# "It from bit" and "Law without law": The route of information theory towards the refoundation of physics

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**Abstract**

We propose an explicit framework for realizing the Wheeler's "It from bit" doctrine that information is the foundation of physics. Based on a Bayesian information-theoretic interpretation of quantum mechanics, we construct a purely information model of universe, providing a new paradigm for quantum gravity. The universe is thereby equated to a gigantic memory that can be specified by its storage capacity and its current entropy. From these sole two inputs, it is possible to infer the physical laws and calculate most physical constants. The theory requires drastic conceptual changes since all physical concepts are necessarily emergent, whether time, space or energy as well as fields, matter and radiation. In return, a large number of standard puzzles, like the information paradox, the asymmetry matter/antimatter, the origin of discrete symmetries or the source of universe expansion become simple platitudes. This version is preliminary.

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# 1 Introduction

Information is at the heart of physics and especially, preservation of information is perhaps the main challenges of cosmology. In so far, can physics be downright identified with information as originally suggested by R. Landauer [1] and J. A. Wheeler [2]? This article tentatively adopts this paradigm by proposing a concrete implementation of the concept within the framework of Bayesian inference theory. Although perfectly definite, the universe is uniquely described in terms of Bayesian probability applied to abstract Boolean variables, that is, in a sense, "law without law"[1].

## 1.1 The quest for information in physics

The early involvement of information in natural sciences took place *avant la lettre* in thermodynamics in the 19th century. Initially viewed as the science of energy, thermodynamics gradually emerged in the 17th and 18th century with the development of steam engines, but the first truly scientific achievement was based on a deep contemplation of the impossibility of perpetual motion by Sadi Carnot formulated in 1824 in a celebrated monograph, *Reflections on the Motive Power of Fire*. The equivalence of work and heat was established by

[1]The meaning we give to this expression is different from that of Wheeler in Ref. [3], who meant that there is indeed a law but it is created at each moment and not determined in advance. By contrast we do mean here that the only "law" is effectively the absence of any physical law, while observation is sustained by a strict Aristotelian logic whose durability constitutes perhaps the only one mystery.

James Joule in 1843 and finally the equivalence of energy and mass by Albert Einstein in 1905.

The indisputable link of thermodynamics with information relates to entropy, initially developed by Rudolph Clausius in 1865 as a purely physical concept. It is the famous Maxwell's demon, conceived by James Clark Maxwell in 1867 that first hinted at a close relation between probability and thermodynamics, formally proposed by Ludwig Boltzmann in 1872 and Josiah Willard Gibbs in 1876. The next significant step, due to Lèo Szilard in 1929 [4], implicitly grasped the concept of information by revisiting critically the Maxwell's demon puzzle.

In 1948, C. E. Shannon [5] founded the very information theory as such, independently of physics while construing the mathematical concept of entropy as a measure of uncertainty in its accurate physical sense. In 1956, Léon Brillouin [6] provided a keen interpretation of the equivalence of information entropy and thermodynamic entropy and coined the term "negentropy" synonymously of information, as opposed to uncertainty.

The final step to identify the two conceptions of entropy is due to Rolph Landauer, who showed in 1961 [1] that any erasure of information is necessarily a dissipative process. Although this is not always fully acknowledged, since erasure of information is by definition an increase of entropy, the essence of the Landauer principle is basically a complete identification of the two conceptions, allowing later C. H. Bennett to definitively resolve the Maxwell's demon puzzle [7]. Closing the debate, this identification has been confirmed experimentally in 2012 [8–11]. The stunning equivalence of two concepts coming from seemingly unrelated fields marks certainly a major milestone towards the comprehension of physics, similar to the equivalence of gravitational and inertial masses in General Relativity.

At this stage, one can argue that thermodynamics goes far beyond a simple theory of energy and actually merges with information theory as a universal concept that governs all physics and ensures its consistency [12]. This opens a new horizon sometimes viewed as the ultimate explanatory principle in physics, with the famous cross slogans, "It from bit" by J. A. Wheeler [2] and "Information is physical" by R. Landauer [13].

Enhancing this perspective, two issues have emerged. The first concerns *Bayesian probability*, which suggests a direct connection between information and quantum physics. In 1957, E. T. Jaynes incorporated the Shannon's concept of entropy in the Bayesian inference theory [14][2] and pointed out in 1989 that the Bayesian model is strongly reminiscent of quantum formalism [15]. In 2002, C. M. Caves, C. A. Fuchs, and R. Schack [16] proposed to understand quantum probability within a Bayesian framework and Fuchs coined the term "QBism" [17] for "Quantum Bayesianism" to describe this conception. Supporting this view, we have shown explicitly that in fact quantum information is nothing but classical information implemented by Bayesian inference theory [18].

The second issue concerns an amazing link between thermodynamics and *General Relativity*. In 1973, Jacob Bekenstein [19] found that a black hole has an entropy proportional to the area of its horizon. This was confirmed by S.W. Hawking [20] in 1975 while J. M. Bardeen *et all* [21] established the black-hole thermodynamics. Next, P. C. W. Davies in 1975 [22] and W. G Unruh in 1976 [23] proved that uniformly accelerated observers experience a thermal bath at a definite temperature. A major turning point was marked in 1995 by Ted Jacobson [24] demonstrating that conversely the laws of General Relativity can be deduced from the thermodynamic equilibrium. Inversely, observation of the spiral galaxy dynamics hinted at limits to the theory of general relativity, expressed by Mordehai Milgrom [25] in the form of a maximum acceleration on the cosmic scale. At last, a surprising consequence of these studies is that the entropy in the universe is not distributed proportionally to the volume but to the border area of space, leading in 1994 to the holographic theory [26] of Leonard Susskind [27] and Gerard 't Hooft [28] and raising a questioning on the very status of spacetime.

Undoubtedly, the slogans by Wheeler and Landauer appear more and more plausible in the scientific community. For instance, Norman Margolus exhibited in 2003 an example in

[2]Surprisingly enough, he stated in 1957 that information and thermodynamics entropies should be distinguished, whereas all his works demonstrate the opposite.

which “Nature” is identified with a computer [29].

### 1.2 Rooting explicitly physics in information

In this article, as an extension of these discoveries, we propose a road to explicitly root physics in information. Beyond the concept of entropy, this requires to identify any physical quantity, e.g, energy, fields, particles, space, time, with an information concept. Such a paradigm has a price at the outset: we have to give up on the principle of reversibility in physics. In addition, a large number of standard notions must be completely revisited, so that this refoundation must be considered at this stage as preliminary. Nevertheless, the results seem extremely attractive and in our view, this approach constitutes a major step towards the theory of quantum gravity, that is a unique framework compatible with all experimental data and encompassing all physics. Such a level of generality requires an ultimate simplicity, what can be posited by the slogan “a law without Law”.

Quantum Bayesian inference theory is the technical tool of the paper and in particular, our model for Qbism [18] is widely a prerequisite, while our guideline is primarily the conventional thermodynamics. From standard physics, we retain some experimental results to calibrate the theory, essentially the value of the Hubble constant to set the age of the universe and that of the Milgrom’s acceleration to determine its storage capacity. Of course, these parameters need to be refined afterwards by comparison with more precise data on their consequences.

We adopt the perspective of the philosopher Karl Popper, according to whom the scope of science is confined on the only falsifiable questions. Since hypotheses based on clues non observable for ever are *ipso facto* non falsifiable, we propose to identify the physical universe with the ensemble of the available information. This means that we deliberately ignore any unobservable entity in an open universe and forgo any specific ontology beyond pure information. For instance, the concepts like multiverses are beyond the scope of this paper. However, excluding ontologies as such does not mean that we rule out various representations where appropriate, including spacetime, particles or fields. Theses useful representations will be regarded as “emergent” from the information framework.

Let us begin with a short draft of the theory which arises to be dramatically simple in its principle.

### 1.3 Refoundation of physics in a nutshell

The universe is conceived as the logbook of all available information, without reference to any preexisting ontology. Inputs are obtained by a question-and-answer procedure and then processed by Bayesian inference.

The universe is thus identified with a gigantic memory. By observing the movement of spiral galaxies, one can deduce that its storage capacity is bounded to about $10^{126}$ bits. This capacity, or rather its square root, is identified with a paramount parameter, namely the total energy of the universe. In accordance with the Bayesian theory of information, evolution is seen as a race towards maximum entropy, so that basically information is not conserved. However, the fact that the present universe is not already in its state of maximum entropy calls for a major constraint, namely a limit of entropy at the present epoch. This is nothing more than a definition of time. As a result, the current universe displays an age, assumed by observation of the Hubble constant to be about 14 Gyrs.

These only two parameters, storage capacity and current entropy, constitute the fundamental prior information. They are sufficient to derive the statistical properties of the universe in its evolution towards the state of maximum entropy. Theses are the “laws of physics”, also called “laws of motion”.

Refoundation of physics is thus to inventory and interpret these statistical properties compared to standard physics. Proceeding by Bayesian inference given the fundamental prior information, this task is just technical implementation of plain information theory. Therefore, from Ref. [18], the universe must be identified with a so-called “Bayesian theater”, based essentially on the standard Hilbert space of quantum mechanics, itself constructed from the initial batch of Boolean variables required by the question and answer procedure.

Of course, there is a lot of contingent additional parameters, for instance, in Wigner words [30], "that we are on the Earth, that there is a Sun around, and so on". They are viewed as initial conditions as opposed to laws of motion. They can be introduced as further updates of the Bayesian prior in our model. But in any case, they are unable to modify the statistical properties of the universe as a whole because they remain in a very limited quantity compared to the gigantic number of variables.

Finally, it happens that most of standard physical notions can be retrieved as particular pieces of information in the Bayesian theater. This is the core of the present article. Beyond time and energy, the most notable concept is space, resulting from the possibility of changing the batch of queries at will in the question-and-answer procedure. It turns out that the increase in entropy over time then results in an expansion of the universe. An unexpected bonus lies in the recovery of the so-called "holographic principle". The differential links between entropy and energy introduce various temperatures interpreted as so many standard fields, like gravity, electromagnetism, strong and weak fields. In particular, gravity governs the universe expansion. Physical massive objects emerge as Boolean formulas on the batch of queries. Remarkably, dark matter and black-holes find a particularly simple information definition, namely, as deterministic and completely random degrees of freedom respectively. Matter and antimatter appear by pair whereas their apparent asymmetry results from particular perception schemes.

The standard model of particles emerges from evolution of the simplest massive objects, identified with the standard fermions. This will be presented quantitatively in a further version of this paper.

# 2 The cosmic Bayesian theater

Already suggested by Einstein in 1956 [31] and explicitly proposed by G.'t Hooft in 1978 [32, 33], it is often accepted that quantum gravity must fall within a discrete framework. Indeed this is already the case of loop quantum gravity, one of the main approaches to quantum cosmology, others being especially string theory and non-commutative geometry. This departs from the standard conception of physics, particularly general relativity but also quantum mechanics, even if the early introduction of quanta by Max Planck in 1901 [34] involves precisely a discretization of light energy. Now, we argue that Quantum Bayesianism must likewise offer a discrete framework for quantum gravity, so that the universe can be identified with a "Bayesian theater", that is the information-theoretic framework sustaining quantum theory [18].

## 2.1 Fundamentals

Consider a particular observer $\mathcal{O}$ who contemplates the cosmos. By definition, the observer collects information through a batch of queries. The best way to handle the responses to these queries is Bayesian analysis. In turn, for a Bayesian observer, the information thus available is the one and only source of knowledge about the universe. We propose therefore to identify the universe with this available information.

**Assumption 1** (Information universe)**.** *For a particular observer $\mathcal{O}$, the universe is the ensemble of the available information.*

As a result, we regard throughout the words "universe" and "observable universe" as synonymous.

### 2.1.1 Degrees of freedom

In standard cosmology, the entropy of all events that have been, are or will be observable, is called "event entropy" and is finite (see below Sec 2.1.2). In a Bayesian framework, this means that the available information as a whole, that is, the storage capacity of the memory is finite. We propose to adopt this result from the start.

**Assumption 2** (Finite universe)**.** *The information contained in the universe is finite.*

Since continuous variables would involve infinite information, Assumption (2) implies that the universe is basically *discrete*.

Now, still in standard cosmology, the universe is evolving. In a Bayesian framework, this means that *information is not conserved*. However, this immediately contradicts the common belief.

**Assumption 3** (Stability)**.** *The information contained in the universe is not stable.*

*Hint.* This is another way of saying that the information universe is evolving even if the concept of stability is not very precise at this stage. Owing to its importance, this assumption will be more widely elaborated later (Sec. 2.2). □

To proceed further, we need to encode this information. Without loss of generality, we adopt a dichotomic gauge, meaning that information is encoded through a finite batch of binary digits[3]. At last, we suppose that the physical data are expressed in Planck units, meaning that the standard gravitational constant $G$, the reduced Planck constant $\hbar$, the speed of light in vacuum $c$ and the Boltzmann constant $k_B$ are normalized to unity at the present epoch, so that the model is formally dimensionless. Occasionally, we will however restore dimensional units for clarity.

Let $N_u$ bits denote the maximum information contained in the universe, that is the storage capacity of the memory. To comply with the uses of information theory, we choose to identify one discrete degree of freedom with one "bit", derived from base 2, although it is common in physics to use instead one "nat", derived from natural logarithms. Nevertheless, when all other variables are formulated in Planck units, entropy in physics is expressed in natural units and not in bits, so that we will adopt the most convenient gauge for entropy where appropriate.

**Definition 1** (Degree of freedom)**.** *A discrete degree of freedom is one dichotomic choice.*

We assume that the data are estimated by Bayesian inference from the observed events. This means that the task of the observer is not to determine the exact truth value of all degrees of freedom, which is totally inaccessible, but only to assign a probability to their occurrence.

According to our interpretation of quantum information [18], this requires that physical data are expressed in quantum formalism, which makes it imperative to use the essential tool of this theory, namely a "Bayesian theater". Basically, a Bayesian theater brings together all compatible batches of Boolean variables to describe the dichotomous choices. We will see just below that each particular batch constitutes an *observation window* and the change of observation window requires to construct the Hilbert space of standard quantum mechanics.

Throughout the article, the reader is assumed to have a minimum familiarity with this tool although we will try to recall the major points before their first use.

**Definition 2** (Cosmic Bayesian theater)**.** *The cosmic Bayesian theater is the Bayesian theater of $N_u$ discrete degrees of freedom perceived by a particular observer $\mathcal{O}$. It is associated with a $d_u$-dimensional Hilbert space, $\mathcal{H}_u(\mathcal{O})$ where $d_u = 2^{N_u}$ and a quantum state defined by a statistical operator $\Pi_u$[4] of rank $r_u \leq d_u$.*

To get an idea, we will propose in Proposition (70) that $N_u \simeq 6.23 \times 10^{125}$, (see also Sec. 2.1.2 just below).

[3] Such a gauge choice greatly simplifies the presentation. In principle, conversion into other gauges could be computed exactly but this would require infinite precision. Yet, infinite information is impossible in a finite universe! As a result, the genuine gauge, if any, is simply unfalsifiable.

[4] To avoid confusion with the classical energy densities, $\rho_m$, $\rho_\Lambda$, etc., used in cosmology, we refer to the standard "density operator" of quantum information usually symbolized by $\rho$ as a "statistical operator" symbolized by $\Pi$. This was the initial terminology employed by von Neumann.

**Observation windows.** The universe is necessarily depicted through a particular set of Boolean variables depending on the observer in terms of location, velocity, orientation, experimental setup, etc., characterizing a particular viewpoint. This corresponds to a specific allocation of $N_u$ Boolean variables to the $N_u$ discrete degrees of freedom. Ideally, one would prefer to choose mutually independent variables. In principle, this is always possible but in practice this is not necessarily granted at the outset. For example, the observer experiments only from Earth or at least from the solar system. Such an allocation is called "observation window" of the Bayesian theater in our information-theoretic interpretation of quantum information Ref. [18]. We will see that the choice of relevant windows is essential to generate the emergence of spacetime.

**Definition 3** (Observation window)**.** *An observation window is the allocation of a specific batch of $N_u$ Boolean variables $\mathsf{X}_j,\ j \in [\![1, N_u]\!]$ to the $N_u$ discrete degrees of freedom of the Bayesian theater.*

Each observation window defines a sample set $\Gamma_u$ of $d_u = 2^{N_u}$ complexions or *classical states*, $\gamma_i \in \Gamma_u$[5], with $i \in [\![1, d_u]\!]$. The classical states can also be regarded as the basic vectors of a real-valued probability space $\mathcal{P}_u$. At the same time, each window specifies a particular basis in the $d_u$-dimensional Hilbert space $\mathcal{H}_u$ associated with the Bayesian theater. In general, we will note its basic vectors $|e_i\rangle$ or $|\gamma_i\rangle$ with $i \in [\![1, d_u]\!]$ in correspondence with the set of classical states. When the $N_u$ variables are mutually independent, *the window is "completely divisible"*, meaning that both the Hilbert space $\mathcal{H}_u$ and the statistical operator $\Pi_u$ are respectively the Kronecker products of $N_u$ 2-dimensional partial Hilbert spaces $\mathcal{H}_j$ and $N_u$ partial $2 \times 2$ statistical operators $\Pi_j$ with $j \in [\![1, N_u]\!]$. In addition, each partial $\Pi_j$ is diagonal, leading to a diagonal global operator $\Pi_u$. The window is then termed *"principal"*.

Notice that the initial allocation of a variable batch in a "source window" requires deciding for each degree of freedom what is the "TRUE" (or 1) truth value and therefore what is the "FALSE" (or 0) truth value. This involves $N_u$ *gauge choices* between the Boolean variables and their negation. In Ref. [18] we have called "discrete Boolean gauge" such an initial choice.

**Definition 4** (Discrete Boolean gauge)**.** *A discrete Boolean gauge is the initial choice for each degree of freedom in the source window of either a specific Boolean variable or of its negation.*

In fact, the only designation of an observer $\mathcal{O}$ must be equated to the attribution of a specific discrete Boolean gauge in the source window, because this sole definition implies necessarily $N_u$ dichotomic choices. We propose to name this gauge the "observer gauge".

**Definition 5** (Observer gauge)**.** *The observer gauge is the specific discrete Boolean gauge implicitly adopted by the observer $\mathcal{O}$ for describing the Bayesian theater.*

This inescapable gauge selection is not so innocuous because the choices are made in one step, which on the one hand implicitly defines the concept of simultaneity and on the other hand involves the implicit creation of $N_u$ bits of information. This will be clarified later with the Mach's principle (Assumption 9).

Now, the object of physics is to determine the likelihood of any event, that is, to assign to every classical state $\gamma_u \in \Gamma_u$ a probability $\mathbb{P}(\gamma_u|\mathcal{O})$ conditional on the observer gauge. As a result, all probability functions like entropy, statistical operators, etc., are implicitly computed with respect to this unique Boolean gauge. For simplicity, unless it could be confusing, we will omit throughout this condition $(|\mathcal{O})$. Now, the probabilities $\mathbb{P}(\gamma_u)$ can be computed by Bayesian inference from a basic prior information and updated [35] by observation where appropriate. However, the status of specific updates is different from that of the initial prior because they can only affect a tiny part of the universe. In other words, the initial prior information determines the physical laws whereas observation depicts particular objects.

[5] To avoid confusion with the density ratios, $\Omega_m$, $\Omega_\Lambda$, etc., used in cosmology, we symbolize the "classical states" $\omega \in \Omega$ in Ref. [18] by $\gamma_u \in \Gamma_u$. The subscript $u$ stands for "universe". When relevant, we will replace $u$ by $t$ (for "time").

By anticipation, in the present article, we will limit the basic prior information to the storage capacity $N_u$ bits of the register (about $6.23 \times 10^{125}$ bits) and the current age of the universe (about $14.45 \times 10^9$ years). This will be clarified in Assumption (8) below.

**Observable.** The information content of the universe can be recovered by means of observable "measurements", each carried out with respect to a specific batch of $N_u$ Boolean variables $\mathsf{X}_i$, $i \in [\![1, N_u]\!]$. Technically, each particular measurement requires a particular observation window called "proper window" of the observable and the result of the measurement is, by definition its Bayesian estimation, that is to say, a *linear combination of the probability of the $d_u$ classical states.* Incidentally, the requirement of a particular window becomes implicit when using the quantum formalism and is even fundamentally the justification for this theory (see Ref. [18]).

As already mentioned, the number of degrees of freedom, $N_u$, is closely related to the standard "event entropy" of the universe.

### 2.1.2 Cosmic event entropy

In conventional thermodynamics, the fate of an isolated system is to end up in a state of maximum entropy. We do assume that this result is valid for the full universe, as first proposed by Jean-Sylvain Bailly in 1777 and supported by William Thomson in 1850. Yet, the idea of a "heat death of the universe" was challenged by Max Planck on the grounds that entropy of an infinite system is inconsistent. In the present model, this objection is no longer valid with regards to Assumption (2). Therefore, the ultimate statistical operator in the Hilbert space, say $\Pi_{\max}$, will be the identity matrix $\mathbb{1}_u$ of dimension $d_u$, normalized to unit trace, $\Pi_{\max} = \mathbb{1}_u/d_u$, whose von Neumann entropy is $S_{\max} = \log_2 d_u = N_u$ bits or $N_u \ln 2$ nats. This ultimate quantum state is invariant by unital channels and irrespective of the window, corresponds to complete randomness in the sample set $\Gamma_u$ (see Ref. [18]).

In standard cosmology [36, 37], the limit of the future observable universe for a particular observer is called the *cosmic event horizon* [38] (CEH). We will define the horizon later (see below Sec. 3.7) but we already propose to identify $S_{\max}$ with the event entropy of standard cosmology.

**Assumption 4** (Cosmic event entropy)**.** *The cosmic event entropy is the ultimate von Neumann entropy $S_{\max} = -\mathrm{Tr}(\Pi_{\max} \log_2 \Pi_{\max})$ of the statistical operator $\Pi_{\max}$.*

This assumption implies immediately the following result:

**Proposition 1** (Number of variables)**.** *The number of binary degrees of freedom of the universe, $N_u$, is the cosmic event entropy $S_{\max}$ expressed in bits.*

*Proof.* We have $S_{\max} = \log_2 d_u = N_u$ bits. □

The present estimation of the cosmic event entropy by C.A. Egan and C. H. Lineweaver [39] is $S_{\max} = 2.6 \times 10^{122}$ nats, which leads to $N_u = 2.6/\ln 2 \times 10^{122} \simeq 3.7 \times 10^{122}$. However, it seems that this estimation leaves out much of the current dark matter in the present model. Based on the velocity of stars in spiral galaxies, we will propose instead $N_u = 6.23 \times 10^{125}$ in Proposition (70) below.

## 2.2 Evolution of the universe

As first proposed by Ludwig Boltzmann [40], it is often conjectured that the arrow of time is derived from thermodynamics [41]. The current model is constructed in accordance with this view.

However, in standard physics, the status of thermodynamic is not very clear and perhaps inconsistent with regard to reversibility. On the one hand, thermodynamics is certainly taken for granted and extensively used in all areas of physics. But on the other hand, standard theories are fundamentally reversible, based on the time symmetry of physical laws in classical physics and conservation of probability in quantum physics. Already in 1874, Loschmidt objected that it must have been impossible to conceive of an irreversible process. Bypassing Loschmidt's objection, Gibbs' entropy [42] is constructed while acknowledging micro-reversibility[6], so that in the end, irreversibility appears as an exogenous feature, ultimately explained in classical physics by the boundary conditions, the complexity of macro-systems and subsequent coarse-graining [35], and in quantum mechanics, by decoherence [44] meaning coupling with claimed "random" unobservable extra degrees of freedom from an open system and leading to deterministic chaos. Be that as it may, irreversibility in standard physics is in fact interpreted as an appearance or even a conspiracy [12], perfectly compatible, for example, with a cyclic evolution of the universe [45]. This leads to paradoxes, and especially the famous loss of information in black-holes, reminiscent of the inconsistency of the aether at the turn of the 19th/20th century.

At any rate, observation is fundamentally irreversible because information captured by an observer accumulates over time[7] (see particularly the illuminating explanation by Jaynes in Ref. [15], Part "Diffusion"). This corresponds to the Gibbs' "thermodynamics irreversibility" which also governs Bayesian inference theory. In other words, an inference problem is fundamentally dissymmetrical between the past and the future, so that the standard conception of physics is simply irrelevant in a Bayesian framework. In the current model based explicitly on the theory of inference, we are therefore committed to taking irreversibility as a starting point, as already expressed by Assumption (3) above.

### 2.2.1 The motor of evolution

In standard physics, evolution is the change of the universe over time. In a Bayesian framework, the state of the universe is the most likely probability distribution taking into account all prior constraints while prohibiting the implicit introduction of any extra hypothesis, i.e., exhibiting the maximum uncertainty compatible with the prior information. Technically, this corresponds to the maximum Shannon entropy. Now, we immediately run into a problem, namely that the most likely distribution is directly the ultimate state $\Pi_{\max}$ of the universe: A steady world at maximum entropy.

Why the universe does not rush in this stable state? At least one reason is that entropy is a continuous function, so that it cannot pass from the minimum, say $S_{\min}$, to the maximum, $S_{\max}$, without going through all intermediate levels. Therefore, if the initial state of the universe is not the stillborn state of maximum entropy, it is possible to observe the intermediate states provided that an extra prior information is imposed, namely that the current entropy of the universe is limited by a bound $S_t \in [S_{\min}, S_{\max}]$. This new prior information is nothing but a definition of time as we will propose formally in Assumption (18) below. In other words, time labels a continuous set of snapshots at intermediate entropy.

As a convenient *convention*, we propose to set the minimum entropy to zero, $S_{\min} = 0$.

**Assumption 5** (Initial entropy)**.** *The minimum entropy at the origin of time $t = 0$ is zero, $S_{\min} = 0$.*

*Hint.* The entropy is conveniently computed with respect to the observer gauge, Definition (5). Since the choice of this gauge can be viewed as creation of information, we can suppose that the entropy is reset to zero on this occasion, that is, $S_{\min} = 0$ at $t = 0$. However, this assumption is rather a convention because it is unfalsifiable. □

The task of the Bayesian observer is now to assign a probability distribution on the sample set $\Gamma_u$ under the sole assumption that the global entropy $S_u$ is bounded above by $S_t$.

**Assumption 6** (Principle of evolution)**.** *The state of the universe is the most likely probability distribution whose entropy $S_u$ is bounded by a monotonically increasing function $S_t$ of a real parameter $t$, so that $S_u(t) \leq S_t$.*

[6] Gibbs' main argument is that "thermodynamic reversibility" has nothing to do with "mechanical reversibility". This claim was later endorsed by Planck and next by Jaynes (see Jaynes [43]).

[7] This is perfectly consistent with the fact that time is irrelevant in a logical statement.

Then, the most likely distribution of maximum entropy corresponds exactly to this bound as we will show just below. In other words the *Bayesian equilibrium* of the universe at time $t$ is attained for $S_u(t) = S_t$. As a result, the increase of entropy over time is the universal cause of evolution. This supports in particular a proposal by Erik Verlinde [46] that gravity forces are of entropic nature, but beyond we must regard the increase of cosmic entropy in its race towards its maximum as the unique impetus of all changes in the universe. In short, *evolution is tautologically driven by the time itself.* To make evolution possible, a hypothesis similar to Assumption (6) is essential in any Bayesian framework, with the conclusion that irreversibility is completely inescapable.

In sharp contrast, evolution over time in standard physics is described by unitary channels and then is reversible. To understand the source of this conflict, note that in quantum theory, the unitarity of the scattering matrix is established by positing as an axiom that both the input and the output are waves vectors, that is *pure states*, i.e., both with zero entropy (see Ref. [18]). Reversibility, based on the conservation of probability, is thus just a consequence of this quasi-universal assumption, which remains nevertheless controversial [47]. In the present model, the hypothesis that the state of the universe is a pure wave function is irrelevant: Input and output are perfectly allowed and even committed to be mixed states so that the proof breaks down. Nevertheless, conservation of probability does obviously still hold. We will show that its physical meaning is found only in the first law of thermodynamics, i.e., conservation of energy (see Definition 24 below).

We will show later that the passage of time is directly expressed by the expansion of the universe. Therefore, we can characterize *standard physics as the approximation in which the expansion of the universe is neglected* and deduce that the entropy of the "standard universe" is then constant, so that time becomes just a parameter.

**Assumption 7** (Standard physics)**.** *Standard physics is a differential description of the universe in which the cosmic entropy is constant and the expansion neglected.*

*Hints.* Except in cosmology, standard physics only concerns a tiny part of spacetime at the scale of the universe. This can be compared to a linearization in the neighborhood of a particular point in differentiable systems. In other words, standard physics must be regarded as a differential description at constant cosmic entropy of a tiny patch of an ideal universe, out of the complexity of real life. □

We will refer to the most likely distribution at entropy $S_t$ as the "instantaneous equilibrium of the universe".

**Definition 6** (Instantaneous equilibrium of the universe)**.** *The instantaneous equilibrium of the universe at entropy $S_t$ is the most likely state whose entropy is bounded by $S_t$.*

Formally, we thus identify this instantaneous state with the statistical operator of the universe $\Pi_u(t)$, which means that the value of any observable is assimilated to its expectation with respect to $\Pi_u(t)$. In Sec (3.4) below, we will propose to equate the parameter $t$ with the standard cosmic time of cosmology.

Now, a definite entropy corresponds to an instantaneous equilibrium of the universe, i.e., $S_u(t) \leq S_t$. Assume that the statistical operator is expressed in a *principal window* by $\Pi_t = Diag(\lambda_1, \lambda_2, \ldots, \lambda_{d_u})$, where the eigenvalues $\lambda_i$ represent the probability of the classical states (See Ref. [18]). Then, we have technically to solve the mere problem

$$\begin{aligned}
&\text{Maximize } \sum_{i=1}^{d_u} -\lambda_i \log \lambda_i \\
&\text{subject to the constraints} \\
&\lambda_i \geq 0 \quad ; \quad \sum_{i=1}^{d_u} -\lambda_i \log \lambda_i \leq S_t \quad ; \quad \sum_{i=1}^{d_u} \lambda_i = 1
\end{aligned} \tag{1}$$

By simple inspection, the objective function is also a constraint. As a result, the problem is degenerate and the solution can be chosen anywhere on the boundary of the attainable domain. In other words, we only have to satisfy to

$$\lambda_i \geq 0 \quad ; \quad \sum_{i=1}^{d_u} \lambda_i = 1 \quad ; \quad \sum_{i=1}^{d_u} -\lambda_i \log \lambda_i = S_t \tag{2}$$

so that any solution to Eq. (2) is formally feasible.

We now arrive to the basic conjecture to refound physics. Beforehand, it is worth quoting E. Wigner who precisely justifies Assumption (8) just below:

E. Wigner: *If we look at the body of our knowledge in physics, we must note that it divides the phenomena into two groups, in fact into two sharply distinguished groups. Into the first group falls our knowledge of what may be called initial conditions, that we are on the Earth, that there is a Sun around, and so on. We maintain that these initial conditions are accidental, that there is no precise theory which would explain them, that there can be no such theory. Into the second category falls our knowledge of the "laws of motion" which tell us how to foresee the future state of the World, assuming that we know the present state. These laws, we are convinced, are exact, of almost unbelievable precision, and of a beauty and simplicity which is much greater and deeper than any that we could have invented* [30].

In the present model, we name "physical laws" the Wigner's "laws of motion" and identify the "initial conditions" with Bayesian updates obtained by further observations, introducing what we call "contingent data". By definition, they are new and therefore totally irreducible to previous data or, in other words, completely "random".

**Assumption 8** (Physical laws)**.** *Physical laws are equated to the statistical properties of the universe, given only its storage capacity and its current global entropy.*

*Hint.* The universe is investigated as a Bayesian theater. The laws of physics describe its general characteristics while occasional objects demand particular observations, which means one-off specific updates of the prior information. Since there are gigantically many variables compared to a very limited number of observations, such one-off specific updates can only provide limited information and are therefore unable to modify the statistical estimation of the universe as a whole. □

Now, the "laws of motion" of Wigner are the statistical properties of the universe. Strictly speaking, they are not "exact" but their accuracy is about $1/\sqrt{N_u}$, that is about 60 significant decimal places. Moreover, their "beauty and simplicity" is none other than that of the binomial probability distribution, as we will show shortly, Proposition (5) below. They can lead to extraordinary phenomena, such as life, because of the gigantic number of variables that make almost certain the occurrence of events *a priori* totally improbable.

Now, the present article aims to compute the statistical properties of the universe irrespective of any update provided by specific observations, that is to say, establish these "laws" as the foundation of physics. *In short, "laws" emerge as the statistical properties of a "no-law" background.*

**Definition 7** (Fundamental prior)**.** *The fundamental prior is composed of the mere condition Eq. (2) without further update.*

Under Assumption (8), that is, conditional on the fundamental prior, Eq. (2) contains all physical laws! First, it allows to define a *snapshot* on the entire register and thus *establishes the concept of simultaneity of the $N_u$ binary variables required to evolve together*, whereas they were regarded hitherto as completely unrelated. By construction, the Bayesian theater is compatible with all statistical distributions of binary variables. Of these, a snapshot is a section with a specific entropy.

**Definition 8** (Snapshot)**.** *A snapshot is a section of the universe characterized by a definite entropy $S_t$.*

The notion of snapshot has introduced the concept of simultaneity. This appears as a Bayesian requirement, in that it is primarily required by the observer to be able to observe. It will be formalized by the so-called "Mach's principle", Definition (9), in the next section.

**Proposition 2.** *In a specific snapshot, the universe is in a particular state expressed in any observation window by a statistical operator* $\Pi_t$*.*

*Proof.* The entropy $S_t$ is necessarily the von Neumann entropy of a specific quantum state. □

Actually, "snapshot" and "quantum state" coincide. When no confusion can occur, a snapshot will be henceforth referred to by its entropy $S_t$.

### 2.2.2 Mach's principle and snapshots of the universe

Newton devised an *absolute space* invariant at any time regardless of its content. This conception was criticized by Mach as problematic, so that the alternative was put forward by Einstein under the name of "Mach's principle" [48]. In the present model, the Mach's principle can be thought as the assertion that a patch of the world at any moment is necessarily defined in relation to the rest of the universe, which implies that all degrees of freedom must be related. We adopt for clarity the following definition:

**Definition 9** (Mach's principle)**.** *The Mach principle is the property of all degrees of freedom to be linked in any snapshot.*

In the present model, the $N_u$ binary variables observed in a principal window are by construction mutually independent and thus completely unrelated, meaning in particular that they can evolve independently. However, in any snapshot, they are by definition grasped as a whole *by the observer*. Therefore, the mere fact of observing a snapshot creates *de facto* a link between all degrees of freedom, yet this link is not intrinsic but only established by the observer.

**Proposition 3.** *The Mach's principle holds in the cosmic theater in that the classical state probabilities* $\lambda_i$ *in a principal window are linked in any particular snapshot* $S_t$ *by Eq. (2). This is the definition of a snapshot and not a property of the variables and expresses that the degrees of freedom are observed as a whole. This does not create new correlations between the variables themselves which remain independent in the principal window.*

*Proof.* The definition of a snapshot demand to collect each variable in a particular state. These states are thus linked to the snapshot. In a specific observation window, this link constitutes a relationship established by the observer between the variables but which does not affect their behavior in any way. In particular, in a principal window, the variables remain mutually independent by definition. This is not a contradiction because the Mach's principle is a property or rather a definition of the snapshot, which does not concern the variables themselves. □

## 2.3 Structure of the Hilbert space

This section essentially provides technical results. Given the fundamental prior information, Definition (7), we construct the statistical operator $\Pi_t$ acting on the cosmic Hilbert space $\mathcal{H}_u$ and investigate the structure of its eigensubspaces $\mathsf{h}_k$. These are routine and tedious calculations useful for later uses. However, the results are widely unexpected.

We finally introduce two major concepts: *the episodic clock*, Sec. (2.3.4), which shows that surprisingly enough the eigensubspaces $\mathsf{h}_k$ can be considered as clock hands for the cosmic time, and the *directional window group*, Sec. (2.3.5), which is a prerequisite for constructing space-time.

In a principal window, consider the batch of $N_u$ independent variables, say $\mathsf{X}_j$ with $j \in [\![1, N_u]\!]$, taking value in $\{0, 1\}$. From Assumption (3) the register is not stable and therefore all variables can potentially jump in an uncontrolled way from $0 \to 1$ or $1 \to 0$. Now, from Mach's principle and Proposition (3), the outcomes of all variables are observed simultaneously in a single snapshot. Therefore, the jumps are observed at some *discrete* snapshots $S_t$. As a result, the functions $S_t \mapsto \mathsf{X}_j(S_t)$ taking value in $\{0, 1\}$ can be defined as right-continuous and for $S_t < S_{\max}$ only a finite number of jumps can occur in the open interval $]0, S_t[$ (technically, there is no "explosion"). Incidentally, when $S_t \to S_{\max}$ the situation turns out to be explosive at $S_{\max}$.

At last, all variables $\mathsf{X}_j$ are equivalent for the Bayesian observer, i.e., are assumed to experience the same process. In particular, they jump from $0 \to 1$ or $1 \to 0$ with equal probability. As a result, the variables are independent Bernoulli random variables and the batch of binary variables is simply subject to a standard *binomial probability distribution*.

**Proposition 4.** *Given the fundamental prior information, the binary variables $\mathsf{X}_j$ considered in a principal observation window are equivalent irrespective of $j$ in any snapshot.*

*Proof.* By hypothesis, the fundamental prior, Definition (7), is entirely provided in the current framework by Eq. (2) in the initial principal window. From the Bayesian principle of indifference, without further update, all variables $\mathsf{X}_j$ or their negation $\overline{\mathsf{X}}_j$ like all their truth values must be treated on the same footing irrespective of $j$ because there are no grounds in the fundamental prior to make any difference. Since the variables are initially chosen in a principal window, the variables are mutually independent and thus the window remains principal and therefore completely divisible (see Ref. [18]). Only updating the prior could make a difference. □

### 2.3.1 The cosmic statistical operator

Consider the complete batch of mutually independent binary variables, $\mathsf{X}_j$ with $j \in [\![1, N_u]\!]$. Since the variables are independent, they define a principal observation window. Each variable $\mathsf{X}_j$ acts on a partial sample set $\Gamma_j$ and generates an individual 2D-probability space $\mathcal{P}_j$ that can be transcribed into a 2D-Hilbert space $\mathcal{H}_j$ (see Ref. [18]). The global Hilbert space, $\mathcal{H}_u$, is the Kronecker product of the $N_u$ partial Hilbert spaces.

Assume that the prior snapshot is the initial snapshot of entropy $S_0 = 0$. Therefore, all variables $\mathsf{X}_j$ are initially in the *same* partial deterministic state of statistical operator $\Pi_{0j} = Diag(1, 0)$ acting on $\mathcal{H}_j$. Consider now a posterior snapshot $S_t$ with the only new hypothesis $S_t > 0$. Then, from Proposition (4), we must still assign the *same* partial statistical operator $\Pi_{tj}$ to the $N_u$ variables. As a result, a principal partial 2D-basis in $\mathcal{H}_j$ defines a $d_u$-principal basis in $\mathcal{H}_u$.

**Proposition 5.** *When the state $\Pi_0$ is the initial deterministic universe of $N_u$ bits in the snapshot $S_0 = 0$ and without further update, the posterior cosmic statistical operator $\Pi_t$ in any snapshot $S_t > 0$ is the Kronecker product of $N_u$* identical 1-*bit states $\Pi_{tj}$ and expressed in a principal window as*

$$\Pi_t = \bigotimes_{j=1}^{N_u} \Pi_{tj} \quad \text{with} \quad \forall j \quad \Pi_{tj} = \begin{bmatrix} q_t & 0 \\ 0 & p_t \end{bmatrix} \quad \text{and} \quad p_t + q_t = 1, \tag{3}$$

*where $p_t \in [0, 1/2]$ is a probability characterizing the transition from 0 to 1 between the prior snapshot $S_0$ and the posterior snapshot $S_t$. The eigenvalues $\lambda_i(p_t)$ of the statistical operator $\Pi_t = Diag(\lambda_i)$ for $i \in [\![1, d_u]\!]$) with $d_u = 2^{N_u}$ in Eq. (2), take only $N_u + 1$ distinct values, respectively $\alpha_k = p_t^k q_t^{N_u - k}$ of multiplicity $d_k = \binom{N_u}{k}$ with $k \in [\![0, N_u]\!]$. As a result*

$$\Pi_t = \sum_{k=0}^{N_u} \alpha_k \times \mathsf{A}_k \quad \text{with} \quad d_k = \binom{N_u}{k} \quad \text{and} \quad \alpha_k = p_t^k q_t^{N_u - k} \tag{4}$$

*where $\mathsf{A}_k$ is the projection operator $\mathcal{H}_u \to \mathsf{h}_k$ of the Hilbert space on the eigensubspace $\mathsf{h}_k$ of dimension $d_k$. The von Neumann entropy $S_t$ of the Kronecker product is equal to $N_u$ times* *the von Neumann entropy $S_{tj}$ of any individual variable. Thus,*

$$S_t = -\sum_{i=1}^{d_u} \lambda_i \log \lambda_i = -\sum_{k=0}^{N_u} d_k \alpha_k \log \alpha_k = N_u \times S_{tj} = -N_u \times (p_t \log p_t + q_t \log q_t). \quad (5)$$

*For $p_t \in [0, 1/2]$ and $S_t \in [0, N_u]$ (in bits), the mapping $S : p_t \mapsto S_t$ is invertible as $S_t \mapsto p_t = S^{-1}(S_t)$.*

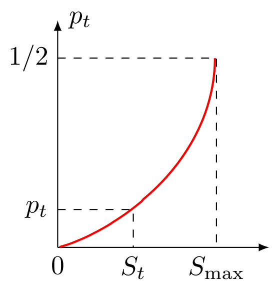
*Proof.* Assume that the prior is the initial snapshot $S_0 = 0$. From Assumption (5), every binary variable $\mathsf{X}_j$ is deterministic, namely, $\mathsf{X}_j = 0$, with zero entropy, so that the individual statistical operators in the prior window are $\Pi_{0j} = Diag(1, 0)$ and the global statistical operator is their Kronecker product $\Pi_0 = Diag(1, 0, 0, \dots, 0)$.

As the variables remain mutually independent, the global state remains the Kronecker product of the individual states. However, the posterior state in no longer deterministic so that the individual posterior states $\Pi_{tj}$ become a statistical combination of $\mathsf{X}_j = 0$ and $\mathsf{X}_j = 1$. While independent, the variables are observed in the same snapshot $S_t$ and thus from Proposition (4) with the same transition probability $p_t$ depending only on $S_t$ irrespective of $j$ as $\Pi_{tj} = Diag(q_t, p_t)$ with $q_t = 1 - p_t$. Initially, we have $p_0 = 0$ and $q_0 = 1$ in the prior snapshot $S_0$ and therefore $p_t \le 1/2$ and $q_t \ge 1/2$ in the snapshot $S_t$. Now, the global posterior statistical operator $\Pi_t$ is the Kronecker product Eq. (3). Expanding the product, we obtain

$$\Pi_t = Diag(\lambda_1, \dots, \lambda_i, \dots, \lambda_{d_u})$$

where $d_u = 2^{N_u}$ and $\lambda_i$ is of the form $\lambda_i = p_t^{\alpha_i} q_t^{\beta_i}$ with $\alpha_i + \beta_i = N_u$ and $\alpha_i, \beta_i \in [\![0, N_u]\!]$. Therefore, there are $n_e = N_u + 1$ distinct products $\alpha_k = p_t^k q_t^{N_u - k}$ repeated $d_k = \binom{N_u}{k}$ times with $k \in [\![0, N_u]\!]$, corresponding to $n_e$ eigensubspaces $\mathsf{h}_k$ of the statistical operator $\Pi_t$. Then, for each index $k \in [\![0, N_u]\!]$ and for all $a \in [\![1, d_k]\!]$, we have $\lambda_i = \alpha_k$ where $i = a + \sum_{\ell=0}^{k-1} d_\ell$. When $k = 0$, $d_0 = 1$, $i = 1$ and $\alpha_0 = \lambda_1 = q^{N_u}$. When $k = N_u$, $d_{N_u} = 1$ and $\alpha_{N_u} = \lambda_{d_u} = p^{N_u}$. Clearly, $\sum_{i=1}^{d_u} \lambda_i = (p_t + q_t)^{N_u} = 1$. The von Neumann entropy $S_t$ of the Kronecker product is also the sum of the $N_u$ individual entropies $S_{tj}$ of the factors $\Pi_j$, Eq. (5). For $S_t = S_{\min} = 0$, we have $p_t = p_{\min} = 0$ by construction and for $S_t \to S_{\max}$, $p_t \to p_{\max} = 1/2$. $\square$

Each binary variable $\mathsf{X}_j$ can fluctuate in an uncontrolled way from $0 \to 1$ and $1 \to 0$ but in the current snapshot, each variable is obviously in one and only one of these states. By definition, we only consider the final balance because any intermediate jump, if any, e.g. $0 \to 1 \to 0$, is not checked and thus simply ignored. We propose to name formally "transition probability" the probability $p_t$ of the transition $0 \to 1$ when the entropy increases from $S_0$ to $S_t$. We propose also to name "binary entropy" the corresponding entropy $S_{tj}$.




**Definition 10** (Transition probability)**.** *The transition probability $p_t \in [0, 1/2]$ is the probability assigned to any binary variable $\mathsf{X}_j$ to jump from 0 to 1 between the prior snapshot $S_0$ and the posterior snapshot $S_t$ without intermediate update.*

Quantitatively, at the present epoch $t_0$, assume that $S_{t_0} = 3.24 \times 10^{122}$ bits and $S_{\max} = 6.23 \times 10^{125}$ bits. The entropy of each binary variable $\mathsf{X}_j$ is $S_{t_0 j} = S_{t_0}/N_u = 2.31 \times 10^{-7}$ bits and thus the transition probability is $p_{t_0} \simeq 0.120 \times 10^{-7}$. Its exact value is extraordinary robust and precise because it is represented by the gigantic number of $N_u$ independent samples.

**Definition 11** (Binary entropy $S_{tj}$)**.** *The binary entropy $S_{tj} = -p_t \ln p_t - q_t \ln q_t$ is the entropy defined by the transition probability $p_t$.*

From Assumption (5), all variables are reset in the initial snapshot $S_0$ and thus start from the same invalid truth value 0. This reset is a shock that will gradually relax to restore balance.

Consider the snapshot of entropy $S_t$. The expected number of valid truth values is $p_t N_u$ and thus the remaining expected number of invalid truth values is $(1-p_t)N_u$. By hypothesis, each variable $\mathsf{X}_j$, $j \in [\![1, N_u]\!]$, is in no way influenced by the other variables, because they are independent, nor by its own current truth value, because it depends on a gauge parameter. Therefore, the jump probabilities from $1 \to 0$ or from $0 \to 1$ are equal, irrespective of the index $j$.

On the other hand, the transition probability, that is the expectation value of $\mathsf{X}_j$, increases from $p_t$ to $p_t + \mathrm{d}p_t$. Therefore, the expected number of valid truth values increases by $N_u \mathrm{d}p_t$. This corresponds to probability increase of the balance of jumps from $0 \to 1$ and those from $1 \to 0$.

**Definition 12** (Jump probability)**.** *The probability of jump is the probability that any binary variable $\mathsf{X}_j$ equal to zero in the snapshot $S_t$ will be 1 in the snapshot $S_t + \mathrm{d}S_t$.*

The jump probability is thus the differential increase of $p_t$ as

$$\mathbb{P}(\mathsf{X}_j(S_t + \mathrm{d}S_t) = 1 \,|\, \mathsf{X}_j(S_t) = 0) = \frac{\mathrm{d}p_t}{\mathrm{d}S_{tj}}\mathrm{d}S_{tj} = \frac{\mathrm{d}S_{tj}}{\ln(q_t/p_t)} \tag{6}$$

### 2.3.2 Parameterizing the universe

The fundamental prior limits to two the number of independent parameters. Let us list some convenient representations.

**Nominal representation $(N_u, p_t)$.** The nominal representation uses $N_u$ and $p_t$. The number $N_u = \log_2 S_{\max}$ is thought as integer but this is not mandatory. Moreover, since it is a gigantic number, it can anyway be approximated by a real-valued variable.

We have

$$\left.\frac{\partial S_t}{\partial N_u}\right|_{p_t} = S_{tj} \quad ; \quad \left.\frac{\partial S_t}{\partial p_t}\right|_{N_u} = N_u \times \ln\frac{1-p_t}{p_t} \quad ; \quad \left.\frac{\partial^2 S_t}{\partial p_t^2}\right|_{N_u} = -\frac{N_u}{p_t(1-p_t)} \tag{7}$$

It is convenient to assign a special name to the coefficient $\log[(1-p_t)/p_t]$, equal to the inverse jump probability density, $\mathrm{d}p_t/\mathrm{d}S_{tj}$, Eq. (6).

**Definition 13** (Entropy increment)**.** *In each snapshot $S_t$, the entropy increment $s_t$ is the coefficient*

$$s_t \overset{(\mathrm{def})}{=} \frac{1}{N_u}\left.\frac{\partial S_t}{\partial p_t}\right|_{N_u} = \frac{\mathrm{d}S_{tj}}{\mathrm{d}p_t} = \ln\frac{1-p_t}{p_t}. \tag{8}$$

Quantitatively, at the current epoch we have $s_{t_0} = 18.235$ nats.

**Basic representation $(S_t, N_u)$.** In general, physics investigates phenomena in a fixed framework as a function of the snapshot. The relevant independent parameters are then $N_u$ and $S_t$.

We have $S_{tj} = S_t/N_u$ while $p_t$ is defined implicitly by $S_{tj} = -p_t \ln p_t - (1-p_t)\ln(1-p_t)$. From Eq. (8), $\mathrm{d}S_{tj} = s_t \mathrm{d}p_t$ and then

$$\begin{aligned}
&\left.\frac{\partial S_{tj}}{\partial N_u}\right|_{S_t} = -\frac{S_t}{N_u^2} \quad ; && \left.\frac{\partial S_{tj}}{\partial S_t}\right|_{N_u} = \frac{1}{N_u} \\
&\left.\frac{\partial p_t}{\partial N_u}\right|_{S_t} = \frac{\mathrm{d}p_t}{\mathrm{d}S_{tj}}\left.\frac{\partial S_{tj}}{\partial N_u}\right|_{S_t} = -\frac{S_t}{s_t N_u^2} \quad ; && \left.\frac{\partial p_t}{\partial S_t}\right|_{N_u} = \frac{\mathrm{d}p_t}{\mathrm{d}S_{tj}}\left.\frac{\partial S_{tj}}{\partial S_t}\right|_{N_u} = \frac{1}{s_t N_u}.
\end{aligned} \tag{9}$$

**Expansion representation $(S_t, S_{tj})$.** The pair $(S_t, S_{tj})$ is another convenient set of independent variables. Then, $N_u = S_t/S_{tj}$ where $S_{tj} = -(p_t \log p_t + q_t \log q_t)$.

$$\frac{\partial N_u}{\partial S_t}\bigg|_{S_{tj}} = \frac{1}{S_{tj}} \quad ; \quad \frac{\partial N_u}{\partial S_{tj}}\bigg|_{S_t} = -\frac{S_t}{S_{tj}^2} \tag{10}$$

For reasons that will appear later, we propose to call this representation the "expansion representation".

**Definition 14** (Expansion representation of the universe)**.** *The expansion representation of the cosmic theater given the fundamental prior is the choice of the entropy $S_t$ and the binary entropy $S_{tj}$ as independent parameters.*

**Information representation $(S_t, I_t)$.** Finally, is also possible to take $S_t$ as an independent variable together with the so-called "negentropy", $I_t = S_{\max} - S_t$.

**Definition 15** (Negentropy)**.** *The negentropy $I_t$ of the universe is $I_t = S_{\max} - S_t$.*

Obviously, at the origin of time $I_0 = N_u \ln 2$ nats and ultimately $I_{t_{\max}} = 0$. Let us call "information representation" this choice of $S_t$ and $I_t$ as independent parameters.

**Definition 16** (Information representation of the universe)**.** *The information representation of the cosmic theater given the fundamental prior is the choice of the entropy $S_t$ and the negentropy $I_t$ as independent parameters.*

We have then $N_u = (S_t + I_t)/\ln 2$ and $S_{tj} = S_t/N_u = S_t \ln 2/(S_t + I_t)$.

$$\frac{\partial N_u}{\partial S_t}\bigg|_{I_t} = \frac{1}{\ln 2} \quad ; \quad \frac{\partial N_u}{\partial I_t}\bigg|_{S_t} = \frac{1}{\ln 2} \tag{11}$$

### 2.3.3 Eigensubspace structure

The gigantic number of dimensions involves special features. In particular, Proposition (10), is nothing but a specific expression of the law of large numbers. Also, the probability $\mathbb{P}(\mathsf{h}_k|p_t)$ can be assimilated to a probability density $\mathrm{p}(\mathsf{h}_k|p_t)$

**Proposition 6.** *The eigensubspaces $\mathsf{h}_k$ collect the classical states with $k$ outcomes $\mathsf{X}_j = 1$ and $N_u - k$ outcomes $\mathsf{X}_j = 0$ in the principal variable batch $\{\mathsf{X}_j\}$. Given the fundamental prior, the probability of drawing from the batch exactly $k$ outcomes with $\mathsf{X}_j = 1$ is $\mathbb{P}(\mathsf{h}_k|p_t) = d_k\alpha_k(p_t)$, where $\alpha_k(p_t)$ is the eigenvalue belonging to $\mathsf{h}_k$ and $d_k$ its multiplicity. As a result, the probability, say $P_t(K)$, of drawing at most $K$ outcomes with $\mathsf{X}_j = 1$ is*

$$P_t(K) = \sum_{k=0}^{K} \mathbb{P}(\mathsf{h}_k|p_t) = \int_0^K \mathrm{p}(\mathsf{h}_k|p_t)\mathrm{d}k$$

*where $k$ is equated with a real-valued number, so that*

$$\mathrm{p}(\mathsf{h}_k|p_t) \stackrel{(\mathrm{def})}{=} \frac{\partial P_t(k)}{\partial k} = \mathbb{P}(\mathsf{h}_k|p_t)$$

*is the probability* density *assigned by the observer to the eigensubspace $\mathsf{h}_k$ for the infinitesimal index interval $(k, k + \mathrm{d}k)$.*

*Proof.* By definition, $\alpha_k(p_t)$ is the probability assigned by the observer to any eigenvector belonging to the eigensubspace $\mathsf{h}_k$ in the snapshot $S_t$ characterized by the transition probability $p_t$. As a result, the probability assigned to the full subspace $\mathsf{h}_k$ is $d_k\alpha_k$ while $\sum_{k=0}^{N_u} d_k\alpha_k = 1$. In standard information theory, irrespective of the snapshot, $k$ is the common Hamming weight of the classical states $\gamma_i$ belonging to the eigensubspace $\mathsf{h}_k$. Since $N_u$ is a gigantic number, it is convenient to equate $k \in [\![0, N_u]\!]$ to a real-valued number $k \in [0, N_u]$ and then compute the sum as an integral. As a result, the discrete probability $\mathbb{P}(\mathsf{h}_k|p_t)$ is also the probability density, $\mathrm{p}(\mathsf{h}_k|p_t)$. □

**Proposition 7.** *Each eigensubspace* $\mathsf{h}_k$ *is invariant by permutation of its basic vectors in the principal window, governed by the standard symmetric group* $\mathrm{S}_{d_k}$ *of degree* $d_k = \binom{N_u}{k}$.

*Proof.* Each permutation of the basic vectors represents a particular realization of the pattern with $k$ outcomes $\mathsf{X}_j = 1$ and $N_u - k$ outcomes $\mathsf{X}_j = 0$. Incidentally, the corresponding entropy is thus $\ln d_k$ nats irrespective of the snapshot $S_t$. □

**Proposition 8.** *Each eigensubspace* $\mathsf{h}_k$ *is invariant under the unitary gauge group* $\mathrm{U}(d_k)$.

*Proof.* The subspace $\mathsf{h}_k$ is a $d_k$-dimensional Hilbert space and therefore globally invariant under any unitary operator acting on it. As a result, from Ref. [18], the unitary group $\mathrm{U}(d_k)$ is a gauge subgroup of the Hilbert space $\mathcal{H}_u$ endowed with the statistical operator $\Pi_t$. Clearly, $\mathrm{U}(d_k)$ is the Lie group derived from the discrete symmetric group $\mathrm{S}_{d_k}$. □

**Proposition 9.** *In any snapshot* $S_t$*, the highest eigenvalue is* $\alpha_0$ *of index* $k = 0$ *and multiplicity* $d_0 = 1$.

*Proof.* From Eq. (4), $\alpha_k(p_t) = p_t^k q_t^{N_u - k}$ with $q_t = 1 - p_t > p_t$. As a result, in any snapshot $S_t$, the maximum eigenvalue is trivially $\alpha_0 = (1 - p_t)^{N_u}$ of index $k = 0$ and multiplicity $d_0 = \binom{N_u}{0} = 1$. Even the highest eigenvalue is minuscule for $p_t > 0$. For instance, at the present epoch, $\alpha_0 \simeq 10^{-10^{117}}$. □

**Proposition 10** (Eigensubspace probability density)**.** *Given the fundamental prior, in any snapshot* $S_t$ *the probability* $\mathbb{P}(\mathsf{h}_k|p_t)$ *assigned by the observer to the eigensubspace* $\mathsf{h}_k$ *for both* $k \gg 1$ *and* $N_u - k \gg 1$ *can be represented by a normal probability density* $k \mapsto \mathrm{p}(\mathsf{h}_k|p_t)$ *described by the Gaussian function*

$$k \mapsto \mathrm{p}(\mathsf{h}_k|p_t) = \frac{1}{\sqrt{2\pi}\,\sigma_t} \exp\Big( - \frac{(k - N_u p_t)^2}{2\sigma_t^2}\Big) \tag{12}$$

*of variance* $\sigma_t^2 = N_u p_t(1 - p_t) = -N_u^2/(\partial^2 S_t/\partial p_t^2)|_{N_u}$*, where* $k$ *is equated with a real-valued number.*

*Proof.* From Eq. (4), irrespective of $p_t$, the dimension of the eigensubspace $\mathsf{h}_k$ is $d_k = \binom{N_u}{k}$. Using the Stirling asymptotic approximation of the factorial $n! \sim \sqrt{2\pi n} \times (n/e)^n$ we obtain for large $k$ and $N_u - k$,

$$\begin{aligned} d_k = \frac{N_u!}{k! \times (N_u - k)!} &= \sqrt{\frac{2\pi N_u}{2\pi k \times 2\pi(N_u - k)}} \times \frac{N_u^{N_u}}{k^k \times (N_u - k)^{N_u - k}} \\ &= \sqrt{\frac{1}{2\pi N_u}} \times \sqrt{\frac{1}{(k/N_u)(1 - k/N_u)}} \times \frac{1}{(k/N_u)^k \times (1 - k/N_u)^{N_u - k}} \end{aligned} \tag{13}$$

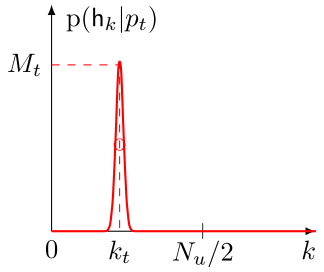
while $\alpha_k$ depends on $p_t$ as $\alpha_k(p_t) = p_t^k (1 - p_t)^{N_u - k}$. Introduce a reduced index $p_k = k/N_u$, so that $p_k \in [0, 1]$ can be thought as a probability. We obtain from a routine computation

$$\begin{aligned} d_k \alpha_k(p_t) &= \sqrt{\frac{1}{2\pi N_u}} \times \sqrt{\frac{1}{p_k(1 - p_k)}} \times \Big[\Big(\frac{p_t}{p_k}\Big)^{p_k} \times \Big(\frac{1 - p_t}{1 - p_k}\Big)^{1 - p_k}\Big]^{N_u} \\ &= M_t \times [\exp(-\mathbb{H}(p_k\|p_t))]^{N_u}. \end{aligned} \tag{14}$$

where $\mathbb{H}(p_k\|p_t) = p_k * \ln(p_k/p_t) + (1 - p_k) * \ln((1 - p_k)/(1 - p_t))$ is the standard *relative entropy* of the binary distribution $(p_k, 1 - p_k)$ with respect to the binary distribution $(p_t, 1 - p_t)$. For $p_k = p_t$, the relative entropy $\mathbb{H}(p_k\|p_t)$ is zero and from Proposition (6),

$$M_t \overset{\text{(def)}}{=} d_k \alpha_k(p_t)\,\big|_{k = p_t N_u} \overset{\text{(def)}}{=} d_k \alpha_{\max}(k) = \frac{1}{\sqrt{2\pi}\,\sigma_t} = \sqrt{\frac{1}{2\pi N_u p_t(1 - p_t)}} \tag{15}$$

is the probability density assigned to the eigensubspace $\mathsf{h}_k$ for $k = p_t N_u$. When $k$ deviates from $p_t N_u$, $d_k \alpha_k$ tends very quickly to zero . Let $|p_k - p_t| \ll 1$. At second order approximation in $(p_k - p_t)$, we obtain $\mathbb{H}(p_k \| p_t)) \simeq (1/2)(p_k - p_t)^2 / p_t(1 - p_t)$. Define the standard deviation $\sigma_t^2 \stackrel{\text{(def)}}{=} N_u p_t(1 - p_t) \simeq N_u p_k(1 - p_k) = 2\pi / M_t^2$, to obtain finally the Gaussian function Eq. (12). □.




We have $\sigma_t / N_u \sim 1/\sqrt{N_u}$, which implies that most of the total probability is concentrated into the eigensubspaces $\mathsf{h}_k$ of adjacent indexes $k \simeq p_t N_u \stackrel{\text{(def)}}{=} k_t$. Let us call $\mathsf{h}_{k_t}$ the "dominant eigensubspace".

**Definition 17** (Dominant eigensubspace)**.** *In any snapshot $S_t$, the dominant eigensubspace is the subspace $\mathsf{h}_{k_t}$ of index $k_t = p_t N_u \leq N_u/2$. The index $k_t$ is called the dominant index.*

In Eq. (12), the total probability is $\sum_{k=0}^{N_u} d_k \alpha_k = 1$. On the other hand, from standard mathematical results on the normal distribution, we have

$$\int_{-\infty}^{+\infty} \frac{1}{\sigma_t \sqrt{2\pi}} \exp\Big(-\frac{(k - k_t)^2}{2\sigma_t^2}\Big) \mathrm{d}k = 1.$$

Neglecting the two distribution tails for $k < 0$ and $k > N_u/2$ respectively, we have with an excellent approximation for the gigantic value of $N_u$,

$$\int_{0}^{N_u/2} \frac{1}{\sigma_t \sqrt{2\pi}} \exp\Big(-\frac{(k - k_t)^2}{2\sigma_t^2}\Big) \,\mathrm{d}k \simeq 1. \tag{16}$$

Quantitatively, at the present epoch, using $N_u = 6.23 \times 10^{125}$ and $p_{t_0} = 1.204 \times 10^{-8}$, the maximum probability density $\mathrm{p}(\mathsf{h}_{k_{t_0}} | p_{t_0}) = M_{t_0} = 4.6 \times 10^{-60}$ is attained for the dominant index $k_{t_0} = N_u p_{t_0} = 1.17 \times 10^{121}$ with a standard deviation of $\sigma_{t_0} = 8.66 \times 10^{58}$ while of course $M_{t_0} \times \sigma_{t_0} = 1/\sqrt{2\pi}$. The origin index, $k = 0$, is separated from $10^{62}$ standard deviations of $k_{t_0}$ and the index $k = N_u/2$ from $10^{66}$ standard deviations.

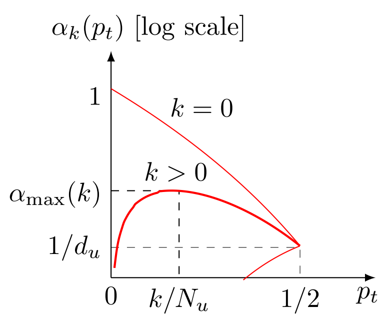
This induces a standard deviation of $\varsigma_{p_{t_0}} = 1.39 \times 10^{-67}$ for the transition probability $p_t$, a standard deviation of $\varsigma_{S_{t_0}} = 5.72 \times 10^{49}$ nats for the snapshot entropy $S_t$ and, by anticipation of Assumption (18), a standard deviation of $\varsigma_{t_0} = 5.8 \times 10^{-56}$ s $= 10^{-12}$ Planck unit of time for the present cosmic time $t_0$. This can be seen as the "thickness" of the present. In other words, one Planck units of time represents currently as many as $10^{12}$ standard deviations! Also, two adjacent indexes $k = k_t \pm 1$ correspond to a minuscule lapse time of about $s_t/(2\pi t) = 3.43 \times 10^{-61}$ Planck time units $= 1.85 \times 10^{-104}$ s.

**Proposition 11.** *The dominant eigensubspace, $\mathsf{h}_{k_t}$, is the eigensubspace $\mathsf{h}_k$ for which the eigenvalue $\alpha_k(p_t)$ is maximum with respect to $p_t$.*

*Proof.* Consider the curve family $p_t \mapsto \alpha_k(p_t)$ indexed by $k$. The maximum $\alpha_{\max}(k)$ of $\alpha_k$ with respect to $p_t$ is obtained for $\mathrm{d}\alpha_k / \mathrm{d}p_t = 0$, where

$$\frac{\mathrm{d}\alpha_k}{\mathrm{d}p_t} = \alpha_k(p_t) \frac{k - N_u p_t}{p_t q_t}, \tag{17}$$

so that when $k \leq N_u/2$, $\alpha_{\max}(k)$ is obtained for $p_t = k/N_u$. In particular for $k = 0$ the maximum is obtained for $p_t = 0$. When $k \geq N_u/2$, $\alpha_{\max}(k) = 1/d_u$ is obtained for $p_t = 1/2$ irrespective of $k$. The curve family is universal in logarithmic scale up to a homothety factor of $N_u$. When the scale is linear, for the actual value of about $N_u \simeq 10^{126}$, the maximum is thus very sharp, like a delta function □



Deriving Eq (17), the second derivative of $\alpha_k(p_t)$ at $p_t = k/N_u$ is

$$\frac{\mathrm{d}^2\alpha_k}{\mathrm{d}p_t^2}\Big|_{p_t=k/N_u} = -\alpha_k(p_t)\frac{N_u}{p_t q_t}\Big|_{p_t=k/N_u} = -\alpha_{\max}(k)\frac{N_u^2}{k(N_u-k)}$$

so that in the close neighborhood of $k/N_u$, the second degree Taylor polynomial approximation reads for $k > 0$

$$\frac{\alpha_k(p_t)}{\alpha_{\max}(k)} = 1 - \frac{N_u^2}{2k(N_u-k)}\left(p_t - \frac{k}{N_u}\right)^2 = 1 - \frac{(p_t - p_k)^2}{2p_k(1-p_k)} \tag{18}$$

where $p_k = k/N_u$ and from Eq. (15),

$$\alpha_{\max}(k) = \frac{k^k(N_u-k)^{N_u-k}}{N_u^{N_u}} = \frac{M_t}{d_k}. \tag{19}$$

**Definition 18** (Tail probability). *The tail probability is the cumulative probability of the eigensubspaces $\mathsf{h}_k$ for all $k \in [\![N_u/2+1, N_u]\!]$.*

The tail distribution is completely negligible except in the vicinity of the ultimate snapshot, because it is composed of eigensubspaces $\mathsf{h}_k$ far away from the dominant subspace.

#### 2.3.4 The episodic clock

It turns out that the eigensubspaces $\mathsf{h}_k$, with $k \in [\![0, N_u/2]\!]$, are key components to interpret the Bayesian theater in physics. They constitute a kind of "clock" embedded in the Hilbert space where the dominant eigensubspace acts as the "clock hand". This is a form of "episodic memory" stored in the statistical operator. The universe started from the eigensubspace $\mathsf{h}_0$ marking the initial position of the "clock hand". Then, in each snapshot $S_t$, the current "clock hand" is represented by the dominant eigensubspace $\mathsf{h}_{k_t}$ with $k_t = N_u p_t$. Its acuity is characterized by the "tiny" standard deviation $\sigma_t$. For instance, quantitatively, at the current epoch, $\sigma_t = 8.66 \times 10^{58}$, which means that $\sigma_t/k_t = 7.41 \times 10^{-63}$. When $N_u \to \infty$ the "clock hand" tends to a Kronecker delta function.

For ease of presentation, we propose the following terminology:

**Definition 19** (Episodic clock-hand). *In the cosmic Hilbert space $\mathcal{H}_u$, irrespective of the snapshot, an episodic clock hand denotes a particular eigensubspace $\mathsf{h}_k$ of the statistical operator $\Pi_t$ with $k \in [\![0, N_u/2]\!]$.*

This definition stresses that the clock hands are independent of the particular snapshot.

**Definition 20** (Episodic distance). *The episodic distance between two episodic clock hand $\mathsf{h}_{k_1}$ and $\mathsf{h}_{k_2}$ is the index gap $|k_2 - k_1|$.*

**Definition 21** (Episodic clock). *The set of all episodic clock hands is termed episodic clock.*

**Definition 22** (Episode). *In any snapshot $S_t$, an episode is the set of events recalled by an episodic clock hand.*

By definition, the current snapshot brings together all the information available to the observer. It turns out that the episodic clock makes it possible to distinguish between the current events, the relics of the past and the clues of the future among the information stored in the statistical operator $\Pi_t$.

In standard information theory, the "Hamming weight" of a binary classical state, e.g., $(0, 1, 0, 0, 1)$, is its number of "1" truth values, e.g. 2.

**Proposition 12.** *In the principal observation window, irrespective of the snapshot, the clock hand $\mathsf{h}_k$ collects all the distinct classical states of Hamming weight $k$.*

*Proof.* By reverse transcription of the Hilbert space into a standard joint probability distribution, the classical states $\gamma_i$ corresponding to $\mathsf{h}_k$ are Boolean conjunctions of $k$ functions $\mathsf{X}_j = 1$ and $N_u - k$ functions $\mathsf{X}_{j'} = 0$. By definition, the "Hamming weight" is thus $k$ and this exhausts the classical states of Hamming weight $k$. □

**Proposition 13.** *In the current snapshot $S_t$ the clock hand $\mathsf{h}_k$ with $k \in [\![0, N_u/2]\!]$ points towards the current episode for $k = k_t$, the relics of past episodes for $k < k_t$ and the clues of future episodes for $k > k_t$.*

*Proof.* Since most of the probability is concentrated in $\mathsf{h}_{k_t}$, this subspace contains most of the current events. From Proposition (12), the eigensubspaces $\mathsf{h}_k$ are fixed irrespective of $S_t$. As a result, in the snapshot $S_t$, they refer to episodes occurring when this subspace $\mathsf{h}_k$ was, is or will be dominant, whether in the past, present, or future. □

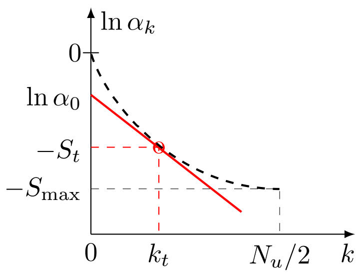
The "future episodes" have in reality a very special status: they are not directly observable because they represent outgoing entities, e.g. outgoing light-rays. Under certain conditions we will propose that they represent antimatter, Assumption (27) below.

Let $p_t \in [0, 1/2]$ denote the transition probability associated to the current snapshot $S_t$ and for any $k \in [0, N_u/2]$ let $\alpha_k(p_t) = p_t^k(1 - p_t)^{N_u - k}$. In the snapshot $S_t$, we note $\alpha_{k_t}$ the "dominant eigenvalue".

**Proposition 14.** *The dominant eigenvalue $\alpha_{k_t}(p_t)$ with $k_t = N_u p_t$ is*

$$\alpha_{k_t}(p_t) = \sup_{p' \in ]0,1/2[} \alpha_{k_t}(p') = e^{-S_t} \tag{20}$$

*where $S_t \in [0, S_{\max}]$ is expressed in nats. Then, $p_{k_t} = p_t = k_t/N_u$, $\alpha_{k_t}(p_{k_t}) = e^{-S_t}$ and from Eq. (15), $d_{k_t} = M_t e^{S_t} = [2\pi N_u p_{k_t}(1 - p_{k_t})]^{-1/2} e^{S_t}$.*



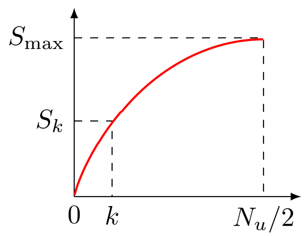

*Proof.* In the snapshot $S_t$, consider the family of straight-lines $k \mapsto \ln \alpha_k(p_t)$ indexed by $p_t$ as

$$\ln \alpha_k(p_t) = k \ln p_t + (N_u - k) \ln(1 - p_t).$$

By a straightforward calculation, when $k = k_t = p_t N_u$, $\ln \alpha_{k_t} = N_u\big(p_t \ln p_t + (1 - p_t) \ln(1 - p_t)\big) = -S_t$. From Eq. (17), the straight-line family envelops the curve $k_t \mapsto -S_t$ (in dashed line), so that Eq. (20) holds. □

Associated to the current snapshot $S_t$ characterized by its transition probability $p_t$, it is easy to establish a correspondence between the indexes $k \in [\![0, N_u/2]\!]$ of the clock hands and an ensemble of discrete snapshots $S_k$. The extra indexes $k \in [\![N_u/2, N_u]\!]$ are associated with the tail of the probability distribution and do not point to a specific snapshot.

*Convention.* For simplicity, when there is no risk of confusion, we use $S_k$ for $S_{t_k}$ where the index $t_k$ should refer to the cosmic time defined later and corresponding to this specific entropy. Notice incidentally that $S_k$ is a snapshot entropy of the full universe and *not* the entropy of the eigensubspace $\mathsf{h}_k$ induced by the current statistical operator $\Pi_t$, which is actually $\ln d_k$ irrespective of the snapshot (see Ref. [18]).




For $k = k_t$, $S_{k_t} = S_t$ is in practice very close to $\ln d_{k_t}$, e.g., $2.248 \times 10^{122}$ nats at the present epoch.

**Proposition 15.** *Irrespective of the current snapshot $S_t$ characterized by the transition probability $p_t$, there is a one-to-one correspondence between every episodic clock hand $\mathsf{h}_k$ with $k \in [\![0, N_u/2]\!]$ and a discrete snapshot $S_k \in [0, S_{\max}]$ as*

$$[\![0, N_u/2]\!] \to [0, S_{\max}] \quad : \quad k \mapsto S_k = -k\ln(k/N_u) - (N_u - k)\ln(1 - k/N_u) \tag{21}$$

*$S_k$ represents the present snapshot $S_t$ for $k = p_t N_u$, a past snapshot for $k < p_t N_u$ and a future snapshot for $k > p_t N_u$.*

*Proof.* The index $k$ is the dominant index in the snapshot $S_k$. With the usual convention $0\ln 0 = 0$, Eq. (21) is always valid even for $k = 0$. Then $S_0 = 0$ and $S_{N_u/2} = N_u \ln 2 = S_{\max}$ respectively. □

A snapshot $S_t$ is a probabilistic combination of eigensubspaces $\mathsf{h}_k$, while every $\mathsf{h}_k$ represents the pattern of Boolean occurrences with exactly $k$ truth values equal to 1. A transition $\mathsf{h}_k \to \mathsf{h}_{k+1}$ is achieved by a single jump of a binary variable, say $\mathsf{X}_j$, from $0 \to 1$. Obviously, the index $k \in [\![0, N_u/2]\!]$ constitutes a discrete sampling of the continuous segment $[0, N_u/2]$. However, because of the gigantic number of variables $N_u$, the discrete set $[\![0, N_u/2]\!]$ is generally indistinguishable from a continuous set $[0, N_u/2]$ with a finite precision. For convenience, we will call the snapshots $S_k$ "episodic snapshots".

**Proposition 16.** *Every snapshot $S_t$ is indistinguishable from an episodic snapshot $S_k$ for some index $k \in [\![0, N_u/2]\!]$ and thus the transition probability $p_t$ is indistinguishable from the rational number $p_k = k/N_u$.*

*Proof.* In the Bayesian theater, all parameters are only probabilistic estimates on a set of $N_u$ samples, that is, with an accuracy of about $1/\sqrt{N_u}$, namely, the standard deviation of the Gaussian distribution. But the separation between two integers implies a precision of about $1/N_u$, which is much more precise and therefore non-significant here. In other words, any snapshot is in fact indistinguishable from it closest snapshot with integer index $k_t = k$. Equivalently, the transition probability $p_t$ is indistinguishable from the rational number $p_k = k/N_u = p_{k_t}$. □

### 2.3.5 Directional window group

In the cosmic Bayesian theater, the source batch of mutually independent variables $\{\mathsf{X}_j\}$ is obviously not the only possible observation window. The other windows are generated from the source window by the so-called "window group" acting on the Hilbert space $\mathcal{H}_u$, that is the full unitary group $\mathrm{U}(d_u)$.

By construction, the source window defined by the batch of independent variables $\{\mathsf{X}_j\}$ is principal and thus *completely divisible* [18], meaning that with this batch, the statistical operator $\Pi_t$ acting on $\mathcal{H}_u$ is the Kronecker product of $N_u$ individual two-dimensional statistical operator $\Pi_{tj}$ acting on individual Hilbert spaces $\mathcal{H}_j$. They are furthermore diagonal in the source window.

*A priori*, any observation window generated from the principal window by the full unitary group $\mathrm{U}(d_u)$ is feasible. Remarkably, it turns out that *the concept of spacetime emerges from a strict limitation to the completely divisible observation windows.* With this restriction, all individual 2-dimensional Hilbert spaces $\mathcal{H}_j$ remain globally invariant. This defines a window subgroup that only acts inside each partial space $\mathcal{H}_j$. For reasons that will appear soon, we term this subgroup the "directional window group".

**Definition 23** (Directional window group)**.** *The directional window group is the subgroup of the full window group $\mathrm{U}(d_u)$ that conserves globally the individual two-dimensional Hilbert spaces $\mathcal{H}_j$.*

**Definition 24** (Directional window)**.** *A directional window is an observation window generated from the principal window by the directional window group.*

Of course, in each 2-dimensional Hilbert spaces $\mathcal{H}_j$, the partial observation windows are in general non-principal. They will be referred to as $\mu$ while the principal window will be marked $\lambda$. By construction, in any snapshot $S_t$ a particular $\mu$-window is derived from the $\lambda$-window by a unitary transition operator $\mathsf{R}_{\mu j}$ acting on $\mathcal{H}_j$ and mapping the $\lambda$-window onto the $\mu$-window and thus the corresponding basic vectors, say $|e_{\mu j}\rangle$, are computed from the principal basic vector $|e_{\lambda j}\rangle$ by the operators $\mathsf{R}_{\mu j}^{-1}$, i.e., .$|e_{\mu j}\rangle = \mathsf{R}_{\mu j}^{-1}|e_{\lambda j}\rangle$. It turns out that these windows can be endowed with a so-called *partial orientation*, namely a *qubit* which will allow to construct spacetime explicitly. Furthermore, the qubit is only defined up to its effective transcription gauge group [18], which will actually represent the exact geometric symmetries as perceived by the Bayesian observer.

**Definition 25** (Orientations of a partial directional window in $\mathcal{H}_j$)**.** *The orientation of a particular window, $\mu$, in a 2-dimensional Hilbert space $\mathcal{H}_j$ is its first basic vector $|e_{\mu j}\rangle$, irrespective of the second basic vector $|f_{\mu j}\rangle$, that is a specific qubit, specified up to its effective transcription gauge group.*

*- Especially, the orientation of the principal window, $\lambda$, is $|e_{\lambda j}\rangle = |0\rangle_j \in \mathcal{H}_j$. We will say that "the partial principal window is pointing in the $|e_{\lambda j}\rangle$ direction".*

*- The orientation of the $\mu$-window is $|e_{\mu j}\rangle = \mathsf{R}_{\mu j}^{-1}|e_{\lambda j}\rangle \in \mathcal{H}_j$, where $\mathsf{R}_{\mu j}$ is a unitary transition operator acting on $\mathcal{H}_j$ mapping the $\lambda$-window to the $\mu$-window. We will say that the partial $\mu$-window is pointing in the $|e_{\mu j}\rangle$ direction.*

**Definition 26** (Uniform directional window)**.** *A directional window $\mu$ is uniform when the $N_u$ partial orientations of the $\mu$-windows are all identical, meaning that irrespective of $j$, assuming $|e_{\lambda j}\rangle = |0\rangle$ we have $|e_{\mu j}\rangle = |e_\mu\rangle$.*

By construction, the principal window is a uniform directional window $|e_{\lambda j}\rangle = |e_\lambda\rangle = |0\rangle$.

**Proposition 17.** *A uniform directional window can be generated from the principal window by a unique transition operator $\mathsf{R}_\mu$ acting in each partial Hilbert space $\mathcal{H}_j$.*

*Proof.* Assuming irrespective of $j$ $|e_{\lambda j}\rangle = |0\rangle$ in the principal window, the first basic vectors $|e_{\mu j}\rangle = |e_\mu\rangle$ are obviously also identical irrespective of $j$. □

For simplicity, under otherwise stated, we will assume that any uniform directional window is so generated. This convention corresponds to a particular transcription gauge for the full Hilbert space $\mathcal{H}_u$ (see Ref. [18]). As a result, the second basic vectors are also identical irrespective of $j$.

Given the fundamental prior, Eq. (2), all partial operators $\Pi_{tj} = \mathrm{Diag}(1 - p_t, p_t)$ expressed in the principal window are identical. As a result, when observed in a uniform directional window $\mu$, the statistical operators $\Pi_{\mu tj}$ are likewise identical in each partial Hilbert space $\mathcal{H}_j$.

**Proposition 18.** *Given the fundamental prior and in a uniform directional window $\mu$, the global statistical operator $\Pi_{\mu t}$ is the Kronecker product of $N_u$ identical partial statistical operators $\Pi_{\mu tj}$ as*

$$\Pi_{\mu t} = (\Pi_{\mu tj})^{\otimes N_u}.$$

*Proof.* Given the fundamental prior, Eq. (2), all partial statistical operators $\Pi_{\lambda tj}$ are identical irrespective of $j$ in the principal window $\lambda$ and thus also $\Pi_{\mu tj}$ expressed in a uniform directional window $\mu$. □

**Definition 27** (Uniform directional group)**.** *The uniform directional group is the subgroup of the directional group that acts identically on all partial spaces $\mathcal{H}_j$.*

**Proposition 19.** *The uniform directional group is the special unitary Lie group* $\mathrm{SU}(2)$*.*

*Proof.* In the uniform directional window the window group acting on any partial subspace $\mathcal{H}_j$ is the Lie group $\mathrm{U}(2) = \mathrm{SU}(2) \ltimes \mathrm{U}(1)$ irrespective of $j$. From Definition (25) the orientation is defined up to the gauge transcription of the first basic vector so that the directional group proper is reduced to SU(2). □

**Proposition 20** (Partial directional statistical operator)**.** *The partial statistical operator expressed in the directional window pointing in the direction* $|e_\mu\rangle = (\alpha, \beta)$ *in each partial Hilbert space* $\mathcal{H}_j$ *is*

$$\Pi_{tj\mu} = \begin{bmatrix} \alpha\alpha^* q_t + \beta\beta^* p_t & \alpha\beta^*(q_t - p_t) \\ \alpha^*\beta(q_t - p_t) & \alpha\alpha^* p_t + \beta\beta^* q_t \end{bmatrix} \tag{22}$$

*Proof.* Consider a uniform orientation window $\mu$ of directional orientation $|e_\mu\rangle$. By hypothesis, $|e_\mu\rangle$ it is the first basis vector in each partial Hilbert space $\mathcal{H}_j$. Let $|e_\mu\rangle = (\alpha, \beta) \in \mathcal{H}_j$ with $\alpha\alpha^* + \beta\beta^* = 1$ denote its coordinates expressed in the principal window $\lambda$. From definition (25), the second basic vectors, say $|f_\mu\rangle$, is arbitrary and can be chosen independently of $j$. For definiteness, let $|f_\mu\rangle = (-\beta^*, \alpha^*)$ be its coordinates expressed in the principal window. The inverse transition matrix from the basis $\lambda$ to the basis $\mu$ is thus

$$\mathsf{R}_{\mu j}^{-1} = \begin{bmatrix} \alpha & -\beta^* \\ \beta & \alpha^* \end{bmatrix} = \mathsf{R}_{\mu j}^\dagger$$

Irrespective of $j$, the partial statistical operator $\Pi_{tj} = \mathrm{Diag}(q_t, p_t)$ in the principal window is expressed in the basis $(|e_\mu\rangle, |f_\mu\rangle)$ as

$$\Pi_{tj\mu} = \mathsf{R}_{\mu j}^\dagger \Pi_{tj} \mathsf{R}_{\mu j} = \begin{bmatrix} \alpha & -\beta^* \\ \beta & \alpha^* \end{bmatrix} \begin{bmatrix} q_t & 0 \\ 0 & p_t \end{bmatrix} \begin{bmatrix} \alpha^* & \beta^* \\ -\beta & \alpha \end{bmatrix}$$

which gives Eq. (22). □

# 3 Emergence of spacetime

Humans perceive the universe as something completely different from an information warehouse. How is it possible? We propose that the fundamental reason lies in their Bayesian perception of information. Indeed, basically, the usual 3D-Euclidean space of the everyday world emerges from the experimental fact that, (surprisingly enough!), several points of view are required to completely describe an object. This is the signature of a Bayesian theater characterized by the need to have several observation windows to be able to specify the full set of observables (see Ref. [18]).

## 3.1 Bayesian perception of Boolean variables

The Bayesian representation of a single Boolean variable is very different from its deterministic representation, simply because it is impossible to express all observables by using a single alternative, namely the variable and its negation. Therefore, each initial query must be complemented by a set of related alternatives using distinct Boolean variables and thus constructing a Bayesian theater. Technically, these alternatives are generated from the source variable by the window group. As a result, *a single degree of freedom becomes necessarily represented by a set of Boolean variables and no longer as a single dichotomic function.*

For the $N_u$ degrees of freedom of the universe, it turns out that given the fundamental prior, Definition (7), the full window group is not necessary. Indeed, the minimum window group required to completely observe all alternatives is reduced to the *uniform directional window group*, Definition (27), namely SU(2). We propose that it founds the standard physical space. Technically, SU(2) is conveniently described as an action group acting on the set of qubits belonging to $\mathbb{C}^2$. In turn, this set of qubits is conveniently represented in $\mathbb{R}^3$ by the so-called "Bloch sphere".

This role of qubits was already conjectured out of any Bayesian framework by a number of authors, pioneered in 1943 by von Weizsäcker and collaborators [49–51] and reinterpreted in 1995 by H. Lyre [52] in the framework of standard quantum information theory.

### 3.2 The Bloch sphere

In standard quantum information, the set of qubits belonging to $\mathbb{C}^2$ is routinely pictured by a *Bloch sphere* $\mathbb{S}^2$ of unit radius in a real-valued three dimensional vector space $\mathbb{R}^3$ endowed with Euclidean metric, $\mathbb{S}^2 \subset \mathbb{R}^3$. To construct this sphere, irrespective of $j$, consider the usual Pauli matrices in the principal window of each $\mathcal{H}_j$ with basis $(|0\rangle_j, |1\rangle_j)$, or simply $(|0\rangle, |1\rangle)$ irrespective of $j$, as

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}; \quad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}; \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

In standard physics, they posit at the same time three spin observables, three generators of the special unitary group SU(2) in its Lie algebra $\mathfrak{su}(2)$ (with the standard conventions of physics) and finally three involutive unitary operators acting on $\mathcal{H}_j$, meaning that they are their own inverse, $\sigma_i^2 = \mathbb{1}_2$. Of course, there are many expressions of Pauli matrices. This particular choice singles out a basis in the principal window that we will call "principal orientation".

From Definition (27), a uniform $\mu$-orientation in the cosmic Hilbert space $\mathcal{H}_u$ is depicted by a unique qubit $|e_\mu\rangle \in \mathcal{H}_j$ irrespective of $j$ in the partial Hilbert spaces $\mathcal{H}_j$. From Definition (25), for each orientation $|e_\mu\rangle = \mathsf{R}_{\mu j}^{-1}|0\rangle_j \in \mathcal{H}_j$, define the three entries of a so-called "Bloch vector" $n_\mu = (n_{\mu i}) \in \mathbb{R}^3$ as the expectations of the Pauli observables $\sigma_i$ with respect to the pure state $|e_\mu\rangle$,

$$n_{\mu i} \overset{\text{(def)}}{=} \langle e_\mu|\sigma_i|e_\mu\rangle, \qquad i \in [\![1,3]\!]. \tag{23}$$

From $\sigma_i^2 = \mathbb{1}_2$, irrespective of $|e_\mu\rangle$, it is easy to verify that $n_{\mu 1}^2 + n_{\mu 2}^2 + n_{\mu 3}^2 = 1$, so that the Bloch vector is a point of a unit 2-sphere $\mathbb{S}^2$ of $\mathbb{R}^3$. Furthermore, it can be seen by routine computation that the qubit $|e_\mu\rangle$ is the eigenvector belonging to the eigenvalue $+1$ of the Hermitian traceless operator $\sigma_\mu = n_{\mu 1}\sigma_1 + n_{\mu 2}\sigma_2 + n_{\mu 3}\sigma_3$, so that $\sigma_\mu|e_\mu\rangle = |e_\mu\rangle$ and $\langle e_\mu|\sigma_\mu|e_\mu\rangle = 1$. Therefore, we have $|e_\mu\rangle\langle e_\mu| = (1/2)(\mathbb{1}_2 + \sigma_\mu)$.

Now, using Eq. (23), we propose to adopt the mapping

$$\mathcal{H}_j \to \mathbb{R}^3: \quad |e_\mu\rangle \mapsto n_\mu = \begin{bmatrix} n_{\mu 1} \\ n_{\mu 2} \\ n_{\mu 3} \end{bmatrix} \tag{24}$$

as the base of the usual three-dimensional perception of space. For instance, we have $|0\rangle \mapsto (0,0,1)$, $|1\rangle \mapsto (0,0,-1)$, $(|0\rangle \pm i|1\rangle)/\sqrt{2} \mapsto (0,\pm 1,0)$ and $(|0\rangle \pm |1\rangle)/\sqrt{2} \mapsto (\pm 1,0,0)$. In particular, the principal direction $|0\rangle$ in the Hilbert space corresponds to the $z$-direction $(0,0,1)$ in $\mathbb{R}^3$.

In mathematics, a complex-valued number in $\mathbb{C}$ can be represented by a point in $\mathbb{R}^2$ so that the set of general qubits of unit norm in $\mathbb{C}^2$ can be identified with a 3-sphere $\mathbb{S}^3$ in a real-valued Euclidean space $\mathbb{R}^4$. Then, the mapping $|e_\mu\rangle \mapsto (n_{\mu i})$, Eq. (24), is a standard "bundle" $\mathbb{S}^3 \to \mathbb{S}^2$ with "fiber" $\mathbb{S}^1$, called "Hopf fibration". The fiber $\mathbb{S}^1$ represents the group U(1) while $\mathbb{S}^2 \subset \mathbb{R}^3$ is by definition the *Bloch sphere*.

Using this mapping, we propose to adopt the following assumption:

**Assumption 9** (Geometry of space). *Given the fundamental prior, Eq. (2), the geometry of space is that of a Bloch sphere, generated from the uniform directional window group by Hopf fibration.*

*Hints.* Every source Boolean variable $\mathsf{X}_j$ is transcribed into a partial Bayesian theater represented by the 2D-Hilbert space, $\mathcal{H}_j$ (see Ref. ([18]). Since the $N_u$ source variables are equivalent, the complex-valued Hilbert spaces $\mathcal{H}_j$ are identical as well. Irrespective of $j$, a convenient real-valued representation is therefore a common Bloch sphere. Its topology is different from that of the usual 3D-Euclidean geometry, but it depicts indeed the now accepted symmetry of space, as stressed by R. Feynman [53]. In non-relativistic physics, the shortcomings of the common Euclidean representation look rather subtle: For instance, it is often stated that the usual space rotational invariance of $2\pi$ should be replaced by $4\pi$, even if it actually requires some contortions. However, we will see that in fact the invariance is not that of a simple rotation of $4\pi$ but the iteration of a rotation of $2\pi$ combined with complex conjugation, Proposition (21). More conclusive, the SU(2) symmetry is proved by the existence of topological insulators [54] and topological photonics [55]. □

**The orientation symmetry group.** What are the basic symmetries of spacetime given the fundamental prior ? We propose that they be constituted by the "transcription gauge group" [18] of the orientation $|e_\mu\rangle$. Technically, in a Bayesian theater representing a single degree of freedom, the transcription gauge group of a qubit is composed of unitary and antiunitary operators.

**Assumption 10** (Orientation symmetries)**.** *The exact symmetries of spacetime are those of the transcription gauge group of the orientations* $|e_\mu\rangle$*.*

*Hints.* By definition, the equivalent transcriptions of the qubit $|e_\mu\rangle$ are indistinguishable for the observer and therefore perceived as an exact symmetry. In other words, the transcription gauge group gathers on its own the basic symmetries of spacetime given the fundamental prior. □

This induces a symmetry group in both the partial Hilbert spaces and the Bloch sphere, which we will call the orientation symmetry group. In order to completely explicit this symmetry structure it is necessary to consider *both sides* of the Bloch sphere. Technically, this expresses a well known property of the morphism $\mathrm{SU}(2) \to \mathrm{SO}(3)$ but remains unconventional because the conjugation symmetry is usually ignored.

**Proposition 21.** *The orientation symmetry of the partial directional window in* $\mathcal{H}_j$ *is generated by the gauge group* $\mathcal{K} \rtimes \mathrm{U}(1)$*, that is the semi-direct product of the complex conjugation group* $\mathcal{K}$ *with the phase group* $\mathrm{U}(1)$*.*

*Proof.* The orientation is specified by a qubit, that is a pure state $|e_\mu\rangle$. In turn, from Ref. [18], the qubit is the transcription in $\mathcal{H}_j$ of a Boolean variable whose effective transcription gauge group is the semi-direct product $\mathcal{K} \rtimes \mathrm{U}(1)$, where $\mathcal{K}$ is the complex conjugation group and $\mathrm{U}(1)$ the phase group. □

*Convention.* The complex conjugation operator expression depends on the observation window. For definiteness, we assume throughout that this operator acting on $\mathcal{H}_j$ is expressed *in the principal window* by the operator $\mathsf{K} \times \mathbb{1}_2 = \mathrm{Diag}(\mathsf{K}, \mathsf{K})$, where $\mathsf{K} : \mathbb{C} \to \mathbb{C} : z \mapsto z^*$ is the standard complex conjugation on the complex field. For simplicity we will note also $\mathsf{K}$ for $\mathsf{K} \times \mathbb{1}_2$ so that $\mathcal{K} = \{\mathbb{1}_2, \mathsf{K}\}$ when expressed in the principal window. □

Let $|e_\mu\rangle = (\alpha, \beta) \in \mathcal{H}_j$ be the orientation of the window $\mu$ expressed in the principal window $\lambda$. Omitting the index $\mu$ of its entries $n_{\mu i}$ for simplicity, the Bloch vector $(n_\mu)$ reads from Eq. (24),

$$n_\mu = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = \begin{bmatrix} 2\,\Re_e(\alpha\beta^*) \\ 2\,\Im_m(\alpha\beta^*) \\ |\alpha|^2 - |\beta|^2 \end{bmatrix}. \tag{25}$$

**Proposition 22.** *Given the fundamental prior, the usual Euclidean space* $\mathbb{R}^3$ *is invariant under the rotation group,* $\mathrm{U}(1)$*, around the principal direction,* $n_3$*.*

*Proof.* The gauge group $\mathrm{U}(1)$ corresponds to a multiplication by a phase factor $e^{i\phi}$ affecting identically $\alpha$ and $\beta$. This corresponds to the phase group of parameter $\phi$. The rotation axis $n_3$ is specified by the initial choice of Pauli matrices used to construct the Euclidean space. By Noether theorem, this will later lead to defining spins and angular momentums. □

**Proposition 23.** *Given the fundamental prior, the usual Euclidean space* $\mathbb{R}^3$ *is invariant under a mirror symmetry with respect to the plane* $(n_1, n_3)$*, that is* $n_2 \mapsto -n_2$*.*

*Proof.* The gauge operator $\mathsf{K} : |e_\mu\rangle \mapsto |e_\mu^*\rangle$ that is $i \mapsto -i$ or $\Im_m(\alpha\beta^*) \mapsto -\Im_m(\alpha\beta^*)$ represents a mirror symmetry with respect to the plane $(n_1, n_3)$ on the Bloch sphere, that is $n_2 \mapsto -n_2$. This will later lead to defining "time reversal". □

On the other hand, a phase factor $e^{i\phi}$ on a single component, say $\beta$, induces a rotation of $\phi$ in the plane perpendicular to $n_3$. Let thus $(\theta, \phi)$ denote the standard spherical coordinates of $n_\mu$ on the sphere $\mathbb{S}^2$.

*Convention.* For definiteness, we adopt the determination $-\pi < \phi \leq \pi$ for the azimuthal angle and $0 \leq \theta < \pi$ for the polar angle. □

Now, we have $n_1 = \sin\theta\cos\phi$, $n_2 = \sin\theta\sin\phi$ and $n_3 = \cos\theta$, while the mirror symmetry $n_2 \mapsto -n_2$ is obtained for $\phi \mapsto -\phi$. Conversely, $\theta = \arccos n_3$ and $\phi = \arctan(n_2/n_1)$.

**The two sides of the Bloch sphere.** It turns out that the Bloch sphere is swept in its entirety from the following series of qubits,

$$|e_\mu\rangle = \alpha\,|0\rangle + \beta\,|1\rangle = \cos\frac{\theta}{2}\,|0\rangle + e^{i\phi}\sin\frac{\theta}{2}\,|1\rangle \in \mathcal{H}_j,$$

with $0 \leq \theta < \pi$ and $-\pi < \phi \leq \pi$. Indeed, from Eq. (25), we recover $n_1 = \sin\theta\cos\phi$, $n_2 = \sin\theta\sin\phi$ and $n_3 = \cos\theta$.

The factor $1/2$ in the polar angle $\theta$ indicates that a single rotation in $\mathbb{S}^3$ corresponds to a double rotation in $\mathbb{S}^2$. Technically, this expresses the well known 2 to 1 morphism of SU(2) on SO(3), potentially covering two complete spheres. Usually, by ignoring the conjugation gauge group, the two spheres are regarded as indistinguishable and the second sphere is simply discarded. By contrast, in the current model, we must make explicit the full orientation symmetry group, which requires distinguishing the two spheres. Therefore, we propose that they represent respectively *both sides of the Bloch sphere*. To deal with, we propose to adopt the following rule:

**Assumption 11** (Correspondence between both sides of the Bloch sphere)**.** *Proper rotations on $\mathbb{R}^3$ and unitary operators on $\mathcal{H}_j$ conserve the same side of the sphere while improper rotations of $\mathbb{R}^3$ and antiunitary operators on $\mathcal{H}_j$ switch both sides.*

*Hints.* Since from Eq. (25) the gauge subgroup U(1) is indifferent, the second sphere must be generated by the conjugation gauge subgroup $\mathcal{K}$. Then, the second side of the sphere arises from the first side by complex conjugation on $\mathcal{H}_j$, that is a *antiunitary operator*. As a result, unitary operators acting on $\mathcal{H}_j$ conserve the same side. They also corresponds to proper rotations of $\mathbb{R}^3$ and thus the side swap represents improper rotations. □

However, we will see that an unconventional consequence is that parity and conjugation cannot be distinguished.

**Proposition 24.** *Every partial Hilbert space $\mathcal{H}_j$ is represented by the two sides of a Bloch sphere up to a gauge group* U(1)*. One side of the sphere is mapped pointwise to the second side by antiunitary operators. Conversely, each point on one side of the Bloch sphere represents a direction in every partial Hilbert space $\mathcal{H}_j$.*

*Proof.* Taking into account the transcription gauge group of qubits, switch both sides of the Bloch sphere by the antiunitary operator $i\sigma_2\mathsf{K}$. We use $i\sigma_2$ rather than $\sigma_2$ because $i\sigma_2$ is real and thus commutes with $\mathsf{K}$. Then,

$$\mathcal{H}_j \to \mathcal{H}_j \quad : \quad |e_\mu\rangle = \begin{bmatrix}\alpha\\ \beta\end{bmatrix} \mapsto i\sigma_2\mathsf{K}|e_\mu\rangle \overset{\text{(def)}}{=} |\psi_\mu^*\rangle = \begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\begin{bmatrix}\alpha^*\\ \beta^*\end{bmatrix} = \begin{bmatrix}\beta^*\\ -\alpha^*\end{bmatrix},$$

so that

$$n_\mu = \begin{bmatrix}n_1\\ n_2\\ n_3\end{bmatrix} = \begin{bmatrix}2\,\Re_e(\alpha\beta^*)\\ 2\,\Im_m(\alpha\beta^*)\\ |\alpha|^2 - |\beta|^2\end{bmatrix} \mapsto n_\mu^* = \begin{bmatrix}-2\,\Re_e(\alpha\beta^*)\\ -2\,\Im_m(\alpha\beta^*)\\ |\beta|^2 - |\alpha|^2\end{bmatrix} = \begin{bmatrix}-n_1\\ -n_2\\ -n_3\end{bmatrix}.$$

This transformation $\mathbb{S}^2 \to \mathbb{S}^2 : n_\mu \mapsto n_\mu^* = -n_\mu$ is an involutive improper orthogonal transformation corresponding to the involutive antiunitary operator $i\sigma_2\mathsf{K}$. The residual gauge group U(1) represents the fiber of the Hopf fibration.

The transformation of each partial statistical probability operator is obtained simply in the principal window by exchanging $p_t$ and $q_t$.

$$\Pi_{tj} = \begin{bmatrix} q_t & 0 \\ 0 & p_t \end{bmatrix} \quad \mapsto \quad \big(i\sigma_2\mathsf{K}\big) \times \Pi_{tj} \times \big(-i\sigma_2\mathsf{K}\big) \quad = \begin{bmatrix} p_t & 0 \\ 0 & q_t \end{bmatrix}.$$

As a result, the antiunitary operator means an exchange of all Boolean variables with their negation. This is nothing but a change of discrete Boolean gauge (Definition 4) in the observer gauge (Definition 5). □

**The Fubini-Study metric.** Actually, the Bloch sphere $\mathbb{S}^2$ can be roughly addressed as a subset of Euclidean space but not completely. Its topology is different and strictly speaking its natural metric [56] is in fact that of a complex projective space called *Fubini-Study metric*, also known as *Bures metric* in quantum physics. This metric $ds^2_{\text{FS}}$ with spherical coordinates $(\theta, \phi)$ is simply

$$ds^2_{\text{FS}} = \frac{1}{4}(\mathrm{d}\theta^2 + \sin^2\theta\ \mathrm{d}\phi^2). \tag{26}$$

Compared to Euclidean metric, the Fubini-Study area of a Bloch sphere is $\pi$ unstead of $4\pi$. It turns out that the factor 4, technically a Gaussian curvature, appears especially in the entropy density expression of the standard Bekenstein black-hole "area law".

**Rescaled Bloch sphere.** The standard Bloch sphere as such does not distinguish between snapshots. In order to represent the current snapshot of specific entropy $S_t$, we propose to introduce a *rescaled Bloch sphere* whose *Fubini-Study area* is by definition normalized to $S_t$ and therefore its Euclidean area is normalized to $4 \times S_t$.

**Proposition 25.** *Given the fundamental prior, a snapshot $S_t$ can be represented by a rescaled Bloch sphere normalized to a Euclidean area $A = 4S_t$ in natural entropy units. Its Euclidean radius $\chi_t$ is thus*

$$\chi_t = \sqrt{\frac{S_t}{\pi}} \tag{27}$$

*Its Fubini-study metrics $ds^2_t$ with spherical coordinates $(\theta, \phi)$ is*

$$ds^2_t = \frac{\chi_t^2}{4}(\mathrm{d}\theta^2 + \sin^2\theta\ \mathrm{d}\phi^2) \tag{28}$$

*so that its Fubini-Study curvature $C_t$ equal to its Fubini-Study square radius is*

$$C_t = \frac{4\pi}{S_t} = \frac{4}{\chi_t^2} \tag{29}$$

*The entropy density is uniform and equal to 1 in Fubini-Study metric and 1/4 in Euclidean metric. This defined a probability density of $1/S_t$ per nat.*

*Proof.* Construct a rescaled Bloch sphere endowed with the standard Fubini-Study metric and normalized to an area $S_t$. Let $\chi_t$ denote its radius. The usual Euclidean area is $A = 4\pi\chi_t^2 = 4S_t$ and thus $\chi_t = \sqrt{A/4\pi} = \sqrt{S_t/\pi}$. The entropy is uniform because the von Neumann entropy does not depend on the observation window. Its density is $S_t/S_t = 1$ in Fubini-Study metric and $S_t/A = 1/4$ in Euclidean metric. The total probability of the universe is 1 by definition over the $S_t$ nats. As a result, the probability density is $1/S_t$ per nat. □

In particular, the ultimate snapshot $S_{\max}$ is represented by a rescaled Bloch sphere of Fubini-Study area $S_{\max} = N_u$ bits $= N_u \ln 2$ nats and radius $\chi_{\max} = \sqrt{N_u \ln 2/\pi}$. The initial snapshot $S_0$ of zero entropy is reduced to a simple geometric point.

**Proposition 26.** *Given the fundamental prior, Eq. (2), the universe evolution from the initial snapshot $S_0$ to the current snapshot $S_t$ can be represented by a foliation of rescaled concentric Bloch spheres of increasing radii, $\chi_{t'} = \sqrt{S_{t'}/\pi}$ with $S_{t'} \in [0, S_t]$, constituting a rescaled Bloch ball of Euclidean area $4S_{t'}$. For the ultimate universe of entropy $S_{\max}$, the radius is maximum but finite and equal to $\chi_{\max} = \sqrt{N_u \ln 2/\pi}$ .*

*Proof.* Construct the set of concentric rescaled Bloch sphere of area $4S_{t'}$. Clearly, the radial distance $\chi_{t'}$ increases with $S_{t'}$ bounded by $S_t \leq S_{\max}$. □

**Definition 28** (The ultimate Bloch ball)**.** *The ultimate Bloch ball is the complete set of rescaled concentric Bloch spheres of maximum radius $\chi_{\max} = \sqrt{N_u \ln 2/\pi}$.*

The Euclidean curvature of the rescaled Bloch sphere is trivially $C_t = 1/\chi_t$ and its minimum is thus $C_{\min} = 1/\chi_{\max} = (\ln 2/\pi)^{1/2} \times 1/\sqrt{N_u}$.

### 3.3 Geometric representation of the universe

The rescaled Bloch spheres are interpreted henceforth as the basis of the universe geometry given the fundamental prior, Eq. (2). They are considered as double-sided surfaces. In Assumption (14) below, the two sides are respectively referred to as inward and outward, or equivalently, as internal and external.

**Definition 29** (Universe-point)**.** *A universe-point is a point of the ultimate Bloch ball on a particular side of a specific rescaled Bloch sphere.*

**Proposition 27.** *At any epoch, every universe-point is either in the past, in the present or in the future. There is no "elsewhere".*

*Proof.* By hypothesis, the current observer contemplates the rescaled Bloch sphere of area $4S_t$ and by definition every universe-point is located on a specific rescaled Bloch sphere of area $4S_{t'}$. Therefore, any universe-point is either in the *present* for $S_{t'} = S_t$, in the *past* for $S_{t'} < S_t$ or in the *future* for $S_{t'} > S_t$. This exhausts all possible universe-points and there is no room for the "elsewhere". □

This contradicts standard physics in which there are extra universe-points located in the conventional "elsewhere" and which are thus separated from the observer by a space-like interval. As a result, they are not observable. They are nonetheless thought to exist at every epoch.

By contrast, in a Bayesian theater, non-observable events are inconsistent and must be ignored: they would have no probability and no entropy! Therefore, for the sake of consistency, we will actually replace the standard "elsewhere" by the past universe-points or rather by their *current relics*. Indeed, even if past events are not observable either, from Proposition (13) their relics are so, and therefore located on some "episodic clock-hand" $\mathsf{h}_k$ with $k < k_t$, Sec. (2.3.4) above. As a result, any standard *space translation* in the principal direction is replaced in the present model by a simple *shift of the episodic clock-hand*. This is of course valid in non-principal directions with some obvious adjustments.

A question arises for the cues of future events. Are they perceptible? In fact, they will be compared to outgoing light rays in Assumption (14) below: their existence is acknowledged but they are not perceptible as such. This will be clarified later (Assumption 29 below).

But what does the universe depict given the fundamental prior? For definiteness, we simply propose that the fundamental prior be equated to the concept of "quantum vacuum".

**Assumption 12** (Quantum vacuum)**.** *The quantum vacuum is the representation of the universe conditionally to the fundamental prior.*

*Hints.* This proposes a precise definition while the standard quantum vacuum is widely puzzling. In any snapshot $S_t$, the void universe is subject to all physical "laws" and contains potentially all particles and their interactions. Technically, it we will be identified later with a so-called "sea of mono-episodic objects", Sec. (4.3.1) □

**Correspondence between space time and Hilbert space.** The current rescaled Bloch sphere represents an image of the past, current and foreseeable episodes of the quantum vacuum.

**Assumption 13** (Light-cone)**.** *The complete set of all universe-points located on the current snapshot $S_t$ represents the standard light-cone of the observer.*

*Hints.* The current snapshot $S_t$ represents the universe-points of the current Bloch sphere. From Proposition (3), *these universe-points can only be on the standard light-cone* because they are neither in the past nor in the future. For simplicity, we will say that they are connected by a "light-ray". □

We use the term "light-rays" for ease of presentation but at this stage, they are not precisely defined. They correspond to the geodesics of the standard pseudo-Riemannian metric of General Relativity. Using the Bousso's terminology [26], *any snapshot is a "light-sheet"*.

*This assumption constitutes a non-standard definition of the "present universe"*, which is thus equated to the standard light-cone and excludes the conventional "elsewhere" of standard physics. This is inevitable in a Bayesian framework because at every time non-observable events have no entropy. A similar concept was previously conjectured by R. Penrose in his "twistor theory" [57].

But with a little hindsight, this conception is actually very close to that of conventional cosmology! The only difference is that we make explicit that past events as such no longer exist and thus only their current relics are observable. In addition, for short distances, relics are indistinguishable from the events proper in standard physics.

**Universe symmetries.** In standard physics, at a particular moment, the flat space symmetry is generated by two groups, the group of rotation SO3 and the group of translation $\mathbb{R}^3$. The other physical entities are generated by the so-called gauge groups.

In the present model, spatial rotations are generated by the directional window group of the Hilbert space $\mathcal{H}_u$, Definition (27), that is SU2. Other symmetries correspond to gauge symmetries of $\mathcal{H}_u$. Spatial translations are not exact symmetries but simply shifts of the "episodic clock", Definition (21) above, and behave similarly as gauge operators.

On the other hand, it is also necessary to characterize the *two sides* of the rescaled Bloch sphere from the observer's point of view. We propose that they depict the two standard half light-cones.

**Assumption 14** (Incoming and outgoing half light-cones)**.** *The internal side of the current rescaled Bloch sphere represents the standard incoming half light-cone and the external side the outgoing half light-cone.*

*Hints.* The incoming light-rays are so in the past half light-cone and the outgoing light rays in the future half light-cone. □

The relics of past events are represented by incoming light rays located on the internal half light cone and recorded on the episodic clock.

**Causality: Status of past, present and future.** It turns out that causality is widely different in the present model and in standard physics. In the current model, time is identified with an increase in entropy, that is, an increase in uncertainty, and so the new increments are unpredictable. This is incompatible with the strict determinism governing standard physics. On the other hand, the events already occurred do limit the field of the possible future. They are recorded by past episodes on the episodic clock. Given the fundamental prior, there is no update and past episodes are summarized in the dominant episode, which therefore corresponds to the initial conditions of standard physics.

**Assumption 15** (Past episodes)**.** *When looking far into infinity the observer discovers the current relics of the past.*

*Hints.* This is clearly what common sense suggests. □

Given the fundamental prior, the foreseeable future is represented by the future episodes on the episodic clock. In terms of binary variables, they are symmetrical of the past episodes with respect to the current episode, that is fewer truth values 1 compared to more truth values 1 in past episodes.

This symmetry is surprising because in everyday life, past relics are perceived while future cues are not. Acknowledging this difference, we propose that future episodes are virtual in the sense of virtual particles in micro-physics. In addition, in the present model, the concept of "perception" itself is not very clear and will be elucidated later, (in short, we will propose that only "mediators" are observed).

**Assumption 16** (Future episodes)**.** *Future episodes are virtual, that is, not really perceived.*

*Hints.* Like outgoing light rays, future events point toward specific cues but are in fact invisible. □

Surprisingly enough, while past episodes represent pieces of ordinary matter, we will propose that future episodes represent antimatter. See Assumption (27) below.

*In summary*, a snapshot $S_t$ represents everything the observer grasps at a specific moment. Past episodes $\mathsf{h}_k$ with $k < k_t$ represent the relics of past events. The dominant episodes $\mathsf{h}_{k_t}$ represent the present. Future episodes $\mathsf{h}_k$ with $k > k_t$ are virtual.

**Time reversal.** Now, the exchange of incoming and outgoing light-rays can be identified with the standard operation of *time reversal*. This has nothing to do with a very problematic reversal of the cosmic time, but is in fact a reversal of the episodic clock.

**Assumption 17** (Time reversal)**.** *The standard operation of time reversal corresponds to an exchange of the incoming and outgoing half light-cones.*

*Hints.* The incoming light-rays so travel in the past half light-cone and the outgoing light rays in the future half light-cone. □

In standard physics, time reversal is an antiunitary operator that reverses light rays. This also corresponds to inversion of the arrows in Feynman's standard diagrams. We propose that this remains true in the present model. As already mentioned, a consequence is that parity and conjugation cannot be distinguished.

**Proposition 28.** *In every partial Hilbert space, $\mathcal{H}_j$, time reversal is expressed in the principal window by the antiunitary operator*

$$\mathsf{T} = i\sigma_2 \mathsf{K} \tag{30}$$

*This means an exchange of all Boolean variables with their negation in the observer gauge (Definition 5). The standard parity and conjugation symmetries, respectively,* $\mathsf{P}$ *and* $\mathsf{C}$ *in isolation, are irrelevant but only their product* $\mathsf{CP} = \mathsf{T}$ *makes sense. The* $\mathsf{CPT}$ *identity is trivial and expresses the involutive property* $\mathsf{T}^2 = \mathbb{1}$.

*Proof.* Consider in $\mathbb{R}^3$ a standard "ray" incoming from the direction $+n_\mu = (n_1, n_2, n_3)$ and therefore pointing towards the direction $-n_\mu = (-n_1, -n_2, -n_3)$. "Reversing time" means changing this ray into a new ray, coming from the direction $-n_\mu$ and therefore pointing towards the direction $+n_\mu$. In other words, times reversal must be identified with both a reflection of the direction $n_\mu$ into $-n_\mu$ and a jump to the other side of the rescaled Bloch sphere. From Proposition (24), this is obtained in particular by the antiunitary operator $\mathsf{T} = i\sigma_2\mathsf{K}$ and means an exchange of all Boolean variables with their negation. Alone, standard parity or conjugation symmetry would be incompatible with Assumption (11). □

**Remark.** Since the topology induced by SU2 is not Euclidean, every direction is in fact represented *twice*. For instance, given that space is isotropic in both half-cones, any direction $n_\mu$ of the internal half-cone can be associated with the same direction $n_\mu$ of the other half-cone.

At last, the common center of all Bloch spheres constitutes a particular universe-point of zero entropy that represents the reset register with a zero truth value for all binary variables. Its current relic is pointed by the episodic clock-hand of index $k = 0$, so that it is perceived by the observer as the location of the origin of the universe.

**Proposition 29.** *For the Bayesian observer, the geometric representation of the universe starts from a unique point corresponding to the common center of all rescaled Bloch spheres.*

*Proof.* The origin of the universe is defined by $S_0 = 0$ and therefore $\chi_0 = 0$. □

We propose to call "center of the universe" this common origin.

**Definition 30** (Center of the universe)**.** *For the Bayesian observer, the center of the universe is the common center of all rescaled Bloch spheres.*

On the other hand, the ultimate Bloch sphere represents a particular snapshot of maximum entropy and transition probability $1/2$. Its current clue in the snapshot $S_t$ is pointed by the episodic clock-hand of index $k = N_u/2$.

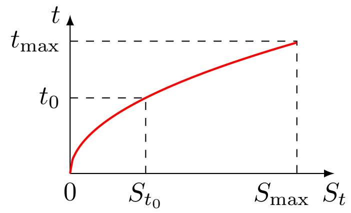
We are now ready to precisely define the cosmic time $t$, that is, to propose a quantitative correspondence with its standard counterpart.

## 3.4 Cosmic time

Standard cosmology is usually based on the Friedmann-Robertson-Walker (FRW) metric, that is an exact solution of Einstein's equations with maximum symmetry. Omitting the details irrelevant to the current model, this solution essentially depends on a *scale factor*, $R(t)$, function of a parameter $t$ called *cosmic time*. In dimensional analysis, the scale factor, $R(t)$, has dimension of length [58]. It is often represented by the dimensionless ratio, $a(t) = R(t)/R(t_0)$ of value 1 at the present epoch.

Now, we propose to identify the *radius* $\chi_t$ of the current rescaled Bloch sphere of area $4S_t$, Eq. (27), with the current *cosmic time* $t$ of standard cosmology. Therefore, the cosmic time is $t = \sqrt{S_t/\pi}$. Of course, this expression is extended beyond the fundamental prior.

**Assumption 18** (Cosmic time)**.**




*The cosmic time $t$ of standard cosmology associated with a snapshot $S_t$ is*

$$t = \sqrt{\frac{S_t}{\pi}}\ . \tag{31}$$

*Hints.* The ultimate Bloch ball is the collection of all rescaled Bloch spheres of entropy $S_t$. Similarly, the standard scale factor $R(t)$ defines a "foliation" of the FRW spacetime into time slices, that is a collection of 3-dimensional spaces at cosmic times $t$, all identical except by their scaling factor. Given the fundamental prior, this suggests to identify each slice at time $t$ with a rescaled Bloch sphere of Euclidean area $4S_t$. No additional factor is needed because the factor 4 has been precisely introduced to be consistent with the Bekenstein bound in the ultimate rescaled Bloch sphere. At last, by convention, this expression is dimensionless and uses Planck units. Restoring the dimensions, while still maintaining $S_t$ in nats, we have

$$t = \sqrt{\frac{\hbar G\, S_t}{c^5 \pi}}\ . \qquad \square$$

Quantitatively at the present epoch $t_0$, with $t_0 = 14.4$ Gyrs, the current particle entropy of the universe is $S_{t_0} = 3.24 \times 10^{122}$ bits.

*Remark.* This departs by several orders of magnitude from the standard estimate $S_{t_0} \sim 10^{106}$ bits but this latter estimation is highly uncertain and seems to miss most of the black-holes [39]. One possible solution could be a review of the size of supermassive black holes in galaxies. $\square$

Finally, the universe is not eternal and $t_{\max} = \sqrt{S_{\max}/\pi} \simeq 633.3$ Gyrs. In addition, with a transition probability $p_{t_0} = 0.120 \times 10^{-7}$ (Definition 10), the current relaxation time can be estimated as $t_0/p_{t_0} = 10^9$ Gyrs $\gg t_{\max}$. The universe is in fact pretty stable and still in its infancy!

By definition, the time flow results from a continuous creation of entropy (or equivalently from a loss of information) of $2\pi t$ per unit time.

$$S_t = \pi t^2 \quad ; \quad \frac{\mathrm{d}S_t}{\mathrm{d}t} = 2\pi t. \tag{32}$$

It is convenient to assign a special name to the time increment $\tau_t$ corresponding to $\mathrm{d}S_t = s_t$, the entropy increment $s_t$, Eq. (8).

**Definition 31** (Time increment)**.** *At cosmic time $t$, the time increment $\tau_t$ is*

$$\tau_t = \frac{s_t}{2\pi t} \tag{33}$$

*where $s_t$ is the entropy increment, Eq. (8).*

Quantitatively, at the current epoch $t_0$, the time increment is $\tau_{t_0} = 3.43 \times 10^{-61}$ Planck time units $= 1.85 \times 10^{-104}$ s.

**Continuous or discrete?** By simple inspection, even though we have defined a time increment $\tau_t$, no discreteness appears in Eqs. (31, 32).

By contrast, the episodic clock stores especially discrete past states of the universe. Indeed, from Eq. (20), the current snapshot of entropy $S_t = S_{k_t}$ is associated with a discrete location $k_t = p_k N_u \in [\![0, N_u/2]\!]$ of the episodic clock-hand, depicting $k_t$ truth value of 1 in the corresponding Boolean variable pattern. From Proposition (30), the time increment $\tau_t$, Eq. (33), corresponds to an additional truth value of 1 in this pattern. It is completely negligible compared to the standard deviation of $\varsigma_{t_0} = 5.8 \times 10^{-56}$ s $= 10^{-12}$ Planck unit of time (see above, just before Proposition 11) and is thus unfalsifiable, that is to say, not perceptible.

The *episodic discreteness itself remains nevertheless a fundamental feature* and was indeed the basis of Planck's derivation of the black-body radiation law [34] as well as the origin in the present model of the standard concept of "particles" in physics.

**Proposition 30.** *The cosmic entropy $S_t$ can be represented by a sum of discrete entropic increments $s_k$ for $k \in [\![1, N_u/2]\!]$ with $s_k = \ln(q_k/p_k)$, $p_k = k/N_u$ and $q_k = 1 - p_k$.*

*Proof.* From Proposition (16), the episodic snapshots $S_t$ form a discrete set $S_k$, with $k \in [\![0, N_u/2]\!]$. Let $p_k = k/N_u$, $q_k = 1 - p_k$ and $S_j(p_k) \overset{\text{(def)}}{=} -p_k \ln p_k - q_k \ln q_k$. Then, from Eq. (21), $S_k = N_u S_j(p_k)$. As a result, for $k > 0$,

$$\frac{\mathrm{d}S_k}{\mathrm{d}k} = \frac{\mathrm{d}S_k}{\mathrm{d}p_k} \times \frac{\mathrm{d}p_k}{\mathrm{d}k} = N_u \frac{\mathrm{d}S_j(p_k)}{\mathrm{d}p_k} \times \frac{1}{N_u} = \ln \frac{1 - p_k}{p_k}.$$

The actual increment $S_{k+1} - S_k \simeq \mathrm{d}S_k$ is about $(\mathrm{d}S_k/\mathrm{d}k) \times \mathrm{d}k$ with $\mathrm{d}k = 1$, and thus from Eq. (8) is precisely equal to the so called "entropic increment", say $s_k = s_t$, for $p_t = p_k$,

$$S_{k+1} - S_k \simeq \left.\frac{\mathrm{d}S_k}{\mathrm{d}k}\right|_{\mathrm{d}k=1} = \ln \frac{q_k}{p_k} = s_k. \qquad \square$$

Quantitatively, at the present epoch, $s_{t_0} = 18.235$ nats compared to $S_{t_0} = 2.248 \times 10^{122}$ nats. From Eq. (33), this corresponds to a completely negligible time lapse of $\tau_{t_0} = s_{t_0}/(2\pi t_0) = 3.43 \times 10^{-61}$ Planck units. Also, two adjacent indexes $k = k_t \pm 1$ correspond to this episodic time lapse $\tau_{t_0}$.

## 3.5 Definition of dark matter and black-holes

At this stage, it is possible to propose a very simple definition of dark matter and black-holes.

The universe begins from the center of the universe, that is a rescaled Bloch sphere of radius $\chi_0 = 0$ and entropy $S_0 = 0$. It ends in the ultimate rescaled Bloch sphere of radius $\chi_{\max} = t_{\max}$ and maximum entropy $S_{\max} = N_u \ln 2$.

Since $S_0 = 0$, the initial sphere of radius zero is simply invisible! In standard physics, this is the characteristic of *dark matter*. The universe is then only composed of pure states, that is deterministic Boolean variables.

Since $S_{\max}$ means a completely random system, the final sphere, in the usual jargon, has *no hairs* anymore. In standard physics, this is the characteristic of *black-holes* and even of a Schwarzschild black-hole, that is a pure black-hole, devoid of any additional feature like charge or momentum. The universe is then only composed of completely random Boolean variables.

For consistency, we therefore propose to define dark matter and black-holes accordingly.

**Assumption 19** (Dark matter)**.** *Dark matter is composed of deterministic degrees of freedom*

**Assumption 20** (Black-holes)**.** *Black-holes are composed of completely random degrees of freedom.*

With this terminology, we get the (tautological) proposition:

**Proposition 31.** *The universe begins as a block of dark matter and ends as a Schwarzschild black-hole.*

Note that given the fundamental prior, there are no *exact* black-holes for $t \neq t_{\max}$ and no *exact* dark matter for $t \neq 0$. Of course, *approximate* black holes and dark matter remain possible and are even ubiquitous when the finite precision is taken into account.

## 3.6 Expansion of the universe

When the area $4S_t$ of the rescaled Bloch sphere increases, its radius, that is the "distance" between the universe center and the observer increases likewise.

**Proposition 32.** *The universe is expanding over time.*

However, the "distance" to the universe center is defined in the Bloch representation and at this stage has no connection with the usual distance of the physical world. Let us interpret the universe radius in standard physics.

**Comoving distance.** By definition, the radius $\chi_t = t$ is both the cosmic time $t$ and the geometric radius of the universe in the Bloch representation. Therefore, we propose to identify this interval $\chi_t$ with its so-called "comoving distance" that is the distance used in the standard FRW metric to express the scale factor.

**Assumption 21** (Comoving distance)**.** *The radius $\chi_t = t$ of the rescaled Bloch sphere is the comoving distance between the universe center and the observer.*

*Hints.* Tautologically, the observer who contemplates the snapshot $S_t$ is indeed comoving with the universe radius. □

**Proposition 33.** *The comoving radius $\chi_t$ of the universe is equal to the cosmic time $t$.*

*Proof.* This is another wording of Assumption (21). □

**Proposition 34** (Rate of expansion)**.** *The FRW universe rate of expansion is constant and equal to 1 at any epoch.*

*Proof.* From Proposition (33) the comoving distance $\chi_t = t$ expands linearly. By definition, the FRW scale factor is $R(t) = t$ and the expansion rate $a(t) = \mathrm{d}R/\mathrm{d}t$ is thus constant and equal to 1. □

**Comparison with standard physics.** Proposition (34) contradicts the standard model of cosmology in which the rate of expansion depends on the cosmic time.

For example, in the standard early universe, in the so-called radiation dominated era, $R(t) \propto t^{1/2}$, while in the following matter-dominated era, $R(t) \propto t^{2/3}$. Finally, the standard model supposes that the expansion is accelerating in the so-called dark-energy-dominated era that holds at the present epoch, with $a(t_0) > 1$. This conjecture was supported by experimental results dating back to 1998 [59, 60] and awarded with a Nobel Prize in 2011.

However, this conclusion is challenged by a more recent update in 2015 by J. T. Nielsen *et al* [61] recording that *data are actually better suited to a constant rate of expansion.* In addition, while these results are obtained by observing the late universe (Ia supernovae), observation of the early universe leads to different conclusions [62, 63]. Therefore, Proposition (34) is in fact compatible with experiment at least at present.

This was previously conjectured by F. Melia [64].

**Definition of light velocity.** From Proposition (34), it is possible to assign a new an especially elegant definition to the light velocity.

**Assumption 22** (Light velocity)**.** *The standard speed of light is the universe expansion velocity.*

*Hints.* By construction, $\mathrm{d}\chi_t/\mathrm{d}t = 1$ is also the normalized light velocity. Turning the logic around, this is a definition of the light velocity, while in reality "light" itself is not defined at this stage. □

**Proposition 35.** *The speed of light is constant at any epoch.*

*Proof.* This is a tautological consequence of Assumption (22) and Proposition (34). □

Turning again the logic around, light-rays are actually located on the episodic clock and thus immobile with respect to the center of the universe, so that it is rather the observer who recedes at light velocity!

**Remote episodes.** Bloch's representation aims to represent the universe geometry but up to now, spatial translations have been left out.

Let us first define precisely what a *remote point* is in the present model. Recall that each cosmic time $t$ is associated with an entropy $S_t$, a transition probability $p_t$ and a clock-hand $\mathsf{h}_{k_t} \subset \mathcal{H}_u$ with $k_t = N_u p_t$. As a result, irrespective of $t$, from Eqs. (20) and (21), a clock-hand $\mathsf{h}_{k'}$ or an index $k'$ alone, represents also a transition probability $p' = k'/N_u$, an entropy $S_{k'} = -N_u\big(p' \ln p' + (1-p')\ln(1-p')\big)$ and a past cosmic time $t_{k'} = \sqrt{S_{k'}/\pi}$. For ease of presentation, we will write indifferently $t'$ or $t_{k'}$.

**Definition 32** (Remote episode)**.** *At time $t$, a past point $\gamma'$ is a triple $(k', \theta', \phi')$ with $k' < N_u p_t$. It depicts a relic whose image is located on the internal side of the rescaled Bloch sphere $S_t$ in the direction $(\theta', \phi')$. In the Hilbert space $\mathcal{H}_u$, it is represented by a clock-hand $\mathsf{h}_{k'}$ and observed from a particular uniform direction window $\mu'$ specified by the Bloch vector of spherical coordinates $(\theta', \phi')$.*

*Future episodes* are represented similarly on the future half light-cone with $k' > k_t$. We need now to specify the physical distance between the observer and a remote episode.

**Comoving distance of a remote episode.** From Proposition (15), a remote episode $(k', 0, 0)$ in the principal window is the relic of an event occurring in the past snapshot $S_{k'} = -k' \ln(k'/N_u) - (N_u - k')\ln(1-k'/N_u)$ and therefore at the cosmic time $t' = \sqrt{S_{k'}/\pi}$. By definition, its comoving distance from the observer measured in the rescaled Bloch sphere at time $t$ is thus $t - t'$.

**Proper distance of a remote episode.** By contrast, the proper distance is measured in the physical world, as opposed to the comoving distance expressed in Bloch's representation. In standard cosmology, the "proper distance" $D$ is measured along a light-ray by a series of variable rulers of unit length $\ell(t)$ undergoing the universe expansion and normalized to unity at the present epoch $t_0$ [58], so that $\ell(t_0) = 1$. The ruler unit length at the cosmic time $t \le t_0$ is thus $\ell(t) = \ell(t)/\ell(t_0) = R(t)/R(t_0)$.

In the current model, we propose to *assign* the proper distance of a remote episode according to the same rule with $\ell(t) = t/t_0$, that is to integrate the fact that the corresponding episode took place in an older and therefore smaller universe.

**Definition 33** (Proper distance)**.** *At the cosmic time $t_2$ and in the principal window, the proper distance $D_{21}$ of a remote episode happened at time $t_1 < t_2$ is the length that would be measured along the radius of the rescaled Bloch sphere with a ruler of variable unit length $\ell(t) = t/t_0$*

$$D_{21} \overset{\text{(def)}}{=} \int_{t_1}^{t_2} \frac{\mathrm{d}t}{\ell(t)} = t_0 \int_{t_1}^{t_2} \frac{\mathrm{d}t}{t} = t_0(\ln t_2 - \ln t_1). \tag{34}$$

In standard physics, the wavelength of light is assumed to exactly follow the universe expansion. Therefore, light undergoes a redshift $z = \frac{R(t_0)}{R(t)} - 1$. It turns out that this is only an approximation in the present model only valid for relatively small intervals $t_0 - t$. Therefore, we will call this ratio "marginal redshift".

**Proposition 36.** *The marginal redshift $z$ of a remote episode occurring at the cosmic time $t$ is*

$$z = \frac{t_0}{t} - 1 \tag{35}$$

*$z$ is positive for $t < t_0$ and negative for $t > t_0$. Therefore, this is genuine "redshift" for $t < t_0$ and in fact a "blueshift" for $t > t_0$.*

*Proof.* By definition, $z = [\ell(t_0) - \ell(t)]/\ell(t)$ and $\ell(t) = t/t_0$. □

At the current time $t_0$, consider in particular the relic of a past remote episode

**Proposition 37.** *In the principal window and at the current time $t_0$, the proper distance of a past episode $(k, 0, 0)$ from the observer is $D_k = t_0(\ln t_0 - \ln t_k)$.*

*Proof.* By definition, $t_k = \sqrt{S_k/\pi}$ where from Proposition (15), $S_k = -k\ln(k/N_u) - (N_u - k)\ln(1 - kN_u)$. Now, just apply Eq. (34) for $t_1 = t_k$ and $t_2 = t_0$. □

**Hubble's radius.** In standard cosmology, Hubble's radius is the distance traveled by light since the origin of time in the absence of expansion of the universe. This formulation is perhaps confusing because it calls for the inconsistent question, what kind of distance? For clarity, we propose therefore the following definition based on Assumption (22):

**Definition 34.** *The Hubble radius is the current comoving universe radius* $\chi_t = t$.

**Finite or infinite?** At last, a popular question is how big is the universe? From Assumption (34), this refers to the radius of the universe expressed in *proper distance*. In standard cosmology, the universe is infinite and flat. Its observable radius is about $60 \times 10^9$ light-years [58], but it is taken for granted that beyond that, there would be an infinite unobservable universe. However, this conclusion remains still debated [63].

By contrast, in a Bayesian theater, any unobservable event is problematic and nay nonexistent. Now, from Eq. (34) with $t_1 = 0$ and irrespective of $t_2$, the center of the universe is at an infinite proper distance from the observer. As a result:

*In comoving distance, the universe radius is finite at any time. In proper distance it is infinite at any time.* That's all!

## 3.7 The holographic principle

The limits of the observable universe are usually depicted in terms of "horizons".

**Horizons.** From V. Rindler [38], "a horizon is a frontier between things observable and things unobservable". A horizon is thus a surface, often characterized by its *proper distance* from the observer. In the Bayesian theater, there is no unobservable event, but there are two limits, respectively the center of the universe and the ultimate Bloch sphere. Therefore, in principle there are two horizons.

The first horizon called *particle horizon* or *cosmological horizon*, is the limit towards the past and represents the interval between the observer and the center of the universe. Its comoving distance is thus the radius $\chi_t = t$ of the rescaled Bloch sphere $S_t$, and its *proper distance* is infinite at any time.

The second horizon called *event horizon*, is the limit towards the future and corresponds to the ultimate Bloch sphere whose comoving distance from the observer is thus $\chi_{\max} - \chi_t = t_{\max} - t$ and therefore from Eq. (34), its *proper distance* is $t_0 \times (\ln t_{\max} - \ln t)$.

However, these two horizons are also specified by their characteristic entropies namely $S_t$ and $S_{\max}$, that is to say the *particle entropy* and the *event entropy*. From this point of view and for consistency, the particle horizon must rather be identified with the current Bloch sphere $S_t$ instead of the center of the universe. We propose to adopt this conception.

**Assumption 23** (Horizons)**.** *The particle horizon is the the current Bloch sphere* $S_t$. *The event horizon is the ultimate Bloch sphere* $S_{\max}$.

Then, the horizons are in fact nothing but the two *snapshots* at the present time $t$ and at the ultimate time $t_{\max}$ respectively. From assumption (13), they are in fact *light-sheets*. It turns out they can also be viewed as *screens* in the holographic theory of L. Susskind [27] and G. 't Hooft [28].

**The World as a Hologram.** According to the holographic theory, the entropy in the universe is distributed proportionally to the area of a two-dimensional screen. Remarkably, this is a natural result of the present model.

**Proposition 38** (Holographic screen)**.** *The entropy of the universe at time t is distributed proportionally to the area of the rescaled Bloch sphere* $S_t$.

*Proof.* The holographic screen is the current the rescaled Bloch sphere $S_t$. From Proposition (25), given the fundamental prior, its entropy distribution is uniform and its density equal to $1/4$. □

This is both the Bekenstein entropy density of the Schwarzschild black-hole horizon and that of the Bousso's "covariant entropy bound" [26] attained here for every snapshot (or light-sheet or holographic screen).

**Proposition 39** (Covariant entropy bound)**.** *The density entropy of any light-sheet $S_t$ is $S_t/A = 1/4$ (in nats) where $A$ is the Euclidean area of the rescaled Bloch sphere.*

*Proof.* From Proposition (25), this characterizes a snapshot. The Bousso's covariant entropy bound appears thus in the current model as a general law of physics. □

# 4 Energy

The notion of energy is fundamental in both standard physics and thermodynamics, as it has proved to be the pivot of the most significant properties and phenomena. In the current model, this remarkable role stems from the combination of the concept of probability with that of the size of the universe. It turns out that every significant quantities can be introduced through their effects on energy. Technically, we have just to compute the differential of energy with respect to the relevant parameters. For instance the concept of *temperature* is introduced when the relevant parameter is entropy, the concept of *force* when the relevant parameter is distance. Other major concepts, like especially *fields*, are introduced by combination of these and will be elaborated in the next section.

Let us first define energy as being an observable.

## 4.1 Definition of energy

In standard physics, according to the first principle of thermodynamics, the universe is endowed with an observable called "energy", or equivalently "mass", whose characteristic feature is to be conserved. It cannot be created nor destroyed. We propose to keep this view.

**Assumption 24** (Energy)**.** *Energy in the universe is an observable $\mathsf{H}_u$ whose value $U_u$ is independent of the current state.*

*Hints.* Technically, conservation of energy means that energy does not depend on the current state. □

As a result, the total mass $U_u$ can only depend on the number of variables, $N_u$. In fact, it explicit value is in part a matter of convention. In order to meet the standard convention, it turns out that we must take gravity into account. This will be achieved latter in Proposition (64) below. At this stage, we leave open its exact expression.

This observable $\mathsf{H}_u$ is traditionally given the name of "Hamiltonian". Since we will encounter a number of variants, we will name this operator "universal Hamiltonian".

**Proposition 40** (Universal Hamiltonian of the cosmos)**.** *The universal Hamiltonian of the whole universe is a homothety operator $\mathsf{H}_u = U_u(N_u) \times \mathbb{1}_{d_u}$, where $\mathbb{1}_{d_u}$ is the identity operator of dimension $d_u = 2^{N_u}$ and the function $U_u(N_u)$ only depends on the number of degrees of freedom, $N_u$.*

*Proof.* The Hilbert space $\mathcal{H}_u$ associated with the universe only depends on the number of degrees of freedom, $N_u$. By hypothesis, $U_u$ does not depend on the statistical operator $\Pi_t$ and thus only depends on $N_u$, so that we can compute $U_u$ with different operators $\Pi_t$. In a basis where $\mathsf{H}_u$ is diagonal, select successively for $\Pi_t$ the projection operators on the basis vectors. This computes the successive diagonal components of $\mathsf{H}_u$ which are all equal to $U_u$ by hypothesis. Conversely, assume that $\mathsf{H}_u = U_u \times \mathbb{1}_{d_u}$. Then

$$U_u \overset{(\text{def})}{=} \langle \mathsf{H}_u \rangle = \mathrm{Tr}(\Pi_t \mathsf{H}_u) = U_u \times \mathrm{Tr}(\Pi_t) \tag{36}$$

does not depend on $\Pi_t$ because $\mathrm{Tr}(\Pi_t) = 1$. □

The universal Hamiltonian is constant over time and hardly ever used in standard physics. From Proposition (40), it commutes with the statistical operator.

Basically, $N_u$ is integer but since it is a gigantic number, it is helpful to treat it as a real valued parameter and $U_u(N_u)$ as a continuous derivable function. Its differential is

$$\mathrm{d}U_u = U_u' \,\mathrm{d}N_u. \tag{37}$$

$U_u'(N_u) = \mathrm{d}U_u/\mathrm{d}N_u$ plays the role of *chemical potential* of the dichotomic variables. By construction, $U_u'$ is independent of the particular statistical operator $\Pi_t$.

**Energy density.** By extension, an energy *density* per unit of entropy can also be defined on the rescaled Bloch spheres.

**Proposition 41.** *Given the fundamental prior, the total universe energy $U_u$ generates a uniform energy density $U_u/4S_t$ of sum $U_u$ on the rescaled Bloch sphere.*

*Proof.* From Proposition (40), the universal Hamiltonian commutes with any window operator. Given the fundamental prior, the orientation windows are defined independently of the statistical operator so that the energy $U_u$ is independent of the orientation. Therefore, its density is evenly distributed on the rescaled Bloch sphere of Euclidean area $4S_t$ and is thus equal to $U_u/4S_t$ at the cosmic time $t$. □

Obviously, beyond the fundamental prior, updates can lead to a non-uniform energy density distribution on the sphere.

## 4.2 Distribution of energy in the universe

From Proposition (40), energy is the observable defined by the operator $\mathsf{H}_u \propto \mathbb{1}_{d_u}$, so that irrespective of the statistical operator *energy and probability distributions are proportional.* Energy is simply a variant of probability but normalized to $U_u$ instead of 1. Therefore, *everything that has a probability has a mass and vice versa.*

### 4.2.1 Energy of parts of the universe

Let $\Pi_t$ denote the statistical operator acting on $\mathcal{H}_u$ given some particular prior. In the $d_u$-dimensional Hilbert space, a relevant partition of the universe is conveniently depicted by a resolution of the identity, that is technically by a standard *positive operator-valued measure* (POVM). For the sake of generality, it is possible extend the definition to *weak* POVMs (see Ref. [18]), i.e., sets of observables $\mathsf{Q}_\iota$ indexed by $\iota$ (iota), not necessarily positive but with positive expectation value, $\langle \mathsf{Q}_\iota \rangle \overset{(\mathrm{def})}{=} \mathrm{Tr}(\mathsf{Q}_\iota \Pi_t) \geq 0$ whose total expectation $\sum \langle \mathsf{Q}_\iota \rangle$ sums to 1. Of course, a strict POVM is also a weak POVM. Let $\mathscr{I} = \{\iota\}$ be the finite ensemble of indexes, with $\iota \in \mathscr{I}$.

$$\forall \iota \in \mathscr{I} : \langle \mathsf{Q}_\iota \rangle \geq 0; \quad \sum_{\iota \in \mathscr{I}} \langle \mathsf{Q}_\iota \rangle = 1 \quad \text{where} \quad \langle \mathsf{Q}_\iota \rangle \overset{(\mathrm{def})}{=} \mathrm{Tr}(\mathsf{Q}_\iota \Pi_t)$$

so that we obtain the weak POVM probability distribution, $\mathrm{p}_\iota$ as

$$\mathscr{I} \to \mathbb{R} : \quad \iota \mapsto \mathbb{P}(\mathsf{Q}_\iota) \overset{(\mathrm{def})}{=} \mathbb{P}(\mathcal{H}_u) \times \mathrm{Tr}(\mathsf{Q}_\iota \Pi_t) = \mathrm{p}_\iota \tag{38}$$

where by definition $\mathbb{P}(\mathcal{H}_u) = 1$.

**Proposition 42** (POVM energy distribution)**.** *Any weak POVM probability distribution on the cosmic Bayesian theater $\mathcal{H}_u$, Eq. (38), determines a distribution of the total energy $U_u$ at time $t$ as*

$$\mathscr{I} \to \mathbb{R} : \quad \iota \mapsto \mathbb{U}(\mathsf{Q}_\iota) \overset{(\mathrm{def})}{=} U_u \times \mathrm{Tr}(\mathsf{Q}_\iota \Pi_t) = U_\iota. \tag{39}$$

*Proof.* Just replace in Eq. (38) the total probability $\mathrm{p}(\mathcal{H}_u) = 1$ by the total energy $U_u$. □

#### 4.2.2 Energy of classical states

Any standard measurement in quantum information can be reduced to a POVM. In particular, a von Neumann measurement in the Hilbert space $\mathcal{H}_u$ results from the partition of the identity into the $d_u$ basic projectors $|\gamma_i\rangle\langle\gamma_i|$ with $i \in [\![1, d_u]\!]$ in correspondence with the set of classical states $\gamma_i \in \Gamma_u$, where $\Gamma_u$ is the sample set of the current observation window. In the principal window, given the fundamental prior, the energy at time $t$ of a classical state $\gamma_i$ is

$$\mathbb{U}(\gamma_i) = U_u \lambda_i$$

where $\lambda_i$ is the corresponding eigenvalue of the statistical operator $\Pi_t$. Let $k$ be the Hamming weight of the classical state $\gamma_i \in \Gamma_u$. Then, from Proposition (12), $|\gamma_i\rangle \in \mathsf{h}_k$ and $\lambda_i = \alpha_k$.

#### 4.2.3 Energy of Boolean functions

In the principal observation window, any Boolean function of the binary variables can be regarded as an element of the sigma algebra over the current sample set $\Gamma_u$ (see Ref. [18]). As a result, the probability of a Boolean function is well defined and equal to a sum of classical state probabilities and therefore its mass is the sum of their individual mass.

**Binary variables.** In particular, each binary variable $\mathsf{X}_j$ is a Boolean function. Given the fundamental prior, the probabilities of the $N_u$ variables in the principal window are equal. Although independent, they are obviously overlapping events. Irrespective of its index $j$, the probability of $\mathsf{X}_j$ at time $t$ is $p_t$ and that of its negation $\overline{\mathsf{X}}_j$ is $q_t$. Therefore, their masses are respectively

$$\mathbb{U}(\mathsf{X}_j) = p_t U_u \quad ; \quad \mathbb{U}(\overline{\mathsf{X}}_j) = q_t U_u. \tag{40}$$

**General Boolean formula.** More generally, from standard Boolean theory, any Boolean function, say $\mathsf{Y}$, expressed in the principal observation window, can be expanded as a disjunction of mutually exclusive classical states as

$$\mathsf{Y} = \sum_\ell \gamma_\ell$$

Therefore, its energy is well defined and equal to

$$\mathbb{U}(\mathsf{Y}) = \sum_\ell \mathbb{U}(\gamma_\ell) = U_u \sum_\ell \lambda_\ell$$

Beyond Boolean functions, it is possible to define the concept of "massive objects".

### 4.3 Massive objects

We propose to represent an object by a *weak effects*, i.e., an observable, say $\mathsf{Q}$, acting on the Hilbert space $\mathcal{H}_u$ with a bounded positive expectation with respect to the current statistical operator $\Pi_t$, that is, $\langle \mathsf{Q}\rangle = \mathrm{Tr}(\mathsf{Q}\Pi_t) \leq 1$. This expectation value is therefore a probability. In turn, it is possible to equate the probability with a mass (up to the factor $U_u$).

**Definition 35** (Massive object). *In the cosmic Bayesian theater $\mathcal{H}_u$, a massive object is described by a weak effect $\mathsf{Q}$, i.e., an observable with a positive expectation value $\langle \mathsf{Q}\rangle$ bounded by 1, that is $\langle \mathsf{Q}\rangle \leq 1$.*

For ease of presentation, we will identify throughout the object with its observable $\mathsf{Q}$.

**Proposition 43.** *The energy of an object $\mathsf{Q}$ is positive and equal to*

$$\mathsf{Q} \mapsto \mathbb{U}(\mathsf{Q}) = U_{\mathsf{Q}} = U_u \times \mathrm{Tr}(\mathsf{Q}\Pi_t) \tag{41}$$

*Proof.* Complement $\mathsf{Q}$ by $\mathsf{Q}' = \mathbb{1}_{d_u} - \mathsf{Q}$. Then $0 \leq \langle \mathsf{Q}'\rangle \leq 1$ and $\langle \mathsf{Q}\rangle + \langle \mathsf{Q}'\rangle = 1$ so that the pair $\{\mathsf{Q}, \mathsf{Q}'\}$ forms a weak POVM. □

### 4.3.1 Episodic objects

Among the massive objects, we are particularly interested in those that can be localized on the episodic clock. Technically, they commute with the statistical operator $\Pi_t$. We will look more specifically at the *projectors* on eigensubspaces of $\Pi_t$, which we propose to call "episodic objects". A precise definition is provided below (Definition 37). As a first step, let us define the concept of "mono-episodic projector".

**Definition 36** (Mono-episodic projector)**.** *A mono-episodic projector* $\mathsf{Q}_k$ *is an orthogonal projection operators acting on a single clock hand* $\mathsf{h}_k$*.*

We will later complement this definition with a particular evolution rule, Proposition (53) below. Let $r_{\mathsf{Q}}$ denote the rank of $\mathsf{Q}_k$. Then, from Proposition (12), by reverse transcription in the principal window, $\mathsf{Q}_k$ represents a Boolean disjunction of $r_{\mathsf{Q}}$ distinct classical states of same Hamming weight $k$. Clearly, there is a vast number of mono-episodic projectors in the universe, a kind of "sea".

**Proposition 44.** *Given the fundamental prior, the quantum vacuum can be regarded as a "sea" of mono-episodic projectors.*

*Proof.* Regard the universe as a massive object, $\mathsf{Q}_u$. At cosmic time $t$, given the fundamental prior, its is possible to decompose $\mathsf{Q}_u$ in a vast number of ways into a set of mono-episodic projectors. From Assumption (12), this "sea" represents the standard *quantum vacuum.* □

As physical objects, the mono-episodic projectors remain potential until the observer updates the Bayesian prior. By contrast, the current universe corresponds to a particular update of the prior obtained especially by astronomical observations.

Whether they are only potential or not, let us now define specific episodic objects as composed of mono-episodic projectors.

**Definition 37** (Episodic object)**.** *At time* $t$*, an episodic object* $\mathsf{Q}$ *on the trajectory* $(k_1, k_2)$ *is a cluster of mono-episodic projectors* $\mathsf{Q}_k$ *of same rank,* $r_{\mathsf{Q}}$*, for each* $k \in [\![k_1, k_2]\!]$

$$\mathsf{Q} = \sum_{k=k_1}^{k_2} \mathsf{Q}_k. \tag{42}$$

At the cosmic time $t$, the mapping $k \to \mathsf{Q}_k$ is thought as the trajectory of a mono-episodic projector of rank $r_{\mathsf{Q}}$ as a function of the episodic index $k$. In the current snapshot of dominant index $k_t$, the most interesting episodic objects are those that exist at the current time, that is when $k_t \in [k_1, k_2]$. Then, the pair $(k, k_t)$ delimits both a cosmic time lapse and a comoving distance. Thus, on the one hand, the mono-episodic projectors $\mathsf{Q}_k$ can be viewed as past relics for $k < k_t$, the current appearance of the object itself for $k = k_t$ and future cues for $k > k_t$. On the other hand, the trajectory describes an extended object with a perceptible part for $k \leq k_t$ and a virtual part for $k > k_t$ (see Assumption 29 below). Obviously, for a definite episodic object *these distinctions slide over time* in other snapshots.

Now, we propose that every massive object of the physical universe be represented by an episodic object.

**Assumption 25** (Massive objects in space-time)**.** *Physical massive objects that can be localized in space-time are represented by episodic objects.*

*Hints.* This includes macro objects and particles. Particles include fermions and composite bosons of the standard model but in the present theory, we will see that it is convenient to treat separately mediators, that is massless field bosons. Basically, this assumption asserts that the properties of particles can be deduced in the context of the fundamental prior. □

Interestingly, note also that the usual concept of massive object corresponds to Boolean formulas.

**Proposition 45.** *Episodic objects depict the Boolean functions of binary variables in the principal window.*

*Proof.* In Bayesian theaters, orthogonal projection operators on eigensubspaces acting on the Hilbert space are the transcription of a Boolean formula (see Ref. [18]). □

**Energy of episodic objects.** From Eq. (41), the mass of the episodic object $\mathsf{Q}$ can be easily computed at the cosmic time $t$.

**Proposition 46.** *Given the fundamental prior, the energy $U_{\mathsf{Q}}$ of the episodic object $\mathsf{Q}$ of rank $r_{\mathsf{Q}}$ on the trajectory $(k_1, k_2)$ at the cosmic time $t$ is equal to*

$$U_{\mathsf{Q}}(t) = U_u \operatorname{Tr}(\mathsf{Q}\Pi_t) = U_u r_{\mathsf{Q}} \times \frac{q_t \alpha_{k_1}(p_t) - p_t \alpha_{k_2}(p_t)}{q_t - p_t} \tag{43}$$

*where $q_t = 1 - p_t$.*

*Proof.* Expand $\mathsf{Q}$ expressed in the principal window, Eq. (42), with $\alpha_k(p_t) = p_t^k q_t^{N_u - k}$, Eq. (4).

$$U_{\mathsf{Q}} = \sum_{k=k_1}^{k_2} U_u \operatorname{Tr}(\mathsf{Q}_k \Pi_t) = U_u r_{\mathsf{Q}} \sum_{k=k_1}^{k_2} \alpha_k(p_t) = U_u r_{\mathsf{Q}} \times q_t^{N_u} \times \sum_{k=k_1}^{k_2} \left(\frac{p_t}{q_t}\right)^k.$$

The sum of the geometric sequence leads to Eq. (43). □

Remarkably, the mass of episodic objects always varies over time.

### 4.3.2 Comoving objects.

From Sec. (2.3.3), the "thickness of the present" is represented by a minuscule time lapse $\varsigma_t$. For example, $\varsigma_t$ is about $10^{-12}$ Planck time unit at the present epoch. This is far too fleeting to be perceived. By contrast, perceptible objects must remain in the range of the observer for a sufficient time lapse. Therefore, they are approximately comoving with the observer. Precisely, we propose that their *mass* be stationary.

**Assumption 26** (Comoving objects)**.** *Comoving massive objects are those whose mass is stationary in time.*

*Hints.* Technically, stationarity means that the derivative of the mass with respect to $t$ or $p_t$ is zero at the moment of observation. □.

Comoving objects must remain comoving for a sufficient time interval. How is it possible? Indeed, given the fundamental prior, mono-episodic projectors $\mathsf{Q}_k$ are fixed on the episodic clock and, from Eq. (17), their masses are not stationary except in the dominant eigensubspace, that is for $k = k_t$. Therefore, a comoving object must be composed of mono-episodic projectors which follow the expansion of the universe.

**Proposition 47.** *A comoving object at time $t$ is a permanently rebuilt cluster of mono-episodic projectors driven by the universe expansion so that its energy remains stationary.*

*Proof.* A comoving object is necessarily a set of mono-episodic projectors *permanently rebuilt*, thanks to a mechanism described below. This reconstruction is driven by the cosmic time that is to say the universe expansion. □

At time $t$, consider a *comoving object* $\mathsf{Q}$, composed of mono-episodic projectors $\mathsf{Q}_k$ of same rank $r_\mathsf{Q}$, Eq. (42), with $k_1 \leq k_t \leq k_2$,

$$\mathsf{Q} = \sum_{k=k_1}^{k_2} \mathsf{Q}_k.$$

It is convenient to regard $\mathsf{Q}$ as a sum $\mathsf{Q}_-$, $\mathsf{Q}_{k_t}$ and $\mathsf{Q}_+$, where $\mathsf{Q}_-$ is the past episodic section and $\mathsf{Q}_+$ the future episodic section, $k_t$ being the dominant index.

$$\mathsf{Q}_- \overset{(\mathrm{def})}{=} \sum_{k<k_t} \mathsf{Q}_k \quad ; \quad \mathsf{Q}_+ \overset{(\mathrm{def})}{=} \sum_{k>k_t} \mathsf{Q}_k \quad \text{so that} \quad \mathsf{Q} = \mathsf{Q}_- + \mathsf{Q}_{k_t} + \mathsf{Q}_+$$

**Proposition 48.** *A comoving object is made up of three sections, $\mathsf{Q}_-$, $\mathsf{Q}_{k_t}$ and $\mathsf{Q}_+$ of masses $U_{\mathsf{Q}_-}$, $U_{\mathsf{Q}_{k_t}}$ and $U_{\mathsf{Q}_+}$ respectively. The derivative of $U_{\mathsf{Q}_{k_t}}$ with respect to $p_t$ is zero while the derivatives of $U_{\mathsf{Q}_-}$ and $U_{\mathsf{Q}_+}$ are opposite.*

*Proof.* Since the object is comoving, the derivative of the total energy $U_\mathsf{Q}$ with respect to $t$ (or $p_t$) is zero. $U_\mathsf{Q}$ is the sum of the energies $U_{\mathsf{Q}_-}$, $U_{\mathsf{Q}_{k_t}}$ and $U_{\mathsf{Q}_+}$ of the three sections. From Eq. (17), the derivative of $U_{\mathsf{Q}_{k_t}}$ with respect to $p_t$ is zero so that the derivatives of $U_{\mathsf{Q}_-}$ and $U_{\mathsf{Q}_+}$ are opposite. □

### 4.3.3 Matter and antimatter

It is now possible to formally introduce the concept of “antimatter”. We simply propose that the antimatter be composed at cosmic time $t$ of mono-episodic projectors whose indexes $k$ are greater than the current dominant index $k_t$.

**Assumption 27** (Matter and antimatter)**.** *At time $t$, matter is made up of mono-episodic projectors $\mathsf{Q}_k$ with indexes $k \leq k_t$ while antimatter is made up of mono-episodic projectors with indexes $k \geq k_t$.*

*Hints.* The standard definition of antimatter is that it is the time reversal of matter. Every mono-episodic projector $\mathsf{Q}_k$ alone is obviously a particular episodic object. It can be associated with another mono-episodic projector $\mathsf{Q}_{k'}$ where $k'$ is chosen to be symmetrical of $k$ with respect to the dominant index $k_t$. Now, each mono-episodic antimatter projector $\mathsf{Q}_k$ with $k \geq k_t$ is therefore the time reversal of another mono-episodic projector of matter $\mathsf{Q}_{k'}$ with $k' \leq k_t$. □

Incidentally, let us recall that in Special Relativity, “Majorana particles” [65, 66] are hypothetical objects (actually “fermions”), identical to their own antiparticles and therefore invariant by time reversal. According to the present model, a Majorana particle is composed of a *single* mono-episodic projector.

**Definition 38** (Majorana particles)**.** *Majorana particles are episodic objects identical to their own antiparticles.*

**Proposition 49.** *At time $t$, Majorana particles are made up of a single mono-episodic projector whose index is the dominant index $k_t$.*

*Proof.* From Assumption (27), their index $k$ is subject to both $k \leq k_t$ and $k \geq k_t$. □

**Proposition 50.** *At time $t$, the mass of a Majorana particle of rank $r_\mathsf{Q}$ is*

$$U_\mathsf{Q} = U_u r_\mathsf{Q} e^{-S_t}, \tag{44}$$

*Proof.* This is Eq. (43) for $k_1 = k_2 = k_t$, using $\alpha_{k_t}(p_t) = e^{-S_t}$, Eq. (20). □

**Proposition 51.** *Given the fundamental prior, every (potential) comoving object is composed of a pair matter-antimatter, namely $\mathsf{Q}_-$ and $\mathsf{Q}_+$, and a Majorana particle, $\mathsf{Q}_{k_t}$, all three of the same rank $r_\mathsf{Q}$.*

*Proof* This is a trivial consequence of Proposition (27). Except for the Majorana object $\mathsf{Q}_{k_t}$, this result essentially joins the standard conception of the quantum vacuum. □

### 4.3.4 Dragging comoving objects

The permanent reconstruction of comoving object is in fact due to the jumps of the binary variables $\mathsf{X}_j$.

**Proposition 52.** *Comoving objects are dragged by the jumps of binary variables* $\mathsf{X}_j$ *from* $0 \to 1$.

*Proof.* In the principal window, mono-episodic projectors $\mathsf{Q}_k$ gather distinct classical states of same Hamming weight $k$, meaning that they depict the conjunction of a number of $k$ binary variables $\mathsf{X}_j$ with a valid truth value (equal to 1) and hence $N_u - k$ with an invalid truth value (equal to 0). Therefore, each time that the mean number of valid truth values in the universe increases by 1, all states of the mono-episodic projectors $\mathsf{Q}_k$ slip by definition to states of $\mathsf{Q}_{k+1}$. This drag is triggered by the jump of just one binary variable $\mathsf{X}_j$ from $0 \to 1$. As a result, it can be viewed as driven by the universe expansion. From Proposition (30), this increment of 1 for $k \to k+1$ corresponds to an entropy increment of $s_t$, Eq. (8), and a time increment of $\tau_t$, Eq. (33). □

In other word, a comoving object is a kind of "strobe entity". It can be compared to a rainbow or interference fringes in a standard wave continuum. Here, the continuum involved is the sea of fixed mono-episodic projectors and the equivalent of phase matching is the mass stationarity of the cluster.

### 4.3.5 Mediators

In order to formally describe the drag mechanism involved in the permanent reconstruction of the mono-episodic projectors, let us introduce the concept of "mediator", used in standard physics of particles.

By definition (Definition 47), the drag can be expressed by slip operators $\mathsf{B}_{k,k+1}$ mapping $\mathsf{h}_k \to \mathsf{h}_{k+1}$. Technically, a specific operator is so attached to each particular mono-episodic projector. Let us call it "mediator emitted by the mono-episodic projector". For the sake of generality, we consider also inverse operators, so that mediators can operate forward or backward.

**Definition 39** (Mediator)**.** *At time* $t$*, a mediator* $k \to k \pm 1$ *emitted by a particular mono-episodic projector* $\mathsf{Q}_k$ *is a slip operator,*

$$\mathsf{B}_{k,k\pm 1} : \mathsf{h}_k \to \mathsf{h}_{k\pm 1}$$

*mapping the episodic clock hand of index* $k$ *into the clock hand of index* $k\pm 1$*. For* $k \to k+1$*, the mediator is called "forward". For* $k \to k-1$*, the mediator is called "reversed".*

**Proposition 53.** *Every jump from* $0 \to 1$ *or* $1 \to 0$ *of any binary variable* $\mathsf{X}_j$ *generates a discrete mediator* $\mathsf{B}_{k,k\pm 1}$ *emitted by each mono-episodic projector* $\mathsf{Q}_k$.

*Proof.* Every eigensubspaces $\mathsf{h}_k$ represents the pattern of Boolean occurrences with exactly $k$ truth values equal to 1. Since the probability of double jumps, e.g., $0 \to 1 \to 0$, is negligible, the mediator expresses a simple jump of just one binary variable from $0 \to 1$ or $1 \to 0$. □

**Mediators represent radiation.** While episodic observables represent massive objects, what can mediators represent in the physical world? In accordance with standard physics, we propose that they represent "radiation". In addition, from the point of view of the Bayesian observer, we propose to distinguish incoming and outgoing radiation.

**Assumption 28** (Physical counterparts of mediators)**.** *Mediators represent radiation. At time* $t$*,*

*-Incoming radiation is represented either by forward mediators emitted by past mono-episodic projectors, or by reverse mediators emitted by future mono-episodic projectors.*

*-Outgoing radiation is represented either by reverse mediators emitted by past mono-episodic projectors, or by forward mediators emitted by future mono-episodic projectors.*

*Hints.* The outgoing radiation is so the time reversal of the incoming radiation. This assumption is consistent with the geometric representation of space time, Assumption (13).

On the other hand, this joints the usual representation of bosons in Feynman diagrams, but an apparent difference is that the mediators are in principle assumed here to be either forward or reversed. In fact, this corresponds to a single interpretation of each diagram, whereas they are usually compatible with both interpretations. However, in practice the distinction does not hold because it would suppose that the episodic clock hands $\mathsf{h}_{k-1}$, $\mathsf{h}_k$ and $\mathsf{h}_{k+1}$ are separated which never happens. □

The observer is unable to perceive the mono-episodic projectors. So what does the observer perceive? We propose that the only phenomena directly perceived by the observer are the incoming radiation.

**Assumption 29** (The observer perception)**.** *The observer does not directly perceive massive objects but only the* incoming *radiation they emit.*

*Hints.* Mono-episodic projectors are fixed on the episodic clock and cannot be perceived because they are too fleeting. To join the everyday experience, we propose that only incoming radiation, for example incoming light rays, is perceived. By contrast, outgoing radiation is not directly perceived. □

**Proposition 54.** *When the transition probability is small enough, $p_t \ll 1$, the vast majority of mediators occurring during a certain time lapse are forward. In particular, during one time increment $\tau_t$, Eq. (33), there is on average only one forward and no reversed mediator.*

*Proof.* At the origin of time, i.e., in the initial snapshot $S_0$, $p_t = 0$, all truth values are "0" and there is no "1". Therefore, at time $t$, for $p_t \ll 1$ there is still only a negligible number of "1". From Proposition (4), the probability of jump is independent of the truth value but the vast majority of truth values are "0". As a result, the vast majority of jumps are forward. In the very short time lapse $\tau_t$, Eq. (33), this number is integer and by definition about 1, that in fact 1. □

**Proposition 55** (Ratio matter-antimatter)**.** *When the transition probability is small enough, the vast majority of objects perceived by the observer are made up of matter, as opposed to antimatter.*

*Proof.* From Proposition (51), matter and antimatter come by pairs. However, from Assumption (28), they are only perceived through the radiation they emit towards the observer, that is, still from Assumption (28), forward mediators for matter and reverse mediators for antimatter. Indeed, this is actually the ratio of forward/reverse mediators, which follows Proposition (54). □

This result is very mysterious in standard physics whereas it is a simple platitude in the present model.

To go on and address the standard concept of particle, we need first to investigate the notion of temperature.

# 5 Temperatures

In standard thermodynamics, temperature expresses the differential link between *energy* and *entropy* specified by some parameters that may vary. In the cosmic Bayesian theater, global entropy is a variant of cosmic time and thus indicates the arrow of evolution. Therefore, the cosmic temperature of massive objects, that is to say the derivative of their energy with respect to the global entropy is the indispensable step to introduce all significant quantities, like *forces* and *fields*.

**Definition 40** (Cosmic temperature)**.** *The cosmic temperature of a massive object is the partial derivative of its energy with respect to the universe entropy.*

A certain number of distinct cosmic temperatures must be distinguished, depending on the object and the phenomenon under investigation, that is to say, technically, the specific parameters involved beyond energy and entropy. We are particularly interested in the characterization of forces and fields by moving massive objects.

**Assumption 30** (Movements)**.** *Given the fundamental prior, the only movements are those produced by the expansion of the universe.*

*Hints.* Given the fundamental prior, this seems the only possibility. □

**Definition 41** (Force)**.** *A force applied to a massive object is a specific way to increase its energy by moving it.*

**Definition 42** (Field)**.** *A field is a specific way to generate forces.*

As a result, beyond entropy, a second indispensable parameter is distance. Possibly, a third parameter of interest may be necessary, according to the particular field involved.

**Global versus local temperature.** A *real* variation of energy is impossible for the full universe because the global energy is constant, thanks to the first principle. Therefore, two domains must be distinguished. The first concerns the *universe as a whole* and demands *virtual* variation of energy. It depicts the expansion of space and leads to the emergence of gravity. The second concerns *local objects*. It describes both the cosmic microwave background and the particles of microphysics, thus leading to the standard fields of physics.

*Convention.* As far as temperature is concerned, unless otherwise stated, we express entropy in *nats* (as opposed to *bits*) to conform to standard physical conventions. □

## 5.1 Temperatures of the universe as a whole

Consider the entire universe at time $t$ specified by $N_u$ dichotomic variables. In the following, we will investigate different temperatures characterized by different partitions of the universe.

### 5.1.1 Information temperature

First, a global environment is defined by the pair $(S_t, I_t)$ of independent parameters where $I_t$ is the negentropy, Definition (15) above. This means partitioning the universe into a pair of coarse-grains, namely its *entropic phase* and the complement, its *negentropic phase*. This corresponds to the so-called *information representation* of the universe, Definition (16).

Starting from $S_{\max} = N_u \ln 2 = S_t + I_t$, we can rewrite Eq. (37) as

$$\mathrm{d}U_u = U'_u \mathrm{d}N_u = \frac{U'_u}{\ln 2} \left(\mathrm{d}S_t + \mathrm{d}I_t\right) \tag{45}$$

Now, it is possible to define the temperature of the universe as a whole *at constant information.*

**Definition 43** (Information temperature of the universe)**.** *The information temperature characterizes the partition of the universe into entropic and negentropic phases.*

These two phases corresponds to the standard "heat" and "work" of conventional thermodynamics, but paradoxically, in cosmology "heat" is generally cold and "work" is mostly hot[8]!

[8]Although it is isolated as a whole, the universe cannot be identified with a microcanonical ensemble because its basic constraint, Eq. (2) is by no means conventional. In particular, the so-called "third principle" or Nernst's theorem, asserting that temperature is minimum for minimum entropy is dramatically flawed. As a result, the standard thermodynamic term "heat" is especially inaccurate

**Proposition 56.** *The information temperature of the universe $T_u$ is constant over time and equal to*

$$T_u \overset{\text{(def)}}{=} \left.\frac{\partial U_u}{\partial S_t}\right|_{I_t} = \frac{U_u'}{\ln 2} \tag{46}$$

*Irrespective of the mass distribution, it only depends on the universe storage capacity $N_u$.*

*Proof.* This is a direct consequence of Eq. (45). Since $U_u(N_u)$ is a function of the sole universe storage capacity $N_u$, $U_u' \overset{\text{(def)}}{=} \mathrm{d}U_u/\mathrm{d}N_u$ only depends on $N_u$. □

Quantitatively, we will see later that $T_u = 4.87 \times 10^{-31}$ K $= 4.2 \times 10^{-35}$ eV.

**Meaning of the information temperature.** At a definite storage-capacity $N_u$, it turns out that this uniform temperature specifies the *minimum expansion temperature* in the universe regardless of the mass distribution (Sec. 5.1.2 just below). The existence of such a minimum is a prominent result that we will take advantage of under Assumption (33) below. At last, $T_u$ can be interpreted as another expression of the storage capacity of the cosmic Bayesian theater in the same way that $N_u$ or $U_u$ do.

### 5.1.2 Expansion temperature

Given the fundamental prior, it turns out that the independent parameters ($S_t$, $S_{tj}$), i.e., global entropy and binary entropy, introduce the universe expansion. Indeed, an infinitesimal increase $\mathrm{d}S_t$ of entropy at constant $S_{tj}$ demands both an increase $\mathrm{d}\chi_t$ of the radius of the rescaled Bloch sphere and a (virtual) increase $\mathrm{d}N_u$ of the number of variables. This defines a temperature that we propose to call "expansion temperature".

**Definition 44** (Expansion temperature)**.** *The expansion temperature $T_t \overset{\text{(def)}}{=} \partial U_u/\partial S_t\big|_{S_{tj}}$ is the derivative of the universe energy $U_u$ with respect to the overall entropy $S_t$ at constant binary entropy $S_{tj}$.*

**Proposition 57.** *The expansion temperature, $T_t$, is equal to*

$$T_t \overset{\text{(def)}}{=} \left.\frac{\partial U_u}{\partial S_t}\right|_{S_{tj}} = \frac{S_{\max}}{S_t}\, T_u, \tag{47}$$

*where $T_u$ is the information temperature. The product $S_t T_t = S_{\max} T_u$ is constant over time and only depends on the storage capacity of the universe $N_u$.*

*Proof.* From Eq. (10) we have $\partial N_u/\partial S_t\big|_{S_{tj}} = 1/S_{tj}$, from Eq. (46), $T_u = U_u'/\ln 2$ and by definition $U_u' = \mathrm{d}U_u/\mathrm{d}N_u$. Then, using $S_{\max} = N_u \ln 2$ and $S_t = N_u \times S_{tj}$,

$$T_t = \left.\frac{\partial U_u}{\partial S_t}\right|_{S_{tj}} = \frac{\mathrm{d}U_u}{\mathrm{d}N_u} \times \left.\frac{\partial N_u}{\partial S_t}\right|_{S_{tj}} = U_u' \times \frac{1}{S_{tj}} = \frac{\ln 2\, T_u}{S_{tj}} = \frac{N_u \times \ln 2\, T_u}{N_u \times S_{tj}} = \frac{S_{\max}}{S_t}\, T_u. \quad \square$$

**Proposition 58.** *The expansion temperature of the universe is infinite at the origin of time and decreases monotonously over time as*

$$T_t = T_u \times \left(\frac{t_{\max}}{t}\right)^2 \geq T_u \tag{48}$$

*The minimum expansion temperature is obtained at time $t_{\max}$ to reach the information temperature $T_u$.*

*Proof.* Use Eq. (31), $S_t = \pi t^2$. □

Quantitatively, at the present epoch, $T_{t_0} = 9.34 \times 10^{-28}$ K $= 8.05 \times 10^{-32}$ eV.

*Remark.* This does not conform to the standard model of cosmology in which there is only one form of "temperature" depending on the era. For instance, it scales as $T \propto t^{-1/2}$ in the radiation-dominated era. □

### 5.1.3 Expansion force

From Assumption (30), given the fundamental prior, any movement is caused by the expansion of the universe. The rescaled Bloch sphere recedes as a whole from the center of the universe.

In classical physics, movements are governed by gravity and inertia. Since there is only a single cause of motion given the fundamental prior, gravity and inertia cannot be distinct phenomena in the present model. This joins the standard equivalence principle of General Relativity.

**Proposition 59** (Equivalence principle)**.** *Gravity and inertia are equivalent.*

*Proof.* This is another wording of Assumption (30). □

This principle is naturally valid beyond the fundamental prior. Then, differential deformations of the rescaled Bloch sphere can lead to local relative movements.

Now, we aim to explain the universe expansion by an entropic force. To this end, let us use the expansion representation of the universe, Eq. (10). From Eq. (37), the virtual increase $\mathrm{d}N_u$ of the number of variables at constant $S_{tj}$ generates an increase of the overall energy $\mathrm{d}U_u = U'_u \mathrm{d}N_u = T_t \mathrm{d}S_t$, where from Eq. (47), $T_t = (S_{\max}/S_t)\, T_u$ is the expansion temperature at the cosmic time $t$.

The increase of energy $\mathrm{d}U_u$ can be identified with a standard thermodynamic *work* produced by the deformation of the rescaled Bloch sphere. Work is the product of a *displacement* and a *force*. Here, the displacement is the infinitesimal increase $\mathrm{d}\chi_t$ so that the expansion at constant $S_{tj}$ defined a "force", say $F_t$, by

$$\mathrm{d}U_u = F_t\, \mathrm{d}\chi_t. \tag{49}$$

However, since the displacement $\mathrm{d}\chi_t$ expresses the differential expansion of the rescaled Bloch sphere, the work $\mathrm{d}U_u$ concerns a *radial displacement* of the *totality of the Bloch sphere.* Although this concept comes naturally in this context, this is not exactly the standard definition of a force, which is usually unidirectional and not radial.

**Definition 45** (Radial expansion force)**.** *The radial expansion force is an isotropic force applied to the entire mass of the universe to produce the isotropic expansion of the Bloch sphere.*

**Proposition 60.** *Given the fundamental prior, the radial expansion force $f_t$ at time $t$ is*

$$f_t \overset{(\mathrm{def})}{=} \frac{1}{2}\frac{\partial U_u}{\partial \chi_t}\Big|_{S_{tj}} \tag{50}$$

*where $\chi_t = t$ is the radius of the rescaled Bloch sphere.*

*Proof.* Since the Bloch sphere is double-sided, each direction is in fact counted twice. Suppressing this duplication involves dividing $F_t$ in Eq. (49) by a factor of 2. Let $f_t \overset{(\mathrm{def})}{=} \frac{1}{2}F_t$ denote this single radial force. □

**Proposition 61.** *The radial expansion force is equal to*

$$f_t = \sqrt{T_t T_u} \times \sqrt{\pi S_{\max}} \tag{51}$$

*Proof.* The radius $\chi_t$ of the rescaled Bloch sphere increases necessarily with the global entropy $S_t = \pi\chi^2$. Then from Eq. (47), $T_t = \left(\partial U_u/\partial S_t\right)_{S_{tj}} = T_u S_{\max}/(\pi\chi_t^2)$ so that

$$2f_t = \frac{\partial U_u}{\partial \chi_t}\Big|_{S_{tj}} = \frac{\partial U_u}{\partial S_t}\Big|_{S_{tj}} \times \frac{\mathrm{d}S_t}{\mathrm{d}\chi_t} = T_t \times 2\pi\chi_t = \frac{T_u S_{\max}}{\pi\chi_t^2} 2\pi\chi_t = \frac{2T_u S_{\max}}{\chi_t}$$

and finally

$$f_t = (\pi T_t) \times \chi_t = \frac{T_u S_{\max}}{\chi_t} \text{ or } f_t = \sqrt{(\pi T_t) \times \chi_t \times \frac{T_u S_{\max}}{\chi_t}} = \sqrt{T_t T_u} \times \sqrt{\pi S_{\max}}. \quad \square \tag{52}$$

### 5.1.4 Gravitational field

The expansion force is global. We propose to identify gravity with its local expression.

**Assumption 31** (Gravity)**.** *Given the fundamental prior, gravity is the force produced locally on massive objects by the radial expansion force, Eq. (51).*

*Hints.* Massive objects have been defined independently of gravity (Definition 35). They move because of the expansion of the universe. □

Precisely, let us define the gravitational field as the force per unit mass.

**Definition 46** (Gravitational field)**.** *Given the fundamental prior, the gravitational field $\kappa_t$ is the radial expansion force applied to a unit of mass.*

Since the deformation of the rescaled Bloch sphere Eq. (49), concerns the displacement of the totality of the universe mass $U_u$, introduce a second parameter $\kappa_t \overset{(\text{def})}{=} f_t/U_u$ or $f_t = U_u \kappa_t$. In classical mechanics, from the second Newton law of motion, $\kappa_t$ represents the *acceleration* of the mass $U_u$ subject to the force $f_t$. In the present model, we propose to use this classical law to compute the "gravitational field".

**Proposition 62.** *The gravitational field $\kappa_t$ is*

$$\kappa_t = \frac{f_t}{U_u}. \tag{53}$$

*Proof.* Use Assumption (31) and Proposition (59). □

**Proposition 63.** *At the cosmic time $t$, the gravitational field $\kappa_t$ is equal to*

$$\kappa_t = \sqrt{T_t T_u} \times \frac{1}{U_u} \sqrt{\pi S_{\max}}. \tag{54}$$

*Proof.* Obvious from Eqs. (51) and (53). □

### 5.1.5 Calibrating the total mass of the universe

So far, the expression of the total mass $U_u$, (Definition 24), has not been precisely calibrated in Sec. (4.1) above, meaning that it is partly a matter of convention. We can take advantage of this freedom to obtain the simplest formula for the gravitational field $\kappa_t$. The corresponding value of $U_u$ will next be calculated in Proposition (64) just below. Now, starting from Eq. (54), we propose to adopt the following expression for $\kappa_t$:

**Assumption 32** (Calibrating the total mass of the universe)**.** *By convention, the mass $U_u$ of the universe as a whole is adjusted so that*

$$\kappa_t = 2\pi \sqrt{T_u T_t} \tag{55}$$

*Hints.* The factor $2\pi$ is somehow arbitrary but we choose to recover the standard surface temperature of the Schwarzschild black hole at the ultimate time $t_{\max}$ (see below, Proposition 71). Indeed, at time $t = t_{\max}$, we so obtain $\kappa_u \overset{(\text{def})}{=} \kappa_{t_{\max}} = 2\pi T_u$. □

Restoring the dimension, we have

$$\kappa_t = \frac{2\pi c k_B}{\hbar} \sqrt{T_u T_t}$$

We can now calibrate the universe mass $U_u$.

**Proposition 64** (Mass of the universe)**.** *The energy $U_u$ of the universe is proportional to the square root of the number of degrees of freedom, $N_u$, as*

$$U_u = \sqrt{\frac{S_{\max}}{4\pi}} = \varepsilon \sqrt{N_u} \quad \text{where} \quad \varepsilon = \left(\frac{\ln 2}{4\pi}\right)^{1/2} \simeq 0.235. \tag{56}$$

*Proof.* From Eq. (54), take into account Eq. (55). □

Restoring the dimension in Eq. (56), we obtain

$$U_u = \sqrt{N_u \ln 2\, \frac{\hbar c}{4\pi G}}. \tag{57}$$

Since $U_u$, $N_u$ and $c$ do not depend on the epoch, so is the $\hbar/G$ ratio. An expression similar to Eq. (57) was previously proposed by I. Haranas and I. Gkigkitzis [67].

Quantitatively, irrespective of the epoch, $U_u = 4.03 \times 10^{54}$ kg.

From Eq. (55), it is possible to interpret the gravitational field as resulting from the convergence of the information and expansion temperatures. As a result, since the expansion temperature depends on the age of the universe, so is the gravitational field.

**Proposition 65.** *The gravitational field decreases proportionally to the cosmic time as*

$$\kappa_t = 2\pi T_u \times \frac{t_{\max}}{t} \tag{58}$$

*Proof.* Replace $T_t$ in Eq. (55) by its expression Eq. (48). □

Quantitatively, at the present epoch we have $\kappa_{t_0} = 0.21 \times 10^{-6}$ m/s$^2$.

### 5.1.6 Unruh's temperature

In standard physics, the Unruh effect [23] establishes a close relationship between acceleration and temperature, namely that an observer subject to a gravitational field $\kappa_t$ experiences a temperature $T_{\text{Unruh}} = \kappa_t/2\pi$. Now, by reversing the logic, we propose that the Unruh temperature generates gravity. Therefore,

$$T_{\text{Unruh}} = \sqrt{T_u T_t} = \kappa_t/2\pi = T_u \times \frac{t_{\max}}{t} \tag{59}$$

This leads to the following definition:

**Definition 47** (Gravitational temperature)**.** *The gravitational temperature is the Unruh temperature, $T_{\text{Unruh}} = \kappa_t/2\pi$, where $\kappa_t$ is the gravitational field.*

**Proposition 66.** *The gravitational temperature is $T_{\text{Unruh}} = \sqrt{T_u T_t}$, where $T_u$ and $T_t$ are respectively the information and the expansion temperatures.*

Quantitatively, at the present epoch, we obtain $T_{\text{Unruh}} = 8.42 \times 10^{-28}$ K, which corresponds to $\kappa_{t_0} = 0.21 \times 10^{-6}$ m/s$^2$. By construction, this value is computed from the Newton gravitational constant $G$.

**Proposition 67** (Information temperature)**.** *The information temperature of the universe is equal to*

$$T_u = \frac{1}{8\pi U_u} = \frac{1}{4}\sqrt{\frac{1}{\pi S_{\max}}} = \frac{1}{4}\sqrt{\frac{1}{\pi N_u \ln 2}}. \tag{60}$$

*Proof.* From Eq. (46), $T_u = U_u'/\ln 2$. Now, from Eq. (56), we have $U_u = \varepsilon\sqrt{N_u}$ so that $U_u' = \varepsilon/(2\sqrt{N_u}) = \varepsilon^2/(2U_u) = \ln 2/(8\pi U_u)$. □

Restoring the dimension, we have

$$T_u = \frac{1}{4}\sqrt{\frac{\hbar c^5}{\pi G k_B^2 N_u \ln 2}}.$$

Quantitatively, irrespective of the epoch, $T_u = 4.87 \times 10^{-31}$ K.

### 5.1.7 Milgrom's acceleration

From Proposition (58), while temperature is not limited above, irrespective of the prior, there is a minimum expansion temperature, namely the information temperature $T_u$. The fact that $T_u > 0$ is another expression of the *finitude of the universe*, i.e., $N_u < +\infty$.

It turns out that it is possible to determine experimentally the value of $T_u$, thus allowing to *compute* $N_u$ from Eq. (60). Indeed, it is known that the velocity of stars in spiral galaxies does not exactly follow General Relativity. A number of remedies has been proposed, including introduction of "dark matter" or more fundamentally, the search for a refinement of General Relativity and Newton dynamics. One of these theories, called "Modified Newtonian dynamics" (MOND), developed by Mordehai Milgrom [25], conjectures the existence of a minimum acceleration. This coincides with the minimum gravitational temperature of the present model. We therefore propose to adopt the Milgrom's interpretation.

**Proposition 68.** *There is a universal minimum acceleration $\kappa_u$, corresponding to the information temperature of the universe $T_u$.*

*Proof.* The minimum expansion temperature, Eq. (48) induces a minimum Unruh temperature, which in turn induces a minimum acceleration $\kappa_u$. □

**Assumption 33** (Milgrom's acceleration)**.** *The universal minimum acceleration $\kappa_u$, corresponds to the standard Milgrom's acceleration of MOND theory.*

*Hint.* The Milgrom's acceleration explains the shift compared to both Newton's law and General Relativity observed on the velocities of stars in spiral galaxies. □

**Proposition 69.** *The Milgrom acceleration is equal to $\kappa_u = 2\pi T_u = 1/(4U_u)$.*

*Proof.* Apply Eq. (55) for $t = t_{\max}$ and use Eq. (60). □

Now, the Milgrom acceleration, that is the information temperature $T_u$ specifies the number of variables $N_u$.

**Proposition 70** (Storing capacity of the Bayesian theater)**.** *The number of binary variables in the universe is $N_u = 6.23 \times 10^{125}$ corresponding to the Milgrom's acceleration $\kappa_u = 1.20 \times 10^{-10}$ $m/s^2$ or the information temperature $T_u = 4.87 \times 10^{-31}$ K.*

*Proof.* From the experimental value of the velocities of stars in spiral galaxies measured by S. S. McGaugh *et all* [68], $\kappa_u = 1.2 \times 10^{-10}$ m/s$^2$, we can compute the number of binary variables in the universe as $N_u = 6.23 \times 10^{125}$. □

This is much higher than the value $10^{122}$ derived from the standard estimate of the event entropy [39].

Consider the ultimate fate of the whole universe. The universe of mass $U_u$ collapses in totality into a Schwarzschild black-hole, as conjectured by Gibbon and Hawking in 1977 [69].

**Proposition 71** (Ultimate Schwarzschild black hole)**.** *In the present model, the normalized parameters of the ultimate Schwarzschild black hole are the following:*

| | Ultimate Schwarzschild black hole | value |
|---|---|---|
| *number of bits* | $N_u$ | $6.23 \times 10^{125}$ |
| *entropy* | $S_{\max} = N_u \ln 2$ | $4.32 \times 10^{125}$ nats |
| *mass* | $U_u = \varepsilon\sqrt{N_u}$ $(\varepsilon = \sqrt{\ln 2/4\pi})$ | $4.03 \times 10^{54}$ kg |
| *comoving radius* | $\chi_{\max} = t_{\max} = 2U_u$ | $633 \times 10^{9}$ light yrs |
| *temperature* | $T_u = 1/(8\pi U_u)$ | $4.87 \times 10^{-31}$ K |
| *acceleration* | $\kappa_u = 1/(4U_u)$ | $1.20 \times 10^{-10}$ m/s$^2$ |

*Proof.* Apply the propositions just above. □

In addition, we find the expression of the so-called "Bekenstein bound" [70], which is the limit $S_{\max}$ of the entropy that can be contained in a physical system to which the total energy is given.

**Proposition 72** (Bekenstein bound)**.** *The event entropy is the Bekenstein bound of the entire universe.*

*Proof.* By simple inspection, the standard entropy of a Schwarzschild black-hole of mass $U_u$, known as "Bekenstein bound" is $S_{\max} = 4\pi U_u^2$ nats. □

### 5.2 Temperatures of parts of the universe

So far we have dealt with the universe as a whole, regarded as a single massive object. We now approach parts of the universe, also considered as massive objects. Their energy depends on the cosmic time $t$ or equivalently the global entropy $S_t$. We can therefore address this dependence at constant storage capacity $N_u$ so that the convenient representation is here the "basic representation", $(N_u, S_t)$, Eq. (9).

In a further version of this paper, we will investigate the different temperatures of comoving objects. We expect them to recover both the standard fields of micro-physics and the CMB temperature.

## 6 Provisional conclusion

In this preliminary version, we have shown that physics can be based on information. This requires a completely new vision of what the universe is and even of what determinism is. At this stage, such a vision can therefore only be conjectural. In a later version, we will investigate the basics of micro-physics in order to find quantitative consistency with the experimental results.

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