On scaling of measurements and parameters for stiffness identification of robots

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Elastostatic calibration of robots is crucial for achieving high-accuracy positioning. Stiffness identification is an important step in elastostatic calibration wherein the stiffness parameters are identified. These stiffness parameters are then used to predict and correct the pose errors due to load applied on the robot's end-effector/platform at any pose in the workspace. Hence, these parameters must be estimated accurately.

For stiffness identification of robots, the relationship between the load applied at the end-effector/platform and its resultant deflection is first parameterized. This is accomplished using an appropriate stiffness parameter model. Redundant deflection measurements are then performed with the robot subjected to a known load. These redundant deflection measurements are then used to estimate stiffness parameters by employing a least squares technique.

Scaling of vectors containing measurements and parameters is crucial for accurate stiffness identification. This report focuses on this aspect. This report is organised as follows: section 1 presents the basic mathematical framework for stiffness identification of robots. Sections 2 and 3 discuss about the necessary scaling of vectors containing measurements and parameters.

1 Mathematical framework for stiffness identification

The relationship between the stiffness parameter set and the measured deflection is written as [1]:

$$A_M\ c = \Delta X_M \tag{1}$$

Here, A_M is called the observation matrix. It is a function of load(s) applied for stiffness identification and the pose(s) at which it is performed. c is the parameter set (compliance parameters) to be estimated and ΔX_M contains the measured deflection(s). The goal is to estimate the parameter vector c by measuring pose deflections ΔX_M at some poses under the influence of known forces. However, measurements are always accompanied with random errors as a result of uncertainty fo the deflection measurement system. Taking these errors into account, equation 1 can be rewritten as,

$$A_M\ (c + \varepsilon_c) = \Delta X_M + \varepsilon_{\Delta X_M} \tag{2}$$

Here, A_M is the $mn \times n_p$ observation matrix, where m is the number of measurements, n is the number of elements in a single deflection vector and n_p is the number of parameters. $\varepsilon_{\Delta X_M}$ is a $mn \times 1$ vector containing the errors in measurement due to uncertainty of the measurement system. $\varepsilon^i_{\Delta X_M}$ is the i^{th} measurement vector (of size $n \times 1$) of $\varepsilon_{\Delta X_M}$ vector. The expectations of $\varepsilon^i_{\Delta X_M}$ and $\varepsilon_{\Delta X_M}$, $E(\varepsilon^i_{\Delta X_M})$ and $E(\varepsilon_{\Delta X_M})$, are zero vectors. ε_c is the error in the estimated parameter set due to $\varepsilon_{\Delta X_M}$. The parameters that give the best fit need to be estimated using a least squares approach.

Appropriate scaling of vectors is necessary to ensure good parameter estimation. Scaling of vectors for parameter estimation has been studied very well in the context of robot geometric calibration [2, 3]. Since parameter identification framework for robot geometric calibration is similar to that of elastostatic calibration, same problems (and solutions) regarding scaling exist.

Two types of matrix scaling need to be performed here [2, Chapter 14]: (a) *task variable scaling*, and (b) *parameter scaling*. (a) concerns scaling of vector containing measured deflections and (b) concerns the scaling of vector containing parameters. Sections 2 and 3 explain these in detail.

2 Task variable scaling

Task variable scaling is performed to ensure that: (a) the elements of the measured deflection error vector are independent [1] and identically distributed [2] (I.I.D), and (b) the units of measurements being used for least squares fitting are same. Taking this into account, equation 2 can be rewritten as,

$$G\ A_M\ (c + \varepsilon_c) = G\ \Delta X_M + G\ \varepsilon_{\Delta X_M} \tag{3}$$

In equation 3, G is the task variable scaling matrix. The method to obtain G is well known when elements of $\varepsilon_{\Delta X_M}$ are independent but don't have identical distribution (different standard deviations). In this case, G must contain the inverse of standard deviations of the corresponding elements of $\varepsilon_{\Delta X_M}$ as its diagonal elements [2]. Let this resulting task variable scaling matrix be called G_U.

$$G_U = \overbrace{\begin{bmatrix} \frac{1}{\mathcal{M}_{11}} & 0 & & & & \cdots & 0 \\ 0 & \frac{1}{\mathcal{M}_{12}} & & & & & \\ & & \ddots & & & 0 & \vdots \\ \vdots & & & \frac{1}{\mathcal{M}_{1n}} & & & \\ & 0 & & & \frac{1}{\mathcal{M}_{21}} & & \\ & & & & & \ddots & 0 \\ 0 & & \cdots & & & 0 & \frac{1}{\mathcal{M}_{mn}} \end{bmatrix}}^{mn \times mn} \tag{4}$$

In equation 4, $\mathcal{M}_{i1} ... \mathcal{M}_{in}$ are the standard deviations of elements of $\varepsilon^i_{\Delta X_M}$ (the i^{th} measurement of the $\varepsilon_{\Delta X_M}$ vector). When G_U is used for task variable scaling in least squares estimation, the least squares estimation method is also referred to as *weighted least squares estimation* [4]. When the elements of $\varepsilon_{\Delta X_M}$ are correlated, it is usually ignored. However, this ignorance is not necessary. Correlated measurements can be dealt with using the *generalized least squares method* [4] in which task variable scaling is dealt with differently. The task variable scaling matrix in this case, G_C, is given by,

$$G_C = \overbrace{\begin{bmatrix} {}_1S^{-1} & 0 & \cdots & 0 \\ 0 & {}_2S^{-1} & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & 0 & {}_mS^{-1} \end{bmatrix}}^{mn \times mn} \tag{5}$$

${}_iS$ is related to $Cov({}^{DU}\varepsilon^i_{\Delta X_M})$ as,

$$Cov(\varepsilon^i_{\Delta X_M}) = {}_iV = {}_iS\ {}_iS^T \tag{6}$$

To obtain ${}_iS$, eigen value decomposition of $Cov(\varepsilon^i_{\Delta X_M})$ needs to be performed. It can then be shown that $(G_C\ \varepsilon_{\Delta X_M})$ is I.I.D and dimensionless.

To prove that $(G_C\ \varepsilon_{\Delta X_M})$ is independent and identically distributed, it needs to be shown that $Cov({}_iS^{-1}\ \varepsilon^i_{\Delta X_M})$ is a diagonal matrix containing same numbers along its diagonal (equal variances). Equations 5 and 6 give

[1] meaning the elements of the vector are uncorrelated

[2] meaning the elements of the vector have same standard deviation

$$Cov({}_iS^{-1}\ \varepsilon^i_{\Delta X_M}) = {}_iS^{-1}\ Cov(\varepsilon^i_{\Delta X_M})\ {}_iS^{-T} \tag{7}$$
$$= {}_iS^{-1}\ {}_iS\ {}_iS^T\ {}_iS^{-T} \tag{8}$$
$$= I \tag{9}$$

It can be seen from equations 7, 8 and 9 that $(G_C\ \varepsilon_{\Delta X_M})$ is independent and identically distributed.

The elements of the j^{th} column of ${}_iS^{-1}$ possess a unit which is the inverse of that of the j^{th} element of $\varepsilon^i_{\Delta X_M}$. Consequently, the resulting measurement vector after scaling, $G_C\ \varepsilon_{\Delta X_M}$, is dimensionless. In order to check this, let us consider a simple case where $\varepsilon^i_{\Delta X_M}$ has two coordinates, one translational and the other rotational. Matrices ${}_iV$ and ${}_iS$ (refer equation 6), in this case, will have the following structure and units:

$${}_iV = \begin{bmatrix} {}_iV_{11}\ (m^2) & {}_iV_{12}\ (mrad) \\ {}_iV_{21}\ (mrad) & {}_iV_{22}\ (rad^2) \end{bmatrix} \tag{10}$$
$$= \begin{bmatrix} {}_iS_{11}\ (m) & {}_iS_{12}\ (m) \\ {}_iS_{21}\ (rad) & {}_iS_{22}\ (rad) \end{bmatrix} \begin{bmatrix} {}_iS_{11}\ (m) & {}_iS_{21}\ (rad) \\ {}_iS_{12}\ (m) & {}_iS_{22}\ (rad) \end{bmatrix} ={}_i S\ {}_iS^T \tag{11}$$

Here, ${}_iV_{pq}$ and ${}_iS_{pq}$ are the p^{th} elements the q^{th} column of matrices ${}_iV$ and ${}_iS$, respectively. ${}_iS^{-1}$, for this case, can be written as,

$${}_iS^{-1} = \frac{1}{({}_iS_{11}\ {}_iS_{22} -\ {}_iS_{12}\ {}_iS_{21})}\ (mrad)^{-1} \begin{bmatrix} {}_iS_{22}\ (rad) & -\ {}_iS_{12}\ (m) \\ -\ {}_iS_{21}\ (rad) & {}_iS_{11}\ (m) \end{bmatrix} \tag{12}$$
$$= \begin{bmatrix} \frac{{}_iS_{22}}{{}_iS_{11}\ {}_iS_{22} - {}_iS_{12}\ {}_iS_{21}}\ (m^{-1}) & -\frac{{}_iS_{12}}{{}_iS_{11}\ {}_iS_{22} -\ {}_iS_{12}\ {}_iS_{21}}\ (rad^{-1}) \\ -\frac{{}_iS_{21}}{{}_iS_{11}\ {}_iS_{22} -\ {}_iS_{12}\ {}_iS_{21}}\ (m^{-1}) & \frac{{}_iS_{11}}{{}_iS_{11}\ {}_iS_{22} -\ {}_iS_{12}\ {}_iS_{21}}\ (rad^{-1}) \end{bmatrix} \tag{13}$$

The resulting i^{th} measured deflection error vector is then given by,

$${}_iS^{-1}\ \varepsilon^i_{\Delta X_M} =$$
$$\begin{bmatrix} \frac{{}_iS_{22}}{{}_iS_{11}\ {}_iS_{22} - {}_iS_{12}\ {}_iS_{21}}\ (m^{-1}) & -\frac{{}_iS_{12}}{{}_iS_{11}\ {}_iS_{22} -\ {}_iS_{12}\ {}_iS_{21}}\ (rad^{-1}) \\ -\frac{{}_iS_{21}}{{}_iS_{11}\ {}_iS_{22} -\ {}_iS_{12}\ {}_iS_{21}}\ (m^{-1}) & \frac{{}_iS_{11}}{{}_iS_{11}\ {}_iS_{22} -\ {}_iS_{12}\ {}_iS_{21}}\ (rad^{-1}) \end{bmatrix} \begin{bmatrix} \varepsilon^{i,1}_{\Delta X_M}(m) \\ \varepsilon^{i,2}_{\Delta X_M}(rad) \end{bmatrix} \tag{14}$$

$\varepsilon^{i,j}_{\Delta X_M}$ is the j^{th} element of $\varepsilon^i_{\Delta X_M}$. As can be seen from equation 14, ${}_iS^{-1}\ \varepsilon^i_{\Delta X_M}$ is dimensionless. Consequently, $G_C\ \varepsilon_{\Delta X_M}$ is also dimensionless. This can be seen in equation 15.

$$G_C\ \varepsilon_{\Delta X_M} = \begin{bmatrix} {}_1S^{-1} & & 0 \\ & {}_2S^{-1} & \\ 0 & & \ddots \end{bmatrix} \begin{bmatrix} \varepsilon^1_{\Delta X_M} \\ \varepsilon^2_{\Delta X_M} \\ \vdots \end{bmatrix} = \begin{bmatrix} {}_1S^{-1}\ \varepsilon^1_{\Delta X_M} \\ {}_2S^{-1}\ \varepsilon^2_{\Delta X_M} \\ \vdots \end{bmatrix} \tag{15}$$

3 Parameter scaling

Parameter scaling is performed to improve the conditioning of the regressor matrix. This problem generally occurs when the parameters being identified have different units with vastly different magnitudes [3]. Parameter scaling is then done to improve the conditioning of the regressor matrix. However, in case of stiffness identification of robots, all parameters have the same unit and the expected magnitudes are generally same. Consequently, parameter scaling is not necessary for stiffness identification of robots.

References

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