Proceedings of the inaugural World Transport Convention
Beijing, China, June 4-6, 2017

Bus Route Design for Different Vehicle Models Considering Environmental Factors

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ABSTRACT

Bus transit plays an important role in commuters' daily life in urban areas. The energy efficiency of bus transit impacts not only on the passengers' travel cost but also the urban environment. The traditional transit vehicles, such as Compressed Natural Gas (CNG) and diesel buses, generate massive Green House Gas (GHG) and pollutants while operating in urban streets. In contrast, zero-emission buses, such as supercharge bus and electric bus, entail environmental friendliness but require huge initial investment for fleet purchase. Conventional bus route design largely ignored the environmental factors. Thus, the air pollutions may be enhanced by the flawed design of bus stop locations because of the frequent and inefficient deceleration and acceleration at stops. In this paper, a multi-period (peak hour and off-peak hour) continuum model will be built to optimize the design of a bus route for different vehicle models (i.e., supercharge bus, electric bus, CNG, and diesel bus). Environmental cost will be explicitly considered. A case study is conducted in Yaan City (China), where the 1st supercharge bus route is in operation. Comparing with current design, the optimal result indicates that the average stop spacing can be further reduced by up to 13%, and the peak hour headway should be further shorted, which results in less access time and waiting time. Furthermore, we examined our results against those using discrete model to verify accuracy. The results show that the outcomes of continuum and discrete model are in the neighborhood (with an error less than 3%).The proposed model and solution method are demonstrated for practical implementation in urban route design.

KEYWORDS: Public Transportation, Route design, Environment pollution, Continuum model, Discrete model

1 INTRODUCTION

The conventional buses, such as Compressed Natural Gas (CNG) and diesel buses, generate huge amount of emissions in terms of CO_2, VOC, and NO_x, which deteriorate the poor urban environment. Clean-energy buses, e.g., electric buses, have been regarded as a new solution to solve the smog problem. It provides a driving force for the national and local government to shift the transit vehicle mode. Many cities in China have spared no effort in developing a new transit networks with clean-energy bus. For instance, Shenzhen has a plan to replace all the city buses, i.e., 16,000in total, with electric buses by the end of 2018 (fastcompany, 2017). In the process of shifting to clean energy, it is crucial for the transit route design considering all technical and economical characteristics of different bus models. Traditional bus route design, however, largely ignored the environmental impacts, which may lead to flawed design with improper bus stop locations that aggravates the environmental pollution by frequent and inefficient deceleration and acceleration.

The theme of this study is related to an optimal design of a single route to yield the optimal stop locations as well as the service headway. In the optimization model, the objective function is to minimize the total system cost, which contains user cost, operator cost, and environmental cost. The model is based on continuum approximation and its accuracy is verified by a discrete approach. A case study is furnished on a bus route in Yaan City (China). The contribution of this study is twofold: First, environmental factors are explicitly considered in transit route design model for different vehicle models. Second, a comprehensive comparison among different vehicle types sheds insights for the choice decision regarding the selection of transit technologies. The remainder of this paper is organized as follows. Next section reviews the relevant literatures. The theoretical model is developed in Section 3, which includes variable definitions and formulations of continuum model and discrete model. In this section, a total cost minimization problem is introduced. Section 4 mainly illustrates the numerical application in Yaan City, China, with explications to the obtained results. Section 5 wraps up with the conclusions and some further extensions.

2 LITERATURE REVIEW

Traditional transit route design (TRD) problems can be mainly solved by 2 categories of methods: the continuum approximation (CA) approach and the discrete approach. The main idea of CA approach is to decide the optimal stop density functions, while discrete approach is aimed to determine the optimal set of stops for a given OD matrix and predicted travel times per link on the route(O.J. Ibarra-Rojas, 2015). Usually, the demand for continuum model is presented as a continuous density function instead of OD matrix (which is convertible via calculus method). CA method offers a more transparent and understandable description of interactions, but usually contains few practical considerations to assure the solvability (R.H Oldfield, 1989) in comparison with the latter. In this part, a review of both continuum models and the discrete models will be elaborated.

2.1 Continuum Approximation

With the advantages of good mathematical properties such as high-efficient computation, continuum approximation has been widely used to tackle TRD problems. Based on the idea that each point along on a route/corridor can be a candidate stop, a continuous stop density function is developed to determine the optimal stop locations (Newell, 1971, 1973, 1979; Hurdel, 1973; Wirashinghe, 1977, 1980,1981; Vaughan, 1986; Daganzo, 2010; Medina, 2012; Gu et al., 2016).Other design parameters include headway, fleet size, vehicle types, timetable, etc. (Byrne 1975; Wirashinghe, 1977, 1980; Sivakumaran et al., 2011). The pioneering research with the usage of CA was derived in the study of Newell (1971). In the work of Newell, a multi-period model has been built to minimize the user cost with a given bus size. After that, Wirashinghe et al. (1977) optimized the rail station spacing and service headway in peak hour to maximize user's benefit. Furthermore, in Wirashinghe's another work in 1980, stop density and service headway have been optimized simultaneously in a rectangular grid feeder bus system for many to one demand pattern to minimize the operator cost and user cost. Chang and Schonfield (1991) applied a multi-period continuum model into a various-demand case (fixed demand and cyclical demand) to minimize the total cost as well as to maximize welfare. The comparison result suggested that the headway of each period depends on the demand over day. In recent years, the continuum model has been represented into a more general and comprehensive way. Medina-Tapia et al. (2012) considered a many to many demand pattern, which varies with positions to determine the optimal stop density function and service headway. Stop capacity constraint and vehicle capacity constraint have been taken into account; a bi-directional stop density function and multi-period headways are obtained. Kim and Schonfeld (2014) integrated the conventional bus service with flexible bus service to provide a probabilistic optimization model. In this model, the effect of timed transfer was considered into the model, and optimal variables including: selected service type for particular region, vehicle size, headway, fleet and slacks time are found by using Generic Algorithm (GA). Mahour Rahimi (2017) used CA to seek the relationship between agency cost, demand and service characteristics (i.e., fleet size, vehicle miles travelled, vehicle hours travelled). In sum, most of the studies optimize the stop density and headway to minimize user cost or total cost (user cost plus operator cost). Among them, few studies have addressed the environmental issues. The only example is Amirgholy et al. (2017), who proposed a continuum model to optimize stop spacing, headway, and fare of transit system by minimizing the sum of user cost, operator cost, and emission cost.

2.2 Discrete Approach

Apart from continuum approximation, a lot of researches tackled the TRD problems based on the discrete approach. The most distinguished difference between the two approaches is that in discrete models, the optimal stops are selected among a given set of candidate stops instead of utilizing a stop density function. The first published work of Vuchic and Newell (1968) investigated a discrete model to determine the number of stations so as to minimize the travellers' total travel time in a studied area. On a basis of "many to one" demand pattern, analytical expression of stop locations is formulated and solved by dynamical programming. Furth and Rahabee (2000) aimed at maximizing ridership and increasing operating speed, thus a function objective was built to minimize the sum of passenger's net walking time, riding time and operating time. Chien and Qin (2004) developed an optimal spacing model to improve passenger's accessibility. A set of demand generation points was settled to find the optimal number of bus stops. Ceder et al. (2015) focused on the effect of uneven topography, applied a bi-level discrete model to a single bus route to optimize stop locations. The geographic factor (i.e., slope) impacts greatly on system's performance.

To the best of authors' knowledge, no previous study has studied the bus route design for clean-energy bus, which differs from that of conventional bus in terms of their service characteristics (e.g., technical performances, operational cost, and emission). This research explores the optimal headways for different periods as well as the optimal stop locations by using continuum approximating. A comparison between continuum model and discrete model is also conducted in the end.

3 METHODOLOGY

Considering a corridor of length L, the bus route runs in two directions (representing eastbound and westbound, respectively). The bus travelling in each direction of corridor stops at every stop. The eastbound/westbound bus stop spacing/density can be unequal to reflect the flexibility of locating curbside bus stops. There are two periods considered in this study: peak hour period and off-peak hour period, in which service headways are different accounting for the demand variation but are equal in both directions. The model parameters are summarized in Table A1 in Appendix A. To facilitate the modelling, a few assumptions are made as follows:

1. The user cost contains three parts of costs: access cost, waiting cost, and in-vehicle cost. For each cost item, the value of time for all passengers is assumed to be the same, regardless the citizen's status, income, etc.
2. Passengers choose the nearest stop to board or alight bus.
3. The idling time at each stop is supposed to increase linearly with the number of boarding and alighting passengers.
4. The operation mode of bus is considered to have three schemes: decelerate at constant deceleration rate, cruising, and accelerate at constant acceleration rate.
5. The average cruising speed varies in different periods, but remains constant within a given period.

In this section, we will firstly present the objective function TC, which is the sum of daily costs incurred by users C_{mu}, operator C_{mo} and the environmental cost C_{mP}. Thus, the formulation of TC is:

$$TC = C_{mu} + C_{mo} + C_{mP} \tag{1}$$

The user cost is defined as the cost that passenger have to access or egress the stop (C_{ma}), wait at the stop (C_{mw}), and stay in vehicle (C_{mv}).

$$C_{mu} = C_{ma} + C_{mw} + C_{mv} \tag{2}$$

The operator cost covers the amortized cost for line (C_{ml}), the amortized and operation cost for stops (C_{ms}), the vehicle kilometers travelled (VKT) related cost (e.g., fuel and maintenance costs) C_{mvk}, and vehicle hours travelled (VHT) related cost (e.g., wages and vehicle depreciation costs) C_{mvh}. The operator cost is thus given by:

$$C_{mo} = C_{ml} + C_{ms} + C_{mvk} + C_{mvh} \tag{3}$$

3.1 Continuum Approximation Models

3.1.1 User Cost in CA models

In the terms of user cost, we will present the access cost C_{ma}, waiting cost C_{mw}, and in-vehicle cost C_{mv}one by one.

The daily access cost can be determined by demand density function $bo_{ri}(x)$and $ali_{ri}(x)$, stop density $\delta_{mr}(x)$ at location *x*, and passenger walking speed v_a. The maximum walking distance is $1/2\delta_{mr}(x)$, as passengers choose the nearest stop to board/alight. Thus, the expected walking distance is $1/4\delta_{mr}(x)$ assuming the demand follows a locally uniform distribution. So the total time spending in accessing the stop at location *x* is $\frac{1}{4 \cdot \delta_{mr}(x) \cdot v_a}$ The formulation of C_{ma} is:

$$C_{ma} = \sum_{r=1}^{2} \sum_{i=1}^{2} \int_0^L \frac{\left(ali_{ri}(x) + bo_{ri}(x)\right) \cdot T_i \cdot \theta_a}{4 \cdot \delta_{mr}(x) \cdot v_a} dx \tag{4}$$

where the product of $\left(ali_{ri}(x) + bo_{ri}(x)\right)$ with $\frac{1}{4 \cdot \delta_{mr}(x) \cdot v_a}$ means the total passengers' time spent in accessing the stop at position *x,* in direction *r,* and during *i* period. The estimated value of access cost at position *x* along the corridor over day is obtained by multiplying $\frac{\left(ali_{ri}(x) + bo_{ri}(x)\right)}{4 \cdot \delta_{mr}(x) \cdot v_a}$ with the value of walking access time (θ_a) and period duration T_i.

The passengers' waiting cost can be formulated as the product of boarding passenger and the average waiting time that equals to half of headway, thus:

$$C_{mw} = \sum_{r=1}^{2} \sum_{i=1}^{2} \int_0^L bo_{ri}(x) \cdot \frac{h_i}{2} \cdot T_i \cdot \theta_w \, dx \tag{5}$$

where, θ_w presents the value of waiting cost.

Passengers' in-vehicle cost contains three parts: cost associated with vehicle travel time (bus is at cruising speed v_{mi}); cost associated with the extra time lost in deceleration and acceleration at each stops; and cost associated with the idling time for passenger to board and alight the bus. The formulation of in-vehicle time is as follows:

$$C_{mv} = \sum_{r=1}^{2} \sum_{i=1}^{2} \int_0^L P_{ri}(x) \cdot T_i \cdot \left(\frac{1}{v_{mi}} + t_d \cdot \delta_{mr}(x) \right) \cdot \theta_v \, dx \tag{6}$$

where, θ_v is the value of in-vehicle time. The product of $P_{ri}(x)$ with $\frac{1}{v_{mi}}$ represents the sum of time experienced by on-board passengers at each point *x* along the corridor. t_d is the total time lost at stops, including the time for acceleration and deceleration at stops, t_{mi}^l, and bus dwell time t_{mri}^d for passengers to board/alight.

$$t_d = t_{mi}^l + t_{mri}^d \tag{7}$$

The extra time lost for deceleration and acceleration t_{mi}^{l} is:

$$t_{mi}^{l} = \frac{v_{mi}}{2} \cdot \left(\frac{1}{a_b} + \frac{1}{d_b}\right) \tag{8}$$

t_{mri}^{d} is related to the dwell time at one stop in direction *r*, period *i*, for *m*^th^ transit mode, which has two parts: one is the time for closing and opening doors, and the other is the boarding and alighting time. We suppose that only one door can be used for boarding and one or more doors for alighting, thus passenger can board and alight simultaneously. So the dwell time depends on the process that takes longer time.

$$t_{mri}^{d} = t_{m0} + max\left(bo_{ri}(x) \cdot h_i \cdot d_r(x) \cdot t_m^b;\ ali_{ri}(x) \cdot h_i \cdot d_r(x) \cdot t_m^a\right)$$
(9)

where, t_{m0} is the dead time for opening and closing the bus doors, t_m^b and t_m^a are the average boarding and alighting time per passenger, respectively.

3.1.2 Operator Cost in CA models

Operator cost in this part is classified into two categories: infrastructure costs (construction of the corridor and stops), which is denoted as $C_{ml}$and C_{ms}; and operational cost, which contains VKT related cost and VHT related cost.

The amortized line cost per day is the multiplication of per day line amortized cost π_{ml} and the total length of corridor:

$$C_{ml} = \pi_{ml} \cdot 2L \tag{10}$$

The cost related to each stop consists of the amortized stop cost per day and a cost for maintaining a stop. π_{ms} represents the unit amortized cost per stop per day, θ_0 is the operation and maintenance cost per stop hour, and *T* presents the span of service, resulting in the following equation:

$$C_{ms} = \sum_{r=1}^{2} \int_{x=0}^{L} \delta_{mr}(x) \cdot (\pi_{ms} + \theta_0 T)\, dx \tag{11}$$

The VKT related cost (e.g., fuel and maintenance) is proportional to total driving distance and the daily bus flow. It can be given by the product of the VKT related cost per vehicle kilometer, denoted as π_{mvk} (\$/vehicle-km), daily bus flow, $\sum_{i=1}^{2} \frac{T_i}{h_i}$ (vehicle/day), and round-trip length, $2L$. Thus:

$$C_{mvk} = \sum_{i=1}^{2} \frac{\pi_{mvk} \cdot 2L \cdot T_i}{h_i} \tag{12}$$

Similarly, the VHT related cost (e.g., wages and vehicle depreciation) is proportional to total driving time and the daily bus flow. It can be presented by multiplying the VHT related cost per vehicle hour, π_{mvh} (\$/vehicle-h), with total travel time**,** which contains the total time lost between stops while bus is at cruising speed v_{mi}, and the total time lost at stops, t_d.

$$C_{mvh} = \sum_{r=1}^{2} \sum_{i=1}^{2} \int_{x=0}^{L} \left(\frac{1}{v_{mi}} + t_d \cdot \delta_{mr}(x)\right) \cdot \pi_{mvh} \cdot \frac{T_i}{h_i} dx \tag{13}$$

3.1.3 Environmental Cost in CA models

The environmental cost can be determined by the multiplication of daily associated emission P_{mrn} (ton) for *n* pollutants (i.e., *n*=1, NO_x; *n*=2, CO; *n*=3, HC), the unit vehicle-related damage cost of pollutant θ_{pn} (\$/ton), and the bus flow $(f_i \cdot T_i)$ over day. f_i is the service frequency $(f_i = \frac{1}{h_i})$. Thus, the total pollutant cost C_{mP} incurred by *n* pollutants is given as:

$$C_{mP} = \sum_{n=1}^{3} \sum_{r=1}^{2} \sum_{i=1}^{2} \int_0^L P_{mrn} \cdot \theta_{pn} \cdot f_i \cdot T_i \, dx \tag{14}$$

P_{mrn} can be divided into two parts: one is the bus emission at stops P_{mrn}^1 , which presents the emission generated by bus deceleration, idling, and bus acceleration at stop. The other is the emission between stops P_{mrn}^2 while bus is travelling at cruising speed. Thus,

$$P_{mrn} = P_{mrn}^1 + P_{mrn}^2 \tag{15}$$

As regards the emission exhausted at stops, it can be determined by summing the emission rates (ton/h) of driving cycles (i.e., decelerate at constant deceleration rate, idling at stops, and accelerate at constant acceleration rate) multiplied by associated time in each driving cycle. Thus, the emission at all stops P_{mrn}^1 is given by:

$$P_{mrn}^1 = \left(e_n(d_m) \cdot \frac{v_{mi}}{d_m} + e_n(I) \cdot t_d + e_n(a_m) \cdot \frac{v_{mi}}{a_m} \right) \cdot \delta_{mr}(x) \, dx \tag{16}$$

where $e_n(d_m)$, $e_n(I)$ and $e_n(a_m)$ present the emission rates of pollutant *n* at deceleration rate d_m, idling, and acceleration rate a_m, respectively.

Similarly, we can formulate the emission between stops, P_{mrn}^2, by multiplying emission rate of pollutant *n* at cruising speed, $e_n(v_{mi})$, with the inverse of the cruising speed of the bus, $\frac{1}{v_{mi}}$.

$$P_{mrn}^2 = e_n(v_{mi}) \cdot \frac{1}{v_{mi}} \tag{17}$$

3.1.4 System Optimization

The following expression allows for the optimization of the multi-period model, where the total cost is a function of the stop density and headway.

Objective function:

$$Min \int_{x=0}^{L} \left\{ \begin{array}{c} \sum_{r=1}^{2} \sum_{i=1}^{2} \frac{(ali_{ri}(x) + bo_{ri}(x)) \cdot T_i \cdot \theta_a}{4 \cdot \delta_{mr}(x) \cdot v_a} + \\ \sum_{r=1}^{2} \sum_{i=1}^{2} bo_{ri}(x) \cdot \frac{h_i}{2} \cdot T_i \cdot \theta_w + \\ \sum_{r=1}^{2} \sum_{i=1}^{2} P_{ri}(x) \cdot T_i \cdot \left(\frac{1}{v_{mi}} + t_d \cdot \delta_{mr}(x) \right) \cdot \theta_v \end{array} \right\}$$

$$+ \left\{ \begin{array}{c} \pi_{ml} \cdot 2L + \sum_{r=1}^{2} \delta_{mr}(x) \cdot (\pi_{ms} + \theta_0 T) + \\ \sum_{i=1}^{2} \frac{\pi_{mvk} \cdot 2L \cdot T_i}{h_i} + \sum_{r=1}^{2} \sum_{i=1}^{2} \frac{\pi_{mvh}}{h_i} \cdot \left(\frac{1}{v_{mi}} + t_d \cdot \delta_{mr}(x) \right) \cdot T_i \end{array} \right\}$$

$$+ \quad \left\{ \sum_{n=1}^{3} \sum_{r=1}^{2} \sum_{i=1}^{m} P_{mrn} \cdot \theta_{pn} \cdot f_i \cdot T_i \right\} dx$$

Subjects to:

$$\begin{cases} max\, Q \le cap_{bus} \cdot f_i \\ \left(ali_{ri}(x) + bo_{ri}(x) \right) \cdot h_i \le cap_{stop} \cdot \delta_{mr}(x) \\ F \ge \sum_{i=1}^{2} T_i \cdot f_i \\ h_i \ge 0, \delta_{mr}(x) \ge 0 \end{cases} \tag{18}$$

The first constraint of the model requires that the hourly bus capacity should satisfy the total hourly passengers' demand. In other words, the bus capacity should be sufficient to load passengers. cap_{bus} represents the capacity of bus in unit of passenger/vehicle.

The second constraint is similar to the former, indicates that the stop capacity cap_{stop} should meet passengers' demand.

The third constraint is the fleet size constraint, which illustrates that the operational fleet size must superior than the actual number of operational buses in the corridor.

In addition, the optimal results of headway and stop density should be positive.

3.1.5 Model Optimization

The objective function has two decision variables (stop density $\delta_{mr}(x)$ and headway h_i), multiple periods (peak hour and off-peak hour) and two directions (eastbound and westbound), which increase the complexity of problems. To propose a solution, two alternative procedures are suggested: firstly, we can obtain the optimal function of stop density by solving the first order condition $\delta_{mr}^*(x) = f(x, h_i)$. We can get the initial function of stop density $\delta_{mr}(x)$ by using a given initial value of headways in different periods,. It should be mentioned that the optimal solution expression of headway also contains stop density , in other words, $h_i^* = f(x, \delta_{mr}(x))$. Thus the second step is to replace $\delta_{mr}(x)$with $\delta_{mr}^*(x)$ we have got in the first step, and we can obtain the first value of h_i^*. The following steps are repeatable; we can finally find the headway and stop density reach convergence. The convergent values of stop density and headway are the optimal solutions.

The expression of optimal stop density in first order condition is:

$$\delta_{mr}^*(x) = \sqrt{\frac{\sum_{i=1}^{2}\left(ali_{ri}(x)+bo_{ri}(x)\right)\cdot T_i\cdot\theta_a}{4v_a\left\{\pi_{ms}+\theta_0 T+\sum_{i=1}^{2} t_d\cdot\left(P_{ri}(x)\cdot T_i\cdot\theta_v+\frac{\pi_{mvh}}{h_i}\cdot T_i\right)+\sum_{n=1}^{3}\sum_{i=1}^{2}\theta_{pn}\cdot f_i\cdot T_i\cdot\left(e_n(d_m)\cdot\frac{v_{mi}}{d_m}+e_n(I)\cdot t_d+e_n(a_m)\cdot\frac{v_{mi}}{a_m}\right)\right\}}} \tag{19}$$

The solution of the optimal headway is:

$$h_i^* = \sqrt{\frac{\pi_{mvk}\cdot 2L\cdot T_i+\sum_{r=1}^{2}\int_0^L\left(\frac{1}{v_{mi}}+\left(t_{mi}^l+t_{m0}\right)\cdot\delta_{mr}(x)\right)\cdot T_i\cdot\pi_{mvh}\,dx+\sum_{n=1}^{3}\sum_{i=1}^{2}\int_0^L P_{mrn}\cdot\theta_{pn}\cdot T_i\,dx}{\sum_{r=1}^{2}\int_0^L bo_{ri}(x)\cdot\frac{T_i}{2}\cdot\theta_w+P_{ri}(x)\cdot T_i\cdot g_{ri}(x)\cdot\theta_v\,dx}} \tag{20}$$

where, $g_{ri} = max\left(bo_{ri}(x)\cdot t_m^b;\ ali_{ri}(x)\cdot t_m^a\right)$. (21)

It should be mentioned that the expressions of the optimal stop density and headway for clean-energy bus don't have the environmental part, and the optimal stop density is:

$$\delta_{mr}^*(x) = \sqrt{\frac{\sum_{i=1}^{2}\left(ali_{ri}(x)+bo_{ri}(x)\right)\cdot T_i\cdot\theta_a}{4v_a\left\{\pi_{ms}+\theta_0 T+\sum_{i=1}^{2} t_d\cdot\left(P_{ri}(x)\cdot T_i\cdot\theta_v+\frac{\pi_{mvh}}{h_i}\cdot T_i\right)\right\}}} \tag{22}$$

The optimal headway is:

$$h_i^* = \sqrt{\frac{\pi_{mvk}\cdot 2L\cdot T_i+\sum_{r=1}^{2}\int_0^L\left(\frac{1}{v_{mi}}+\left(t_{mi}^l+t_{m0}\right)\cdot\delta_{mr}(x)\right)\cdot T_i\cdot\pi_{mvh}\,dx}{\sum_{r=1}^{2}\int_0^L bo_{ri}(x)\cdot\frac{T_i}{2}\cdot\theta_w+P_{ri}(x)\cdot T_i\cdot g_{ri}(x)\cdot\theta_v\,dx}} \tag{23}$$

3.2 Discrete Metrics

To obtain more accurate results, the above results form CA models are converted into discrete metrics. In the first place, we discretize the bi-directional stop density function by locating stops when the integral of its left boundary and right boundary is 1. For instance, the location of the first stop $x_1$is:

$$\int_0^{x_1} \delta_{mr}(x)\, dx = 1 \tag{24}$$

The others are calculated similarly. After we get all the stop locations, it is easy to find the stop shed lines between each stop's coverage market. The stop shed lines present the attractiveness of a stop. To simplify, we suppose that the center line of each link is the right/ left boundary of a stop, that is:

$$L_s = \frac{x_{s-1}+x_s}{2} \tag{25}$$

$$R_s = \frac{x_s+x_{s+1}}{2} \tag{26}$$

Based on the assumption that passengers chose the nearest stop to board/ alight, passengers will board/alight at stop *s* only when they are within $[L_s, R_s]$. Therefore, the volume of demand at stop *s* is the integral of the continuous density function of boarding demand between stop shed lines:

$$b_s = \int_{L_s}^{R_s} bo_{ri}(x)\, dx \tag{27}$$

The alighting demand at stop *s* is:

$$a_s = \int_{L_s}^{R_s} ali_{ri}(x)\, dx \tag{28}$$

The on-board flow passing stop *s* to stop *s*+1 is:

$$P_{ri} = \sum_{s=1}^{s} b_s - \sum_{s=1}^{s} a_s \tag{29}$$

3.2.1 User cost in discrete metrics

After converting the bus route design into discrete metrics, user cost is rewritten by:

$$\widetilde{C_{mu}} = \widetilde{C_{ma}} + \widetilde{C_{mw}} + \widetilde{C_{mv}} \tag{30}$$

where, the discrete metrics access cost $\widetilde{C_{ma}}$, waiting cost $\widetilde{C_{mw}}$, and in-vehicle cost $\widetilde{C_{mv}}$ are given as follows:

$$\widetilde{C_{ma}} = \sum_{s=1}^{s} \int_{L_s}^{R_s} \left(bo_{ri}(x) + ali_{ri}(x)\right) \cdot \frac{\left|\frac{R_s+L_s}{2} - x\right|}{v_a} \cdot \theta_a \cdot T_i dx \tag{31}$$

In the equation above, $\frac{\left|\frac{R_s+L_s}{2} - x\right|}{v_a}$ presents the average accessing time per passenger walking from position *x* to the nearest stop *s*.

The passengers' waiting cost is the sum of waiting cost at stops, for example, at stop *s*, the passengers' waiting cost is presented as the integral of boarding demand over the stop's coverage market multiplied by average waiting time $\frac{h_i}{2}$, and the value of waiting cost θ_w:

$$\widetilde{C_{mw}} = \sum_{s=1}^{S} \int_{L_s}^{R_s} bo_{ri}(x) \cdot \frac{h_i}{2} \cdot \theta_w \cdot T_i\ dx \tag{32}$$

Passengers' in-vehicle cost is given by summing the total time cost at stops (including bus's acceleration, deceleration, and boarding/alighting time lost at stops) for on-board passengers depart from stop s to stop $(s+1)$ and on the corridor while bus is travelling at cruising speed.

$$\widetilde{C_{mv}} = \sum_{s=1}^{s-1} P_{ri} \cdot \left(\frac{R_{s+1}+L_{s+1}-R_s-L_s}{2v_{mi}} + \left(t_{mi}^{l} + h_i \cdot max(b_s(x) \cdot t_m^b;\ a_s(x) \cdot t_m^a) \right) \right) \cdot \theta_v \cdot T_i \tag{33}$$

3.2.2 Operator cost in discrete metrics

The per-day operator cost in discrete metrics is the sum of line cost $\widetilde{C_{ml}}$, stop cost $\widetilde{C_{ms}}$, the VKT related cost $\widetilde{C_{mvk}}$, and the VHT related cost $\widetilde{C_{mvh}}$, and is reformulated as:

$$\widetilde{C_{mo}} = \widetilde{C_{ml}} + \widetilde{C_{ms}} + \widetilde{C_{mvk}} + \widetilde{C_{mvh}} \tag{34}$$

The expressions for each cost item are presented as follows, with N_r representing the number of stops in direction r:

$$\widetilde{C_{ml}} = \pi_{ml} \cdot 2L \tag{35}$$

The daily stop cost is the sum of stop amortization cost and stop maintenance cost θ_0, multiplying with N_r in each direction.

$$\widetilde{C_{ms}} = \sum_{r=1}^{2} (\pi_{ms} + \theta_0 T) \cdot N_r \tag{36}$$

The VKT related cost is the same as that in CA model:

$$\widetilde{C_{mvk}} = \sum_{i=1}^{2} \frac{\pi_{mvk} \cdot 2L \cdot T_i}{h_i} \tag{37}$$

While the total VHT related cost in discrete model is the sum of VHT related cost at stops and on the corridor.

$$\widetilde{C_{mvh}} = \sum_{s=1}^{S} \sum_{r=1}^{2} \sum_{i=1}^{2} \frac{T_i \cdot \left(\frac{L}{v_{mi}} + \left(t_{mi}^{l} + h_i \cdot max\left(b_{s+1}(x) \cdot t_m^b;\ a_{s+1}(x) \cdot t_m^a\right) \right) \cdot N_r \right)}{h_i} \cdot \pi_{mvh} \tag{38}$$

3.2.3 Emission cost in discrete metrics

In terms of emission cost, it can be identified as the emission cost at stops and between stops over day. The emission cost at stops/ between stops can be formulated as the product of the associated emission during different driving cycles (i.e., deceleration, acceleration, idling, and cruising) with emission cost, θ_{pn} (\$/ton), and bus flow $(f_i \cdot T_i)$. The emission emitted at stops is:

$$\widetilde{P_{mrn}^1} = \sum_{n=1}^{3} \sum_{r=1}^{2} \sum_{i=1}^{2} \left(e_n(d_m) \cdot \frac{v_{mi}}{d_m} + e_n(I) \cdot t_d + e_n(a_m) \cdot \frac{v_{mi}}{a_m} \right) \cdot N_r \tag{39}$$

The associated emission between stops is:

$$\widetilde{P_{mrn}^2} = \sum_{n=1}^{3} \sum_{r=1}^{2} \sum_{i=1}^{2} e_n(v_{mi}) \cdot \frac{L}{v_{mi}} \tag{40}$$

Thus, the entire mass of pollutants exhausted per bus in a round trip is:

$$\widetilde{P_{mrn}} = \widetilde{P_{mrn}^1} + \widetilde{P_{mrn}^2} \tag{41}$$

The total emission cost along the corridor over day is given as following:

$$\widetilde{C_{mP}} = \widetilde{P_{mrn}} \cdot \theta_{pn} \cdot f_i \cdot T_i \tag{42}$$

4 NUMERICAL APPLICATION

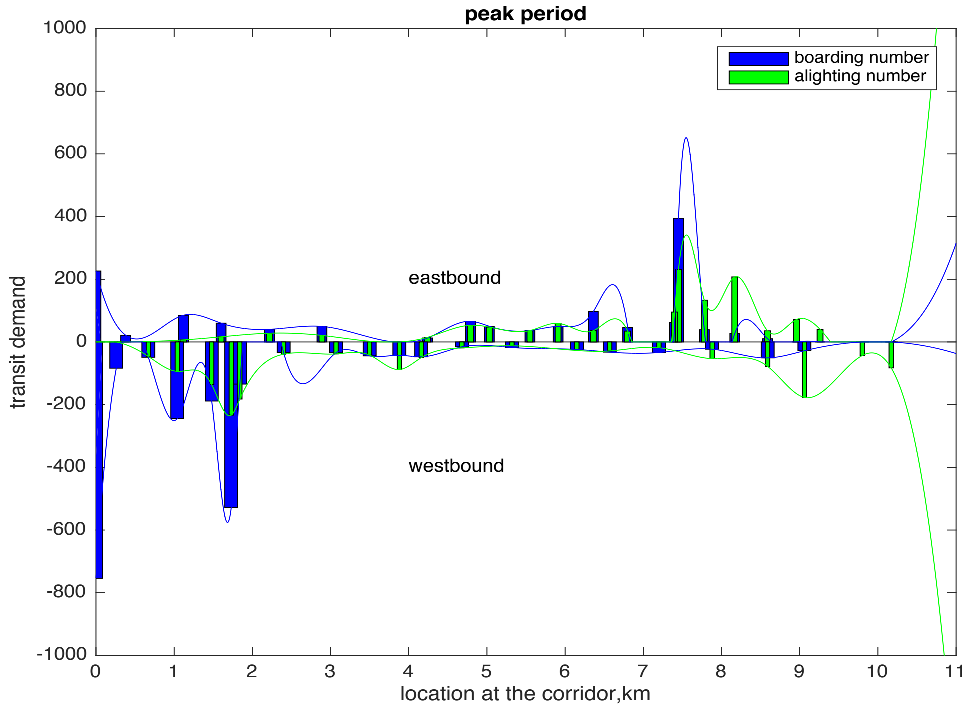
Both the continuum and discrete models have been successfully applied to the 7th bus route in Yaan (City), Chengdu. The studied route is approximately 11 kilometers. The bus average speed on this route is 40 km/h in peak hour time and 50 km/h in off-peak hour time, given by Transportation Agency.

In terms of vehicle types, two types of conventional bus are considered: 12m CNG bus and 12m diesel bus. The emission rates of CO, HC, NO_x at different driving cycles are based on the practical research data in previous studies (Qu. 2015)and (H.Y.Tong et al. 2000) as shown in Table 2. The pollutants' social costs of CO, HC, NO_x are 2000$/ton, 3000$/ton, 10000$/ton, respectively, using median value from the survey of Su Song (2017) at Chengdu. All the optimization procedures are programmed in MATLAB. The boarding/alighting data was given by Transportation Agency so as all the operator costs (i.e., maintenance cost and construction cost), which are shown in details in Table A2 in Appendix A.

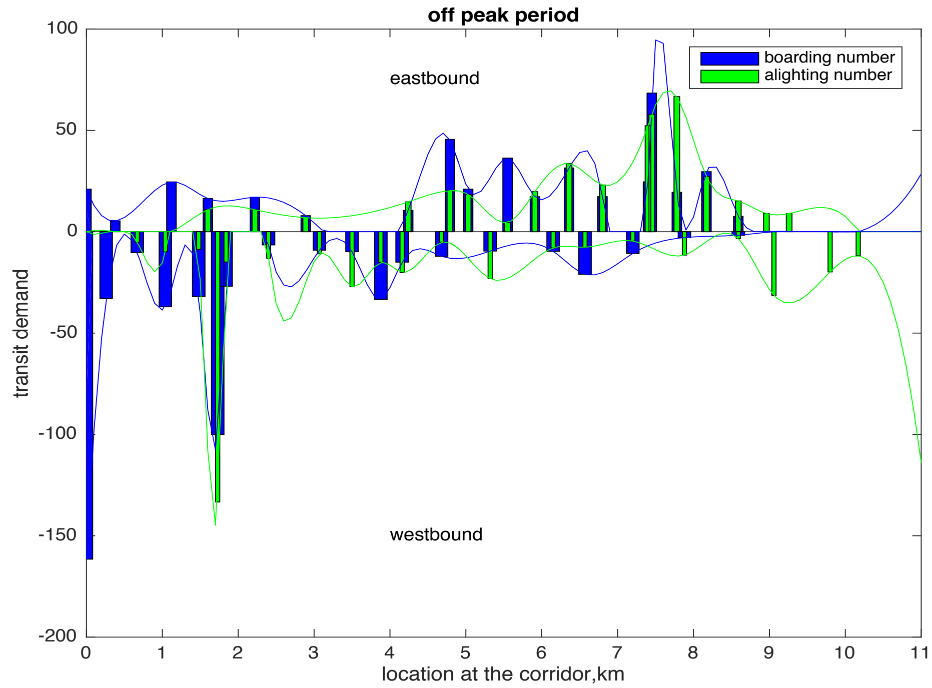
Table 1 Emission Rate of Pollutants at Driving Cycles

12m CNG bus				
Pollutant	Idling	Acceleration	Deceleration	Constant velocity
NO_X(g/s)	0.00357	0.01518	0.00708	0.01152
HC(g/s)	0.00118	0.00342	0.00214	0.00267
CO (g/s)	0.02114	0.04725	0.03027	0.03628
12m Diesel Bus				
NO_X(g/s)	0.00404	0.01394	0.01165	0.01062
HC (g/s)	0.00115	0.00236	0.00196	0.00136
CO (g/s)	0.0075	0.01263	0.01092	0.00816

Using the on-and-off counts data of a bus route in Yaan, we transformed the demand data into continuous functions $bo_{ri}(x)$ and $ali_{ri}(x)$ by using curve fitting tool in Matlab. The results are presented in Figure 1.



(a) Peak period demand data



(b) Off-peak period demand data

Figure 1 Empirical demands and demand functions

The service headways are segmented into 4 time periods: 7:00-17:00, the current headway is 7 min; 17:00-19:00, the service headway is 12min; 19:00-20:00, the value of headway is 15min; 20:00-21:00, the service headway is 20min. The values of walking time, riding time and waiting time are set at 4.09$/h, 1.64$/h, 2.73$/h (Medina-Tapia, et al. 2012), respectively. For passenger's access and egress speed, a standard value 3.6km/h was used. A constant lost for opening doors at each stop is 2s.

In this part, four major transit modes are taken into consideration, including two kinds of clean-energy bus (i.e., Supercharge and Lithium-ion battery electricity bus) and two traditional bus models (i.e., CNG bus and Diesel bus). The technology parameters including average acceleration rate and deceleration rate differ from each other: for supercharge bus, the average acceleration rate/deceleration rate is 0.5 m/s^2 and -2 m/s^2; for CNG bus is 1 $m/s^2$and -1 m/s^2 accordingly (Qu. 2015); for Lithium-ion bus is 0.71 $m/s^2$and -0.68 m/s^2, (Gao et al 2011); and for diesel bus is 0.42 $m/s^2$and -0.4 m/s^2, measured by Liu et al (2010).

4.1 Optimization Result Analysis

The results of the discretization of the bi-directional stop density function for four transit modes are shown below in Table 2. As we can see, the optimal average spacing for CNG and diesel bus is equal: 478m for the eastbound; and 458m for the westbound, while the average spacing for clean-energy bus is larger.

Table 2 Number of Stops and Average Spacing

Vehicle type	Number of stops (eb)	Number of stops (wb)	Average spacing (eb)	Average spacing (wb)
Supercharge	22	23	500m	478m
CNG	23	24	478m	458m
Lithium-ion	21	23	524m	478m
Diesel	23	24	478m	458m

Here we take the example of Supercharge bus route design in eastbound as presented in Figure 2. The cycles on the stop density curves represent the optimal location of stops resulting from the discretization, and the dash lines present the coverage area of each stop's market. The integral of each coverage area is equal to one, which means a stop is contained in this area.

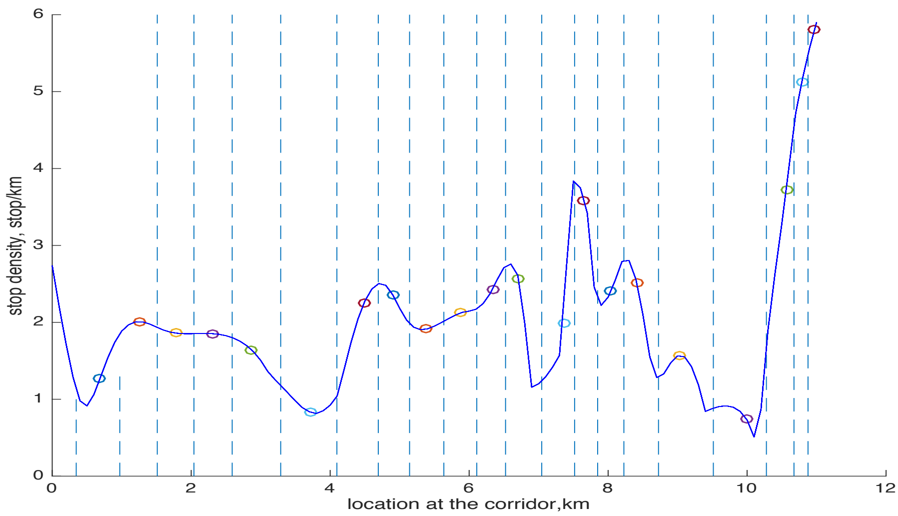


Figure 2 : Supercharge Bus Optimal Stop Location in Eastbound

The existing route has 21 stops in the east bound and 22 stops in the west bound, where the average spacing is 523m/stop in the east and 500m/stops in the west, accordingly. The distance between stops is decreased by up to 13% comparing with the current stop design.

Table 3 presents the optimal headway/frequency in peak/off-peak hour as well as the observed headway/frequency of four transit modes in optimal/current system. It can be found that the optimal system has a shorter headway in peak hour period than the given headway, which lead to less waiting time for passengers. On the other side, in the off-peak hour, the optimal headway increased, resulting in a decrease in operator cost. The peak-hour headway is further decreased by 60%, which can better feed the peak-hour demand.

Table 3 Optimal Headway/ Observed Headway

Periods	Observed headway (min)	Observed frequency	Optimal headway (min)	Optimal frequency
Supercharge				
Morning peak	7	8 veh/h	4.70	12 veh/h
Evening peak	12	5 veh/h	4.70	12 veh/h
Off-peak	9.64 (weighted)	6 veh/h	9.71	6 veh/h
CNG				
Morning peak	7	8 veh/h	5.14	11 veh/h
Evening peak	12	5 veh/h	5.14	11 veh/h
Off-peak	9.64 (weighted)	6 veh/h	10.81	5 veh/h
Lithium-ion				
Morning peak	7	8 veh/h	5.57	10 veh/h
Evening peak	12	5 veh/h	5.57	10 veh/h
Off-peak	9.64 (weighted)	6 veh/h	11.68	5 veh/h
Diesel Bus				
Morning peak	7	8 veh/h	6.04	9 veh/h
Evening peak	12	5 veh/h	6.04	9 veh/h
Off-peak	9.64 (weighted)	6 veh/h	12.89	4 veh/h

4.2 Accuracy of CA models

To verify accuracy, we examined our results based on continuum model against that using discrete model. The results are shown in Table 4. It is seen that the outcomes of continuum and discrete models are in the neighborhood (with an error less than 3%). The total cost results for four different transit modes indicate that supercharge bus is the most cost-effective transit mode. The cost of diesel bus is the highest, due to the high fuel cost and maintenance cost.

Table 4 Cost Comparison between Continuum and Discrete Model

Supercharge		
	Continuum model	Discrete model
User cost	\$4697.4	\$4579.6
Operator cost	\$8107.8	\$7942
Total cost	\$12805	\$12522
Difference%	2.2%	
CNG		
User cost	\$4762.4	\$4629.7
Operator cost	\$8078.8	\$7922.7
Pollutant cost	\$72.9	\$69.6
Total cost	\$12914	\$12622
Difference%	2.3%	
Lithium-ion		
User cost	\$5011.9	\$4907.7
Operator cost	\$8472.9	\$8304.2
Total cost	\$13212	\$13485
Difference%	2.1%	
Diesel bus		
User cost	\$5049.1	\$4913.2
Operator cost	\$8374.6	\$8216
Pollutant cost	\$67.2	\$65.2

Total cost	\$13491	\$13194
Difference%	2.2%	

5 CONCLUSION AND FUTURE EXTENSION

In this study, a multi-period continuum approximation is developed to identify the optimal stop location and service headway in a bi-directional corridor. This model is highly efficient in solving transit design optimization problems. The optimal bi-directional stop densities and optimal multi-period headways can be obtained simultaneously. The numerical application in Yaan City 7th avenue indicates that the average stop spacing is reduced by up to 13%, compared with the current design. In addition, the optimal peak-hour headway is further shorted than the current headway by 13 % in morning peak hours, 60% in the evening peak hours. The optimal headway in off-peak hours becomes longer. These changes lead to a decrease in passenger's average waiting time in peak hours as well as saving in operator cost during off-peak hours. From the perspective of operator, the proposed model, after discretizing the optimal stop density function, represents a more proper configuration than current stop design. The comparison between the continuum approximation results and the discrete metrics shows good accuracy. Furthermore, the comparison among the total costs of different transit modes indicates that the supercharge bus is the most financial friendly mode while being environmental friendly. As for future extension, this model can be represented into a more comprehensive way. For example, the congestion effect can be taken into account, and more detailed driving cycles can be added into emission model.

6 ACKNOWLEDGEMENTS

This study is funded by the National Nature Science Foundation of China (NSFC 51608455). The author thanks the ShuTong Transportation Agency for providing the useful data for case study.

APPENDIX A

Table A1 Parameters Definitions and Values used in Calculation

Parameters	Units	Definitions	Values
$bo_{ri}(x)$	Pass/km-hour	Number of passengers who board at x, in direction r, period i	-
$ali_{ri}(x)$	Pass/km-hour	Number of passengers who alight at x, in direction r, period i	-
b_s	Pass/hour	Number of passengers who board at stop s	-
a_s	Pass/hour	Number of passengers who alight at stop s	-
$P_{ri}(x)$	Pass/hour	Passenger load for buses at point x in direction r of corridor in period i	-
$C_{ma}/\widetilde{C_{ma}}$	\$/day	Access cost per day for m^{th} mode	-
$C_{mw}/\widetilde{C_{mw}}$	\$/day	Waiting cost per day for m^{th} mode	-
$C_{mv}/\widetilde{C_{mv}}$	\$/day	In-vehicle cost per day for m^{th} mode	-
$C_{ml}/\widetilde{C_{ml}}$	\$/day	Line cost per day for m^{th} mode	-
$C_{ms}/\widetilde{C_{ms}}$	\$/day	Stop cost per day for m^{th} mode	-
$C_{mvk}/\widetilde{C_{mvk}}$	\$/day	Vehicle km cost per day for m^{th} mode	-
$C_{mvh}/\widetilde{C_{mvh}}$	\$/day	Vehicle hour cost per day for m^{th} mode	-

$C_{mu}/\widetilde{C_{mu}}$	\$/day	User cost per day for m^{th} mode	-
$C_{mo}/\widetilde{C_{mo}}$	\$/day	Operation cost per day for m^{th} mode	-
$C_{mP}/\widetilde{C_{mP}}$	\$/day	Pollutant cost per day for m^{th} mode	-
θ_a	\$/hour	Value of access cost	4.09
θ_w	\$/hour	Value of waiting cost	2.73
θ_i	\$/hour	Value of in-vehicle cost	1.64
θ_{pn}	\$/ton	Average damage cost for pollutant *n*	-
θ_0	\$/stop hour	Maintenance cost per stop hour	1
π_{ml}	\$/km/day	Unit line amortized cost for m^{th} mode	-
π_{ms}	\$/station/day	Unit stop amortized cost for m^{th} mode	-
π_{mvk}	\$/veh/km	Cost of service per vehicle kilometre for m^{th} mode	-
π_{mvh}	\$/veh/hour	Cost of service per vehicle hour for m^{th} mode	-
$\delta_{mr}(x)$	Stop/km	Stop density function for m^{th} mode	-
h_i	hour	Headway in period *i*	-
f_i	Vehicle/hour	Frequency in period *i*	-
T_i	hour	Duration of period	[4, 10]
T_w	hour	Average waiting time per passenger	
v_a	Km/h	Velocity of average walking speed	
v_{mi}	Km/h	Cruising speed for m^{th} mode in *i* period	
a_m	Km/h^2	Average acceleration rate of bus for m^{th} mode	
d_m	Km/h^2	Average deceleration of bus for m^{th} mode	
t_m^b	Hour per passenger	Average boarding time per passenger for m^{th} mode	-
t_m^a	Hour per passenger	Average alighting time per passenger for m^{th} mode	-
t_d	hour	Total delay at stop	
t_{mi}^l	hour	Extra time lost of acceleration and deceleration for m^{th} mode in period *i*	-
t_{mri}^d	hour	Dwell time of direction *r* in period *i* for m^{th} mode	-
$P_{mrn}/\widetilde{P_{mrn}}$	ton/hour	Total emission of pollutant *n* in direction *r*	-
$P_{mrn}^1/\widetilde{P_{mrn}^1}$	ton/hour	Emission of pollutant *n* at stops in direction *r*	-
$P_{mrn}^2/\widetilde{P_{mrn}^2}$	ton/hour	Emission of pollutant *n* between stops in direction *r*	-
$e_n(d_m)$	ton/hour	Emission rate of pollutant *n* at deceleration	
$e_n(I)$	ton/hour	Emission rate of pollutant *n* at idling	
$e_n(a_m)$	ton/hour	Emission rate of pollutant *n* at acceleration	
$e_n(v_{mi})$	ton/hour	Emission rate of pollutant *n* at cruising speed	
F	Vehicle	Fleet size	-

L	Km	Corridor length	11

Table A2 Cost Parameters of Four Transit Modes

Supercharge Cost Parameters		
Parameters	Value	Comment
Infrastructure Costs		
Infrastructure line cost ($/km)	$1,805,440	Derived fromSivakumaran et al (2014)
$I-L infrastructure cost ($/km/h)	$10	Derived from Sivakumaran et al (2014).
$I-S infrastructure station cost ($/station/h)	$0.47	Derived from Gu et al (2016), with additional construction cost 2,590,000 yuan for charging facilities.
Operating Costs (Distance)		
Maintenance cost per veh-km	$0.012	Shu Tong Transportation Agency
Fuel cost per km ($/km)	$0.088	Electricity price 2017, 0.6yuan/km
$v, Cost per veh-km ($/veh-km)	$0.1	
Operating Costs (Time)		
Employees per vehicle	1.5	
Average wage ($/h)	$5	Yaan City average wage standard
Labour cost per hour	$7.5	
Vehicle cost ($)	$257,353	
Vehicle lifespan (years)	8	
Depreciation per hr ($/hr)	$6.3	Assumed straight-line depreciation, work 14 hr per day
$M Cost per veh-hr ($/veh/hr)	$13.8	
CNG Cost Parameters		
Parameters	Value	Comment
Infrastructure Costs		
Infrastructure line cost ($/km)	$1,805,440	Derived from Sivakumaran et al (2014)
$I-L infrastructure cost ($/km/h)	$10	Derived from Sivakumaran et al (2014)
$I-S infrastructure station cost ($/station/h)	$0.35	Derived from Gu et al (2016)
Operating Costs (Distance)		
Maintenance cost per veh-km	$0.02	Shu Tong Transportation Agency
Fuel cost per km ($/km)	$0.238	CNG price 2018
$v, Cost per veh-km ($/veh-km)	$0.258	
Operating Costs (Time)		
Employees per vehicle	2	
Average wage ($/hr)	$5	Yaan City average wage standard
Labor cost per hour	$10	
Vehicle cost ($)	$73,529	
Vehicle lifespan (years)	8	
Depreciation per hr ($/hr)	$1.8	Assumed straight-line depreciation, work 14 hr per day
$M Cost per veh-hr ($/veh/hr)	$11.8	
Lithium-ion battery Pure Electricity Bus Cost Parameters		
Parameters	Value	Comment
Infrastructure Costs		

Infrastructure line cost ($/km)	$1,805,440	Derived from Sivakumaran et al (2014)
$I-L infrastructure cost ($/km/h)	$10	Derived from Sivakumaran et al (2014).
$I-S infrastructure station cost ($/station/h)	$0.58	Derived from Gu et al (2016), with an additional cost 5,000,000yuan for supplement equipment
Operating Costs (Distance)		
Maintenance cost per veh-km	$0.242	Shu Tong Transportation Agency
Fuel cost per km ($/km)	$0.088	Electricity price 2017
$v, Cost per veh-km ($/veh-km)	$0.33	
Operating Costs (Time)		
Employees per vehicle	1.7	
Average wage ($/hr)	$5	Yaan City average wage standard
Labour cost per hour	$10.5	
Vehicle cost ($)	$235,294	
Vehicle lifespan (years)	8	
Depreciation per hr ($/hr)	$5.76	Assumed straight-line depreciation, work 14 hr per day
$m Cost per veh-hr ($/veh/hr)	$16.26	
Diesel Bus Cost Parameters		
Parameters	Value	Comment
Infrastructure Costs		
Infrastructure line cost ($/km)	$1,805,440	Derived from Sivakumaran et al (2014)
$I-L infrastructure cost ($/km/h)	$10	Derived from Sivakumaran et al (2014).
$I-S infrastructure station cost ($/station/h)	$0.35	Derived from Gu et al (2016)
Operating Costs (Distance)		
Maintenance cost per veh-km	$0.025	Shu Tong Transportation Agency
Fuel cost per km ($/km)	$0.512	Diesel price 2017, 3,48yuan/km
$v, Cost per veh-km ($/veh-km)	$0.537	
Operating Costs (Time)		
Employees per vehicle	2	
Average wage ($/hr)	$5	Yaan City average wage standard
Labor cost per hour	$10	
Vehicle cost ($)	$73,529	
Vehicle lifespan (years)	8	
Depreciation per hr ($/hr)	$1.8	Assumed straight-line depreciation, work 14 hr per day
$M Cost per veh-hr ($/veh/hr)	$11.8	

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