

Many-Body Physics with Individually-Controlled Rydberg Atoms

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Over the last decade, systems of individually-controlled neutral atoms, interacting with each other when excited to Rydberg states, have emerged as a promising platform for quantum simulation of many-body problems, in particular spin systems. Here, we review the techniques underlying quantum gas microscopes and arrays of optical tweezers used in these experiments, explain how the different types of interactions between Rydberg atoms allow a natural mapping onto various quantum spin models, and describe recent results that were obtained with this platform to study quantum many-body physics.

I. MANY-BODY PHYSICS WITH SYNTHETIC MATTER

Many-body physics is the field that studies the behavior of ensembles of interacting quantum particles. This is a broad area encompassing almost all condensed matter physics, but also nuclear and high-energy physics. Despite the immense successes obtained over the last decades, many phenomena observed experimentally still do not have a fully satisfactory explanation. At the origin of the difficulty lies the exponential scaling of the size of the Hilbert space with the number of interacting particles. In practice, the best known *ab-initio* methods allow calculating the evolution of less than 50 particles. To investigate relevant questions involving a much larger number of particles (after all even 1 mg of usual matter contains already 10^{18} atoms!), one must rely on approximations, and the art of solving the many-body problem largely relies on mastering them. However, using approximations is not always possible and it may be hard to assess their range of validity. One approach to move forward was suggested by Richard Feynman [1] and consists in building a synthetic quantum system in the lab, implementing a model of interest for which no other way to solve it is known. The model may be an approximate description of a real material, but it can also be a purely abstract one. In this case, its implementation leads to the construction of an artificial many-body system, which becomes an object of study in its own. One appealing feature of this approach is the ability to vary the parameters of the model in ranges inaccessible otherwise, thus providing a way to better understand their respective influence. For example, if one is interested in the influence of interatomic interactions on the phase of a given system, synthetic systems become interesting as they allow varying their strength in a way which is usually impossible in real materials. The approach introduced by Feynman is usually referred to as quantum simulation [2, 3], but it can be viewed more generally as exploring many-body physics with synthetic systems: in the same way chemists design new materials exhibiting interesting properties (such as magnetism, superconductivity...), physicists assemble artificial systems and study their properties, with the hope to observe new phenomena.

For a long time, this idea remained theoretical as the experimental control over quantum objects was not advanced enough. The situation changed radically in the last 20 years with the development of experimental techniques allowing to control the quantum state of individual quantum objects, be they atoms, molecules, ions, photons, or even artificial atoms such as quantum dots, superconducting circuits or excitons in semi-conductors, to name a few [3]. For all these platforms, physicists designed sets of tools allowing the control of individual “atoms”, as well as the ability to tune their interactions. This led to the idea of programmable quantum simulation where all the parameters of the Hamiltonian one wants to implement are tunable. But this synthetic systems can also be viewed as machines able to prepare quantum states useful for many applications. For example, they can generate large entangled states, whose correlations are useful to beat the standard quantum limit, hence leading to clocks or sensors with enhanced precision [4]. In the long-term, they could lead to quantum computers, with each “atom” carrying a quantum bit [5, 6]. Interestingly, machines able to implement spin models could be useful to answer computationally hard problems well beyond physics, such as combinatorial optimization problems (one prominent example being the traveling salesman’s problem). Many of these optimization problems can be recast as Ising models [7], that most quantum simulators implement naturally. By varying the parameters on the experiment, one could drive the system into a state encoding the solution of the problem.

Among all the platforms being developed (many of them have been reviewed recently [8–12]), this article will focus on ensembles of individual atoms trapped in optical lattices or in arrays of microscopic dipole traps separated by a few micrometers. In this platform, the atoms are almost fully controllable by optical addressing techniques. To make them interact at distances larger than a micrometer, they are excited to Rydberg states, i.e. states with large principal quantum numbers n [13, 14]. When in this state, they feature two important properties. First, their lifetime, scaling as n^3 , is much longer than for low lying transitions (typically in the $100 \mu\text{s}$ range for $n \approx 50$). Second, they exhibit large dipole moments between states n and $n - 1$ with opposite parity, scaling as n^2 . This leads to large interaction strengths V , corresponding to frequencies $V/h \gtrsim 1 \text{ MHz}$ for $n \approx 50$ at distances around $5 \mu\text{m}$. As we will see in this review, these ensembles of interacting atoms naturally implement spin models, one of the simplest (and probably most thoroughly studied) many-body systems.

After introducing the concept of Rydberg blockade and the techniques used to prepare and manipulate arrays of single atoms, we describe the various types of interactions at play between Rydberg atoms. We then review quantum simulation experiments dealing with the Ising and XY spin models, and conclude by discussing the perspectives opened by the recent developments of the field.

II. RYDBERG BLOCKADE

The study of Rydberg atoms played an important role in the early days of atomic physics and in the development of quantum mechanics. A second ‘golden age’ of Rydberg physics started when tunable lasers became available in the 1970’s, where their strong coupling to electromagnetic fields made Rydberg atoms an ideal test-bed for understanding atom-light interactions, culminating with the birth of cavity quantum electrodynamics [15]. At this stage, interactions between Rydberg atoms, although observed as early as 1981 in dense Rydberg gases [16], did not play a crucial role. This changed at the end of the 1990’s, when progress in laser cooling of atoms allowed for the realization of *frozen* Rydberg gases [17, 18], in which thermal motion is negligible over the timescales where interactions take place. Soon after, it was proposed that the strong interactions between Rydberg atoms could be harnessed to implement fast and robust quantum gates between neutral atoms [19, 20]. The key ingredient for this implementation is the so-called Rydberg blockade (see Box 1), where the interaction prevents the simultaneous Rydberg excitation of two nearby atoms. This allows for conditional logic, as the excitation of a second atom is governed by the excitation of a first one [21].

However, at the time, the control of neutral atoms at the individual level was still in its infancy [22], and only a few groups took up the challenge to demonstrate the Rydberg blockade between individually controlled atoms. This was finally achieved in 2009 [23–26]. In the meantime, many groups had observed clear effects of the Rydberg blockade in large ensembles of atoms without individual control (see [27] and references therein). One soon realized that the theoretical description of these systems naturally mapped onto that of the quantum Ising model [28–32], one of the simplest models used to describe quantum magnetism. This suggested that systems of neutral atoms in the Rydberg blockade regime could be used for quantum simulation, provided individual control of large number of atoms would be available.

In parallel, the progress in the manipulation and detection of individual neutral atoms has made tremendous progress, either by using quantum gas microscopes [33, 34] or by creating arrays of optical tweezers [35–38]. Combined with Rydberg excitation to induce controllable interactions between the atoms, this provides an almost ideal platform to realize quantum spin models, as we will see below.

Box 1 | The Rydberg blockade

The strong interactions between atoms excited to Rydberg state can be exploited to suppress the simultaneous excitation of two atoms and to generate entangled states, in a regime called *Rydberg blockade*. Consider a resonant laser field coherently coupling the ground state $|g\rangle$ and a given Rydberg state $|r\rangle$, with a Rabi frequency Ω (Figure B1.a). In the case of two atoms separated by a distance R (Figure B1.b), the doubly excited state $|rr\rangle$ is shifted in energy by the quantity C_6/R^6 due to the van der Waals interaction (all the other pair states have an energy nearly independent of R). We assume that the blockade condition $\hbar\Omega \ll C_6/R^6$ is fulfilled, i.e. $R \ll R_b$ where the *blockade radius* is defined by $R_b = (C_6/\hbar\Omega)^{1/6}$. Then, starting from the ground state $|gg\rangle$, the system evolves to the collective state $|\psi_+\rangle = (|gr\rangle + |rg\rangle)/\sqrt{2}$ with a coupling $\sqrt{2}\Omega$. The coupling to $|rr\rangle$ is now non-resonant and thus suppressed. This leads to a collective Rabi oscillation at the frequency $\sqrt{2}\Omega$ between $|gg\rangle$ and the entangled state $|\psi_+\rangle$.

The above considerations can be extended to an ensemble of N atoms all included within a blockade volume. In this case, at most one Rydberg excitation is possible, leading to collective Rabi oscillations with an enhanced frequency $\sqrt{N}\Omega$ between the collective ground state $|g \dots g\rangle$ and the entangled state $\sum_i |g \dots g r_i g \dots g\rangle / \sqrt{N}$ where the Rydberg excitation is shared among all the atoms. In the case of a system whose size is larger than the blockade radius (Figure B1.c), several Rydberg atoms can be excited, but their positions will be strongly correlated due to the blockade constraint, giving rise to a complex many-body dynamics.

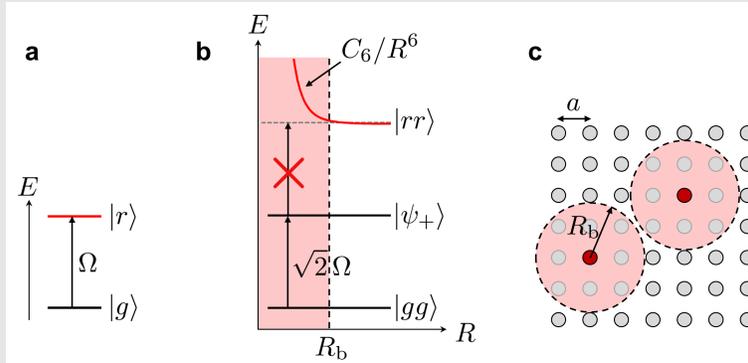


Figure B1 | The Rydberg blockade. **a:** The ground and Rydberg states $|g\rangle$ and $|r\rangle$ are coupled by a resonant laser with Rabi frequency Ω . **b:** For two atoms separated by a distance $R < R_b$, the collective ground state $|gg\rangle$ is coupled only to $|\psi_+\rangle = (|gr\rangle + |rg\rangle)/\sqrt{2}$, but not to $|rr\rangle$, which is shifted out of resonance by the van der Waals interaction. **c:** In a large ensemble of atoms, e.g. a regular array with spacing a , an atom excited in $|r\rangle$ (red dot) prevents the excitation of all the atoms contained in a sphere of radius R_b .

III. ARRAYS OF INDIVIDUAL ATOMS

The first experimental platform that has allowed the control of ordered assemblies of neutral atoms at the single-particle level became available in 2010 and is the *quantum gas microscope* [39] (Figure 1a). This “top-down” approach relies on the loading of a two-dimensional ultracold atom cloud, typically a Bose-Einstein condensate —although fermions can also be used— into an optical lattice, *i.e.* the periodic optical potential obtained by interfering several laser beams. Atoms can tunnel between neighboring sites of the lattice, and when the on-site contact interaction between the atoms overcomes the kinetic energy given by the tunneling rate, the system undergoes a superfluid to Mott insulator transition [40]. Deep in the Mott phase, the system is characterized by a fixed number of atoms per site, which can be exactly one for an appropriate choice of parameters. In order to obtain single-site resolution when imaging the atomic fluorescence, a high numerical aperture microscope objective is required, as two neighboring sites are separated by typically 500 nm [33, 34]. In this way, one realizes two-dimensional square arrays of up to a few hundred single atoms, with filling fractions that can exceed 95%. Individual control of the atoms can be achieved by applying local light shifts tailored with a spatial light modulator (SLM) such as a Digital Micro-mirror Device [41].

More recently, a novel, “bottom-up” platform has emerged based on arrays of optical tweezers (Figure 1b). Trapping of a single laser-cooled atom in a tightly focused dipole trap, or optical tweezers, was demonstrated as early as 2001 [22]. An optical dipole trap with a tight focus of about $1\ \mu\text{m}$ is immersed in a magneto-optical trap (MOT). In the course of its random motion, an atom from the MOT cloud enters the optical tweezers and gets trapped there. Since it is still under the illumination of the near-resonant MOT beams, it continuously scatters fluorescence light, that can be collected on a sensitive camera, thus signaling the presence of an atom in the tweezers. If now a second atom enters the trap, very fast light-assisted collisions give rise to the almost immediate loss of both atoms, and the tweezers is empty again. Therefore the number of atoms in the microtrap is either zero or one: this is a *single-atom source*, albeit non deterministic since one cannot predict when the trap is occupied. As the same random event (namely the random entering of an atom in the trap) induces the transitions from one to zero atom and from zero to one atom, the occupation probability is $p \sim 1/2$.

The next step is to produce arrays of microtraps. One method relies on a spatial light modulator [35, 42], which imprints an appropriate phase pattern on the trapping beam before focusing, thus allowing the realization of almost arbitrary arrays of traps in the focal plane of the objective. Other methods use arrays of microlenses [43, 44] or interference techniques [45]. However, for a long time, the stochastic loading of microtraps limited the use of this platform to just a few atoms, since the probability of having all N traps simultaneously filled decreases as $1/2^N$. Careful engineering of the light-assisted collisions in order to lose just one atom of the pair [46, 47] was shown to enhance p to values up to ~ 0.9 , but the probability p^N of an N -trap array to be defect-free still decreases very fast with N . This problem was circumvented simultaneously in 2016 by three groups. The idea is to start from a large array with $2N$ traps, load it randomly with $\sim N$ atoms, take an image of this configuration, and finally actively sort the atoms into an ordered, target configuration [48]. This was achieved by two methods. In the first one, loaded tweezers are dynamically moved using acousto-optic deflectors to assemble one-dimensional chains [38] (Figure 1c) as well as two [49] and three-dimensional [36] arrays by slowly varying the phase pattern of the SLM creating the array. In the second method, atoms are moved one at a time using a moving optical tweezers to catch and release atoms within a fixed two-dimensional array produced by a SLM [37] or microlenses [50]. More recently, the latter technique was extended to the assembling of three-dimensional arrays [51]. The assembling approach offers a fast repetition rate of the experiment (a few per

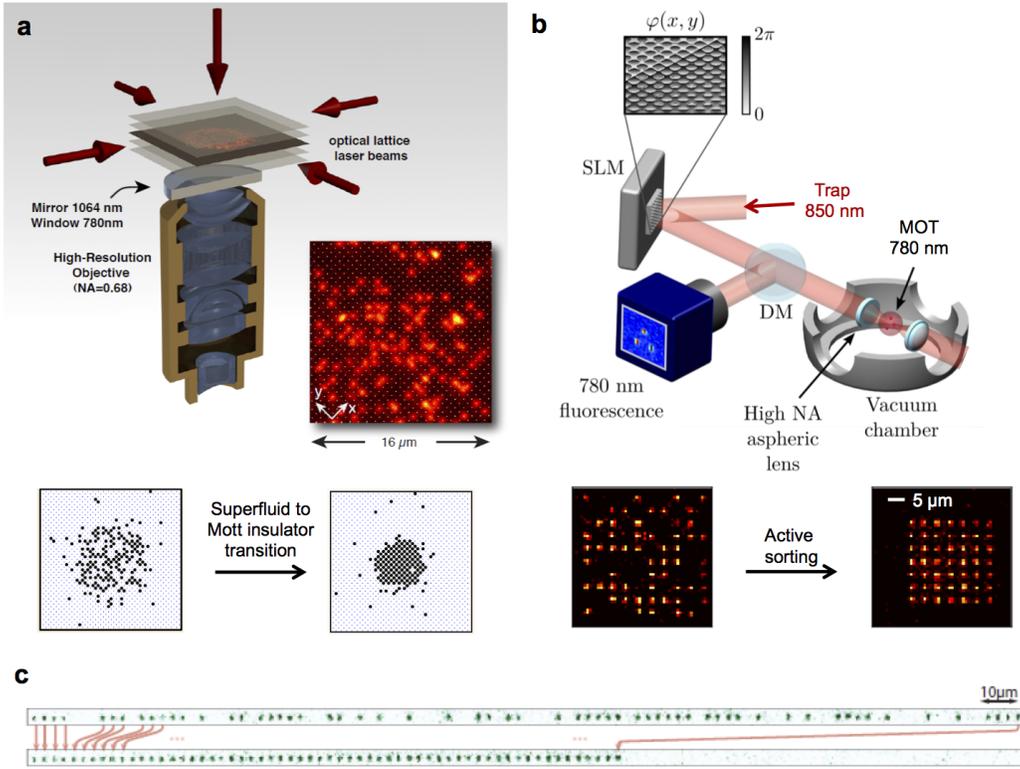


FIG. 1: **Experimental platforms for realizing arrays of individually controlled neutral atoms.** **a:** In a quantum gas microscope, a high-numerical-aperture objective is used to observe the fluorescence of ultracold atoms trapped in an optical lattice obtained by interfering several laser beams. To achieve a filling of exactly one atom per site, one drives the superfluid to Mott-insulator transition [34]. **b:** In the tweezers array platform, a spatial-light modulator (SLM) imprints an appropriate phase on a trapping beam prior to focusing with a high-numerical-aperture lens, resulting in arrays of traps with almost arbitrary geometries [35]. Single, laser-cooled atoms are loaded in the optical tweezers from a magneto-optical trap, resulting in a random loading array at half filling, which can be actively reordered into a target array using a moving optical tweezers [37]. Bottom: single shot fluorescence image of an array of traps before and after assembly. **c:** Alternatively, in one dimension, the tweezers can be generated using an acousto-optic deflector fed with multiple radio-frequency tones, which allows rearranging the atoms in a single step [38].

second), filling fractions in excess of 98% even in large arrays, and a great flexibility in geometry; the atom number reached so far is around one hundred atoms. This sorting of atoms has also been applied in two and three-dimensions in optical lattices with large spacing between sites [52, 53].

IV. INTERACTIONS BETWEEN RYDBERG ATOMS AND MAPPING ONTO SPIN SYSTEMS

The large electric dipole moment of atoms excited to Rydberg states leads to strong dipole-dipole interactions between them [54–56], even for atoms separated by micrometer-large distances as in the lattices and arrays described above. Two types of interaction naturally occur between two Rydberg atoms (see Box 2). In the context of quantum simulation, this leads to a mapping onto different spin models.

Let us consider first the case where the two atoms are placed in the *same* Rydberg state. There, the dipole-dipole interaction leads to the van der Waals interaction, which induces an energy shift of the pair state $|rr\rangle$ scaling as C_6/R^6 . This shift occurs only when the two atoms are both excited to the Rydberg state. If we now map the ground and Rydberg state $|g\rangle$ and $|r\rangle$ of each atoms onto a spin 1/2 following $|\downarrow\rangle = |g\rangle$ and $|\uparrow\rangle = |r\rangle$ the Hamiltonian of an ensemble of atoms driven by a coherent laser (Rabi frequency Ω , frequency detuning δ) is [28]

$$H = \frac{\hbar\Omega}{2} \sum_i \sigma_x^i - \hbar\delta \sum_i n_i + \sum_{i<j} V_{ij} n_i n_j, \text{ with } V_{ij} = \frac{C_6}{R_{ij}^6}. \quad (1)$$

Here n_i is the operator counting the number of Rydberg excitation at site i , and σ_x is the usual Pauli matrix. As n_i is related to the σ_z Pauli matrix by $n_i = (\sigma_z^i + 1)/2$, Eq.(1) has the form of the quantum Ising model, with a transverse field $B_\perp \propto \Omega$, a

longitudinal field $B_{||} \propto -\delta$ and Ising couplings J_{ij} decaying as $1/R_{ij}^6$, with the distance. In practice, for the alkali atoms used so far, the laser excitation leading to the Rabi frequency Ω is often achieved on a two-photon transition, with one of the laser in the infra-red and the other one in the blue, far-detuned with respect to an intermediate state in order to avoid spontaneous emission [57, 58]. In this way, the excitation is coherent to a good approximation. The Hamiltonian of Eq.(1) assumes that the excitation laser covers uniformly the atomic array, but owing to the single-site addressability, the detunings and Rabi frequency can be made site dependent by adding local laser control [59]. Finally, it is also possible to realize the quantum Ising model by using a technique called Rydberg dressing and encoding the two spin states in ground-state long-lived levels [60]. In this case, the couplings J_{ij} have a soft-core spatial dependence (see Box 2).

We now consider the second case where the atoms are prepared in two *different* Rydberg states that are dipole-coupled, such as $|nS\rangle$ and $|nP\rangle$, separated by a transition frequency typically in the 10 GHz range. There, the dipole-dipole interaction gives rise to a coherent exchange of the internal states of the atoms and the interaction potential scales like C_3/R^3 . The mapping onto a spin-1/2 model is then $|\downarrow\rangle = |nS\rangle$ and $|\uparrow\rangle = |nP\rangle$. Microwave radiation can be used to manipulate the spin and thus acts as an external magnetic field. The Hamiltonian for a system of atoms then reads:

$$H = \frac{\hbar\Omega_{\mu w}}{2} \sum_i \sigma_x^i - \frac{\hbar\delta_{\mu w}}{2} \sum_i \sigma_z^i + \sum_{i \neq j} \frac{C_3}{R_{ij}^3} \left(\sigma_+^i \sigma_-^j + \sigma_-^i \sigma_+^j \right), \quad (2)$$

which is the XY spin Hamiltonian with transverse and longitudinal fields given by the Rabi frequency $\Omega_{\mu w}$ and the detuning $\delta_{\mu w}$ of the microwave field.

Both the Ising and the XY Hamiltonians have been extensively studied in the last 60 years in various contexts, such as magnetism and excitation transport. However many important open questions remain the subject of active research, such as the nature of the phase diagram when the spins are placed in arrays featuring geometrical frustration, the dynamics of the system after a quench (i.e. the sudden variation of one parameter of the Hamiltonian), the role of disorder in the couplings, their combination with situations where topology plays a role. . . Besides, as explained in the introduction, many combinatorial optimization problems can be mapped onto spin models [7], and their interest is therefore beyond the traditional realm of many-body physics. All these questions can be studied using the Rydberg platforms described here, as we now show.

Box 2 | Interactions between Rydberg atoms

Two atoms separated by a distance R much larger than the size of the electronic wavefunction interact mainly *via* the electric dipole-dipole Hamiltonian $\hat{V}_{dd} \sim \hat{d}_1 \hat{d}_2 / (4\pi\epsilon_0 R^3)$ where \hat{d}_i is the electric dipole moment of atom i . The effect of this Hamiltonian on a pair of Rydberg atoms depends on how the pair is prepared.

In the most common case, the two atoms are excited in the same Rydberg level, for instance $|nS\rangle$. In this case, \hat{V}_{dd} has no effect at first order in perturbation theory, as an atomic state has a vanishing average electric dipole moment. The effect of the interaction is thus of second order: the pair state $|nS, nS\rangle$ is coupled via \hat{V}_{dd} to other pair states of opposite parity differing in energy from $|nS, nS\rangle$ by a quantity Δ (Figure B2a) with a matrix element $V \propto d_1 d_2 / R^3$. This gives rise to a van der Waals shift of the considered pair state scaling as $V^2 / \Delta \propto C_6 / R^6$ [61]. The C_6 -coefficient is on the order of d^4 / Δ , and thus scales roughly as n^{11} since $d \sim n^2$ and $\Delta \sim n^{-3}$. The dependence on distance of the van der Waals interaction was directly measured for the cleanest system of a pair of single atoms at controlled positions in [62], as was its angular dependence in [63].

The van der Waals interaction between two Rydberg atoms is huge: it can reach tens of MHz for atomic separations of several micrometers. However Rydberg states have a lifetime on the order of a few hundreds of microseconds. In order to reach much longer lifetimes, at the expense of reducing the interaction strength, the idea of *Rydberg dressing* has been proposed [64–68]. It consists in driving off-resonantly the transition from the ground to the Rydberg state, in a regime where the Rabi frequency is smaller than the detuning. The Rydberg state population remains negligible, but the ground state being weakly admixed with the Rydberg state, a pair of ground-state atoms acquire sizable interactions, with a long-distance tail decaying as $1/R^6$ beyond the Rydberg blockade radius, and a flat-top interaction at shorter distances. Figure B2b shows this *soft-core potential*, measured on a pair of single atoms trapped in optical tweezers, for two different detunings of the dressing laser [60].

On the contrary, when the two atoms are prepared in two different, dipole-coupled Rydberg states, such as $|nS\rangle$ and $|nP\rangle$, the pair state $|nS, nP\rangle$ is directly coupled to the same-energy state $|nP, nS\rangle$ by \hat{V}_{dd} (Figure B2c). This gives rise to new eigenstates $|nS, nP\rangle \pm |nP, nS\rangle$ with energies $\pm C_3 / R^3$. A pair of atoms initially prepared in $|nS, nP\rangle$ will coherently evolve into $|nP, nS\rangle$ and back, with a “flip-flop” oscillation frequency $\propto R^{-3}$ (Figure B2.c, adapted from [69]). Moreover the interaction is anisotropic, varying as $1 - 3 \cos^2 \theta$ with θ the angle between the internuclear axis and the quantization axis (inset of figure B2.c).

Finally, online calculators are now available to compute the interactions in all regimes, including in the presence of external electric and magnetic fields [55, 56].

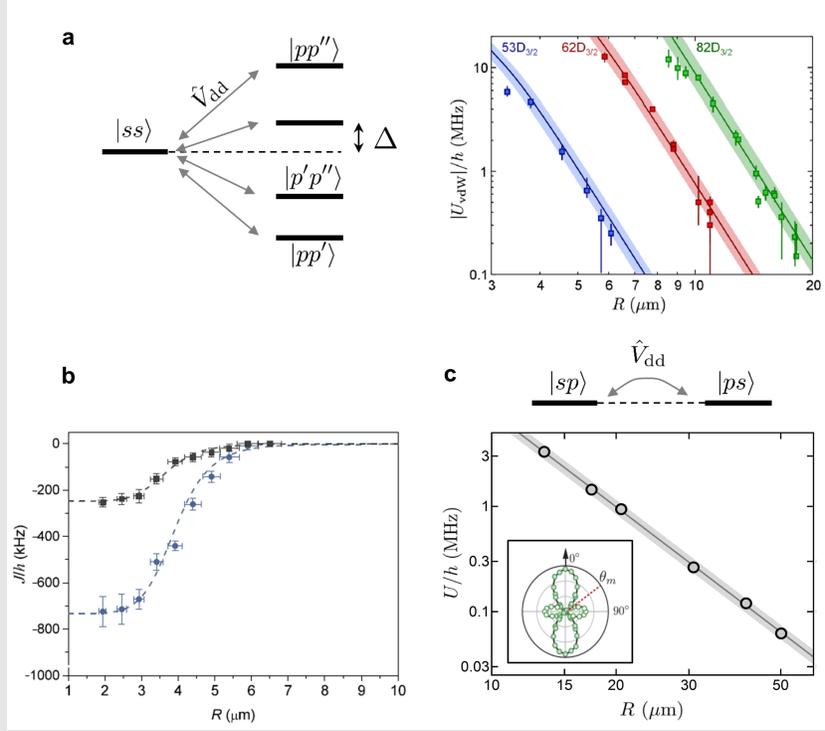


Figure B2 | Interactions between Rydberg atoms. **a:** The van der Waals interaction between two identical Rydberg states, for instance $|nS\rangle$ states, arises due to non-resonant dipolar interactions with other dipole-coupled Rydberg states. The C_6 coefficient varies extremely fast with the principal quantum number n , as illustrated on this measurement for two atoms separated by R , excited to nD states [62]. **b:** Soft-core potential between Rydberg dressed atoms, measured for a pair of single atoms, as a function of the atomic separation [60]. **c:** The resonant dipole-dipole interaction arises when two atoms are in different, dipole-coupled Rydberg states, such as $|nS\rangle$ and $|n'P\rangle$. It varies as C_3/R^3 with the distance [69], with the angular dependence $C_3 \propto 1 - 3 \cos^2 \theta$ typical of the dipole-dipole interaction (inset, adapted from [70]).

V. QUANTUM SIMULATION OF THE ISING MODEL

To study many-body systems experimentally one can, for example, vary suddenly one parameter of the Hamiltonian and study the resulting dynamics of the closed many-body system. One can also prepare the ground state using an adiabatic variation of the parameters of the Hamiltonian, and study its properties. The experiments performed on Rydberg quantum simulators in the Ising model regime used these two approaches.

Let us first discuss qualitatively the generic phase diagram of the quantum Ising model described by Eq.(1), at zero temperature and for spins placed on a chain or on two-dimensional square arrays. We first consider the case of nearest neighbor couplings only ($V_{i,i+1} = V$ and 0 otherwise) and assume $V > 0$, so that the interactions favor anti-ferromagnetic ordering. The phase diagram consists of two regions, a paramagnetic and an anti-ferromagnetic one, separated by a quantum phase transition, as represented in Fig. 2a. Two limiting cases are easy to understand: for $\Omega, \delta \gg V$, the ground state is paramagnetic, *i.e.* the spins align along the effective magnetic field; for $\Omega = 0$, the phases results from the minimization of the energy of the classical configuration. When we relax the constraint of nearest neighbor couplings only (as is the case for a van der Waals interaction) the diagram exhibits several phases around the line separating the para- from the antiferromagnetic phases. For example, on a chain, if $V_{i,i+1}, V_{i,i+2} \gg \delta \gg \Omega \gg V_{i,i+3}$, the ground state corresponds to one excitation separated by two ground state atoms (Z_3 symmetry). This situation corresponds to $R_b = 2a$, with a the spacing between atoms. Similarly, $V_{i,i+1}, V_{i,i+2}, V_{i,i+3} \gg \delta \gg \Omega \gg V_{i,i+4}$ leads to a phase with Z_4 -symmetry, and so on. By controlling the detuning δ and Rabi frequency Ω , one can explore the phase diagram of this Ising model.

The sudden variation of a parameter in the Hamiltonian (also called a quench), is the easiest method to implement experimentally. In the case of the Rydberg platform, all the experiments realized so far suddenly applied the Rydberg excitation laser mimicking the transverse magnetic field, usually at resonance ($\delta = 0$), after having prepared the atoms in their ground state (*i.e.* spin $|\downarrow\rangle$). They then measure two quantities relevant to the study of spin systems. First, the average magnetization, *i.e.* the average number of atoms excited to the Rydberg states, or equivalently in the spin state $|\uparrow\rangle$. Second, the spin-spin correlation

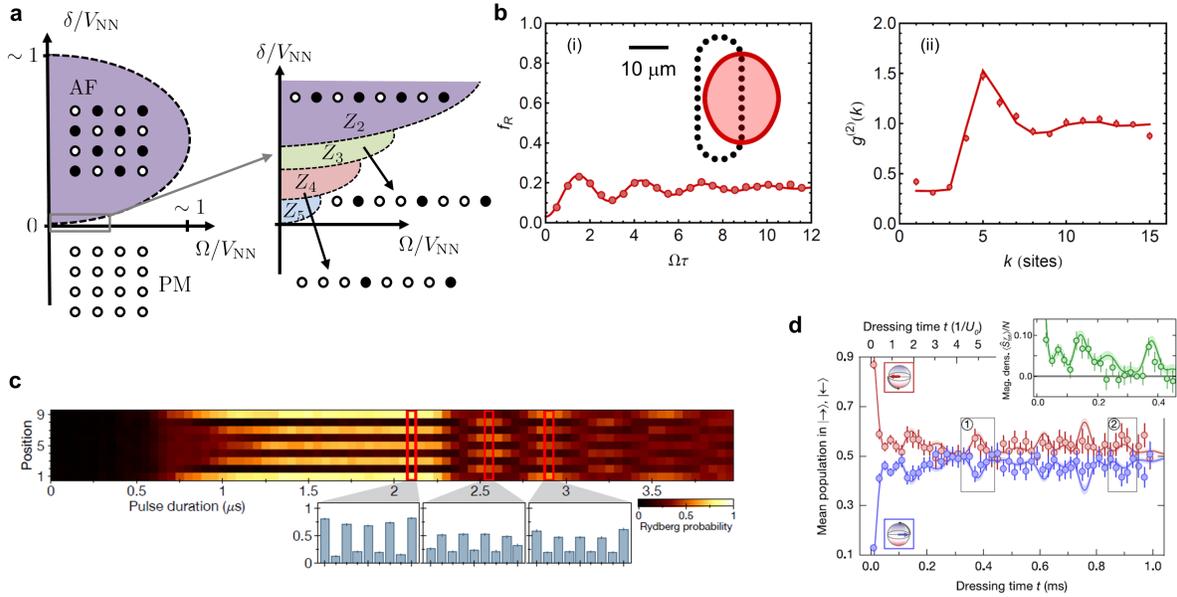


FIG. 2: **Quantum quench experiments for the Ising model.** **a:** Schematic phase diagram of the quantum Ising model for a 2d square or a 1d chain of atoms interacting via the van der Waals interaction, showing the paramagnetic (PM) and antiferromagnetic (AF) phases. Here $V_{\text{NN}} = C_6/a^6$, with a the spacing between atoms (we take $C_6 > 0$). The right side is a zoom of the phase diagram around the critical point ($\Omega = 0, \delta = 0$), for the 1d chain. **b:** Quantum quench in a one-dimensional Ising magnet with periodic boundary conditions, starting with all atoms in $|\downarrow\rangle$, for $R_b/a \simeq 4$ [71]. The shaded area represents the blockade volume. The magnetization of the chain (i) and the spin-spin correlation function (ii) show clear effects of the Rydberg blockade. The solid lines are simulations of the Schrödinger equation without any adjustable parameter. **c:** In [72], a quasi-adiabatic sweep was used, in the regime $R_b/a \simeq 1.5$, to prepare an ‘antiferromagnet’ $|\uparrow\downarrow\uparrow\downarrow\cdots\rangle$. This state was then suddenly quenched (at $2.2 \mu\text{s}$) by driving it on resonance, resulting in surprisingly long-lived collective oscillations between $|\uparrow\downarrow\uparrow\downarrow\cdots\rangle$ and $|\downarrow\uparrow\downarrow\uparrow\cdots\rangle$. **d:** Rydberg dressing can also be used to study quenches of Ising magnets. In [73], a chain of 10 atoms all initially prepared in $|\leftarrow\rangle = (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$ was suddenly subjected to the Ising Hamiltonian, giving rise to a dynamical evolution showing collapse and revivals of the magnetization along x .

function, which is the probability to find a Rydberg excitation at site j when one is already present at site i . The measurement of this correlation function is only made possible owing to the fact that the quantum gas microscope and the tweezer array platforms allow for single site readout of the atomic state. The group of I. Bloch and C. Gross at the Max-Planck Institute in Garching was the first to implement this method using their newly built quantum gas microscope [74]. They operated in a regime where the blockade radius R_b was much larger than the inter-site distance $a \sim 500 \text{ nm}$. The group could observe the effect of the blockade as the fact that Rydberg excitations appeared separated typically by a distance R_b . They could also follow the dynamics of the appearance of the excitation and compare it to the theoretical prediction of the Ising model. Later, the same group has also demonstrated the \sqrt{N} enhanced coupling for an ensemble of atoms all within a blockade radius for up to 200 atoms [75].

The case $R_b \gg a$, as explored by the Max Planck group, is an extreme situation where the interaction dominates all the energy scales in the problem. It is also interesting to explore the case where the interaction energy between neighbouring atoms is on the order of the energy scale associated with the transverse field, *i.e.* operating at $R_b \sim a$. The tweezer arrays is naturally in this regime and our group at Institut d’Optique was the first to explore this situation [71]. As an example, we considered a one-dimensional chain of ~ 20 atoms with periodic boundary conditions (see Fig. 2b). After suddenly turning on the excitation laser, we observed the dynamics and measured the pair correlation functions. We compared them to the solution of the Schrödinger equation for this many-body system, including all the experimental imperfections. The agreement with the data is very good, and this theory-experiment comparison can be considered as a benchmark of this Rydberg quantum simulator in a regime where *ab-initio* calculations are still possible. In particular, the appearance of a steady state regime for the magnetization at long time results from the beating of all the eigen-frequencies of this interacting system. The behavior of the pair-correlations function (also observed in two-dimensions by the Max Planck group [74]) with its suppression at short distance and its oscillatory behavior at larger distance is reminiscent of the pair correlation function of a liquid of hard rods with an effective size of the particles of R_b . We repeated this quench experiment in a 7×7 arrays [76], and compared the observed dynamics to a theory, which had to be approximate as the number of particles involved is too large to allow for *ab initio* calculations. These studies have been refined by the group of J. Ahn in Korea, who considered the appearance of a steady state at long evolution times as an evidence of thermalization of the system [77, 78]. Finally the group of M. Lukin at Harvard also studied an out-of-equilibrium situation, but contrarily to the experiments described above, they started from the ground state of the many-body system [72]

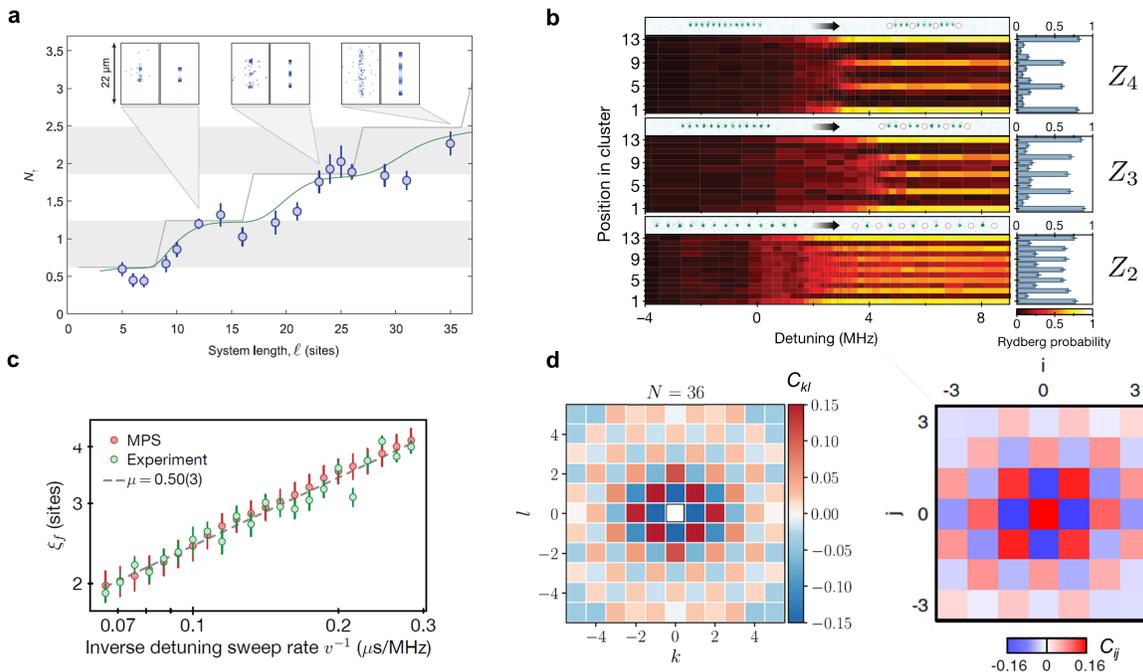


FIG. 3: | **Quasi-adiabatic sweeps experiments for the Ising model.** **a:** “Crystallization” of Rydberg excitations in a quasi one-dimensional chain, in a regime where the blockade radius is much larger than the lattice spacing a . When the chain length is increased, the number of excitations increases in a step-like manner [79]. **b:** Adiabatic preparation of the ground state of a 13-atom chain for various values of R_b/a , giving an excitation every second site, every third site, or every fourth site, corresponding to different ordered phases Z_n of the system [72]. **c:** Antiferromagnetic correlation length ξ obtained in a chain of 51 atoms as function of the detuning sweep rate [80]. The slope gives access to critical exponents characterizing the quantum phase transition. **d:** Antiferromagnetic spin-spin correlation function in two dimensional arrays of atoms after adiabatic preparation for the experiment of [83] (left) and [84] (right).

obtained from an adiabatic preparation detailed below. By suddenly changing the detuning of the laser so as to cross a quantum phase transition, they observed non trivial dynamics with long-lived collective oscillations that they were able to model (see Fig. 2c).

A second method to study many-body systems consists in preparing the ground state of the system. In order to do so, one starts from the state where all atoms are prepared in their ground state: $|\psi_{\text{ini}}\rangle = |ggg\dots\rangle$. By sweeping the Rabi frequency $\Omega(t)$ and detuning $\delta(t)$ on a time scale long with respect to the inverse of the energy gap with the first excited state, the systems is driven adiabatically in the ground state of the interacting system for a given final value of Ω and δ . Once again, based on a suggestion by T. Pohl [85], the group of I. Bloch was the first to demonstrate this approach in the regime $R_b \gg a$ [79]: they observed a controlled number of excitations separated by R_b in one and two dimensions, a situation reminiscent of a crystal. The regime $R_b \sim a$, was then explored by three groups. The group of M. Lukin in Harvard investigated the one-dimensional case [72] with up to 51 atoms. In their case, they varied the ratio R_b/a between 2 and 4, so that they could access several Z_n phases. The Institut d’Optique group [83] and the group of W. Bakr in Princeton [84] explored the two-dimensional case using respectively atoms in arrays of tweezers and in optical lattices. Both groups observed the appearance of antiferromagnetic correlations in their system. The Institut d’Optique group also studied the propagation of these correlations during the adiabatic ramp of the parameters. Let us note that this adiabatic approach becomes harder and harder as the system size increases, as the energy gap vanishes at the quantum phase transition.

Of course, the transition between the quenched and adiabatic regimes is continuous. The group of Harvard studied what happens at the quantum phase transition when varying the rate at which the parameters are varied [80]. In doing so they found out that if the ramp is too fast, defects with respect to the ideal Z_n symmetry appears. These defects can be studied according to a model introduced by Kibble and Zurek in the 80’s [81, 82]. In particular, the correlation length obtained as a function of the speed at which the parameters are ramped follows a universal law with a critical exponent that can be extracted from the experiment. They found an experimental value in excellent agreement with the theoretical prediction from the Ising model. Their experiment nicely illustrates that synthetic quantum systems can be used to measure with high precision the properties of quantum phase transitions.

As pointed out at the beginning of this review, Rydberg atoms have lifetimes in the 100’s of microseconds, a time long enough

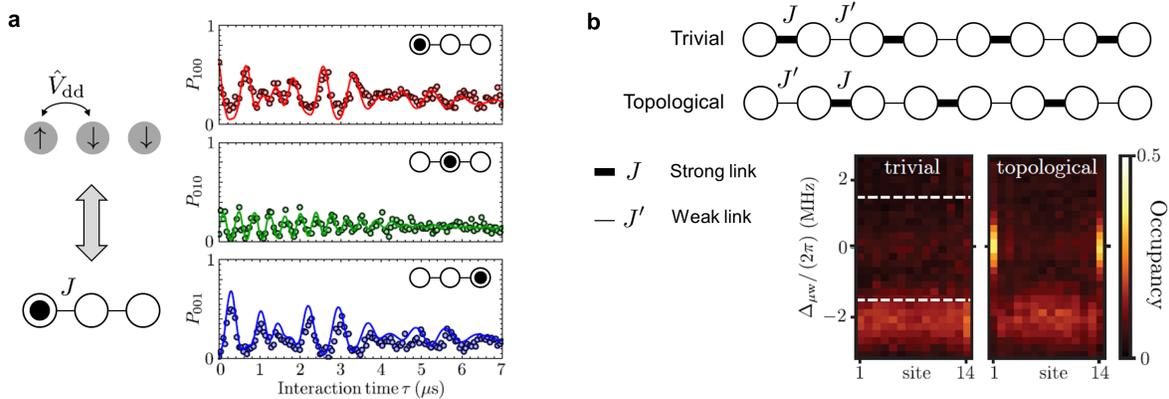


FIG. 4: **Quantum simulation of the XY model.** **a:** Mapping between the XY model and the propagation of a particle from one site to another in a chain. **b:** Illustration in a chain of 3 atoms, where the initial state is $|PSS\rangle$: evolution of the probability that the particle “ P ” is on the left, center or right site, as a function of time. One observes quasi revivals. From [69]. **c:** Implementation of the Su-Schrieffer and Heeger model using Rydberg atoms. The model consists of a chain with an alternance of strong and weak couplings between neighboring sites. A finite chain of even number of sites exists in two different configurations (called normal or topological), with the weak or the strong link at the edges. **d:** Microwave spectroscopy of the Su-Schrieffer-Heeger model implemented on a chain of Rydberg atoms: probability of exciting the atoms from Rydberg state nS to nP as a function of the frequency of the microwave, and for different positions in the chain. The topological configuration features two at zero energy, corresponding to atoms located at the two edges of the chain. From [70].

to observe interaction-driven dynamics. However some features such as revivals in the dynamics, occurring at long times for large systems, may be hard to observe, as the atoms decay to their ground states before the revival occurs. The Rydberg dressing introduced earlier was proposed as an alternative to circumvent this problem [64–68]. First experiments in this direction were performed in the group of I. Bloch and C. Gross in Garching [73, 86]. There, the spin is encoded in two hyperfine ground states of Rb atoms. The atoms arranged along a chain (containing around 10 atoms) are initially all prepared in a superposition state $\propto |\uparrow\rangle + |\downarrow\rangle$. A laser is then switched on to admix the state $|\uparrow\rangle$ with a Rydberg state $|r\rangle$, thus making the atoms interact. The system evolves under the influence of the interactions. After some time, the laser is switched off and the state of each atom is read out. One can then compute the average magnetization and the correlations. The group was able to observe revivals in the magnetization, as shown in Fig. 2(d). Although this result is very encouraging as it shows the potential of Rydberg dressing, it remains a bit unclear at the present time how large the system sizes can be, as unwanted losses appear during the dressing when the number of atoms grows [86–88].

VI. QUANTUM SIMULATION USING RESONANT EXCHANGE INTERACTIONS

As explained above, the resonant dipole-dipole interaction between two Rydberg states naturally realizes the XY spin-1/2 model. Besides its interest for the study of quantum magnetism, this model is also useful to describe transport properties in many situations. Let us think for example about a chain of spin-1/2 particles all prepared in their state $|\downarrow\rangle$ and able to interact by the XY interaction. If one now flips the spin of one of them to $|\uparrow\rangle$, this spin excitation will propagate along the chain under the influence of the exchange interaction, in the same way that a single particle can tunnel between neighboring sites in a lattice. This transport of excitations driven by the resonant dipole interaction is for example the process that takes place in photo-synthesis, where the energy deposited by light in a light-harvesting cell is carried towards a reaction center [89].

This example suggests using a different language to describe the transport of spin excitations under the influence of the resonant dipole interaction. Let us now view the $|\downarrow\rangle$ -state as the absence of particle (the “vacuum”), and rename it $|0\rangle$. The state $|\uparrow\rangle$ now corresponds to the presence of a particle, that we call $|1\rangle$. In a chain, the state $|0000\dots\rangle$ corresponds to no particle on any site, while, for example $|0100\dots\rangle$ indicates the situation where one particle is present on the second site of the chain. We can also consider the situation where two excitations are present in the chain, e.g. by preparing the state $|0110\dots\rangle$. Importantly, these particles interact very strongly: as one atom cannot carry more than one excitation, it is not possible to find two particles on the same lattice site. Therefore spin excitations behave as artificial particles with infinite on-site interactions, i.e. with a hard-core constraint. It turns out that the spin excitations have the same commutation relations as those of bosons. Therefore the problem of an ensemble of two-level Rydberg atoms interacting by the resonant dipole-dipole interaction can be mapped either onto a spin-1/2 XY model, or equivalently onto a system of hard-core bosons [90]. But one should keep in mind that the resonant dipole interaction that drives the transport of an excitation leads to a *single-particle* problem when considering a single excitation. What makes the excitations interact is the fact that the atoms only have two-level, and not the fact that the atoms

carrying the excitations interact.

The first experiments performed with Rydberg atoms in a laser-cooled gas were actually a study of the transport of excitations driven by the dipole-dipole interaction in a situation where the atoms' positions are frozen [17, 18]. Since then, several experiments refined our understanding of the dynamics of the propagation in atomic ensembles with random positions [92, 93]. The case where the atoms are placed in regular arrays with individual control has been much less studied experimentally, and, so far, only two experiments, performed in our group, have used the resonant dipole-dipole interaction to perform a quantum simulation.

The first one was a proof-of-principle experiment demonstrating the propagation of a spin excitation in a chain of 3 atoms [69]. Preparing three atoms all in the same $|nD\rangle$ Rydberg state, we flipped the state of the first one into a state $|n'P\rangle$ and observed how the “ P ” excitation moved in the chain. The results, shown in Fig. 4(a), indicate that despite the simplicity of the situation, the dynamics is already non trivial: this comes from the fact that the two energy scales in the problem, C_3/a^3 and $C_3/(8a^3)$, lead to three incommensurate eigenvalues of the Hamiltonian.

The goal of the second experiment was to implement the Su, Schrieffer and Heeger (SSH) model, initially developed in the late 1970's to explain the conductivity of some organic polymers [94, 95]. In its simplest setting, it consists of a one-dimensional chain of sites that are coupled by an alternation of strong and weak links and where an excitation can hop in the chain. Since then, the SSH model has been recognized as one of the simplest examples of a system exhibiting topological properties. Let us consider the two configurations of a finite chain represented in Fig. 4b: either the chain ends up with the strongest link J , or with the weakest one J' . One can show that in the first configuration, the single-particle spectrum consists of two bands with width J' , separated by an energy gap $|J - J'|$. On the contrary, in the second configuration, two states at zero energy appear in the middle of the gap, and correspond to states localized on the edge of the chain. The fact that they have zero energy is rather intuitive in the extreme case where $J' = 0$, as adding a particle on each edge does not cost energy. It turns out that this remains true even when $J' \neq 0$. The two configurations correspond to two different topological phases of the system: it is impossible to vary the ratio J/J' and continuously transform one configuration into the other without closing the gap [96]. Our group has implemented this model in a chain of Rydberg atoms, allowing us to study both single-particle properties, but also the genuinely many-body properties arising from the hard-core constraint [70]. Microwave spectroscopy was used to measure the single-particle energy spectrum (see Fig. 4). We have entered the many-body regime by preparing the ground state of the chain comprising $N/2$ excitations (where N is the number of sites), using an adiabatic preparation in the spirit of the one we used to prepare the antiferromagnetic correlations in the Ising model. We have characterized this many-body ground state in the topological configuration, and found it displayed a characteristic robustness with the respect to the breaking of certain symmetries of the Hamiltonian. The prepared state is probably the first experimental realization of a type of topological order for bosons introduced in 2012 [97], called a symmetry-protected topological phase, which is the only one that exists in one dimension.

VII. PERSPECTIVES

Finally, we discuss short and longer-term perspectives on the use of Rydberg atoms for quantum simulation. The field is rapidly evolving, and here we merely identify a few emerging directions of research.

An obvious trend in the short term, on the technical side, is (i) to improve the fidelity of the simulations, and (ii) to scale up the number of atoms in the arrays. The first objective requires not only, at the single-particle level, to understand the limitations of the Rydberg excitation schemes [98] and to overcome them, for instance using different schemes for the two-photon transitions [99], but also, at the two-atom level, to optimize the mapping of the complex level structure of interacting Rydberg atoms onto simple two-level systems [76]. In addition, the possibility to scale up the number of atoms to several hundreds is one the crucial assets of Rydberg arrays when compared to other platforms. To do so, the recently demonstrated use of gray-molasses loading of optical tweezers [100] opens up exciting prospects as, for a given trap depth, the required optical power per trap is strongly reduced, and, at the same time, the loading probability p is significantly enhanced. Cryogenic platforms such as the ones recently developed for trapped ion chains [101] could help increase the atomic trapping lifetime and thus help in scaling up the atom number.

The second short-term prospect is the extension of the techniques that have so far been applied to alkali atoms to new atomic species with two valence electrons. Arrays of single Strontium [102–104] and Ytterbium [105] atoms have been reported recently. Although so far no quantum simulation has been performed with these novel systems, the richer internal structure of these species might allow new ways to manipulate, control and probe them [106, 107].

A longer-term goal would consist in using arrays of *circular* Rydberg states for quantum simulation. Not only would they allow to implement more complex spin models, such as the the Heisenberg Hamiltonian, in a more natural way, but they may also, with their much longer lifetimes, open up the possibility to study long-time dynamics with Rydberg quantum simulators [108].

Finally, one exciting prospect for Rydberg array quantum simulators is the fact that their applications may extend to a much broader class of problems than that of the mere implementation of spin Hamiltonians inspired by quantum magnetism, as they could be used to study optimization problems by quantum annealing [109, 110]. While for general optimization problems

Rydberg arrays may have limitations comparable to other platforms, they could be for instance particularly adapted to solving a classical combinatorial problem in graph theory, namely finding the Maximum Independent Set of a graph [111].

A hybrid, closed-loop approach, combining a Rydberg quantum simulator with increased degree of control and a classical computer, could be used to implement *variational quantum simulation*. There, the quantum machine is used for efficiently generating many-body quantum states depending on a small number of variational parameters and measuring the average value of non-trivial observables, while classical hardware is used to optimize these parameters, making it possible to find in an interactive way, e.g., the ground state of spin Hamiltonians that cannot be realized physically with the platform at hand [112]. This type of architecture blurs the distinction between programmable quantum simulators and noisy, intermediate scale quantum computers [113], for which Rydberg atom arrays are also a promising platform [114, 115].

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