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IFAC PapersOnLine 51-24 (2018) 1199–1204

Interval Observer Design for Actuator Fault Estimation of Linear Parameter-Varying Systems

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Abstract: This work is devoted to fault estimation of discrete-time Linear Parameter-Varying (LPV) systems subject to actuator additive faults and external disturbances. Under the assumption that the measurement noises and the disturbances are unknown but bounded, an interval observer is designed, based on decoupling the fault effect, to compute a lower and upper bounds for the unmeasured state and the faults. Stability conditions are expressed in terms of matrices inequalities. A case study is used to illustrate the effectiveness of the proposed approach.

Keywords: Fault estimation, Interval observer, LPV system, Unknown input observer.

1. INTRODUCTION

Industrial systems are subject to several kinds of faults, such as sensors, actuators or component malfunction, which may lead to performance degradation or even serious human damages. Therefore, it is important to improve system and human safety and reliability. As a result, during the past decades, fault estimation has been received a great attention; it allows one to determine the size, location and dynamic of the fault.

In reality, most of industrial systems are nonlinear and Linear Parameter-Varying (LPV) models can be used to approximate a large class of nonlinear systems (Shamma [2012]). Many researches have been carried out about LPV systems in the filed of fault estimation. For LPV systems, the discrete fault estimation problem has been dealt within many works. In (Abdullah and Zribi [2013]), a fault estimation and compensation scheme for LPV systems, under actuator and sensor faults is presented. A robust fault estimation and compensation for LPV systems under actuator and sensor faults is proposed in (Seron and De Doná [2015]). In (Luspay et al. [2015]), fault estimation for discrete-time LPV systems under noisy scheduling measurements is proposed taking into account stochastic and additive scheduling parameter ambiguities.

An effective approach based on fault decoupling principle can be used to estimate the fault. This is obtained by designing an Unknown Input Observer (UIO). Over the past decades, a great deal of research effort has been devoted to the UIO theory. In (Sun et al. [2012]), a linear time invariant model-based robust fast adaptive fault estimator with unknown input decoupling is proposed to estimate faults. An UIO design procedure is generalized to LPV descriptor system is presented in (Hamdi et al. [2012]). UIO for LPV systems with parameter varying output equation is developed in (Ichalal et al. [2015]).

Usually, an unknown input observer is designed to estimate the fault. However, in presence of uncertainties, classical observers may not be efficient to solve the estimation problem. In this case, interval observer is proposed to compute the set of all the admissible values and provides certain lower and upper bounds for the estimate at each instant of time in the presence of bounded uncertainties. Several works have investigated interval observers based on cooperative systems theory (Raïssi et al. [2010]), (Mazenc et al. [2014]), (Lamouchi et al. [2016]) and (Lamouchi et al. [2017]). Few works have considered unknown input interval observers. In (Gucik-Derigny et al. [2014]), (Gucik-Derigny et al. [2016]), an interval state and unknown inputs estimation for linear time-invariant continuous-time systems is studied. The work in (Xu et al. [2017]) proposes an UIO for LPV systems based on set theoretic approach. Recently, a new method developed in (Robinson et al. [2017]) has addressed the problem of unknown input interval state estimation for Linear Time-Invariant systems. To the best of our knowledge, fault estimation problem in discrete-time LPV systems using unknown input interval observer has not been yet fully tackled.

Motivated by the previous explanations and inspired by the method in (Robinson et al. [2017]), an extension of fault estimation to a class of discrete-time LPV systems in a set-membership framework is proposed in this paper. The idea consists in decoupling the fault effect on the state using transformation of coordinates. Then, an unknown input interval observer is designed to compute a lower and upper bounds for the state and the fault.

This paper is organized as follows. Preliminaries are given in Section 2. In Section 3, the problem statement is presented. Section 4 proposes an interval observer for state and fault estimation. Numerical simulations are presented in Section 5 to show the efficiency of the proposed approach. Finally, the paper is concluded in Section 6.

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10.1016/j.ifacol.2018.09.699

2. PRELIMINARIES

A system described by $x_{k+1} = Ax_k + u_k$, with $x_k \in \mathbb{R}^n$ and $A \in \mathbb{R}^{n\times n}$, is nonnegative if and only if the matrix A is elementwise nonnegative, $u_k \geq 0$ and $x_{k_0} \geq 0$. A matrix $A \in \mathbb{R}^{n\times n}$ is nonnegative if all its elements are nonnegative. Given a matrix $M \in \mathbb{R}^{m\times n}$, define $M^+ = max\{0, M\}$, $M^- = max\{0, -M\}$ (similarly for vectors). For two vectors $x_1, x_2 \in \mathbb{R}^n$ or matrices $M_1, M_2 \in \mathbb{R}^{n\times n}$, the relations $x_1 \leq x_2$ and $M_1 \leq M_2$ are understood elementwise. The symbol $|.|$ denotes vector or corresponding induced matrix Euclidean norm. For a measurable and locally essentially bounded input $u : \mathbb{N} \to \mathbb{R}$, the symbol $\|u\|_{[t_0,t_1)}$ denotes its L_∞-norm where $\|u\|_{[t_0,t_1)} = sup\{|u_t|, t \in [t_0,t_1)\}$, $\|u\| = \|u\|_{[0,+\infty)}$. The set of all inputs u with the property $\|u\| < \infty$ is denoted by $\mathscr{L}_\infty$.

Lemma 1. (Chebotarev et al. [2013]) Let $\underline{x}, x, \overline{x} \in \mathbb{R}^n$ if $\underline{x} \leq x \leq \overline{x}$ then,

$$\underline{x}^+ \leq x^+ \leq \overline{x}^+ \text{ and } \overline{x}^- \leq x^- \leq \underline{x}^- \quad (1)$$

Similarly, let $\underline{A}, A, \overline{A} \in \mathbb{R}^{m\times n}$, if $\underline{A} \leq A \leq \overline{A}$ then

$$\underline{A}^+ \leq A^+ \leq \overline{A}^+ \text{ and } \overline{A}^- \leq A^- \leq \underline{A}^- \quad (2)$$

Lemma 2. (Chebotarev et al. [2013]) Let $x \in \mathbb{R}^n$ be a vector such that $\underline{x} \leq x \leq \overline{x}$ for some $\underline{x}, \overline{x} \in \mathbb{R}^n$.

(1) If $A \in \mathbb{R}^{m\times n}$ is a constant matrix, then

$$A^+\underline{x} - A^-\overline{x} \leqslant Ax \leqslant A^+\overline{x} - A^-\underline{x} \quad (3)$$

(2) If $A \in \mathbb{R}^{m\times n}$ is a matrix satisfying $\underline{A} \leq A \leq \overline{A}$ for some $\underline{A}, \overline{A} \in \mathbb{R}^{m\times n}$, then

$$\begin{aligned}\underline{A}^+\underline{x}^+ - \overline{A}^+\underline{x}^- - \underline{A}^-\overline{x}^+ + \overline{A}^-\overline{x}^- &\leq Ax \\ \leq \overline{A}^+\overline{x}^+ - \underline{A}^+\overline{x}^- - \overline{A}^-\underline{x}^+ + \underline{A}^-\underline{x}^-&. \end{aligned} \quad (4)$$

3. PROBLEM STATEMENT

Consider the following discrete-time LPV system:

$$\begin{cases} x_{k+1} = (A + \Delta A(\rho))x_k + Bu_k + w_k \\ y_k = Cx_k + v_k \end{cases} \quad (5)$$

where $x_k \in \mathbb{R}^n$ is the state, $u_k \in \mathbb{R}^q$ is the input, $y_k \in \mathbb{R}^p$ is the output; w_k, v_k are respectively the bounded disturbance and noise. The vector of scheduling parameters $\rho \in \Pi$ is considered unknown and only the set of its admissible values Π is given. $\Delta A : \Pi \to \mathbb{R}^{n\times n}$ is a known piecewise continuous matrix function.

We assume in the following that an actuator fault can be modelled by an additive term in the system (5). Therefore, the faulty system dynamics are given by:

$$\begin{cases} x_{k+1} = (A + \Delta A(\rho))x_k + Bu_k + Ff_k + w_k \\ y_k = Cx_k + v_k \end{cases} \quad (6)$$

where $F \in \mathbb{R}^{n\times q}$ is a known matrix and $f_k \in \mathbb{R}^q$ is the fault vector.

In the following, it is assumed that the matrix $\Delta A(\rho)$ belongs into the interval $[\underline{\Delta A}, \overline{\Delta A}]$. The value of the scheduling vector ρ is not available for measurement but it is easy to compute $\underline{\Delta A}$ and $\overline{\Delta A}$ for a given set Π and a known function $\Delta A : \Pi \to \mathbb{R}^{n\times n}$. The disturbance w_k and the measurement noise v_k are bounded by two known sequences such that $\underline{w}_k \leqslant w_k \leqslant \overline{w}_k$ and $\underline{v}_k \leqslant v_k \leqslant \overline{v}_k$.

Assumption 1. $\underline{\Delta A} \leqslant \Delta A(\rho) \leqslant \overline{\Delta A}$ for all $\rho \in \Pi$ and some known $\underline{\Delta A}, \overline{\Delta A} \in \mathbb{R}^{n\times n}$. □

Assumption 2. There exist $\overline{w}$, $\underline{w}$, $\overline{v}$ and $\underline{v}$ such that: $\underline{w} \leqslant w_k \leqslant \overline{w}$ and $\underline{v} \leqslant v_k \leqslant \overline{v}$ are satisfied $\forall\, k \in \mathbb{N}$. □.

The methodology presented below is based on two steps. First, an estimation of the state bounds $\underline{x}_k, \overline{x}_k \in \mathbb{R}^n$ is described. The second step consists in estimating a lower and upper bounds $\underline{f}_k, \overline{f}_k \in \mathbb{R}^q$ for the actuator fault f_k.

4. MAIN RESULTS

The following assumption is required.

Assumption 3. C is full row rank matrix and F is a full column rank matrix. □

Under Assumption 3, there exists a transformation of coordinates $z_k = H^T x_k$, $H \in \mathbb{R}^{n\times n}$ such that $F = H\,[\,R_0\ \ 0\,]^T K_0^T$ with $R_0 \in \mathbb{R}^{q\times q}$ and $K_0 \in \mathbb{R}^{q\times q}$.

The system (6) can be rewritten in the coordinates z as:

$$\begin{cases} z_{k+1} = (\tilde{A} + \Delta\tilde{A}(\rho))z_k + \tilde{B}u_k + \begin{bmatrix} R_0 \\ 0 \end{bmatrix} \tilde{f}_k + \tilde{w}_k \\ y_k = \tilde{C}z_k + v_k \end{cases} \quad (7)$$

with $H = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}$, $\tilde{A} = H^T AH = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \tilde{A}_{21} & \tilde{A}_{22} \end{bmatrix}$,

$\Delta\tilde{A}(\rho) = H^T \Delta A(\rho) H = \begin{bmatrix} \Delta\tilde{A}_{11}(\rho) & \Delta\tilde{A}_{12}(\rho) \\ \Delta\tilde{A}_{21}(\rho) & \Delta\tilde{A}_{22}(\rho) \end{bmatrix}$,

$\tilde{B} = H^T B = \begin{bmatrix} \tilde{B}_1 & \tilde{B}_2 \end{bmatrix}^T$, $\tilde{C} = CH = \begin{bmatrix} \tilde{C}_1 & \tilde{C}_2 \end{bmatrix}$, $\tilde{f}_k = K_0^T f_k$, $\tilde{w} = H^T w = \begin{bmatrix} \tilde{w}_{1,k} & \tilde{w}_{2,k} \end{bmatrix}^T$.

H^T is assumed to be bounded, then, it follows that $|\tilde{w}| \leq \overline{\tilde{w}}$ where $\overline{\tilde{w}}$ is a constant positive vector.

The system (7) can be rewritten as:

$$\begin{cases} z_{1,k+1} = (\tilde{A}_{11} + \Delta\tilde{A}_{11}(\rho))z_{1,k} + (\tilde{A}_{12} + \Delta\tilde{A}_{12}(\rho))z_{2,k} \\ \qquad\quad + \tilde{B}_1 u_k + R_0\tilde{f}_k + \tilde{w}_{1,k} \\ z_{2,k+1} = (\tilde{A}_{21} + \Delta\tilde{A}_{21}(\rho))z_{1,k} + (\tilde{A}_{22} + \Delta\tilde{A}_{22}(\rho))z_{2,k} \\ \qquad\quad + \tilde{B}_2 u_k + \tilde{w}_{2,k} \\ y_k = \tilde{C}_1 z_{1,k} + \tilde{C}_2 z_{2,k} + v_k \end{cases} \quad (8)$$

$\tilde{C}_1$ is supposed to be a full column rank matrix (Hou and Muler 1992) and can be decomposed as: $\tilde{C}_1 = N\,[\,R_1\ \ 0\,]^T K_1^T$ with $N = [\,N_{11}\ \ N_{12}\,]$ and $\tilde{y}_k = N^T y_k$. Then, the measurement equation can be decomposed as:

$$\begin{cases} \tilde{y}_{1,k} = R_1 K_1^T z_{1,k} + N_{11}^T \tilde{C}_2 z_{2,k} + N_{11}^T v_k \\ \tilde{y}_{2,k} = N_{12}^T \tilde{C}_2 z_{2,k} + N_{12}^T v_k = C_2 z_{2,k} + N_{12}^T v_k \end{cases} \quad (9)$$

As $\tilde{y}_{1,k} = G_s^T \tilde{y}_k$ with $G_s^T = \left[\, I_q \ \ O_{q\times(p-q)} \,\right]$, the expression of z_1 is extracted from (9):

$$z_{1,k} = E(y_k - \tilde{C}_2 z_{2,k} - v_k) \quad (10)$$

with $E = K_1 R_1^{-1} G_s^T N^T$. By replacing this expression of $z_{1,k}$ in the second equation of (8) we obtain:

$$\begin{cases} z_{2,k+1} = A_2 z_{2,k} + \Delta\tilde{A}_2(\rho) z_{2,k} + B_2 u_k + D_2(\rho) y_k \\ \qquad\quad - D_2(\rho) v_k + \tilde{w}_{2,k} \\ \tilde{y}_{2,k} = C_2 z_{2,k} + N_{12}^T v_k \end{cases} \quad (11)$$

with $A_2 = \tilde{A}_{22} - \tilde{A}_{21} E\tilde{C}_2$, $\Delta\tilde{A}_2(\rho) = \Delta\tilde{A}_{22}(\rho) - \Delta\tilde{A}_{21}(\rho) E\tilde{C}_2$, $B_2 = \tilde{B}_2$, $D_2(\rho) = \tilde{A}_{21}E + \Delta\tilde{A}_{21}(\rho)E$, and $C_2 = N_{12}^T \tilde{C}_2$.

Assumption 4. The pair (A_2, C_2) is detectable. □

Remark 1. It is worth to note that it is not always possible to compute a matrix L such that $A_2 - LC_2$ is nonnegative. This restrictive condition can be relaxed by means of a change of coordinates. It has been shown in (Efimov et al. [2013]) that there always exists an transformation matrix P such that $R = P(A_2 - LC_2)S$, with $S = P^{-1}$, is nonnegative. □

The system (11) can be described in the coordinates $r_2 = Pz_2$ as follows:

$$\begin{cases} r_{2,k+1} = Rr_{2,k} + P\Delta\tilde{A}_2(\rho)Sr_{2,k} + PB_2u_k + M(\rho)y_k \\ \qquad -M(\rho)v_k + P\tilde{w}_{2,k} \\ \tilde{y}_{2,k} = C_2P^{-1}r_{2,k} + N_{12}^T v_k \end{cases} \tag{12}$$

where $M(\rho) = P(D_2(\rho) + LN_{12}^T)$. In the following, an interval observer design is proposed to estimate the state and the actuator fault.

4.1 State Estimation

Consider the following observer structure for (12):

$$\begin{cases} \overline{r}_{2,k+1} = R\overline{r}_{2,k} + PB_2u_k + \overline{\varphi}(\overline{r}_{2,k}, \underline{r}_{2,k}) + \overline{\Gamma}(y_k^+, y_k^-) - \\ \qquad \underline{\vartheta}(\overline{v}, \underline{v}) + \overline{\Delta}(\overline{w}_2, \underline{w}_2) \\ \underline{r}_{2,k+1} = R\underline{r}_{2,k} + PB_2u_k + \underline{\varphi}(\overline{r}_{2,k}, \underline{r}_{2,k}) + \underline{\Gamma}(y_k^+, y_k^-) - \\ \qquad \overline{\vartheta}(\overline{v}, \underline{v}) + \underline{\Delta}(\overline{w}_2, \underline{w}_2) \end{cases} \tag{13}$$

with

$$\begin{cases} \overline{\varphi}(\overline{r}_{2,k}, \underline{r}_{2,k}) = \overline{\sigma}^+ \overline{r}_{2,k}^+ - \underline{\sigma}^+ \overline{r}_{2,k}^- - \overline{\sigma}^- \underline{r}_{2,k}^+ + \underline{\sigma}^- \underline{r}_{2,k}^- \\ \underline{\varphi}(\overline{r}_{2,k}, \underline{r}_{2,k}) = \underline{\sigma}^+ \underline{r}_{2,k}^+ - \overline{\sigma}^+ \underline{r}_{2,k}^- - \underline{\sigma}^- \overline{r}_{2,k}^+ + \overline{\sigma}^- \overline{r}_{2,k}^- \\ \overline{\sigma} = (P^+\overline{\Delta\tilde{A}}_2 - P^-\underline{\Delta\tilde{A}}_2)S^+ - (P^+\underline{\Delta\tilde{A}}_2 - P^-\overline{\Delta\tilde{A}}_2)S^- \\ \underline{\sigma} = (P^+\underline{\Delta\tilde{A}}_2 - P^-\overline{\Delta\tilde{A}}_2)S^+ - (P^+\overline{\Delta\tilde{A}}_2 - P^-\underline{\Delta\tilde{A}}_2)S^- \\ \overline{\Delta\tilde{A}}_2 = \overline{\Delta\tilde{A}}_{22} - (\underline{\Delta\tilde{A}}_{21}(E\tilde{C}_2)^+ - \overline{\Delta\tilde{A}}_{21}(E\tilde{C}_2)^-) \\ \underline{\Delta\tilde{A}}_2 = \underline{\Delta\tilde{A}}_{22} - (\overline{\Delta\tilde{A}}_{21}(E\tilde{C}_2)^+ - \underline{\Delta\tilde{A}}_{21}(E\tilde{C}_2)^-) \\ \overline{\Gamma}(y_k^+, y_k^-) = \overline{M}y_k^+ - \underline{M}y_k^-, \underline{\Gamma}(y_k^+, y_k^-) = \underline{M}y_k^+ - \overline{M}y_k^- \end{cases} \tag{14}$$

and

$$\begin{cases} \overline{M} = (P^+\overline{D}_2 - P^-\underline{D}_2) + PLN_{12}^T \\ \underline{M} = (P^+\underline{D}_2 - P^-\overline{D}_2) + PLN_{12}^T \\ \overline{D}_2 = \tilde{A}_{21}E + (\overline{\Delta\tilde{A}}_{21}E^+ - \underline{\Delta\tilde{A}}_{21}E^-) \\ \underline{D}_2 = \tilde{A}_{21}E + (\underline{\Delta\tilde{A}}_{21}E^+ - \overline{\Delta\tilde{A}}_{21}E^-) \\ \overline{\vartheta}(\overline{v}, \underline{v}) = \overline{M}^+\overline{v}^+ - \underline{M}^+\overline{v}^- - \overline{M}^-\underline{v}^+ + \underline{M}^-\underline{v}^- \\ \underline{\vartheta}(\overline{v}, \underline{v}) = \underline{M}^+\underline{v}^+ - \overline{M}^+\underline{v}^- - \underline{M}^-\overline{v}^+ + \overline{M}^-\overline{v}^- \\ \overline{\Delta}(\overline{w}_2, \underline{w}_2) = P^+\overline{w}_2 - P^-\underline{w}_2 \\ \underline{\Delta}(\overline{w}_2, \underline{w}_2) = P^+\underline{w}_2 - P^-\overline{w}_2 \end{cases} \tag{15}$$

Since $\Delta A(\rho)$ is assumed to be bounded such that $\underline{\Delta A} \leqslant \Delta A(\rho) \leqslant \overline{\Delta A}$, it follows that $\underline{\Delta\tilde{A}} \leqslant \Delta\tilde{A}(\rho) = H^T \Delta A(\rho) H \leqslant \overline{\Delta\tilde{A}}$, where $\overline{\Delta\tilde{A}} = ((H^T)^+\overline{\Delta A} - (H^T)^-\underline{\Delta A})H^+ - ((H^T)^+\underline{\Delta A} - (H^T)^-\overline{\Delta A})H^-$, $\underline{\Delta\tilde{A}} = ((H^T)^+\underline{\Delta A} - (H^T)^-\overline{\Delta A})H^+ - ((H^T)^+\overline{\Delta A} - (H^T)^-\underline{\Delta A})H^-$. Therefore,

$$\overline{\Delta\tilde{A}} = \begin{bmatrix} \overline{\Delta\tilde{A}}_{11} & \overline{\Delta\tilde{A}}_{12} \\ \overline{\Delta\tilde{A}}_{21} & \overline{\Delta\tilde{A}}_{22} \end{bmatrix}, \underline{\Delta\tilde{A}} = \begin{bmatrix} \underline{\Delta\tilde{A}}_{11} & \underline{\Delta\tilde{A}}_{12} \\ \underline{\Delta\tilde{A}}_{21} & \underline{\Delta\tilde{A}}_{22} \end{bmatrix}$$

Denote by $\overline{e}_{r_{2,k}} = \overline{r}_{2,k} - r_{2,k}$ and $\underline{e}_{r_{2,k}} = r_{2,k} - \underline{r}_{2,k}$ the interval estimation errors; their dynamics follow:

$$\begin{cases} \overline{e}_{r_{2,k+1}} = R\overline{e}_{r_{2,k}} + \overline{\phi}_k + \overline{\delta}_k \\ \underline{e}_{r_{2,k+1}} = R\underline{e}_{r_{2,k}} + \underline{\phi}_k + \underline{\delta}_k \end{cases} \tag{16}$$

where

$$\begin{cases} \overline{\phi}_k = \overline{\varphi}(\overline{r}_{2,k}, \underline{r}_{2,k}) - P\Delta\tilde{A}_2(\rho)Sr_{2,k} \\ \underline{\phi}_k = P\Delta\tilde{A}_2(\rho)Sr_{2,k} - \underline{\varphi}(\overline{r}_{2,k}, \underline{r}_{2,k}) \\ \overline{\delta}_k = \overline{\Gamma}(y_k^+, y_k^-) - M(\rho)y_k - \underline{\vartheta}(\overline{v}, \underline{v}) + M(\rho)v_k \\ \qquad +\overline{\Delta}(\overline{w}_2, \underline{w}_2) - P\tilde{w}_{2,k} \\ \underline{\delta}_k = M(\rho)y_k - \underline{\Gamma}(y_k^+, y_k^-) - M(\rho)v_k + \overline{\vartheta}(\overline{v}, \underline{v}) \\ \qquad +P\tilde{w}_{2,k} - \underline{\Delta}(\overline{w}_2, \underline{w}_2) \end{cases} \tag{17}$$

The functions $\overline{\phi}$ and $\underline{\phi}$ are globally Lipschitz. Using Lemma 6 in (Zheng et al. [2016]), it follows that for $\underline{e}_{r_{2,k}} \leq e_{r_{2,k}} \leq \overline{e}_{r_{2,k}}$ and for a chosen submultiplicative norm $|.|$, there exist positive constants a_1, a_2, a_3, b_1, b_2 and b_3 such that:

$$\begin{cases} |\overline{\phi}_k| \leqslant a_1|\overline{e}_{r_{2,k}}| + a_2|\underline{e}_{r_{2,k}}| + a_3 \\ |\underline{\phi}_k| \leqslant b_1|\overline{e}_{r_{2,k}}| + b_2|\underline{e}_{r_{2,k}}| + b_3 \end{cases} \tag{18}$$

Theorem 1. Assume that Assumptions 1-4 are satisfied, R is nonnegative and the initial state $r_{2,0}$ verifies $\underline{r}_{2,0} \leq r_{2,0} \leq \overline{r}_{2,0}$. Then, for all $k \in \mathbb{R}_+$ the state $r_{2,k}$ solution of the system (12) satisfies:

$$\underline{r}_{2,k} \leq r_{2,k} \leq \overline{r}_{2,k} \tag{19}$$

In addition, if there exist matrices $P_1 \in \mathbb{R}^{n\times n}, P_1 = P_1^T \succ 0$, $W \in \mathbb{R}^{n\times n}, W = W^T \succ 0$, $Q \in \mathbb{R}^{n\times n}, Q = Q^T \succ 0$ and constants $\gamma > 0$, such that the following matrix inequality is verified:

$$\Psi = \begin{bmatrix} \Upsilon & G^TP_1 & G^TP_1 \\ P_1G & P_1 - \gamma I_n & P_1 \\ P_1G & P_1 & P_1 - W \end{bmatrix} \prec 0, \tag{20}$$

$$\Upsilon = G^TP_1G - P_1 + \gamma\alpha^2 I_n + Q, \; G = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix},$$

with $\alpha = 2\max\{(a_1 + b_1), (a_2 + b_2)\}$. Then $\overline{e}_{r_{2,k}}, \underline{e}_{r_{2,k}} \in \mathscr{L}_\infty^n$. □

Proof. Since R is assumed to be nonnegative, and by construction $\overline{\phi}_k$, $\underline{\phi}_k$, $\overline{\delta}_k$ and $\underline{\delta}_k$ are nonnegative, then if $\overline{r}_{2,0}$ and $\underline{r}_{2,0}$ are chosen such that $\overline{e}_0$ and $\underline{e}_0$ are nonnegative, the dynamics of interval estimation errors $\overline{e}_{r_{2,k}}$ and $\underline{e}_{r_{2,k}}$ stay nonnegative for all $k \in \mathbb{N}$.

For the stability analysis, we introduce the auxiliary system:

$$\xi_{r_{2,k+1}} = G\xi_{r_{2,k}} + \phi(\xi_{r_{2,k}}) + \delta_k \tag{21}$$

where $\xi_{r_{2,k}} = \begin{bmatrix} \underline{e}_{r_{2,k}} \\ \overline{e}_{r_{2,k}} \end{bmatrix}$, $\phi_k = \begin{bmatrix} \underline{\phi}_k \\ \overline{\phi}_k \end{bmatrix}$, $\delta_k = \begin{bmatrix} \underline{\delta}_k \\ \overline{\delta}_k \end{bmatrix}$, $\delta_k \in \mathscr{L}_\infty^{2n}$. Using equation (18), it is clear that $|\phi(\xi_{r_{2,k}})| \leqslant \alpha|\xi_{r_{2,k}}|$. The constant α does not depend on a_3 and b_3. For brevity, the calculus of $|\phi(\xi_{r_{2,k}})|$ is omitted.

To establish the stability of the system (13), consider the positive definite quadratic Lyapunov function:

$$V_k = \xi_{r_{2,k}}{}^T P_1 \xi_{r_{2,k}} \tag{22}$$

The increment of ΔV is given by:

$$\begin{aligned}
\Delta V &= V_{k+1} - V_k \\
&= \xi_{r_{2,k}}^T G^T P_1 G \xi_{r_{2,k}} - \xi_{r_{2,k}}^T P_1 \xi_{r_{2,k}} + \xi_{r_{2,k}}^T G^T P_1 \phi(\xi_{r_{2,k}}) \\
&+ \phi^T(\xi_{r_{2,k}}) P_1 G \xi_{r_{2,k}} + \phi^T(\xi_{r_{2,k}}) P_1 \phi(\xi_{r_{2,k}}) \\
&+ 2\xi_{r_{2,k}}^T G^T P_1 \delta_k + 2\delta_k^T P_1 \phi(\xi_{r_{2,k}}) + \delta_k^T P_1 \delta_k \\
&\leq \xi_{r_{2,k}}^T (G^T P_1 G - P_1) \xi_{r_{2,k}} + \xi_{r_{2,k}}^T G^T P_1 \phi(\xi_{r_{2,k}}) \\
&+ \phi^T(\xi_{r_{2,k}}) P_1 G \xi_{r_{2,k}} + \phi^T(\xi_{r_{2,k}}) P_1 \phi(\xi_{r_{2,k}}) \\
&+ 2\xi_{r_{2,k}}^T G^T P_1 \delta_k + 2\delta_k^T P_1 \phi(\xi_{r_{2,k}}) + \delta_k^T (P_1 - W) \delta_k \\
&+ \gamma\alpha^2 \xi_{r_{2,k}}^T \xi_{r_{2,k}} - \gamma\phi^T(\xi_{r_{2,k}}) \phi(\xi_{r_{2,k}}) + \xi_{r_{2,k}}^T Q \xi_{r_{2,k}} \\
&- \xi_{r_{2,k}}^T Q \xi_{r_{2,k}} + \delta_k^T W \delta_k \\
&\leq \begin{bmatrix} \xi_{r_{2,k}} \\ \phi(\xi_{r_{2,k}}) \\ \delta_k \end{bmatrix}^T \Psi \begin{bmatrix} \xi_{r_{2,k}} \\ \phi(\xi_{r_{2,k}}) \\ \delta_k \end{bmatrix} - \xi_{r_{2,k}}^T Q \xi_{r_{2,k}} + \delta_k^T W \delta_k
\end{aligned}$$

Then, the system (13) is ISS stable and $\xi_{r_{2,k}}$ is bounded. □

Since $r_2 = P z_2$, it follows that $\underline{z}_{2,k} \leq z_{2,k} \leq \overline{z}_{2,k}$ with:

$$\begin{cases} \overline{z}_{2,k} = S^+ \overline{r}_{2,k} - S^- \underline{r}_{2,k} \\ \underline{z}_{2,k} = S^+ \underline{r}_{2,k} - S^- \overline{r}_{2,k} \end{cases} \quad (23)$$

As $x_k = H z_k$ and based on (10), it follows that:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} E(y_k - \tilde{C}_2 z_{2,k} - v_k) \\ z_{2,k} \end{bmatrix} \quad (24)$$

Therefore

$$\begin{cases} x_{1,k} = H_{11} E y_k + E_1 z_{2,k} + H_{12} z_{2,k} - E_2 v_k \\ x_{2,k} = H_{21} E y_k + E_3 z_{2,k} + H_{22} z_{2,k} - E_4 v_k \end{cases} \quad (25)$$

The bounds of the state x_k are given by:

$$\begin{cases} \overline{x}_{1,k} = H_{11} E y_k + H_{12}^+ \overline{z}_{2,k} - H_{12}^- \underline{z}_{2,k} + (-E_1)^+ \overline{z}_{2,k} \\ \quad -(-E_1)^- \underline{z}_{2,k} + (-E_2)^+ \overline{v} - (-E_2)^- \underline{v} \\ \underline{x}_{1,k} = H_{11} E y_k + H_{12}^+ \underline{z}_{2,k} - H_{12}^- \overline{z}_{2,k} + (-E_1)^+ \underline{z}_{2,k} \\ \quad -(-E_1)^- \overline{z}_{2,k} + (-E_2)^+ \underline{v} - (-E_2)^- \overline{v} \\ \overline{x}_{2,k} = H_{21} E y_k + H_{22}^+ \overline{z}_{2,k} - H_{22}^- \underline{z}_{2,k} + (-E_3)^+ \overline{z}_{2,k} \\ \quad -(-E_3)^- \underline{z}_{2,k} + (-E_4)^+ \overline{v} - (-E_4)^- \underline{v} \\ \underline{x}_{2,k} = H_{21} E y_k + H_{22}^+ \underline{z}_{2,k} - H_{22}^- \overline{z}_{2,k} + (-E_3)^+ \underline{z}_{2,k} \\ \quad -(-E_3)^- \overline{z}_{2,k} + (-E_4)^+ \underline{v} - (-E_4)^- \overline{v} \end{cases} \quad (26)$$

with $E_1 = H_{11} E \tilde{C}_2$, $E_2 = H_{11} E$, $E_3 = H_{21} E \tilde{C}_2$ and $E_4 = H_{21} E$.

Theorem 2. Assume that the conditions of Theorem 1 are satisfied and $\underline{x}_0 \leq x_0 \leq \overline{x}_0$. Then, for all $k \in \mathbb{Z}_+$ the estimates $\underline{x}_k = \begin{bmatrix} \underline{x}_{1,k} \\ \underline{x}_{2,k} \end{bmatrix}$ and $\overline{x}_k = \begin{bmatrix} \overline{x}_{1,k} \\ \overline{x}_{2,k} \end{bmatrix}$ satisfies:

$$\underline{x}_k \leq x_k \leq \overline{x}_k. \quad (27)$$

where $\underline{x}_k$ and $\overline{x}_k$ are given by (26). □

Proof. Introducing the observation errors for the state x_k:

$$\begin{cases} \overline{e}_{x_1,k} = \overline{x}_{1,k} - x_{1,k} \\ \underline{e}_{x_1,k} = x_{1,k} - \underline{x}_{1,k} \\ \overline{e}_{x_2,k} = \overline{x}_{2,k} - x_{2,k} \\ \underline{e}_{x_2,k} = x_{2,k} - \underline{x}_{2,k} \end{cases} \quad (28)$$

Then, we obtain:

$$\begin{cases} \overline{e}_{x_1,k} = (H_{12}^+ + (-E_1)^+) \overline{e}_{z_2} - (H_{12}^- + (-E_1)^-) \underline{e}_{z_2} \\ \quad +(-E_2)^+ (\overline{v} - v_k) - (-E_2)^- (\overline{v} + v_k) \\ \underline{e}_{x_1,k} = (H_{12}^+ + (-E_1)^+) \underline{e}_{z_2} - (H_{12}^- + (-E_1)^-) \overline{e}_{z_2} \\ \quad +(-E_2)^+ (\overline{v} + v_k) - (-E_2)^- (\overline{v} - v_k) \\ \overline{e}_{x_2,k} = (H_{22}^+ + (-E_1)^+) \overline{e}_{z_2} - (H_{22}^- + (-E_1)^-) \underline{e}_{z_2} \\ \quad +(-E_2)^+ (\overline{v} - v_k) - (-E_2)^- (\overline{v} + v_k) \\ \underline{e}_{x_2,k} = (H_{22}^+ + (-E_3)^+) \underline{e}_{z_2} - (H_{22}^- + (-E_3)^-) \overline{e}_{z_2} \\ \quad +(-E_4)^+ (\overline{v} + v_k) - (-E_4)^- (\overline{v} - v_k) \end{cases} \quad (29)$$

Similarly to Proof 1, the observation errors (29) are nonnegative. Therefore, $\underline{x}_k \leq x_k \leq \overline{x}_k$, $\forall\, k \geq k_0$. By defining $\xi_x = \begin{bmatrix} \overline{e}_{x_1}^T & \underline{e}_{x_1}^T & \overline{e}_{x_1}^T & \underline{e}_{x_1}^T \end{bmatrix}^T$ and the matrices

$$\mathfrak{F} = \begin{bmatrix} H_{12}^+ + (-E_1)^+ & H_{12}^- + (-E_1)^- \\ H_{12}^- + (-E_1)^- & H_{12}^+ + (-E_1)^+ \\ H_{22}^+ + (-E_3)^+ & H_{22}^- + (-E_3)^- \\ H_{22}^- + (-E_3)^- & H_{22}^+ + (-E_3)^+ \end{bmatrix},$$

$$\mathfrak{I} = \begin{bmatrix} (-E_2)^+ - (-E_2)^- \\ -(-E_2)^- + (-E_2)^+ \\ (-E_4)^+ - (-E_4)^- \\ -(-E_4)^- + (-E_4)^+ \end{bmatrix}$$

Therefore, $\xi_x \leq \mathfrak{F} \xi_{z_2} + 2\mathfrak{I}\overline{v}$. We have $\xi_{z_2} = \mathfrak{R} \xi_{r_{2,k}}$ with $\mathfrak{R} = \begin{bmatrix} R^+ & -R^- \\ -R^- & R^+ \end{bmatrix}$. It follows that $\xi_x \leq \mathfrak{F}\mathfrak{R}\xi_{r_{2,k}} + 2\mathfrak{I}\overline{v}$. Since $\xi_{r_{2,k}}$ is bounded, it can be deduced that ξ_x is bounded as well. □

4.2 Fault Estimation

From equation (9), the fault vector can be expressed as:

$$\begin{aligned} f_k = K_0 R_0^{-1} [& z_{1,k+1} - \tilde{A}_{11} z_{1,k} - \Delta\tilde{A}_{11}(\rho) z_{1,k} - \tilde{A}_{12} z_{2,k} \\ & - \Delta\tilde{A}_{12}(\rho) z_{2,k} - \tilde{B}_1 u_k - \tilde{w}_{1,k}] \end{aligned} \quad (30)$$

Replacing z_1 with its expression in (10), equation (30) becomes:

$$\begin{aligned} f_k = K_0 R_0^{-1} [& E y_{k+1} + G_1 z_{2,k+1} + (-E) v_{k+1} - G_2(\rho) y_k \\ & + G_3 z_{2,k} + G_4(\rho) z_{2,k} + G_2(\rho) v_k - \tilde{B}_1 u_k - \tilde{w}_{1,k}] \end{aligned} \quad (31)$$

with $G_1 = -E\tilde{C}_2$, $G_2(\rho) = (\tilde{A}_{11} + \Delta\tilde{A}_{11}(\rho))E$, $G_3 = \tilde{A}_{11} E \tilde{C}_2 - \tilde{A}_{12}$ and $G_4(\rho) = \Delta\tilde{A}_{11}(\rho) E \tilde{C}_2 - \Delta\tilde{A}_{12}(\rho)$. The bounds of the actuator fault vector are given by:

$$\begin{cases} \overline{f}_k = K_0 R_0^{-1} [E y_{k+1} - \underline{\Gamma}_f(y_{k+1}^+, y_{k+1}^-) + \overline{\varphi}_1(\overline{z}_{2,k+1}, \underline{z}_{2,k+1}) \\ \quad + \overline{\vartheta}_1(\overline{v}, \underline{v}) + \overline{\varphi}_2(\overline{z}_{2,k}, \underline{z}_{2,k}) + \overline{\vartheta}_2(\overline{v}, \underline{v}) + \overline{\varphi}_3(\overline{z}_{2,k}, \underline{z}_{2,k}) \\ \quad - \tilde{B}_1 u_k - \underline{\overline{w}}_1] \\ \underline{f}_k = K_0 R_0^{-1} [E y_{k+1} - \overline{\Gamma}_f(y_{k+1}^+, y_{k+1}^-) + \underline{\varphi}_1(\overline{z}_{2,k+1}, \underline{z}_{2,k+1}) \\ \quad + \overline{\vartheta}_1(\overline{v}, \underline{v}) + \underline{\varphi}_2(\overline{z}_{2,k}, \underline{z}_{2,k}) + \underline{\vartheta}_2(\overline{v}, \underline{v}) + \underline{\varphi}_3(\overline{z}_{2,k}, \underline{z}_{2,k}) \\ \quad - \tilde{B}_1 u_k - \overline{\overline{w}}_1] \end{cases} \quad (32)$$

with

$$\begin{cases} \underline{\Gamma}_f(y_{k+1}^+, y_{k+1}^-) = \overline{G}_2 y_{k+1}^+ - \underline{G}_2 y_{k+1}^- \\ \underline{\Gamma}_f(y_{k+1}^+, y_{k+1}^-) = \underline{G}_2 y_{k+1}^+ - \overline{G}_2 y_{k+1}^- \\ \overline{\varphi}_1(\overline{z}_{2,k+1}, \underline{z}_{2,k+1}) = G_1^+ \overline{z}_{2,k+1} - G_1^- \underline{z}_{2,k+1} \\ \underline{\varphi}_1(\overline{z}_{2,k+1}, \underline{z}_{2,k+1}) = G_1^+ \underline{z}_{2,k+1} - G_1^- \overline{z}_{2,k+1} \\ \overline{\varphi}_2(\overline{z}_{2,k}, \underline{z}_{2,k}) = G_3^+ \overline{z}_{2,k} - G_3^- \underline{z}_{2,k} \\ \underline{\varphi}_2(\overline{z}_{2,k}, \underline{z}_{2,k}) = G_3^+ \underline{z}_{2,k} - G_3^- \overline{z}_{2,k} \\ \overline{\varphi}_3(\overline{z}_{2,k}, \underline{z}_{2,k}) = \overline{G_4}^+ \overline{z}_{2,k}^+ - \underline{G_4}^+ \overline{z}_{2,k}^- - \overline{G_4}^- \underline{z}_{2,k}^+ + \underline{G_4}^- \underline{z}_{2,k}^- \\ \underline{\varphi}_3(\overline{z}_{2,k}, \underline{z}_{2,k}) = \underline{G_4}^+ \underline{z}_{2,k}^+ - \overline{G_4}^+ \underline{z}_{2,k}^- - \underline{G_4}^- \overline{z}_{2,k}^+ + \overline{G_4}^- \overline{z}_{2,k}^- \end{cases} \tag{33}$$

and

$$\begin{cases} \overline{\vartheta}_1(\overline{v}, \underline{v}) = (-E)^+ \overline{v} - (-E)^- \underline{v} \\ \underline{\vartheta}_1(\overline{v}, \underline{v}) = (-E)^+ \underline{v} - (-E)^- \overline{v} \\ \overline{\vartheta}_2(\overline{v}, \underline{v}) = \overline{G_2}^+ \overline{v}^+ - \underline{G_2}^+ \overline{v}^- - \overline{G_2}^- \underline{v}^+ + \underline{G_2}^- \underline{v}^- \\ \underline{\vartheta}_2(\overline{v}, \underline{v}) = \underline{G_2}^+ \underline{v}^+ - \overline{G_2}^+ \underline{v}^- - \underline{G_2}^- \overline{v}^+ + \overline{G_2}^- \overline{v}^- \\ \overline{G}_4 = (\overline{\Delta \tilde{A}}_{11} (E\tilde{C}_2)^+ - \underline{\Delta \tilde{A}}_{11} (E\tilde{C}_2)^-) - \underline{\Delta \tilde{A}}_{12} \\ \underline{G}_4 = (\underline{\Delta \tilde{A}}_{11} (E\tilde{C}_2)^+ - \overline{\Delta \tilde{A}}_{11} (E\tilde{C}_2)^-) - \overline{\Delta \tilde{A}}_{12} \\ \overline{G}_2 = \tilde{A}_{11} E + (\overline{\Delta \tilde{A}}_{11} E^+ - \underline{\Delta \tilde{A}}_{11} E^-) \\ \underline{G}_2 = \tilde{A}_{11} E + (\underline{\Delta \tilde{A}}_{11} E^+ - \overline{\Delta \tilde{A}}_{11} E^-) \end{cases} \tag{34}$$

Theorem 3. Assume that the conditions of Theorem 1 are satisfied. Then, for all $k \in \mathbb{Z}_+$ the estimates $\overline{f}_k$ and $\underline{f}_k$ satisfies:

$$\underline{f}_k \leq f_k \leq \overline{f}_k. \tag{35}$$

where $\underline{f}_k$ and $\overline{f}_k$ are given by (32). □

Proof. The lower and upper observation errors of the fault vector are given by: $\overline{e}_{f,k} = \overline{f}_k - f_k$ and $\underline{e}_{f,k} = f_k - \underline{f}_k$. Then, we obtain:

$$\begin{cases} \overline{e}_{f,k} = K_0 R_0^{-1} [G_1^+ \overline{e}_{z_{2,k+1}} - G_1^- \underline{e}_{z_{2,k+1}} + G_3^+ \overline{e}_{z_{2,k}} - G_3^- \underline{e}_{z_{2,k}} \\ \quad + \overline{\varphi}_3 - \varphi_3 + \overline{\delta}_{f,k}] \\ \underline{e}_{f,k} = K_0 R_0^{-1} [G_1^+ \underline{e}_{z_{2,k+1}} - G_1^- \overline{e}_{z_{2,k+1}} + G_3^+ \underline{e}_{z_{2,k}} - G_3^- \overline{e}_{z_{2,k}} \\ \quad - \underline{\varphi}_3 + \varphi_3 + \underline{\delta}_{f,k}] \end{cases} \tag{36}$$

with $\varphi_3 = G_4(\rho) z_{2,k}$, $\overline{\delta}_{f,k} = (-E)^+(\overline{v} - v_{k+1}) - (-E)^-(\overline{v} + v_{k+1}) + (\overline{\tilde{w}}_1 + \tilde{w}_1) + \overline{\vartheta}_2 - G_2 v_k - \underline{\Gamma} + G_2 y_k$, $\underline{\delta}_{f,k} = (-E)^+(\overline{v} + v_{k+1}) - (-E)^-(\overline{v} - v_{k+1}) + (\overline{\tilde{w}}_1 - \tilde{w}_1) + G_2 v_k - \underline{\vartheta}_2 + \overline{\Gamma} - G_2 y_k$. With the same reasoning as in the proof of Theorem 1, it is deduced from (36) that the fault observation errors are positive. Since $\overline{e}_{z_2}$, $\underline{e}_{z_2}$, v and $\tilde{w}_1$ are bounded, then $\overline{f}$ and $\underline{f}$ are bounded as well.

The functions $\overline{\varphi}_3$ and $\underline{\varphi}_3$ are globally Lipschitz. Using Lemma 6 in (Zheng et al. [2016]), it follows that for $\underline{e}_{z_2,k} \leq e_{z_2,k} \leq \overline{e}_{r,k}$ and for a chosen submultiplicative norm $|.|$, there exist positive constants c_1, c_2, c_3, d_1, d_2 and d_3 such that:

$$\begin{cases} |\overline{\varphi}_k - \varphi_k| \leqslant c_1 |\overline{e}_{z_2,k}| + c_2 |\underline{e}_{z_2,k}| + c_3 \\ |\underline{\varphi}_k - \varphi_k| \leqslant d_1 |\overline{e}_{z_2,k}| + d_2 |\underline{e}_{z_2,k}| + d_3 \end{cases} \tag{37}$$

If we define $\xi_f = \begin{bmatrix} \overline{e}_f \\ \underline{e}_f \end{bmatrix}$, $\mathfrak{F}_1 = \begin{bmatrix} |G_1^+| & |-G_1^-| \\ |-G_1^-| & |G_1^+| \end{bmatrix}$, $\mathfrak{F}_2 = \begin{bmatrix} |G_3^+| & |-G_3^-| \\ |-G_3^-| & |G_3^+| \end{bmatrix}$, $\mathfrak{F}_3 = \begin{bmatrix} c_1 & c_2 \\ d_1 & d_2 \end{bmatrix}$, $\overline{\tilde{\delta}} = |\overline{\delta}_{f,k}| + c_3$ and $\underline{\tilde{\delta}} = |\underline{\delta}_{f,k}| + d_3$. It follows that

$$|\xi_f| \leq |K_0 R_0^{-1}| (\mathfrak{F}_1 + \mathfrak{F}_2 + \mathfrak{F}_3) |\xi_z| + \begin{bmatrix} \overline{\tilde{\delta}}_{f,k} \\ \underline{\tilde{\delta}}_{f,k} \end{bmatrix} \tag{38}$$

As $|\xi_{z_2}| = |\mathfrak{R}| |\xi_{r_{2,k}}|$, we obtain

$$|\xi_f| \leq |K_0 R_0^{-1}| (\mathfrak{F}_1 + \mathfrak{F}_2 + \mathfrak{F}_3) |\mathfrak{R}| |\xi_{r_{2,k}}| + \begin{bmatrix} \overline{\tilde{\delta}}_{f,k} \\ \underline{\tilde{\delta}}_{f,k} \end{bmatrix} \tag{39}$$

As $\xi_{r_{2,k}}$ is bounded, it follows that ξ_f as well. □

5. NUMERICAL EXAMPLE

Consider an LPV system described by:

$$\begin{cases} x_{k+1} = (A + \Delta A(\rho)) x_k + B u_k + F f_k + w_k \\ y_k = C x_k + v_k \end{cases} \tag{40}$$

where: $A = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & -1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $F = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$

$$\Delta A(\rho) = \begin{bmatrix} 0.1\rho_{1,k} & 0.5 & 0.1 \\ 0.1 & 0.2 & 0.4 \\ 0.3 & 0.5 & 0.05\rho_{2,k} \end{bmatrix}.$$

with $\rho_{1,k} = sin(0.2\,k)$ and $\rho_{2,k} = cos(0.1\,k)$. The disturbance v_k and the measurement noise w_k are uniformly distributed signals assumed to belong to the interval $[-0.01\ 0.01]$ and $[-0.01\ 0.01]$, respectively. The actuator fault is simulated by $f_k = cos(0.5\,k)$.

Since F is full column rank matrix and C is a full row rank matrix, Assumption 3 is satisfied and a transformation of coordinates $z_k = H^T x_k$ is required with $H = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $K_0 = 1, R_0 = 1$. Then, the system (40) can be described in the coordinates z by equation (8).

The matrix $\tilde{C}_1$ can be decomposed as: $\tilde{C}_1 = N \, [\, R_1 \;\; 0 \,]^T K_1^T$, with $N = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$, $R_1 = 1, K_1 = 1$.

For $L = [-0.3 \quad -2.1]^T$, $A_2 = \begin{bmatrix} 0 & 0 \\ -1 & -1 \end{bmatrix}$ and $C_2 = [0 \quad 1]$, the matrix $A_2 - LC_2 = \begin{bmatrix} 0 & 0.3 \\ -1 & 1.1 \end{bmatrix}$ is not nonnegative. Thus, using a transformation of coordinates $P = \begin{bmatrix} 10 & -5 \\ -10 & 6 \end{bmatrix}$, the matrix $R = P(A_2 - LC_2)P^{-1} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.6 \end{bmatrix}$ is nonnegative. Then, all the conditions of Theorem 1 are satisfied with $\alpha = 0.057$ and the matrix inequality (20) is feasible.

The initial states are chosen as $x_0 = [1\ 1\ 1]^T$, $\overline{x}_0 = [1.5\ 1.5\ 1.5]^T$, $\underline{x}_0 = [0.5\ 0.5\ 0.5]^T$. The state and fault estimation bounds are computed using (26) and (32), respectively, and the results are given in Figure 1, Figure 2 and Figure 3.

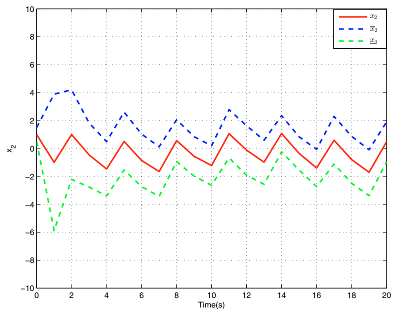

Fig. 1. Evolution of the second component of state x_2.

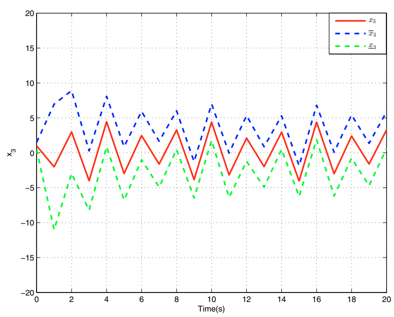

Fig. 2. Evolution of the third component of state x_3.

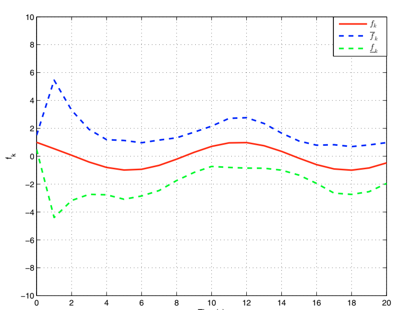

Fig. 3. Evolution of the fault f_k.

The simulation results show that the upper and the lower bounds of the state and fault estimation converge to an interval despite the presence of measurement noise and disturbances.

6. CONCLUSION

A methodology for actuator fault estimation for discrete-time LPV systems has been proposed in this paper. The actuator fault is considered as an unknown input. An unknown input interval observer is designed to estimate the system state and the fault under the assumption that the exogenous disturbances and the measurement noises are bounded. Based on Lyapunov theory, stability conditions are given in terms of matrices inequality. Simulation results show the robustness of the proposed approach.

The design of Active Fault Tolerant Control and its interaction with fault identification module will be investigated in further works.

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