importance of inputs after the constructions of metamodels, based on how easily the estimation will get wrong if some certain inputs get disturbed. With the explosion of data size on the Internet, these data-based methods become highly recommended. Instead of traditional mathematical tools, data mining and deep learning have become the main tools for data analysts either in industry or in academia. The current RF algorithm in use is almost the same as it was proposed [12], and has been validated to be parametrically robust [13], and compatible for various data types [14]. Apart from informational industries, the use of RF can be found, among others, in the following domains: ecology [15], medicine [16], transport [17], etc.. The function of ranking inputs importance for RF models do not really have any analytical basis such as a formula, but it gives some most direct indications on how serious problem the uncertainty of the inputs can result in.

With both the analytical GSA methods and data-based Deep Learning methods capable for uncertainty identification, some interesting comparison can then be made. Regarding the acoustic background of the datasets in this paper, some publications can be referred to: application of FAST on analytical models of sound transmission [18] and application of RF on numerical datasets of sound emission [19] for example. Conducted from these research cases, some impressive properties can be drawn on FAST and RF. In the aspect of theoretical basis, FAST seems to be more analytically solid while RF gives a more practical definition of sensitivity indicator. In the aspect of scientific applications, FAST is a tool of preliminary study while RF is a metamodeler. In terms of advantage/disadvantage, FAST is fast yet too statistical, and RF is functionally strong but without a convincing theoretical basis. Witnessing the pros and cons of both algorithms, a comparison between them on their uncertainty quantification abilities could be full of interest to improve the performance of both.

The present paper is organized as follows: In sec. 2, the basis of variance-based GSA are presented, along with some theoretical background on FAST method; sec. 3 gives an overview of Random Forest method and its OOB validation process for feature selection; numerical evaluations are thus designed in sec. 4 with the vibroacoustic model and its related variables briefly introduced; sec. 5 provides an overview of both uncertainty quantification results, some graphical comparison can directly be made upon; lastly sec. 7 and sec. 8 make a final conclusion of this paper and point out some possible improvements in further researches.

2. Variance-based Global Sensitivity Analysis

In this section, basic definitions and analytical expressions of the variance-based GSA algorithms including FAST will be presented. Sensitivity analysis is *the study of how uncertainty in the output of a model (numerical or otherwise) can be apportioned to different sources of uncertainty in the model input* [20], with multiple methods been developed in the last 50 years. Some most widely used ones, including FAST, belong to the ANOVA (ANalysis Of VAriance) class.

ANOVA denotes a group of SA methods based on a same system of sensitivity indexes S. For a model $Y = f(x_1, x_2, ..., x_n)$ with $f : \mathbb{R}^n \to \mathbb{R}$, it has been proved by Sobol' [9] that the total variance of the output $V(Y)$ can be uniquely decomposed into the sum of conditional variances as following under several conditions:

$$V(Y) = \sum_i V_i(x_i) + \sum_i \sum_{j>i} V_{ij}(x_i, x_j) + \sum_i \sum_{j>i} \sum_{l>j} V_{ijl}(...) + ... + V_{123...n}(x_1, ..., x_n), \quad (1)$$

and the sensitivity indexes are defined as $S_u = V_u/V(Y)$, $u \subseteq \{1, ..., n\}$.

A more compact definition of the first order sensitivity index is given by:

$$S_i = \frac{V_{X_i}(E_{X_{\sim i}}(Y|X_i))}{V(Y)}, \quad (2)$$

where $X_{\sim i}$ means all the inputs except X_i. The index S_i represents the ratio of variance of the output Y explained by the input X_i, which can thus be reduced if X_i remain fixed. For systems with uncorrelated inputs, $\sum S_i \leq 1$ is always true. When $\sum S_i = 1$, the system is called an additive system.

2.1. Fourier Amplitude Sensitivity Test (FAST)

In practice, the definition formula (2) can not be used directly due to the reason of its low computational efficiency. Therefore, a more compact algorithm called Fourier Amplitude Sensitivity Test (FAST) is suggested. Since it was firstly computed by McRae et al. [21], FAST has always been regarded as one of the best methods in the area of global sensitivity analysis in efficiency benchmarks among multiple ANOVA-based GSA methods [22].

The basic methodology of FAST is to estimate the total variance and the conditional variance throughout a Fast Fourier Transform of a periodically re-ordered dataset. With the order M defined as the minimum interference order (usually set as $M = 4$), the total variance $V(Y)$ is approximated using the Fourier coefficient A_j, B_j:

$$V(Y) \approx 2 \sum_{j=1}^{(N-1)/2} (A_j^2 + B_j^2), \tag{3}$$

where N is the total sampling number chosen to accord to the value of M and

$$A_j = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x_1, x_2, ..., x_n) \cos(js) ds,$$

$$B_j = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x_1, x_2, ..., x_n) \sin(js) ds.$$

In FAST, the discrete sampling vector $\mathbf{X_i}$ is generated by

$$x_i^{(j)} = \frac{1}{2} + \frac{1}{\pi} \arcsin(\sin(\omega_i s^{(j)} + \phi_i)), \tag{4}$$

where ω_i is the characteristic frequency particularly chosen for each $\mathbf{X_i}$, depending on the value of M. ϕ is a set of random numbers generated for this quasi-random sampling process. Then the V_i used for the estimation of S_i is approximated by:

$$V_i \approx 2 \sum_{j=1}^{M} (A_{j\omega_i}^2 + B_{j\omega_i}^2). \tag{5}$$

The proof of results uniqueness and model convergence can be found in [23].

2.2. Difficulties during interpretation of sensitivity indices

The variance-based sensitivity indices are rigorously defined with analytical decomposition and have many good properties, but still under many restrictions. One of the drawbacks of SA in applications is how to properly interpret the sensitivity indices under different industrial backgrounds. Being simple and fast, SA has been promoted into a variety of different domains [24], to give a preliminary study of variables in global view. But not every domain of scientific research takes statistical conceptions into their uncertainty indication methodologies. In such cases, theoretical explanation of sensitivity indices may not be straightforward to interpret among researchers.

3. Overview of Random Forest in the aspect of uncertainty quantification

This section aims at plotting the basic structure of Random Forest method and to introduce its importance selection feature. For a classification or regression problem of a set of observations, RF method builds a large collection of trees by a so-called CART (Classification And Regression Trees) method. One particular point in CART algorithm is to build binary trees. To build a CART tree, one attribute (predictive or explanatory variable) is selected by some criterion and is placed at the root of the tree. The selected variable divides the observations in hand into two groups with a given variance for each group. CART method chooses the variable that minimizes the weighted sum of these variances. The same procedure is recursively applied to each group. There may be many conditions to stop partitioning a node. When a node is no more partitioned, it is called a leaf node.

Noted that for RF algorithm to build one CART, an important point is the subsampling (with replacement) of data points during the tree construction. Another point is to choose a subset of attributes to be used in that tree. This type of data operation is called 'bagging', thus its specific cross-validation algorithm is named as Out-Of-'Bag' (OOB) validation. The OOB validation cannot only estimate the error rate of RF estimation, but also evaluate the relative importance of the inputs concerning their influence on the outputs. As presented in Fig. 1, the original training dataset **D** is divided into several subsets using bagging process and multiple CARTs are constructed based on these subsets. These decision trees will give individual estimation results upon given testing datasets and finally the RF will take an overall consideration for a final model estimation.

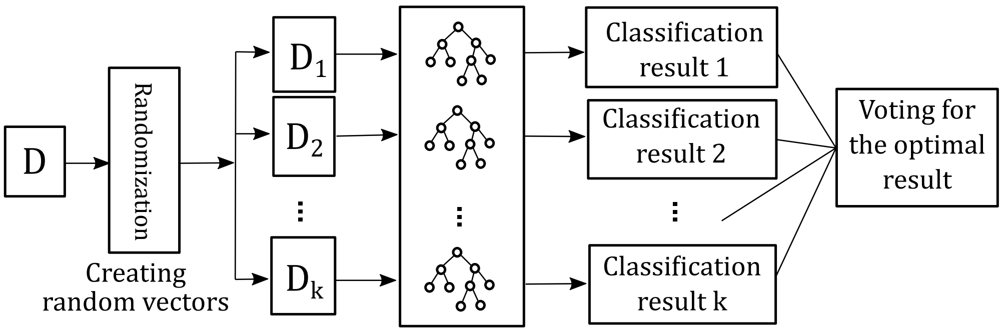

Figure 1: A simple diagram of Random forest estimation process [25].

Both the variance-based sensitivity indices and this OOB validation-based variable importance indicators can represent how the uncertainty of each input can disturb the output value. One based on theoretical decomposition and another based on numerical experimental observations. Therefore, it raises the interest of comparing these methods for a deeper inspection.

3.1. Classification And Regression Tree (CART)

CART is one category of decision trees capable to treat continuous inputs and outputs for regression problems. In an RF model, several dozens of trees are required to obtain convergent results. For the purpose of 'randomize' the 'forest', each CART is randomly parameterized.

Firstly, each CART randomly selects about 60% of the total training data as its own training data, and the rest will later be used in the OOB validation phase. Such convention of taking about 60% of the total training data for each CART has a statistical explanation. For a dataset S with n elements, if n randomly drawn are taken with replacement, the expectation of total unique elements to be drawn when n become large enough is:

$$(1 - (1 - 1/n)^n)n \approx (1 - e^{-1})n \approx 0.632n. \tag{6}$$

It is also called "0.632 rule" in bootstrapping. The fact that different trees use different training samples makes a good basis for generating trees with different 'backgrounds'.

Secondly, to ensure the efficiency, all CARTs used in RF are binary trees, which means that at each node the training data will be divided into two parts satisfying the least square criterion. Normally, all the input datasets need to be screened in order to find the optimized split point of decision trees. While for RF, in order to avoid local optimums, only limited randomly picked inputs are chosen to be screened for each split. Normally for a dataset with K features, $\sqrt{K}$ features will be chosen for a classification case and $K/3$ features will be chosen for a regression case. In this way the trees can have a much larger variety of 'personalities' without being dominated by certain variables with dominant influence on the output.

Lastly, in cases of continuous output, the nodes are considered to be enough converged and become a leaf node when split training samples already meet the convergence criterion. The criterion can either be a certain amount of depth of the tree, a small margin of value bounds, or a rather low threshold of group variance. All these criteria have been proved to be statistically stable and capable to reduce the total variance. Logically each CART will take average of leaf node sample values as a single-tree estimation value, then the whole RF will again calculate the average of all these estimation values to make a final decision of classification or prediction. These procedures makes an RF with 'democratic' semi-continuous outputs.

3.2. Out-Of-Bag (OOB) validation

The OOB validation refers to the techniques of using unbagged samples as validation sets for each CART, giving an all-over percentage of correct estimation P, where higher value generally means better approximation. The importance selection mechanism is exactly based on this notation P of OOB validation. Each time all the sampling values of a certain feature x_i will be randomly permuted, so that this value will become some irrelative noise value. The difference $SOOB_i = P - P(x_i \text{ randomly permuted})$ is defined in this case the importance factor of x_i. Or it can also be called 'OOB-based sensitivity index'. A greater value of this index means more serious estimation error would be made if this input gets disturbed, so more importance should be given to quantify its uncertainty.

This 'sensitivity index' has very direct practical meaning as the probability of causing RF estimation error when facing uncertainty. After normalization, each $SOOB_i$ represents the percentage of wrong estimations caused by the uncertainty of x_i. Compared to FAST, though the reliability of this index can hardly be proven in analytical means, much fewer restrictions of application is one important advantage for this method.

OOB validation is an essential component of RF, while it rarely serves independently as an uncertainty indicator, possibly because of its low calculation efficiency. But theoretically it works on all types of samples that FAST is capable to treat, and supports better the models with correlated datasets and strong non-linearities. Compared with ANOVA and FFT, the logistic binary trees are perfectly compatible with sampling continuity.

4. Experimental design and datasets generation

This section details the methodology of uncertainty quantification methods applied on vibroacoustic models with all input-output variables justified. The datasets being used in this research are retrieved from a former study on acoustic sandwich materials. Being a model with 13 inputs and 1 output, each dataset contains 20 000 samples in form of $y = f(x_1, x_2, ..., x_{13})$.

For an acoustic model, the frequency response, as the model output, must be repetitively evaluated on a continuous frequency interval. 100 frequencies are taken in this test to fit a curve from 100Hz to 10 000Hz. Under each frequency, the 20 000 $\{x_1, x_2, ..., x_{13}\}$ input vectors are the same while their corresponding y values are different.

In addition, for this simple acoustic models, the y value can be obtained either by analytical model or Wave Finite-Element (WFE) model. These two models give slightly different results and are also taken into comparison in this research. So in total there are two $20000 \times 13 \times 100$ 3d-matrices of input values x and two 20000×100 matrices of output values y.

4.1. Acoustic transmission loss of sandwich panels

The Transmission Loss (TL) is a very important criterion of material's sound isolation capacity. This value generally represents how much the power of the sound wave can be decreased after travelling through the piece of material.

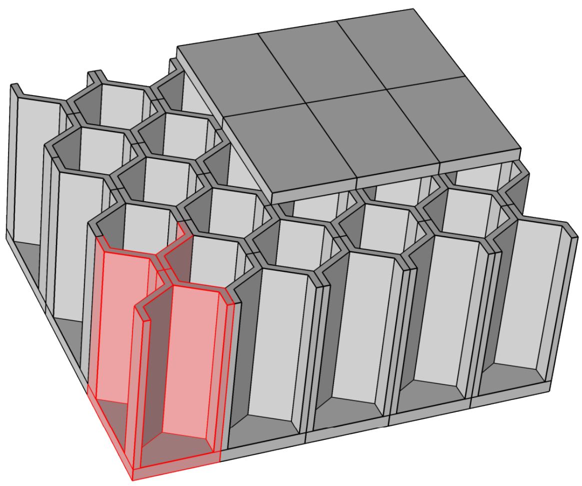

Figure 2: A model of sandwich panel with honeycomb structures.

In this research, TL corresponds to the output of model $f(x)$ and the input vector $\{x_1, x_2, ..., \mathbf{x_{13}}\}$ is composed with 13 variables concerning mechanical and geometrical properties of honeycomb sandwich composite materials. For the sake of simplicity, all the inputs are set to be uniformly distributed with a 20% variance around their mean values, as described in Table 1:

Table 1: Table of variables and their distributions (**U**niform[min*value*, max*value*] and **E**xponential[min*value*, max*value*])

Notation	Variable	Value distribution	Unit
Inputs			
E_c	Core material Young's modulus	$U[13.6, 20.4]$	GPa
E_s	Face-plate Young's modulus	$U[56, 84]$	GPa
h_c	Core layer thickness	$U[16, 24]$	mm
h_s	Face-plate thickness	$U[0.8, 1.2]$	mm
l	Non-vertical meso-structural length	$U[2.08, 3.12]$	mm
l_h	Vertical meso-structural length	$U[2.08, 3.12]$	mm
t	Meso-structural wall thickness	$U[0.08, 0.12]$	mm
η	Structural damping factor	$U[0.004, 0.006]$	-
θ	Meso-structural angle	$U[24, 36]$	deg
ν_c	Core material Poisson's ratio	$U[0.272, 0.408]$	-
ν_s	Face-plate Poisson's ratio	$U[0.16, 0.24]$	-
ρ_c	Core material density	$U[2160, 3240]$	kg/m^3
ρ_s	Face-plate density	$U[2440, 3760]$	kg/m^3
Output			
TL	Acoustic Transmission Loss	-	dB
Parameter			
f	Frequency	$Exp[100, 10000]$	Hz

The mean values of these variables are mostly obtained from experiments while some other parameters such as air sound speed are fixed as constant.

4.2. *Choice between analytical and WFE models*

In traditional case of sandwich panels with isotropic and homogeneous core material, analytical models[26] can give a fast and accurate estimation of its vibroacoustic properties. But with fast development of manufacturing techniques, more and more delicate core meso-structures have been developed[27], Finite-Element models[28] are required to avoid the error generated in the homogenization.

Their relation is very like the one between FAST and RF. The analytical one is robust and computationally more efficient while the numerical one is sometimes more powerful and suitable for statistical presentation.

5. SA results and their comparison

This section forms the core part of paper by giving analysis on uncertainty quantification results, but before that, some parametric details of the algorithms are to be detailed in this section.

FAST is a non-parametric distribution-based method and RF is a parametric sample-based method. Therefore, the methodology is to generate the samples by FAST, getting the sensitivity indices, and then to re-use these samples in RF.

According to the former introduction on RF, in this test the bagging percentage of samples is 65% with 4 variables randomly picked up each time at node split. For the configuration of other parameters including the number of trees and several others, for the sake of calculation efficiency, they are set to minimized values with confirmation of RF model convergence. As the RF process will be repeated tens of thousands times in FAST loops, the burden of calculation resources must be considered in priority.

According to the ANOVA decomposition S_i is theoretically normalized with a sum smaller than 1, but $SOOB_i$ has no mathematical restrictions. In order to facilitate the visual comparison of the two indices, the values of $SOOB_i$ are also normalized by $\sum S_i$.

5.1. *Analytical model dataset*

In this subsection the analytical TL estimation model of honeycomb sandwich material is evaluated by the two methods FAST and RF. The frequency-based sensitivity indices obtained by FAST is shown in Fig. 3, and the variable importance sorted by RF OOB validation is shown in Fig. 4.

Visually, the results shown in the two graphs are in good agreement, not only because Fig. 4 is normalized with the value of Fig. 3, but also the values of each sensitivity indices are in the same trend of frequency evolution. In general, both of the graphs highlight the influence of material densities (mass) on structural TL performance at low frequency and present the trend that the thickness of the panel become more or less dominant at mid-high frequency. Generally, the two methods give similar estimations on the list of most influential variables, within the specified frequency band.

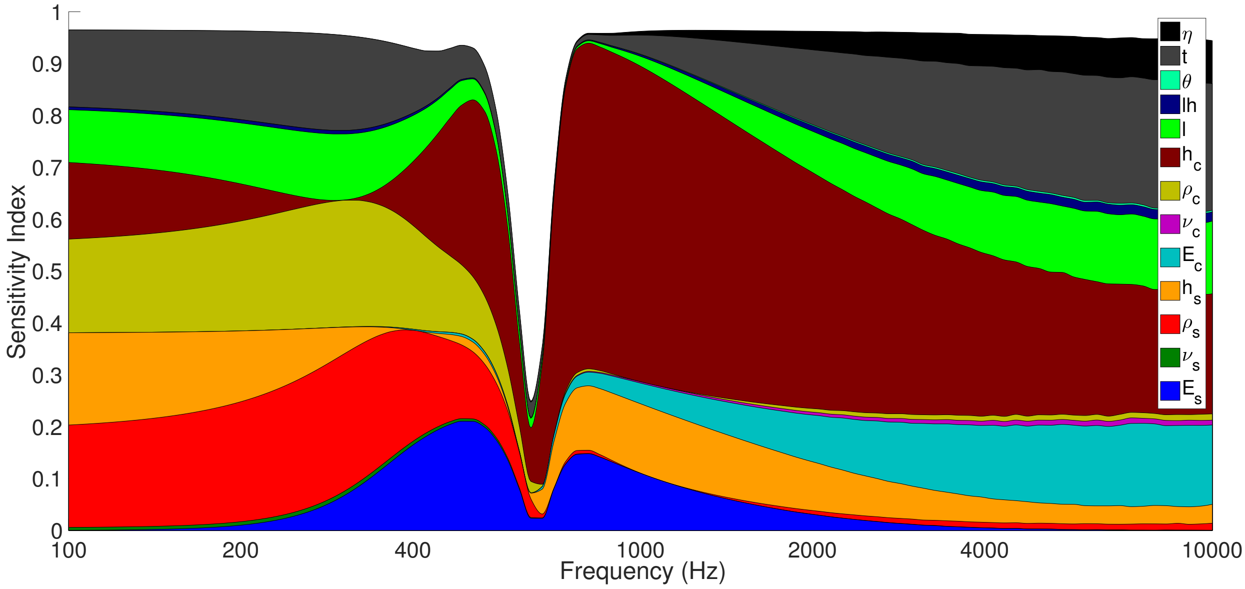

Figure 3: Sensitivity indices obtained by FAST, samples from analytical model.

Some graphical details can also be found in both results. Around 350Hz, the sensitivity index of h_c gradually decreases and then re-increase into the most influential parameter, similarly for h_s at 400Hz. Further analysis shows that at 340Hz, TL and h_c turn from negative correlation to positive correlation. At that point, TL is almost irrelevant to h_c, no matter other parameters' values are. A similar property can also be observed for h_s, but as h_s has a much smaller value than h_c, its evolution is less evident. Noteworthy, around 700Hz, the sum of variance-based sensitivity indices becomes suddenly very small. This frequency corresponds to the coincidence (i.e. critical) frequency of the panel, where the acoustic model gets insensitive to inputs' uncertainties at this point. Under this frequency, a small sum of S_i indicates that large error is present and FAST can no longer guarantee the effectiveness of its SA results. Under the methodology of sensitivity analysis, it means that the uncertainty of model output is quantified but unable to be apportioned to model inputs. At similar frequency slightly below 700Hz, the RF OOB error curve also reaches a peak above 50%, meaning horrible approximation by RF, so the results of OOB importance selection can neither be trusted.

The most obvious defect for this results comparison is that the curve in 4 is not as smooth as the one in 3, one possible explanation lies in the random permutation of the variables. In real 'random' case, a 1% error and a -20% error will eventually make a difference. Besides, the fact that all inputs have taken uniform distribution instead of Gaussian distribution may also have an impact on these oscillations.

5.2. *WFE model dataset*

As for the SA results upon samples generated by the Wave Finite Element model, Fig. 5 refers to the results obtained by FAST and Fig. 6 refers to the results obtained by RF OOB validation.

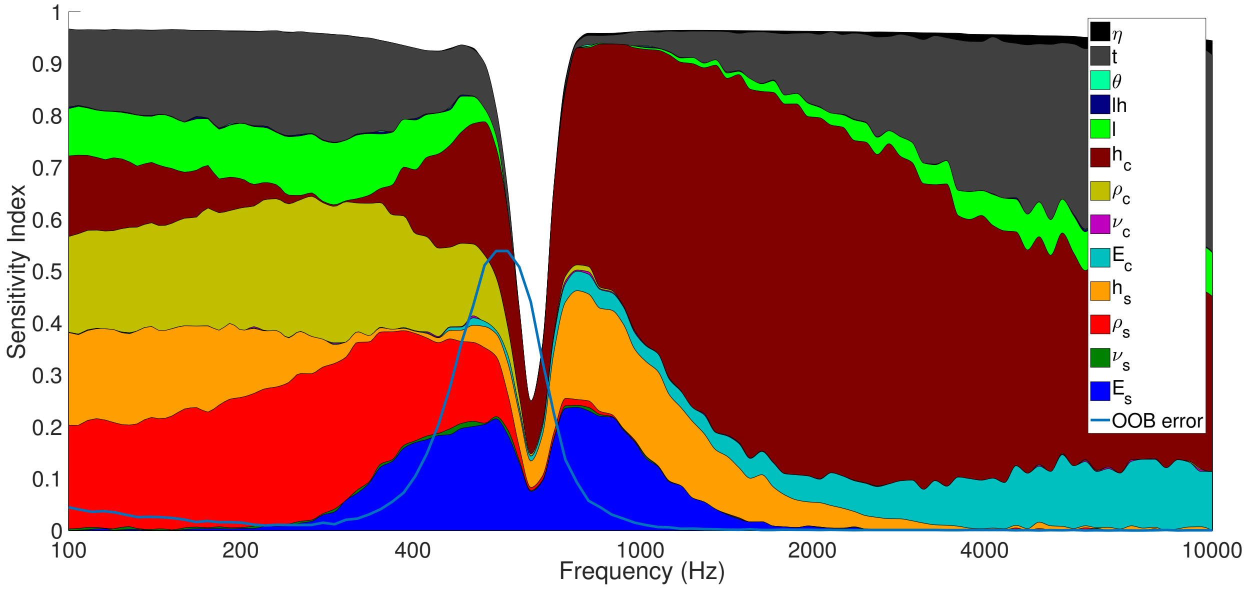

Figure 4: Variable importance sorted by RF OOB validation and normalized with the results of FAST, samples from analytical model.

The comparison between Fig. 5 to Fig. 3 shows that the two models reflect eventually to the same problem, although a minor difference can be observed. Most of the particular points mentioned in last part can also be found in these graphs, such as the critical frequency, the point of property transfer and even the fact that E_s is only influential around the critical frequency.

Also, from the comparison between Fig. 6 and Fig. 4, no similar unsmoothness can be found in common. This detail shows that the unsmoothed curves are likely to be caused by random errors, rather than some systematic singular points. Slight vibration on the curve can even also be observed in Fig. 5 at high frequency.

It is very nice to see that Fig. 6 (RF on WFE datasets) is much more graphically similar to Fig. 5 (FAST on WFE datasets) rather than Fig. 4. This gives a strong proof of the numerical agreement between these two sets of uncertainty quantification results. Though they are obtained by two methods with completely different definitions, one in analytical aspect and the other from experimental observation.

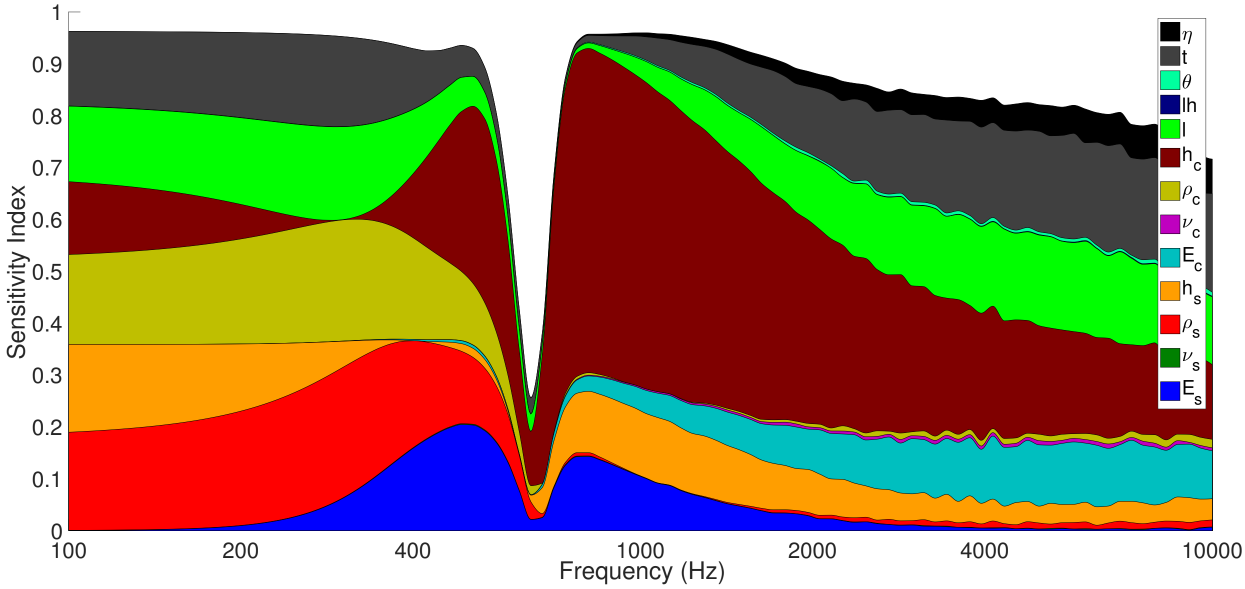

Figure 5: Sensitivity indices obtained by FAST, samples from WFE model.

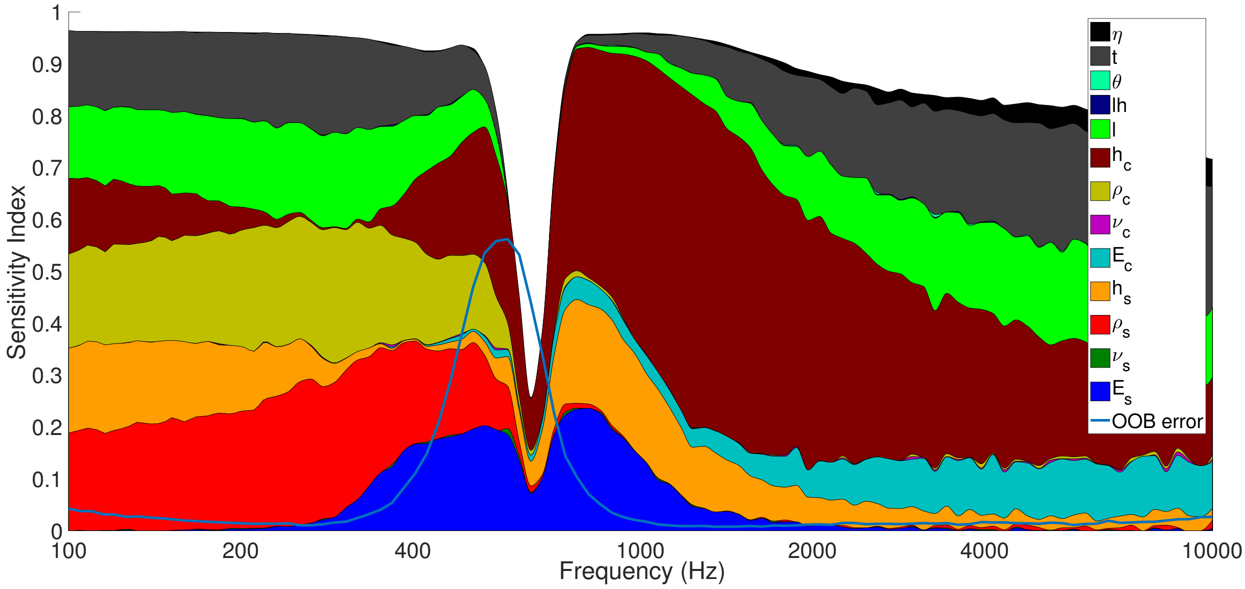

Figure 6: Variable importance sorted by RF OOB validation and normalized with the results of FAST, samples from WFE model.

6. Supplementary test case on Tuned Mass Damper (TMD) systems

In order to test the rigidity of both methods, another comparison of their application on Tuned Mass Damper (TMD) system is given in this section. TMD systems are classical solutions for the absorption of vibration energy in huge-volume structures, a typical one can be presented in Fig. 7.

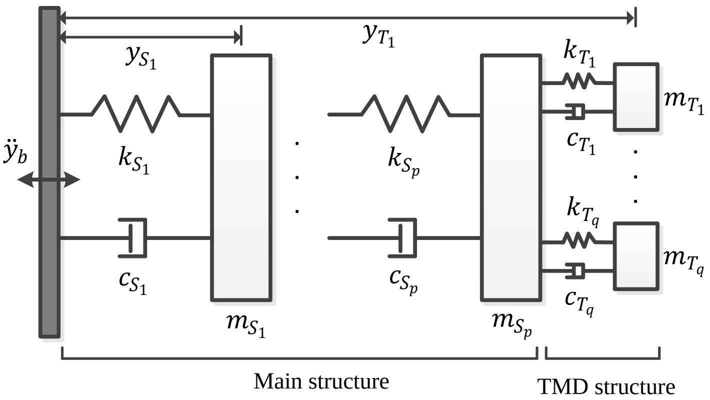

Figure 7: A TMD system composed of a main structure with p DOFs (Degrees Of Freedom), and a TMD structure with q DOFs.

In case of the TMD system excited by a base acceleration $\ddot{y}_b$, the structural response is determined by the following motion equation[29]:

$$\mathbf{M}\ddot{\mathbf{y}}(t) + \mathbf{C}\dot{\mathbf{y}}(t) + \mathbf{K}\mathbf{y}(t) = -\mathbf{M}\mathbf{r}\ddot{y}_b \tag{7}$$

where the state space $\mathbf{y} = \left[y_{s_1}, \ldots, y_{s_p}, y_{T_1}, \ldots, y_{T_q}\right]^T$, $\mathbf{r} = [1, 1, \ldots, 1]^T_{1\times(p+q)}$. $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ the damping matrix, $\mathbf{K}$ the stiffness matrix.

6.1. Testing preset

In this test, the simplest form of TMD system is chosen for the sake of computational cost, with only 2 DOFs and 1 TMD resonator. Uncertainties are given to 6 of these structural parameters: $\mathbf{X} = [m_{s_1}, c_{s_1}, k_{s_1}, m_{s_2}, c_{s_2}, k_{s_2},]$, for this system excited by a white noise with a power spectral density of $015m^2/s^3$, and the maximum displacement response of y_{s_1} is studied as model output.

2000 independent datasets of 6 inputs and 1 output are generated by the classic solution using Finite Element Analysis (FEA), and will be evaluated separately by FAST and RF to get a comparison. Also, as the maximum structural displacement is calculated upon Gaussian noise excitation, multiple noise samples are also prepared to avoid their random effects on SA results.

Table 2: Structural parameters

Item	$m(kg)$ the mass	$c(N \cdot m/s)$ the damping factor	$k(N/m)$ the stiffness
S_1	$N[4.6, 1]$	$N[62, 10]$	$N[6500, 300]$
S_2	$N[4.6, 1]$	$N[62, 10]$	$N[6500, 300]$
TMD	1.38	38.997	1.8327

6.2. *SA results upon TMD system*

Shown in Fig. 8, 10 sets of SA results on different white noise excitation are evaluated by FAST and RF, together drawn on stack bars.

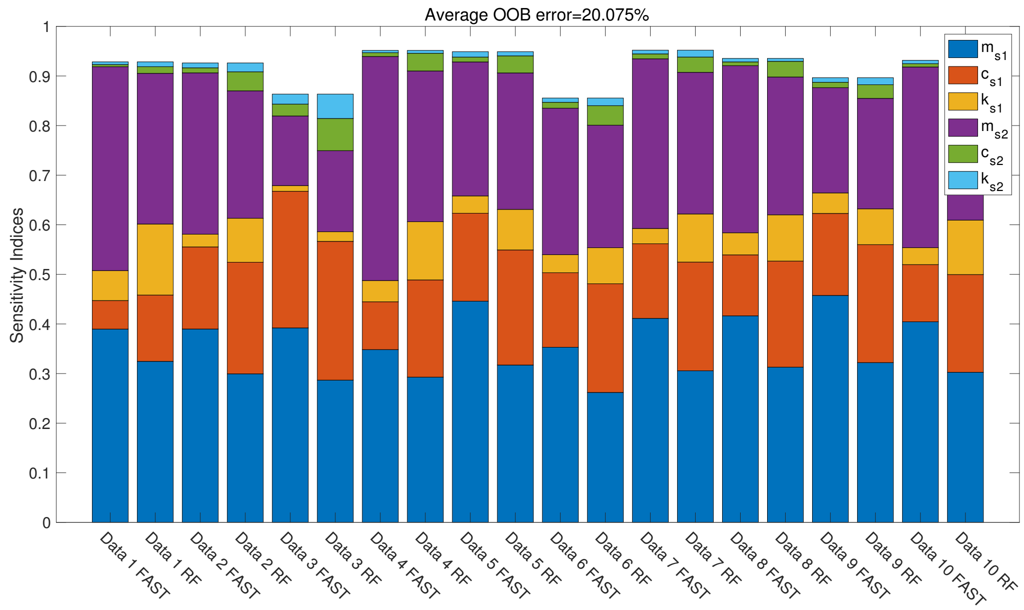

Figure 8: SA results on TMD system using FAST and RF.

Firstly, as can mentioned, the average OOB error reaches almost 20%, which means the approximation of RF is not quite perfect. And looking at the sum of first order SIs, it also has a 10%—20% gap from 100%, showing the FAST estimation also has some uncertainty. This might be caused by randomly generated Gaussian noise, where the uncertainty is not considered in this test.

Then, looking at the values of SIs obtained by FAST and RF, both methods indicate that m_{s_1} and m_{s_2} are the most sensitive parameters, which is often the case for vibroacoustic problems. Statistically the parameters for DOF-1 have slightly greater sensitivity than those of DOF-2, this is probably due to the definition of model output, the maximum displacement of DOF-1 structure.

As for the comparison between FAST and RF, some differences are observed, but both give similar order of parametric importance. Generally FAST tend to under estimate the sensitivity of m_{s_1} and m_{s_2} and overestimate c_{s_1}. In previous studies, FAST has often been found to tend to underestimate SIs, while the overestimate is quite difficult to explain.

7. Discussion

Random Forest, like other data mining tools, is quite 'subjective' towards different industrial cases. There are always sampling datasets suitable or not really suitable for the algorithm. In this research, though the OOB error curve remains very steadily low, except at the critical frequency, some questionable estimations can be found as depicted in Fig. 9:

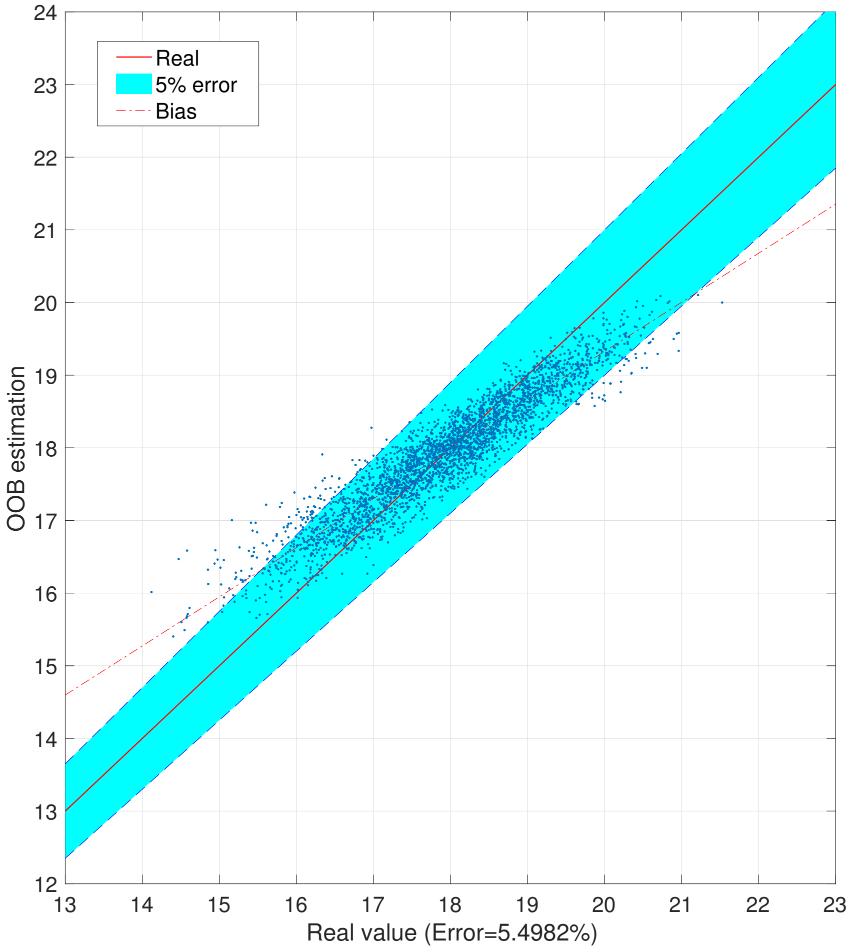

Figure 9: A mapping of estimated values towards original sampling values.

The fact that the bias ordinarily occurs can be intuitively explained. As RF gives an estimation made by a large group of regressive decisioners, the results will always tend to become closer to the global average value. An example of how these phenomena can ruin the estimation can be seen in Fig. 10:

The quasi-continuous output design of the RF and the CARTs yields to make estimation results to vary very

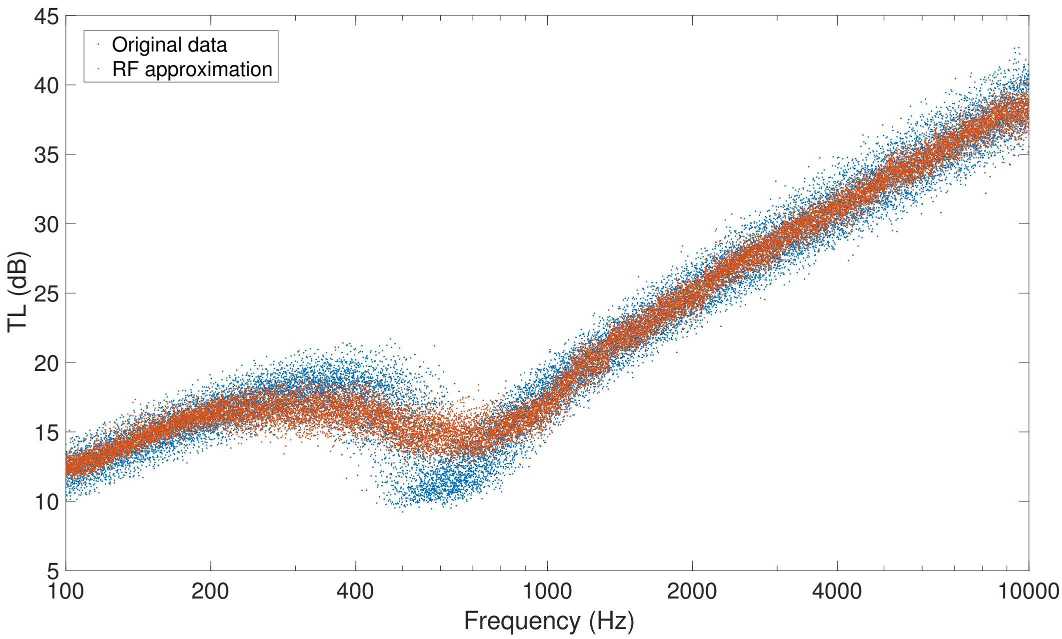

Figure 10: An example of how RF estimations (orange points) mistreat function discontinuity and lost extreme values(blue edge).

smoothly along the variation of some certain inputs. This property is responsible some serious distortion towards the discontinuity of original data.

Meanwhile, the error could be reduced as the values are graphically distributed along a biased line, as shown in Fig. 9. Therefore, a good opportunity actually exists to improve its estimation using some geometrical corrections such as rotation or zooming on certain ax.

8. Conclusion

This paper presents a comparison between two uncertainty quantification algorithms of different categories, namely FAST and RF, with their applications on vibroacoustic models. FAST is a classic statistic global sensitivity analysis method, with well-established theory basis and high calculation efficiency, while sometimes its results can be difficult to interpret in industrial cases. Random Forest is an upcoming data mining based regressionier and classifier, capable to build metamodels in various cases. With the OOB design, RF is also capable to tell the importance of each variable, but its selection feature is still intuitively defined and can not be recognized when not using RF. These are two different methods with different theoretical structures while both capable to achieve the goal of uncertainty quantification.

By comparing their numerical experiment results it is demonstrated that the two differently defined sensitivity indicators S_i and $SOOB_i$ can numerically reach a great agreement. Such results show a potential of numeric tools being applied together in specific cases. The variance-based sensitivity indices can hardly be explained in engineering word, which is the advantage of RF OOB variable importance indicator. A run of constructing and evaluating a RF takes comparably long time but FAST can save the time and even give a reasonable proof for the results. The potential of combining the advantages of each tools may worth investigation for researchers and engineers.

Furthermore, with a vibroacoustic background, the special properties of sandwich composite panel also helps to find out some weakness of the mentioned algorithms. More study on this problem can make further improvements on the accuracy of both algorithms.

9. Conflict of interest statement

On behalf of all authors, the corresponding author states that there is no conflict of interest.

10. Replication of Results

Results presented in this paper can be replicated by applying standard FAST and Random Forest algorithms on vibroacoustic models that estimate the TL of sandwich panels. The model for the homogenization of the honeycomb structure can either be the analytical model (Gibson-Malek) or the WFE model (J.-L. Christen).

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