Optical Turbulence and Outer Scales above Dome C in Antarctica

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ABSTRACT

Dome C in Antarctica is a particular astronomical site when considering the optical turbulence conditions. From the first winterover campaign performed in 2005 at Dome C, the set of 34 meteorological balloon profiles has been analyzed. The meteorological balloons were equipped with microthermal sensors used to sense the vertical profile of the optical turbulence intensity C_n^2 . The C_n^2 median profile, mean temperature and mean horizontal wind speed are given. The C_n^2 median profile is characterized by a very strong and thin turbulent surface layer. The surface layer height is defined. The median outer scale profile at Dome C is computed using the Tatarski definition. The von Kármán outer scale is also deduced. The integrated parameters as Fried parameter r_0 , coherence time τ_0 , isoplanatic angle θ_0 and the spatial-coherence outer scale $\mathcal{L}_0$ used to define astronomical site quality, are computed at 8 m above the ground and above the turbulent surface layer.

Keywords: atmospheric effects, interferometry, site testing, outer scale

1. INTRODUCTION

Dome C on the Antarctic plateau has many advantages for Astronomy. At 3233 m of altitude, the dry, cold and clear air and with the possibility to observe without discontinuity during several months, placed Dome C as a potentially interesting astronomical site. Ground based telescopes and high angular resolution technics, like adaptive optics or interferometry, are highly concerned by atmospheric turbulence as it drastically reduce the angular resolution of telescopes.

2. OPTICAL TURBULENCE

Since 1960, the effects of atmospheric turbulence on wave propagation, called optical turbulence, has been studied. The turbulent process as suggested by Richardson in 1922,¹ may be described by a flow of energy: a small fraction of the ambient kinetic energy of the flow is converted into turbulence energy. The energy is injected through a large eddy at the size of the outer length scale L_0 which is then break up into smaller eddies. The input energy is progressively divided and redistributed to smaller and smaller scales. The energy cascade continues until the eddies are comparable in sizes to the inner scale length l_0 , below which the energy is dissipated by viscosity in heat. The inertial range is defined between the energy input region and the energy-loss region $l_0 < \rho < L_0$. In this range, Kolmogorov in 1941² assumed that the velocity fluctuations are both isotropic and homogeneous and established the so well known 2/3 scaling law for the fluctuations of the velocity component. The structure function of the velocity is expressed as a function of the distance ρ between two points and ϵ the energy dissipation rate [m^2s^{-3}]:

$$D_v(\rho) = \langle (v(r) - v(r + \rho))^2 \rangle = c(\epsilon\rho)^{2/3} = C_v^2\rho^{2/3}, \quad (1)$$

where c is a dimensionless constant and C_v^2 is the velocity structure constant. The same analysis may be applied to describe the turbulent mixing of a conservative and passive scalar as made by Obukhov in 1949³ on the refractive-index. The refractive-index structure constant C_n^2 is defined by:

$$D_n(\rho) = C_n^2\rho^{2/3}, \quad (2)$$

and C_n^2 is the relevant parameter which quantify the optical turbulence strength.

The refractive index of air is a function of the absolute temperature T , the atmospheric pressure p and the partial pressure of water vapor e :

$$n = 1 + \frac{79.10^{-6}}{T} \left(p + \frac{4800e}{T} \right) \quad (3)$$

Since T is not a conservative and passive scalar, the potential temperature θ has to be considered:

$$\theta = T \left(\frac{1000}{P} \right)^{0.286}, \quad (4)$$

which is the temperature of parcels of air displaced adiabatically at 1000 Hpa. Like in most of astronomical site the effect of the humidity may be neglected and the index structure constant C_n^2 is related to the temperature structure constant C_θ^2 according to the Gladstone formula:

$$C_n^2 = C_\theta^2 \left(\frac{79.10^{-6}P}{T^2} \right)^2 \quad (5)$$

3. C_N^2 PROFILE ABOVE DOME C

The structure function of temperature fluctuations $D_\theta(\rho)$ may be computed using microthermal sensors as developed by M. Azouit and J.Vernin.⁴ These sensors may be fixed on a meteorological balloon in order to retrieve the C_n^2 profile in the atmosphere. This technic has been used during the first winterover in 2005 at Dome C in order to study the vertical profile of the optical turbulence. Between March 2005 and October 2005, 34 balloons were launched. Balloon profiles are resampled to a 10 m vertical resolution in the boundary layer (the first kilometer above the ground) and 100 m in the free atmosphere. The C_n^2 median profiles and the mean profiles of the potential temperature $\langle \theta \rangle$, mean horizontal wind speed $\langle V_h \rangle$ and mean direction $\langle Dir.(V_h) \rangle$ are computed over all the campaign (fig. 1). Different seasons as Autumn, Winter and Spring may be considered to study the seasonal effects on the parameters as published by Trinquet et al 2008.⁵

In the Boundary Layer, usually defined as the first kilometer above the ground. The mean wind speed is less than 3 ms^{-1} in the first few meters and increases up to 8 ms^{-1} at 100 m. The mean wind speed at the ground level may be variable from year to year, but as studied by Aristidi et al 2005,⁶ with a median value of 2.9 ms^{-1} , Dome C is one of the lowest mean ever recorded for an observatory. Even in the case of low wind speed, the median C_n^2 profile shows a clear excess of optical turbulence in the first 30 m above the ground. The thickness of this turbulent surface layer h_{sl} is defined by the part of the boundary layer containing 90 % of the total boundary optical turbulence:

$$\frac{\int_{8m}^{h_{sl}} C_n^2(z) dz}{\int_{8m}^{1km} C_n^2(z) dz} < 0.9. \quad (6)$$

The first point is fixed at 8 m, to be at the same level used for the DIMM monitor⁷ and to discard the turbulence very near the ground. During the 2005 campaign, h_{sl} fluctuates between 10 m and 517 m (fig. 2). The median value is 33 m and the 75% percentile is 42 m. The C_n^2 median profile may be considered as constant in the surface layer and then decreases exponentially following the relations:

$$8m < h < 33m : \quad C_n^2 \simeq 3.10^{-14} \quad (7)$$

$$33m < h < 100m : \quad C_n^2(h) = 2.110^{-6} h^{-5.6} \quad (8)$$

$$100m < h < 1000m : \quad C_n^2(h) = 2.410^{-16} h^{-0.7} \quad (9)$$

$$(10)$$

In the free atmosphere, the median C_n^2 profile is quite similar than for mid-latitude sites. An excess of optical turbulence is observed above 20 km but only 3 flights reached this altitude. Most of the time the balloons

Figure 1. Median $\langle C_n^2 \rangle$, mean potential temperature $\langle \theta \rangle$, mean horizontal wind speed $\langle V_h \rangle$ and mean wind direction $\langle \text{Dir.} \rangle$ profiles (dots), computed from the balloons launched from the 34 balloon profiles launched during the 2005 at Dome C. Standard deviations (error bars) for the $\langle C_n^2 \rangle$ (in logarithmic scale), $\langle \theta \rangle$ and $\langle V_h \rangle$. N is the number of valid flights at the corresponding altitude. The C_n^2 dashed curve is the median profile for mid-latitude sites. In the main plot starting at 3250 m over the sea level, the mean value is computed over a range of 100 m every 100 m. In the subplot, starting at 5 m over the ground, the mean value is computed over a range of 10 m every 10 m.

Figure 2. Dome C surface layer thickness (left), as defined by eq.6, during the 2005 campaign. Each flight number is printed near the marker. The cumulative distribution (right) is the probability to be above the surface layer at a given elevation. The upper vertical scale is fixed to 100 m for convenience. Flight 569 is out of scale (517 m).

exploded when the temperature decreases below -70°C . The mean wind speed is particularly low when compared with mid-latitude sites. At Dome C, up to 14 km above sea level, the mean wind speed is below 20 m s^{-1} . Above, the mean wind speed increases significantly in winter as quoted by Geissler and Masciadri 2006.⁸ This acceleration contributes to decrease the mean coherence time from 8 ms in Autumn to 5 ms in Winter.⁵

4. OUTER SCALE PROFILE AT DOME C

The outer scale of the turbulence L_0 defined the upper bound of the Kolmogorov inertial subrange in which the turbulence is fully developed, homogeneous and isotropic. In this range a systematic difference of a passive and conservative scalar as θ does not affect the amplitude of fluctuations $D_{\theta}(\rho) \gg \Delta \bar{\theta}^2$. Tatarski in 1961⁹ defined the outer scale L_{0T} the distance over which the field of fluctuations of θ cannot be considered locally isotropic and the fluctuational difference is no more negligible:

$$C_{\theta}^2 L_{0T}^{2/3} = a^2 (\text{grad} \bar{\theta})^2 L_{0T}^2, \quad (11)$$

where a is a constant ($a^2 \approx 2.8$). According to this expression, the outer scale L_{0T} as defined by Tatarski may be deduced from C_{θ}^2 profiles and θ profiles. From the Dome C measurements we estimate L_{0T} for each profiles and then the median profiles (fig. 3). From the median profile it is clear that most of the time L_{0T} is smaller than 1 m and smaller than outer scales usually retrieved from other sites. In the boundary layer, from the ground up

Figure 3. Median profile of the Tatarski outer scale L_{0T} , as defined by equation 11, computed from the 2005 Dome C campaign. The dashed curve is the median profile for mid-latitude sites.

to 100 m, L_{0T} decreases rapidly from 1 m to few centimeters. In the free atmosphere two main slabs with large L_{0T} are noticeable, in the troposphere between 5 km to 8 km, and in the stratosphere above 20 km. These two slabs are at the same levels than the turbulent slabs observed in the C_n^2 median profile.

5. SPATIAL RESOLUTION AT DOME C

The contribution of the optical turbulence on the wave front is described by a set of spatial, angular and temporal parameters. The atmospheric diffraction-limited equivalent pupil r_0 , as defined by Fried 1966, is related to the C_n^2 profile for a plane wave crossing the atmosphere along the zenith angle γ and at the wave length λ :

$$r_0^{-5/3} = 16.7\lambda^{-2}\cos(\gamma)^{-1} \int_0^\infty C_n^2(h)dh. \quad (12)$$

The resolving power of a telescope of diameter D is limited by D when $r_0 < D$ and is limited by r_0 when $r_0 > D$. The seeing, the angular resolving power of the telescope for a long exposure time, considering the turbulent wave, is related to the Fried parameter r_0 by:

$$r_0 = 0.98\lambda/\epsilon_0. \quad (13)$$

The isoplanatic angle θ_0 , as defined by Roddier et al 1982, is the angular domain in which phase fluctuations induced by optical turbulence are still correlated:

$$\theta_0 = 0.058\lambda^{6/5} \left[\int_0^\infty h^{5/3} C_n^2(h) dh \right]^{-3/5}. \quad (14)$$

θ_0 is more sensitive to high turbulent layers and it is roughly the field of view in which the adaptive optics remains efficient. Temporal properties of the turbulent wave front may be quantified by the coherence time τ_0 . It is the time during which the phase fluctuations are still correlated. τ_0 is function of the C_n^2 profile and of the horizontal wind speed V of the turbulent layers:

$$\tau_0 = 0.058\lambda^{6/5} \left[\int_0^\infty |V(h)|^{5/3} C_n^2(h) dh \right]^{-3/5}. \quad (15)$$

τ_0 sets the upper limit of adaptive optics bandwidth. Effects of the optical turbulence over large pupil diameter or interferometers have been studied by Borgnino 1990¹⁰ which defined the spatial-coherence outer scale $\mathcal{L}_0$. The first definition used the Tatarski outer-scale L_{0T} in order to limit the low spatial frequencies of the phase fluctuations:

$$\mathcal{L}_0^{-1/3} = \frac{\int_0^\infty L_0(h)^{-1/3} C_n^2(h) dh}{\int_0^\infty C_n^2(h) dh}, \quad (16)$$

so that the variance of the beam at the focus of a telescope of diameter D is:

$$\sigma^2 \approx 0.18\lambda^2 r_0^{-5/3} [D^{-1/3} - \mathcal{L}_0^{-1/3}]. \quad (17)$$

In a second publication, Borgnino 1992 considered the von Kármán spectra as a relevant expression to describe the saturation of the phase structure function out of the inertial range ($\rho > L_0$) and gives a second definition of the spatial-coherence outer scale $\mathcal{L}_{0VK}$ using the von Kármán outer scale L_{0VK} . The structure function for temperature fluctuations is then :

$$D_\theta(\rho) = \int_0^\infty [1 - \text{sinc}(kr)] \Phi_\theta(k) k^2 dk, \quad (18)$$

where $\Phi_T(k)$ is the three-dimensional temperature fluctuations spectrum:

$$\Phi_\theta(k) = 0.033C_\theta^2 \left[k^2 + \left(\frac{2\pi}{L_{0VK}} \right)^2 \right]^{-11/6}. \quad (19)$$

One has to pay attention on the fact that the outer scale defined by von Kármán L_{0VK} is different of the Tatarski L_{0T} . Tatarski considered the intersection of the two asymptotes $D_\theta(\rho \ll L_0)$ and $D_\theta(\rho \gg L_0)$:

$$D_\theta(L_{0T}) = aC_\theta^2 L_{0T}^{2/3} = \Delta\theta^2, \quad (20)$$

and the von Kármán structure function has the asymptotic value $D_\theta(\rho \gg L_0)$:

$$D_\theta(\rho \gg L_0) = 8\pi \int_0^\infty K^2 \Phi_\theta(k) dk = 0.307 C_\theta^2 L_{0VK}^{2/3}. \quad (21)$$

So a relation between them may be deduced as:

$$L_{0VK} = \alpha L_{0T}, \quad (22)$$

$$\alpha = \left(\frac{a}{0.307} \right)^{3/2} \quad (23)$$

It is clear that L_{0VK} overestimates the inertial subrange as defined by Tatarski by a factor $\alpha = 12.7$ in which the classical theory of the wave propagation in a turbulent media may be applied. α has been experimentaly

estimated $\alpha \approx 13$ by simultaneous measurements of the spatial coherence outer scale $\mathcal{L}_0$ between the GSM monitor, using the von Kármán spectra, and C_n^2 balloon profiles using the Tatarski definition.¹¹ A factor has to be introduced in the von Kármán spectra so that the same outer scale L_{0T} may be considered as published by Reinhardt in 1972¹² and focused by Greenwood and Tarazano in 2008:¹³

$$\Phi_\theta(k) = 0.033C_\theta^2 \left[k^2 + \left(1.071 \frac{2\pi}{L_{0T}} \right)^2 \right]^{-11/6}. \quad (24)$$

Table 1 gives the statistical values of the considered integrated parameters for Dome C deduced from the balloon profiles.

Table 1. Statistics of the integrated wave front parameters as deduced from the balloon profiles of the 2005 Dome C campaign. Median values and quartiles ($q_{\pm}^+$) are given. Two different elevations above the ground h_0 are considered. As only 3 flights are available to estimate θ_0 with less than 10% of error, only the θ_0 median value is given.

h_0 [m]	ϵ_0 [']	θ_0 [']	τ_0 [ms]	$\mathcal{L}_{0T}$ [m]	$\mathcal{L}_{0VK}$ [m]
8	1.4 $\begin{pmatrix} 1 \\ 1.6 \end{pmatrix}$	2.7	5.7 $\begin{pmatrix} 3 \\ 9 \end{pmatrix}$	0.6 $\begin{pmatrix} 0.4 \\ 1.0 \end{pmatrix}$	7.8 $\begin{pmatrix} 4.6 \\ 13.0 \end{pmatrix}$
33	0.4 $\begin{pmatrix} 0.3 \\ 0.6 \end{pmatrix}$	2.7	6.8 $\begin{pmatrix} 5 \\ 12 \end{pmatrix}$	0.3 $\begin{pmatrix} 0.2 \\ 0.4 \end{pmatrix}$	3.2 $\begin{pmatrix} 2.3 \\ 5.1 \end{pmatrix}$

6. CONCLUSIONS

Dome C is noticeable by the very strong and thin optical turbulent surface layer. To improve significantly the seeing conditions, telescopes at Dome C should be elevated over the turbulent surface layer. Another strategy could be to develop a specific adaptive optic. As the surface layer is so thin, a single adaptive optic system should be able to improve and damp significantly the seeing over a large field of view. The free atmosphere turbulence is also specific at Dome C with small outer scales and no main turbulent layers. Thin turbulent layers occurred in two turbulent slabs in the troposphere between 5 km to 8 km and in the mid-stratosphere above 20 km. The mean wind speed is particularly low in the troposphere and during the beginning of the winter. This explain the large mean value of coherence time. As the upper part of the atmosphere has been sensed only during the beginning of the winter, a better statistics should be provided by the Single Star Scidar SSS from the 2006 campaign.

ACKNOWLEDGMENTS

The authors wish to thank all the people who have contributed to this campaign, the summer camp logistics and the winterover team of Concordia. This program has been supported by the polar institutes IPEV and PNRA. We thank also the Association of Universities for Research in Astronomy (AURA) for funding the Vaisala station and the US Air Force (contract F61775-02-C002) for part of the microthermal payloads. Part of this work has also been funded by the French Agence Nationale de la Recherche under contract title “CASDOA” (ANR-05-BLAN-0033-01).

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