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A new method for quantifying the phylogenetic redundancy of biological communities

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Author contribution statement CR formulated the idea. CR, GB and SP developed the methodology. GB and SP wrote the R code. CR, MC and BC analyzed the data. CR wrote the manuscript; other authors provided editorial advice.

Compliance with ethical standards

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Abstract The increasing use of phylogenetic methods in community ecology recognizes that accumulated evolutionary differences among species mirrors, to some extent, ecological processes. The scope of this work is thus to propose a new method for the measurement of community-level phylogenetic redundancy, which takes into account the branching pattern of the underlying phylogeny. Like for functional redundancy, a measure of phylogenetic redundancy can be described as a normalized measure in the range [0–1] that relates the observed community-level phylogenetic diversity to the value of a hypothetical assemblage with the same abundance distribution of the focal community in which all species had independent evolution. Therefore, phylogenetic redundancy can be interpreted as the diversity decrease that is obtained by taking into account the evolutionary relationships among species in the calculation of diversity. The behavior of the proposed method, for which we provide a simple R function called 'phyloredundancy', was evaluated with published data on Alpine plant communities along a primary succession on a glacier foreland in northern Italy. As shown by our results, the method accounts for the length of shared branches in the phylogeny, producing a coherent framework for describing the evolutionary relationships within a species assemblage. Being based on classical diversity measures, which have been used in ecology for decades, it also has vast potential for future research in phylogenetic community ecology.

50 **Keywords** Branch length; Lineage abundance; Phylogenetic diversity; Species features; Ultrametric
51 trees.

52 **Introduction**

53 Measures of biodiversity are central for exploring the complex ecological processes that drive
54 species coexistence and community assembly. Early work on biodiversity attempted to summarize
55 different aspects of community structure based on either presence and absence scores or on species
56 abundances, thus assuming that all species are equally and maximally distinct. More recently, the
57 recognition that the species ecological differences constitute an important facet of biodiversity has
58 received increasing attention (Webb 2000; Petchey and Gaston 2002). As stressed by Cadotte et al.
59 (2013): “Mechanisms influencing patterns of community assembly act on the ecological similarities
60 (or redundancies) and differences (or complementarities) of organisms and not on the number of
61 species”. In this light, the ecological differences between species are usually summarized based
62 either on their functional or phylogenetic distances. The pairwise species differences can be then
63 combined at the community level to summarize the average degree to which the species differ in
64 terms of their evolutionary history or functioning (Cadotte et al. 2013).

65 When several species perform similar functions, functional redundancy may enhance community
66 stability, thus providing insurance against the loss of ecosystem functions induced by external
67 factors, such as disturbance or biotic erosion (Yachi and Loreau 1999). Likewise, evolutionary
68 relationships may affect ecological processes because the branching patterns of phylogenies mirrors
69 the accumulation of multidimensional phenotypic differences among species (Felsenstein 1985;
70 Harvey and Pagel 1991; Vane-Wright et al. 1991; Faith 1992), which may in turn describe the
71 overall complexity of community functioning better than a limited set of functional traits (Cadotte
72 et al. 2010; Tucker et al. 2016).

73 However, while several methods have been proposed for summarizing functional redundancy
74 (Micheli and Halpern 2005; Mouillot et al. 2013, 2014; Guerrero et al. 2014; Ricotta et al. 2016),
75 the calculation of phylogenetic redundancy has received far less attention. Like for functional
76 redundancy, phylogenetic redundancy is defined as the diversity decrease that is obtained by taking
77 into account the evolutionary relationships among species in the calculation of diversity. This
78 definition makes phylogenetic redundancy clearly distinct from other notions of redundancy, such
79 as e.g. sample redundancy (*sensu* Garcillán et al. 2003).

80 The aim of this paper is thus to propose a new method for the measurement of community-level
81 phylogenetic redundancy taking into account the branching pattern of the underlying phylogeny.
82 The performance of the proposed method was evaluated with published data on Alpine plant
83 communities along a primary succession on a glacier foreland in Italy.

84 **Methods**

85 Imagine an assemblage (or community, plot, sample, relevé, quadrat, etc.) composed of N
86 species with relative abundances p_i ($i = 1, 2, \dots, N$) with $0 < p_i \leq 1$ and $\sum_i p_i = 1$, together with a
87 symmetric $N \times N$ dissimilarity matrix whose elements d_{ij} summarize the pairwise functional
88 dissimilarities between species i and j with d_{ij} in the range $[0-1]$, such that $d_{ij} = d_{ji}$ and $d_{ii} = 0$.

89 Ricotta et al. (2016; see also de Bello et al. 2016) proposed to summarize functional redundancy
90 R as the complement of community uniqueness U , such that $R = 1 - U = 1 - Q/S$ where Q is Rao's
91 (1982) quadratic diversity $Q = \sum_{ij} p_i p_j d_{ij}$ (i.e. the expected dissimilarity between two individuals
92 of the assemblage selected at random with replacement) and S is Simpson's diversity index
93 $S = 1 / \sum_i p_i^2$ (i.e. the probability that two individuals randomly selected from the assemblage will
94 belong to different species).

95 Considering all species as equally and maximally distinct (i.e. for $d_{ij} = 1$ for all $i \neq j$ and $d_{ii} = 0$)
96 we have that $Q = S$. Hence, according to Ricotta et al. (2016), community-level functional
97 uniqueness (the complement of redundancy) can be summarized as the normalized ratio of two
98 diversity measures with different levels of 'ecological abstraction': the Rao index, which takes the
99 species dissimilarities into account for the calculation of functional diversity and the Simpson
100 index, which considers all species equally and maximally dissimilar. For details, see Ricotta et al.
101 (2016).

102 In principle, the same approach can be used for calculating community-level phylogenetic
103 redundancy after rescaling the species phylogenetic distances d_{ij} to the unit range. However, a
104 primary difference between functional and phylogenetic methods is that phylogenetic methods are
105 fundamentally related to a tree-like structure which represents the evolutionary relationships among
106 species (Pavoine 2016). Therefore, suitable measures of phylogenetic redundancy should take into
107 account the branching pattern of phylogenetic trees.

108 Faith (1992) suggested that branch lengths can be expressed in terms of species features, which
109 can be interpreted as "an evolutionarily conserved trait that may be unique to a particular species or
110 shared by many species" (Nipperess et al. 2010). As a result, the total branch length of the
111 phylogeny is basically a measure of the total amount of evolutionary history embodied in an
112 assemblage. In practice, the branch lengths may be set proportional to some measure of
113 evolutionary change, such as the amount of functional or morphological differentiation from a
114 common ancestor, or the number of base changes at a given locus (Chao et al. 2010). If the branch

lengths are proportional to time of divergence, all species (terminal nodes) are at the same distance from the root node and the tree is called ultrametric.

A general approach for taking into account the tree-like structure of phylogenies in the calculation of community diversity has been recently proposed by Pavoine et al. (2009) and Chao et al. (2010). Here, we will consider only ultrametric trees reporting time of divergence among species, which usually represent the major source of phylogenetic information in community ecology. For details on how to extend this approach to non-ultrametric trees, see Appendix 1. Given a species assemblage and the associated ultrametric tree in Figure 1, let T be the age of the root node. We can define non-overlapping evolutionary periods slicing the tree at each node so that there are no internal nodes between two slices. Starting from the root node, the node ages can be labeled t_k ($k = 0, 1, \dots, K$) so that $t_0 = T$ (the age of the root node) and $t_K = 0$ (the present-day time).

Two subsequent slices delimit an evolutionary period $\Delta_k = |t_k - t_{k-1}|$ such that $\sum_k \Delta_k = T$. Each evolutionary period contains M lineages ($N \leq M$) with relative abundances π_{km} ($m = 1, 2, \dots, M$) such that $0 < \pi_{km} \leq 1$ and $\sum_m \pi_{km} = 1$. These lineage abundances are obtained as the sum of the relative abundances of all descendent species in the phylogenetic tree. In terms of features, the length of a lineage reflects the number of the features associated to that lineage, while the lineage abundance reflects the abundance of these features (Pavoine 2016).

Based on the lineage abundances π_{km} we can calculate a ‘lineage diversity’ D_k for each node age t_k of the phylogeny. The lineage diversity at time t_K is the traditional present-day diversity calculated from the species abundances p_i (i.e. assuming all species are equally and maximally distinct). The community-level phylogenetic diversity D_P is then obtained averaging the diversities D_k over the corresponding time intervals of interest:

$$D_P = \sum_k D_k \times \tau_k \quad (1)$$

where $\tau_k = \Delta_k / T$ is the duration of the k -th evolutionary period after rescaling the tip-to-root length to unit, and Δ_0 (and hence τ_0) = 0 by definition. Unlike traditional diversity measures, D_P incorporates information about the branching pattern of the phylogeny and its lineage abundances. If the diversity measure used for calculating phylogenetic diversity is species richness, the product $D_P \times T = \sum_k D_k \times \Delta_k$ turns out to be Faith’s (1992) phylogenetic diversity (PD). That is, the sum of the branch lengths of all species in the phylogeny, which is basically a measure of the total amount of evolutionary history embodied in the assemblage since the time of the most recent common

147 ancestor. This relationship reinforces the interpretation of D_P as a measure of ‘phylogenetic
148 diversity.

149 Due to Dalton’s principle of transfers, for any schur-concave diversity index such as species
150 richness, the Shannon, or the Simpson diversity and their species equivalents (Hill 1973; Jost 2007),
151 the present-day diversity D_K is always greater than or equal to any other lineage diversity D_k . For
152 details, see Patil and Taillie (1982). Accordingly, like in Ricotta et al. (2016), we can obtain a
153 normalized index of phylogenetic uniqueness U taking the ratio of two diversity measures: average
154 community-level phylogenetic diversity D_P , which takes into account the branching pattern of the
155 underlying phylogeny, and traditional present-day diversity D_K , which considers all species as
156 equally and maximally dissimilar, such that $U = D_P / D_K$. Likewise, phylogenetic redundancy, can
157 be simply calculated as:

158
159
$$R = 1 - U = 1 - D_P / D_K \quad (2)$$

160

161 Phylogenetic redundancy R thus summarizes the diversity decrease that is obtained by taking
162 into account the branching pattern of the phylogenetic tree in the calculation of diversity.
163 Redundancy will be small when there are many deep branches, each well represented in the present-
164 day assemblage, attaining its minimum value $R = 0$ for a star phylogeny where all terminal branches
165 originate from a single polytomy at the root node. By contrast, redundancy will be large when all
166 branches emerge recently such that the present-day assemblage is composed of phylogenetically
167 close species (Chao et al. 2010; Tucker and Cadotte 2013). Put another way, phylogenetic
168 redundancy tells us how much the real phylogenetic diversity departs from that of a star phylogeny
169 in which all species had independent evolution.

170

171 **Case study**

172 Starting from the assumption that, during evolution, traits are on average more conserved than
173 random, in community ecology phylogenetic metrics have been often considered a proxy for
174 differences in functional traits. According to this hypothesis, phylogenetically close species are
175 generally assumed to be more functionally similar than more distantly related species (Freckleton et
176 al. 2002; Ackerly 2009; Srivastava et al. 2012; Gerhold et al. 2015; Letten and Cornwell 2015).
177 Therefore, the relevance of the new index of phylogenetic redundancy was evaluated comparing
178 phylogenetic changes in plant communities along a primary succession on the foreland of the Rutor

179 glacier (northern Italy) with the functional changes obtained for the same data by Ricotta et al.
180 (2016).

181 The study area is located between 2400 and 3000m on moraine deposits, which are mainly
182 composed of metamorphic schists. Climate is continental with very cold winters and low annual
183 precipitation. The glacier terminus had its maximum expansion during the Little Ice Age (1820–
184 1864). In the past century, the Rutor glacier had more limited oscillations, the most relevant of
185 which ended in 1933; for additional information on the study site, see Caccianiga et al. (2006).

186 Fifty-nine vegetation plots of approximately 25 m² were sampled by Caccianiga et al. (2006) on
187 the glacier foreland above the local tree limit. Based on the age of the moraine deposits, the plots
188 were assigned to three different successional stages: 17 early-successional plots (sampled in areas
189 exposed after 1933), 32 mid-successional plots (sampled in areas exposed between 1820 and 1933)
190 and 10 late-successional plots (sampled in areas exposed before 1820). For each plot, species
191 abundances were measured with a five-point ordinal scale transformed to ranks. All data are
192 available in Ricotta et al. (2016, Appendix S2).

193 Based on Grime's (1974, 2001) functional classification of plants as competitors (C), stress-
194 tolerators (S) and ruderals (R), Ricotta et al. (2016; Table 2) showed that vegetation dynamics along
195 the succession were shaped by a progressive increase in community-level functional redundancy
196 from the early to the late-successional stages. Here, we will analyze the phylogenetic redundancy of
197 the same data. First, we calculated the community-level phylogenetic diversity and redundancy of
198 each plot according to Eq. 1 and 2, respectively. Next, we tested for significant differences among
199 the phylogenetic diversity and redundancy values of the three successional stages using a
200 permutational t-test with 9999 permutations.

201 The phylogenetic tree was extracted from the Daphne phylogeny (Durka and Michalski 2012), a
202 dated phylogeny of a large European flora comprising most Alpine species of Italy. For calculating
203 phylogenetic redundancy, we used two diversity measures with different sensitivity to common
204 versus rare lineages: the Shannon index $H = \sum_i p_i \log p_i$ and the Simpson index S , which is more
205 sensitive to common lineages compared to H (Jost 2007; Pavoine 2016). Note however that since
206 Ricotta et al. (2016) defined functional redundancy as $1 - Q/S$, where Q is the Rao quadratic
207 diversity, the use of the Simpson index for calculating phylogenetic diversity provides the most
208 direct term of comparison between functional and phylogenetic redundancy. Note also that the
209 ordinal scale used for measuring the species abundances (see Ricotta et al. 2016, Appendix S2) is

210 disproportionately sensitive to rare species. Therefore, the Simpson index compensates to some
211 extent the overweighting of low species abundances induced by the ordinal measurement scale.

212

213 **Results**

214 Figure 2 shows the phylogenetic tree of the species used in this study, together with the plots of
215 the mean Shannon and Simpson lineage diversities D_k of each successional stage vs. node age t_k . All
216 calculations were done with the new *R* function ‘phyloredundancy’ available in Appendix 2; an
217 electronic version of the phylogeny is available in Ricotta et al. (2015, Appendix A). Table 1 shows
218 the mean community-level diversity and redundancy values for the three successional stages
219 identified along the primary succession on the foreland of the Rutor Glacier. In good agreement
220 with the functional redundancy values calculated by Ricotta et al. (2016) and irrespective of the
221 diversity index used, a general tendency towards an increase in phylogenetic redundancy from the
222 early to the late-successional stages is observed.

223 As shown in Figure 2, for all successional stages, the major decrease in lineage diversity from
224 node age t_k to t_{k-1} occurs at deep evolutionary times. For the early- and mid-successional stages,
225 which are mainly colonized by ruderal forbs, such as *Oxyria digyna* and *Tussilago farfara*, the most
226 important change in lineage diversity along the nodes of the phylogenetic tree is associated to
227 diversification within the eudicots at 118 Ma. By contrast, for the late-successional stage, which is
228 dominated by stress-tolerant graminoids, such as *Carex curvula*, or *Festuca halleri*, the main
229 change in D_k is related to differentiation within the monocots at 81 Ma. Hence, the increase in
230 functional redundancy along the succession observed by Ricotta et al. (2016) is accompanied by a
231 parallel increase in phylogenetic redundancy induced by a compositional shift from ruderal early-
232 successional forbs to stress-tolerant late-successional graminoids (Caccianiga et al. 2006).

233

234 **Discussion**

235 The growing application of phylogenetic methods in ecological studies recognizes that
236 accumulated evolutionary differences among species may reflect ecological processes. In this paper,
237 we thus introduced a new measure of community-level phylogenetic redundancy, which is
238 calculated by normalizing phylogenetic diversity in the range [0-1]. While phylogenetic diversity is
239 an absolute measure that takes into account the species abundances and the branching pattern of the
240 underlying phylogeny, phylogenetic redundancy is a relative measure that relates the observed
241 phylogenetic diversity to the value of a maximally dissimilar model assemblage with the same

242 abundance distribution of the focal community. In this sense, phylogenetic redundancy differs from
243 the classical notion of ‘phylogenetic evenness’ *sensu* Villéger et al. (2008) because the latter has
244 been used to quantify the regularity of the distribution of species dissimilarities *and* abundances in
245 phylogenetic space (see also Tucker et al. 2016).

246 Unlike other indices of phylogenetic diversity, which are based on pairwise species distances, the
247 proposed measures account for the length of shared branches in the phylogeny. On one hand, this is
248 a more natural approach to the calculation of phylogenetic diversity, since a tree representation is
249 the most logical way for describing the evolutionary relationships within a species assemblage. On
250 the other hand, this method may become problematic for comparing phylogenetic and functional
251 diversity, since a tree-based hierarchical representation of the functional relationships within a set of
252 species (see Petchey and Gaston 2002) is not biologically justified (Pavoine 2016; Villéger et al.
253 2017).

254 The proposed method for calculating phylogenetic diversity and redundancy has vast potential
255 for future research in phylogenetic community ecology as it is based on classical diversity
256 measures, which have been used in ecology for decades. Therefore, due to their well-known
257 properties, the practitioner can select the measure that best matches the requirements of his/her
258 specific question. For example, at broad spatial scales, species abundances are often unknown or
259 irrelevant, such that species occurrences are usually represented as presence/absence scores. By
260 contrast, at local scales, the ecological interactions among species are thought to reflect both their
261 abundances and evolutionary relationships. In this case, data on species abundances become a
262 relevant piece of ecological information.

263 In this framework, unlike most tree-based measures of phylogenetic diversity (but see Cadotte et
264 al. 2010; Tucker et al. 2016), the proposed method enables the incorporation of information on
265 species abundances in a very flexible manner. At the same time, since D_P is an additive quantity, it
266 is possible to decompose phylogenetic diversity into the contributions of the single evolutionary
267 periods which shape the phylogeny. For example, in our case study the increased evolutionary
268 relatedness of the late successional species is most likely due to the increasing dominance of
269 graminoids which progressively substituted early-successional ruderal forbs. On the other hand, one
270 might assume that the increasing phylogenetic redundancy along the successional process is simply
271 a ‘sampling effect’ due to larger species richness in late successional stages. However, for the
272 alpine vegetation of the Rutor glacier, the largest species richness is associated to the mid-
273 successional plots (see Ricotta et al. 2016, Table 2), while the index used for calculating

274 phylogenetic redundancy is generally independent of species richness (for a detailed discussion on
275 the sensitivity of R to species richness, see Appendix 3). Therefore, at least for our case study,
276 phylogenetic redundancy is not directly related to species richness.

277 From a more technical perspective, since the calculation of redundancy is done after
278 normalization of the tip-to-root length T of the phylogeny to the unit interval (see Eq. 1), we
279 recommend adapting the way the data are normalized to the question at hand. For example, if our
280 aim consists of comparing the redundancy of sites within a region, the normalization of T in the
281 range [0-1] should be done at the regional level. In this way, all sites are compared using the same
282 phylogeny, but different vectors of species abundances. On the other hand, if normalization is done
283 separately on each local phylogeny, the resulting redundancy values are not expressed in the same
284 units, thus hampering the comparison among data sets. From a biological viewpoint, if we imagine
285 that branch lengths are related to the number of features shared by two species (Faith 1992; Pavoine
286 et al. 2016), the way of normalizing the height of the phylogeny can be interpreted as a
287 normalization by the theoretical maximum number of features shared by two species in our
288 assemblage. This raises the more general question of developing methods for scaling branch lengths
289 to increase the ecological significance of phylogenies (Clarke and Warwick 1999; Blomberg et al.
290 2003; Cadotte et al. 2013; Letten and Cornwell 2015).

291 To conclude, in the last decades, ecologists have recognized that ecological similarities between
292 species are driven to some extent by patterns of common ancestry (Cadotte et al. 2010). As a
293 consequence, the concept of biodiversity has increasingly become a complex multifaceted notion
294 that incorporates functional and phylogenetic differences. Therefore, to be really useful in
295 ecological research, the properties of biodiversity indices must be unambiguously understandable
296 by ecologists. We believe that the proposed method provides a relatively simple way for
297 summarizing a key metric of community structure, such as phylogenetic redundancy, thus offering
298 new insights into the mechanisms that drive community composition through a variety of spatial
299 scales.

300

301 **Supporting Information**

302 **Appendix 1.** How to extend the calculation of phylogenetic redundancy to non-ultrametric trees.

303 **Appendix 2.** R script for the calculation of phylogenetic redundancy.

304 **Appendix 3.** On the sensitivity of phylogenetic redundancy to species richness.

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395 **Table 1.** Mean (SD) phylogenetic diversity and redundancy values for the three successional stages
396 identified on the foreland of the Rutor Glacier calculated both with the Shannon and the Simpson
397 diversity coefficients. Pairwise comparisons of index differences between the successional stages
398 were done with one-way permutational t-tests (9999 permutations). For each index, numbers
399 followed by the same letter do not differ significantly ($P > 0.05$; Holm-corrected values).

	Early-successional stage (17 plots)	Mid-successional stage (32 plots)	Late-successional stage (10 plots)
Shannon Diversity			
Phylogenetic diversity D_P	0.675 (0.064) a	0.673 (0.060) a	0.600 (0.069)
Present-day diversity D_K	1.030 (0.154) a	1.134 (0.094)	1.018 (0.136) a
Phylogenetic Redundancy R	0.336 (0.086)	0.405 (0.047) a	0.408 (0.050) a
Simpson Diversity			
Phylogenetic diversity D_P	0.706 (0.035) a	0.710 (0.025) a	0.671 (0.039)
Present-day diversity D_K	0.893 (0.039) a	0.918 (0.018)	0.891 (0.035) a
Phylogenetic Redundancy R	0.208 (0.040) a	0.226 (0.025) ab	0.247 (0.035) b

406 **Figure 1.** Schematic ultrametric phylogeny composed of five species with relative abundances $p_1, \dots,$
 407 p_5 . Let T be the age of the root node. We can delimit four non-overlapping evolutionary periods Δ_k
 408 by slicing the tree at each node so that there are no internal nodes between two slices. Starting from
 409 the root node, the node ages are labeled t_k ($k = 0, 1, \dots, K$) such that $t_0 = T$, $t_K = 0$ (the present-day
 410 time), and $\Delta_k = |t_k - t_{k-1}|$. Each evolutionary period contains M lineages with relative abundances
 411 π_{km} ($m = 1, 2, \dots, M$). The lineage abundances are obtained as the sum of the relative abundances of
 412 all descendent species in the phylogenetic tree. For example, $\pi_{11} = p_1 + p_2 + p_3$, $\pi_{12} = \pi_{23} = p_4 + p_5$,
 413 and $\pi_{K1} = p_1$.

418 **Figure 2.** Ultrametric phylogeny of the species used in this study, together with the plots of the
 419 mean Shannon and Simpson lineage diversities D_k of each successional stage vs. node age t_k . The
 420 root node ($T = 147$ million years Ma) and the nodes corresponding to the major changes in lineage
 421 diversity along the phylogenetic tree are shown with a black dot. For details, see the main text.
 422
 423
 424

Appendix 1. How to extend the calculation of phylogenetic redundancy to non-ultrametric trees.

Given an assemblage composed of N species, we will restrict our analysis to dichotomic diversity measures D expressed as the average rarity $R(i)$ of the species relative abundances $p_i = (p_1, p_2, \dots, p_N)$, such as the Shannon or the Simpson diversity:

$$D = \sum_i p_i \times R(p_i) \quad (1)$$

where $R(p_i)$ is a decreasing function of the relative abundances p_i (see Patil and Taillie 1982). Note that according to Eq. (2), the contribution of species i to overall diversity depends only on the numerical value of p_i . Patil and Taillie (1982) called this property ‘dichotomy’ because the contribution of i to D would be unchanged if the other species were grouped into a single complementary category.

Also, according to Patil and Taillie (1982), a diversity measure is said ‘schur-concave’ if D increases when abundance is transferred from a species to a another strictly less abundant species. This property is known in econometrics as Dalton’s (1920) principle of transfers. In its very essence, given two species i and j with relative abundances $p_i > p_j \geq 0$, diversity is increased if the quantity δ is transferred from p_i to p_j so long as the transfer does not reverse the ranking of the two abundances: $p_i - \delta > p_j + \delta$.

Chao et al. (2010) showed that for an ultrametric tree, the phylogenetic diversity D_P in Eq. (1) of the main text can be also calculated as follows. Consider a rooted ultrametric phylogeny with B branches and N species as tips with relative abundances p_i . Let L_j be the length (i.e. duration) of branch j ($j = 1, 2, \dots, B$), N_j be the set of species descending from branch j and p_{ij} be the relative abundance of the i -th descendant of branch j so that $a_j = \sum_{i \in N_j} p_{ij}$ is the relative abundance of all species descending from branch j . Then, phylogenetic diversity D_P can be obtained by calculating a diversity index of choice of the general form:

$$D_P = \sum_j \frac{L_j}{T} a_j \times R(a_j) \quad (2)$$

where T is the age of the root node. The so-obtained phylogenetic diversity D_P is identical to the phylogenetic diversity obtained with the procedure described in the main text, being at the same time formally similar to the weighted diversity measures proposed by Guiasu (1971) and Guiasu and Guiasu (2012). See also Chiu et al. (2014) and Pavoine (2009, 2016).

Phylogenetic redundancy can be then calculated as:

$$R = 1 - D_P / D \quad (3)$$

where D is the present-day diversity calculated from the N species relative abundances p_i . A necessary requirement to obtain a meaningful measure of phylogenetic redundancy in the range 0–1, is that $D_P \leq D$. That is, phylogenetic diversity D_P cannot be greater than the present-day diversity D , which is calculated assuming that the N species are maximally distinct from each other. From an evolutionary point of view, this is tantamount saying that D , is calculated from a star phylogeny in which all species had independent evolution.

Figure 1. (a) Hypothetical non-ultrametric phylogeny with three species with relative abundances p_1 , p_2 and p_3 and unequal distances t_i from tips to root. (b) Non-ultrametric phylogeny with the same set of species of (a) in which all species evolved independently from the root node while preserving the unequal distances t_i from tips to root. (c) Ultrametric phylogeny with the same set of species of (a) in which all species assumed the maximum distance t_{max} from tips to root.

More generally, for any tree, be it ultrametric or not, D_P can be calculated as (Chao et al. 2010):

$$D_P = \sum_j \frac{L_j}{\bar{T}} a_j \times R(a_j) \quad (4)$$

where $\bar{T} = \sum_j L_j a_j = \sum_j L_j \sum_{i \in N_j} p_{ij} = \sum_i p_i t_i$ and t_i is the distance from tip i to root on the phylogenetic tree.

We can compare the phylogenetic diversity D_P of any non ultrametric tree, such as that in Figure 1a, to the diversity assumed by the same set of species if all species evolved independently from the root node while preserving the unequal distances t_i from tips to root (Figure 1b):

$$D_{Star} = \frac{1}{\bar{T}} \sum_j L_j \sum_{i \in N_j} p_{ij} R(p_{ij}) \quad (5)$$

Equivalently, D_{Star} can be written as follows:

$$D_{Star} = \frac{1}{\bar{T}} \sum_i t_i p_i R(p_i)$$

Because the index of diversity $D = \sum_i p_i \times R(p_i)$ is schur-concave, and because it is dichotomic *sensu* Patil and Taillie (1982), then for any j we have:

$$\left(\sum_{i \in N_j} p_{ij} \right) \times R \left(\sum_{i \in N_j} p_{ij} \right) \leq \sum_{i \in N_j} p_{ij} R(p_{ij})$$

This yields $D_P \geq D_{Star}$ (6)

505 With ultrametric trees, the distances t_i from tips to root are constant ($t_i = T$) and hence
 506 $D_{Star} = \sum_i p_i R(p_i)$. D_{Star} is thus species diversity (D).

507
 508 With non-ultrametric trees, however, D_{Star} still contains phylogenetic information: the distances
 509 from tip to root (t_i) and their mean $\bar{T}$. As an alternative, to compare phylogenetic diversity with
 510 species diversity, we can consider the following formulas:
 511

$$512 \quad D_{\Pi} = \frac{1}{t_{\max}} \sum_j L_j a_j \times R(a_j) \quad (7)$$

513
 514 for phylogenetic diversity, where $t_{\max}$ is the maximum of t_i over i , and
 515

$$516 \quad D_{\Sigma} = \frac{1}{t_{\max}} \left[\sum_j \sum_{i \in N_j} L_j p_{ij} R(p_{ij}) + \sum_i (t_{\max} - t_i) p_i R(p_i) \right] \quad (8)$$

517
 518 for species diversity.
 519

520 D_{Σ} corresponds to the star phylogeny with equal, maximum distances from tips to roots (Figure
 521 1c). Because the index of diversity $D = \sum_i p_i \times R(p_i)$ is schur-concave, and because it is
 522 dichotomic *sensu* Patil and Taillie (1982)
 523

$$524 \quad \sum_j L_j a_j \times R(a_j) \leq \sum_j \sum_{i \in N_j} L_j p_{ij} R(p_{ij})$$

525
 526 This yields
 527

$$528 \quad D_{\Pi} \leq \frac{1}{t_{\max}} \left[\sum_j \sum_{i \in N_j} L_j p_{ij} R(p_{ij}) \right] \leq D_{\Sigma} \quad (9)$$

529
 530 Note that, if the phylogeny is ultrametric, $D_{Star} = D_{\Sigma}$ and $D_P = D_{\Pi}$.
 531
 532
 533

534 References

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 548 applications to rockfish (genus: *Sebastes*) in the Southern California Bight. *Ecology Letters* 12:
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 550

551 **Appendix 2.** R function ‘`phyloredundancy`’
552
553 This program is free software: you can redistribute it and/or modify it under the terms of the GNU General
554 Public License <http://www.gnu.org/licenses/>.
555
556 **Disclaimer:** users of this code are cautioned that, while due care has been taken and it is believed accurate,
557 it has not been rigorously tested and its use and results are solely the responsibilities of the user.
558
559 **Description:** the function ‘`phyloredundancy`’ calculates community-level phylogenetic redundancy taking into
560 account the branching pattern of the underlying phylogeny.
561
562 **Dependencies:** “`vegan`” and “`ape`” libraries
563
564 **Usage:** `phyloredundancy(community, tree, index=c("richness","shannon","simpson"))`
565
566 **Arguments**
567
568 ***community*:** a community data frame with plots as rows, species as columns, and species abundance as
569 entries.
570
571 ***tree*:** an object of class ‘`phylo`’. Only ultrametric trees are allowed. `Phyloredundancy` checks if the
572 phylogenetic tree is ultrametric with the function “`is.ultrametric`” of the “`ape`” library. If the tree is not
573 ultrametric, a warning message is displayed.
574
575 ***index*:** `Phyloredundancy` computes phylogenetic species richness (“`richness`”) or the phylogenetic
576 analogue of the Shannon or Simpson diversity (“`shannon`” or “`simpson`”, respectively); “`richness`” is
577 assumed as the default value.
578
579 **Outputs:**
580
581 An object of class “`data.frame`” is returned containing the following statistics:
582
583 ***D_K*:** the present-day diversity of all plots in the data frame.
584
585 ***D_P*:** the phylogenetic diversity D_P of all plots in the data frame obtained averaging the diversities D_K over the
586 corresponding K -th evolutionary periods after rescaling the tip-to-root length to unit (for details, see the main
587 text).
588
589 ***U*:** phylogenetic uniqueness calculated as the ratio between phylogenetic diversity and present-day diversity
590 D_P / D_K .
591
592 ***R*:** phylogenetic redundancy, calculated as $1-U$.
593
594
595 **Function Syntax**
596
597 `phyloredundancy<-function(community, tree, index= "richness")`
598 `{`
599 `require(vegan)`
600 `require(ape)`
601 `if (!inherits(community, "data.frame")){`
602 `if(!inherits(community, "matrix"))`
603 `stop("Object community must be of class data.frame or matrix")`
604 `}`
605 `if (any(community < 0))`
606 `stop("Negative value in community")`
607 `if (any(rowSums(community) < 1e-16))`
608 `stop("Empty plots in community")`
609 `if (!inherits(tree, "phylo"))`
610 `stop("tree must be of class phylo")`
611 `if (ncol(community)!=length(tree$tip.label))`
612 `stop("Species should be the same and ordered in the same way in the 'community' and 'tree' objects")`

```

613 else if (is.null(colnames(community)))
614   warning("Species should ordered in the same way in the 'community' and 'tree' objects")else
615   community <- community[, tree$tip.label]
616   if (is.ultrametric(tree, tol = 0.001)== FALSE)
617     warning("the phylogenetic tree is not ultrametric")
618   if(!index[1]%in%c("shannon", "simpson", "richness"))
619     stop("index must be shannon or simpson or richness")
620   dis_phylo<-cophenetic.phylo(tree)
621   clusters<-hclust(as.dist(dis_phylo))
622   groups<-cutree(clusters, h=clusters$height)
623   colnames(groups) <- paste("g", 1:ncol(groups), sep="")
624   groups_matrix<-data.frame(groups)
625   norm_dist<-c(0,clusters$height/max(clusters$height))
626   delta<-diff(norm_dist)
627   all_delta<-c(delta,0)
628   if (index %in% c("simpson", "shannon")){
629     index_base <- apply(community, 1, function(v) diversity(v, index=index))
630   funmerge <- function(w)
631     {
632       merged<- aggregate(t(community) ~ w, FUN=sum)
633       cleaned <- t(merged[,-1])
634       return(apply(cleaned, 1, function(v) diversity(v, index=index)))
635     }
636   index_trees <- apply(groups_matrix, 2, funmerge)
637   all_index_plot <- cbind.data.frame(index_base,index_trees)
638 }
639 if (index == "richness"){
640   index_base <- apply(community, 1, function(v) specnumber(v))
641   funmerge <- function(w)
642     {
643       merged<- aggregate(t(community) ~ w, FUN=sum)
644       cleaned <- t(merged[,-1])
645       return(apply(cleaned, 1, function(v) specnumber(v)))
646     }
647   index_trees <- apply(groups_matrix, 2, funmerge)
648   all_index_plot<-data.frame(index_base,index_trees)
649 }
650 all_branch_index<- t(t(all_index_plot) * all_delta)
651 mean_index_tree<-rowSums(all_branch_index)
652 phylo_uniq<-mean_index_tree/ index_base
653 phylo_redundancy<-1-phylo_uniq
654 outputs<-data.frame(index_base,mean_index_tree,phylo_uniq,phylo_redundancy)
655 colnames(outputs)<-c("Dk", "Dp", "Phylogenetic Uniqueness", "Phylogenetic Redundancy")
656 return(outputs)
657 }

```

658
659 ***Example with simulated data - requires the 'ape' and the 'TreeSim' libraries***

```

660
661 library(ape)
662 library(TreeSim)
663 phy <- sim.bd.taxa(n=10, 1, 1, 0)[[1]] #generates an ultrametric phylogenetic tree with 10 tips (assumed as
664 species) under a pure birth model
665 com_matrix<-t(data.frame(replicate(20, rpois(10, 1)))) #generates a community matrix with ten species and
666 20 plots. Strictly positive abundance values are randomly assigned.
667 colnames(com_matrix)<-phy$tip.label
668 phyloredundancy(com_matrix, phy, index="richness") #calculates the phylogenetic redundancy of the
669 simulated dataset.
670
671
672
673

```

Appendix 3. On the sensitivity of phylogenetic redundancy to species richness.

Imagine an assemblage composed of N species with species relative abundances $p_i = (p_1, p_2, \dots, p_N)$, and let R be a measure of community-level phylogenetic redundancy $R = 1 - U = 1 - D_p / D_K$, where U is phylogenetic uniqueness, D_p is phylogenetic diversity, and D_K is traditional present-day diversity defined as in the main text.

While all measures of present-day diversity are by definition sensitive to species richness, in most cases the ratio D_p / D_K removes this effect. Irrespective of the index of present-day diversity D_K and of the number of species N , redundancy attains its minimum value for a star phylogeny for which $D_p = D_K$ and hence $R = 0$. However, in some cases, the maximum value of redundancy may depend on species richness.

If present-day diversity is measured as species richness, such that $D_K = N$, phylogenetic diversity D_p becomes Faith's (1992) index after normalization of the tip-to-root length of the phylogeny to the unit interval $[0, 1]$. It follows that the average phylogenetic diversity per species D_p / D_K can range from $1/N$ (for the extreme case where a long branch of length $1 - \varepsilon$ with $\varepsilon \ll 1$ leads to a polytomy of N branches of length ε), to 1 (for a star phylogeny; see Figure 1).

Figure 1. (a) Hypothetical star phylogeny with three species S_1 , S_2 and S_3 . (b) Phylogeny with the same species richness of (a) in which a long branch of length $1 - \varepsilon$ (with $\varepsilon \ll 1$) leads to a polytomy of three branches of length ε . When species abundances are considered for the calculation of D_p and D_K , the phylogenetic redundancy of (b) tends to 0 for ε tending to 0.

Therefore, for $D_K = N$, $0 \leq R \leq (N - 1) / N$. In this case, to make phylogenetic redundancy independent of species richness, we may rescale R to the unit interval. Alternatively, when dealing with species presence/absence data, instead of species richness N , we can use the index $N - 1$ or $\log N$ for summarizing D_p and D_K (see Hill 1973 and Patil and Taillie 1982). With index $N - 1$, the extreme scenario where a long branch of length $1 - \varepsilon$ leads to a polytomy of N branches of length ε will lead to $D_p = (1 - \varepsilon) \times 0 + \varepsilon \times (N - 1)$ which tends to 0 when ε tends to 0. With $\log N$, it will lead to $D_p = (1 - \varepsilon) \times \log(1) + \varepsilon \times \log(N)$ which also tends to 0 when ε tends to 0.

708
 709 To the contrary, when species abundances are considered, like for the Shannon or the Simpson
 710 index, redundancy R becomes independent of species richness. For a given value of D_K , the
 711 extreme scenario where a long branch of length $1-\varepsilon$ leads to a polytomy of N branches of length ε
 712 (with $\varepsilon \ll 1$) will lead to $D_p = D_K \times \varepsilon + (1-\varepsilon) \times 0$ and thus to $D_p/D_K = 0$ and $R = 1$. Indeed with
 713 the Shannon or the Simpson index the diversity associated to the branch of length $1-\varepsilon$ is zero. In
 714 that case, when ε tends to 0, the range of variation for uniqueness and redundancy is $[0, 1]$.
 715
 716

717 **References**

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