Controlling anthropogenic and natural seismicity: Insights from active stabilization of the spring-slider model

Ioannis Stefanou

¹Université Paris-Est, Laboratoire Navier, CNRS UMR 8205, IFSTTAR, ENPC, Paris, France

Key Points:

- Stability conditions and steady-state of the frictional slider are determined using double-time scale asymptotics and general Coulomb-type friction.
- We show that the system is stabilizable by fluid pressure control. A stabilizing controller is designed and tested.
- Two examples show that anthropogenic and natural earthquakes could be in theory prevented.

keywords: induced seismicity, stability, control theory, earthquake control

Abstract

We present a theoretical study focusing on exploring the possibility of controlling anthropogenic and natural seismicity. We actively control the pressure of injected fluids using a negative-feedback control system. Our analysis is based on the spring-slider model for modeling the earthquake instability. We use a general Coulomb-type rheology for describing the frictional behavior of a fault system. This model leads to a non-autonomous system, whose steady-state and stability are studied using a double-scale asymptotic analysis. This approach renders the dominant order of the system time invariant. Established tools from the classical mathematical theory of control are used for designing a proper stabilizing controller. We show that the system is stabilizable by controlling fluid pressure. This is a central result for industrial operations. A stabilizing controller is then designed and tested. The controller regulates in real-time the applied pressure in order to assure stability, avoid unwanted seismicity and drive the system from unstable states of high potential energy, to stable ones of low energy. The controller performs well even in the absence of complete knowledge of the frictional properties of the system. Finally, we present two numerical examples (scenarios) and illustrate how anthropogenic and natural earthquakes could be, in theory, prevented.

1 Introduction

Given the current intense human activity for conventional and unconventional energy production (e.g. oil and gas), renewable energies (e.g. geothermal) and potential environmental friendly methods related to climate change (e.g. CO₂ sequestration), avoiding anthropogenic seismicity is a challenging topic. Even though evidence and proof of the origin of reported seismicity and its relation with human activities will always have a degree of uncertainty, it is nowadays generally accepted that humans can induce or trigger earthquakes. Anthropogenic earthquakes are usually of small to moderate magnitude, i.e. less than $M_w = 4$ (moment magnitude). However, they usually exceed the acceptable limits set by the authorities and can cause damage. Moreover, there are several cases where induced earthquakes had important magnitudes (no distinction is made here between triggered and induced earthquakes and both terms are used interchangeably for simplicity). Some examples of induced earthquakes, among several others, are the $M_w = 5.3$ at Trinidad, Colorado earthquake (EQ) in US due to wastewater injection (Rubinstein & Mahani, 2015), the $M_w = 5.7$ at Prague, Oklahoma EQ in US prob-

ably due to wastewater injection (Keranen, Savage, Abers, & Cochran, 2013; McGarr, 2014) and the $M_w = 3.6$ at Basel geothermal project in Switzerland (Cornet, 2016; Deichmann & Giardini, 2009). In fact the number and the importance of induced seismicity events were such that the United States Geological Survey (USGS) incorporated them in the 2014 United States National Seismic Hazard Model (Petersen et al., 2015).

Here we address the general question of earthquake control: Is it possible to control and avoid anthropogenic and natural earthquakes and how? We address this question using a mathematically rigorous framework and keeping complexity to a minimum degree. In particular, we investigate the conditions under which a simplified, seismogenic fault system can be stabilized by automatic fluid pressure adjustment/control. The concept that pore pressure increase due to fluid injection can stimulate fault slip is well established, nowadays (Frohlich, 2012; Healy, Rubey, Griggs, & Raleigh, 1968; Hubbert & Rubey, 1959; Raleigh, Healy, & Bredehoeft, 1976). However, the question of EQ control and EQ risk mitigation is still open and challenging.

Evidence of EQ control from field experiments is very limited. We refer to the seminal field experiment of Raleigh et al. (1976) in the 70's in Rangely, Colorado, US, where earthquakes could be turned off and on by varying the pore pressure. Another example of EQ control is in Dale, New York, US (Fletcher & Sykes, 1977), where earthquakes of magnitude -1 to 1.4 formed a cluster about 650m across near the bottom of a 426m injection well. The earthquake activity was arrested when the top hole pressure dropped below 5 MPa. More recent field experiments involve the well monitored tests by Cappa, Scuderi, Collettini, Guglielmi, and Avouac (2019); Guglielmi, Cappa, Avouac, Henry, and Elsworth (2015) performed at 252m depth within the LSBB underground laboratory in France. In these field experiments aseismic slip was systematically preceding seismic slip, giving concrete evidence that slip can be also aseismic.

Here we follow a theoretical approach in order to get useful insight of the controllability of a fault system and the possibility of injecting fluids in a way that guarantees aseismic slip (definitions of the terms seismic and aseismic slip, as used here, are given in Section 2.2). Our analysis is based on the classical spring-slider model. We adopt a general frictional law that depends on slip and rate of slip. Additionally to rock elasticity (spring) we consider also radiational damping. This is performed by accounting for the rock viscosity through a damper in Kelvin-Voigt configuration (see also Wang (2017)).

We consider that the rock is saturated and that our system is well oriented for slip in the ambient stress-field. Both physics and geometry of the system are kept as simple as possible. The role of heterogeneities, of pore fluid diffusivity and special hydrological conditions are not considered in the present work. Consequently, we don't focus on EQ rupture and propagation in details, but only on average (over the fault's length) using the spring-slider model. The above mentioned aspects, as well as the observability of the real system and other techno-economical aspects of EQ control, exceed the scope of the present article and they are explored in the frame of the ongoing ERC project "Controlling earthquakes - CoQuake" (see <http://coquake.com>).

We give particular emphasis on the study of the stability of the system which is constantly driven by the far-field tectonic velocity. The term *stability* is used here only in the sense of Lyapunov stability (i.e. the system remains close to its equilibrium state under small perturbations from it; for a rigorous mathematical definition of Lyapunov stability we refer to (Lyapunov, 1892; Stefanou & Alevizos, 2016)). Due to the far field tectonic velocity and the general rheology considered for friction, the system is non-autonomous (i.e. it depends explicitly on time) and the classical methods of Lyapunov stability cannot be directly applied. For this purpose, we use a double-time scale asymptotic analysis that a) eliminates the secular terms (i.e. growing, unbounded terms in time), b) provides the steady-state of the system describing its slow, creep-like motion and c) allows to derive the (in-)stability conditions, i.e. determine when slow, creep-like slip is possible and when an earthquake takes place. Based on these results we prove mathematically that the system is *stabilizable* by fluid pressure control. This is a major result. An opposite conclusion would mean that EQ control is impossible, implying inevitable risks for on-going, large-scale industrial applications.

Following these mathematical developments and using the *classical mathematical theory of control* we design a *stabilizing controller (compensator)* and we investigate the conditions for which the controller can stabilize the system, even when complete knowledge on the evolution of the frictional properties of the system is not available. The ability of the controller to stabilize the system and avoid unwanted seismicity, is then illustrated through two scenarios of fluid injection at 5km depth. The first one refers to an injection project, where an injection under constant pressure rate is planned. It is shown that the controller automatically stabilizes the system and avoids the anticipated earthquake event of $M_w = 3.8$. The second scenario concerns a fault system with higher rup-

ture area, able to give earthquakes of $M_w = 5.8$. In this second scenario, we drive the system from its initial, high energy, unstable state to a new, stable one. This is done by automatically adjusting the fluid pressure such as to assure a constant slow slip rate. In this way no seismicity is observed and the earthquake is avoided. Finally, the robustness of the designed controller to successfully mitigate seismicity is challenged by perturbing the friction properties of the system.

The paper is organized as follows. In Section 2 we present the basic modeling assumptions of the spring-slider analog and we study it using double-scale asymptotics in time. The steady-state motion is approximated by a power series of the orders of the slow time scale, for which we calculate explicitly the dominant one. The necessary and sufficient conditions for stable and unstable steady-state motion are determined. This asymptotic analysis renders the dominant order of the system time invariant and allows us to study it further in Section 3 using the classical mathematical theory of control. The linearization of the system is investigated in paragraph 3.2 and its stabilizability in paragraph 3.3. A stabilizing controller is designed in paragraph 3.4. Asymptotic tracking and robustness of the controller are discussed next. Finally, in Section 4 we present two numerical examples, i.e. the aforementioned scenarios of EQ control.

2 Steady-state and stability conditions for the spring-slider analog

Consider the spring-slider model of Figure 1a. This is the classical paradigm and starting point for studying the dynamic instability of earthquake nucleation in a mathematically simplified manner (see Burridge and Knopoff (1967); Reid (1910)). In this model, the block represents the mobilized rock mass, m , during an earthquake event. With the spring, k , we model the elastic deformation of the surrounding to the fault rock. This allows to account for the progressive elastic energy build-up due to the far field tectonic movement, δ_∞ . The far field tectonic movement is of the order of some cm's per year, contrary to the seismic slip that can rise up to one meter per second. The coefficient of the elastic spring is proportional to the effective shear elastic modulus of the surrounding to the fault rocks, G , and inversely proportional to the fault length, L , i.e. $k \propto \frac{G}{L}$. This scaling is retrieved from elasticity theory (Palmer & Rice, 1973). With effective shear modulus we mean the apparent/averaged shear modulus over a region that extends at a distance L in the direction and perpendicular to the fault (see Figure 1b). This region includes the damaged area that extends from some meters to some kilometers around

the fault (Okubo et al., 2019). Additionally, we consider the effective viscosity of the surrounding rock, C . This viscosity is represented through an equivalent dashpot of coefficient η , connected in a Kelvin–Voigt configuration as shown in Figure 1.

Figure 1. Spring-slider analog.

With the term *fault* we mean the region of the rock system that is under pronounced localized shear deformation and opposes to the movement of the block, with a frictional force F . A fault is usually modeled as a mathematical plane due to its very small thickness, h , compared to the other characteristic lengths of the problem. Nevertheless, its thickness is linked with the softening response of the system, i.e. the reduction of friction in function of slip, rate of slip and other variables related to multiphysical couplings. This region of extreme shearing is usually consisted of ultracataclastic materials and it has a complex structure (Ben-Zion & Sammis, 2003; Brodie, Fettes, Harte, & Schmid, 2007) due to various physico-chemical phenomena that take place during pre- and co-seismic slip (see Anthony and Marone (2005); Rattez, Stefanou, and Sulem (2018); Rattez, Stefanou, Sulem, Veveakis, and Poulet (2018a, 2018b); Reches and Lockner (2010); Scuderi, Collettini, and Marone (2017); Tinti et al. (2016), among others). As a result, the apparent friction, F , does not depend only on the extent and the rate of slip, $\delta, v = \dot{\delta}$, but also on the evolution of the microstructural network, the grain size, the presence and pressure of interstitial fluids, the temperature, time (state) and the reactivation of chemical reactions (Brantut and Sulem (2012); Veveakis, Alevizos, and Vardoulakis (2010); Veveakis, Stefanou, and Sulem (2013), among others).

As far as it concerns the triggering of the dynamic instability studied here, the knowledge of the constitutive description of the apparent friction is central. Depending on the way that the fault's apparent friction evolves with shearing, an earthquake can be nu-

cleated or arrested. The transition of the system from a potential unstable state to a stable one can be studied mathematically.

Let the apparent fault friction depend on δ and $\dot{\delta}$, i.e. $F = F(\delta, \dot{\delta})$. The spring-slider analog is described by the following equation:

$$m\ddot{\delta} = -F_r(\delta, \dot{\delta}) + k(\delta_\infty - \delta) + \eta(v_\infty - v) \quad (1)$$

where δ_∞ is the far-field displacement due to the far-field velocity v_∞ . Notice that the above expression considers the dependence of friction on the degree of freedom of the system (slip) and its first derivative (rate of slip) and not on any internal, state variables (see rate and state friction laws). Nevertheless, it is often possible to eliminate these internal variables and express friction as a function of slip and rate of slip only. Later, in paragraph 2.3, the apparent fault friction will depend on the fluid pressure too, which will allow to control seismic slip.

It is worth emphasizing that the above equation is a non-autonomous, non-linear dynamic system, whose stability and steady state cannot be directly studied using the classical Lyapunov methods (Brauer & Nohel, 1969). Stability of this system has been studied in the literature by several researchers who considered it ad-hoc as autonomous by either neglecting the far field velocity or by applying a constant force. Nevertheless, this is a strong assumption that can lead, in general, to incorrect results regarding stability (Brauer & Nohel, 1969). Here we follow a different approach that allows us to asymptotically approximate the steady-state movement of the block and study its (Lyapunov) stability.

Due to the fact that the far-field tectonic movement is many orders of magnitude slower than seismic slip (v_∞ is a very small quantity) the steady-state solution of the spring-slider can be asymptotically approximated using the double-scale approach presented in the following paragraph.

2.1 Double-scale asymptotic analysis

Equation (1) can be written as follows:

$$\ddot{y} = -a(y, \dot{y}) + c(\varepsilon t - y) + d(\varepsilon - \dot{y}), \quad (2)$$

where $\ddot{y}$ and $\dot{y}$ denote, respectively, the second and first order derivatives with respect to time, t , of the unknown function $y = y(t)$, ε is a small parameter ($\varepsilon \ll 1$) and a ,

192 c, d are of $O(\varepsilon^0)$. The above equation is dimensionless y expresses normalized displace-
 193 ment and the exact expressions of coefficients are given in Section 4 (Eq.(41)), where the
 194 theory is applied to specific scenarios of earthquake control.

We introduce a new variable $\tau = \varepsilon t$, which defines a long time scale because it is not negligible when τ is of order ε^{-1} or larger. In this way the system is expressed in terms of two time scales, a slow one, τ , and a fast one, t :

$$\ddot{y} = -a(y, \dot{y}) + c(\tau - y) + d(\varepsilon - \dot{y}). \quad (3)$$

This double scale approach allows to seek solutions of y , which are functions of both variables t and τ , treated as independent (Bender & Orszag, 1999). It is worth emphasizing that expressing y as a function of two variables is an artifice to remove secular effects. The exact solution $y(t)$ is a function of t alone. We assume the following perturbation expansion for the unknown function $y(t)$:

$$y(t) = y_0(t, \tau) + \varepsilon y_1(t, \tau) + \varepsilon^2 y_2(t, \tau) + O(\varepsilon^3). \quad (4)$$

Using the chain rule for differentiation, we obtain:

$$\frac{dy(t)}{dt} = \frac{\partial y_0}{\partial t} + \varepsilon \left(\frac{\partial y_0}{\partial \tau} + \frac{\partial y_1}{\partial t} \right) + \varepsilon^2 \left(\frac{\partial y_1}{\partial \tau} + \frac{\partial y_2}{\partial t} \right) + O(\varepsilon^3) \quad (5)$$

and

$$\frac{d^2 y(t)}{dt^2} = \frac{\partial^2 y_0}{\partial t^2} + \varepsilon \left(2 \frac{\partial^2 y_0}{\partial \tau \partial t} + \frac{\partial^2 y_1}{\partial t^2} \right) + \varepsilon^2 \left(\frac{\partial^2 y_0}{\partial \tau^2} + 2 \frac{\partial^2 y_1}{\partial \tau \partial t} + \frac{\partial^2 y_2}{\partial t^2} \right) + O(\varepsilon^3). \quad (6)$$

We assume that $a(y, \dot{y})$ can be expanded in power series in terms of ε :

$$a(y, \dot{y}) = a(y_0, \frac{\partial y_0}{\partial t}) + \varepsilon \left[\frac{\partial a}{\partial y} \Big|_{(y_0, \frac{\partial y_0}{\partial t})} y_1 + \frac{\partial a}{\partial \dot{y}} \Big|_{(y_0, \frac{\partial y_0}{\partial t})} \left(\frac{\partial y_0}{\partial \tau} + \frac{\partial y_1}{\partial t} \right) \right] + O(\varepsilon^3). \quad (7)$$

Inserting the above equations into Eq.(2) and collecting powers of ε , we obtain the following cascade problems:

$$\begin{aligned} \varepsilon^0 : \quad & \frac{\partial^2 y_0}{\partial t^2} = -a(y_0, \frac{\partial y_0}{\partial t}) + c(\tau - y_0) - d \frac{\partial y_0}{\partial t} \\ \varepsilon^1 : \quad & \frac{\partial^2 y_1}{\partial t^2} = - \left(\frac{\partial a}{\partial y} \Big|_{(y_0, \frac{\partial y_0}{\partial t})} + c \right) y_1 - \left(\frac{\partial a}{\partial \dot{y}} \Big|_{(y_0, \frac{\partial y_0}{\partial t})} + d \right) \frac{\partial y_1}{\partial t} - \left(\frac{\partial a}{\partial \dot{y}} \Big|_{(y_0, \frac{\partial y_0}{\partial t})} + d \right) \frac{\partial y_0}{\partial \tau} - 2 \frac{\partial^2 y_0}{\partial t \partial \tau} + d \\ \varepsilon^2 : \quad & \dots \end{aligned} \quad (8)$$

2.2 Slow dynamics and Lyapunov stability

We will first study the first of Eqs.(8). Setting $q(t) = \frac{\partial y_0(t, \tau)}{\partial t}$ and $r(t) = y_0(t, \tau)$ we obtain the following first order, non-linear system of ODE's:

$$\begin{aligned}\dot{q} &= -a(r, q) + c(\tau - r) - d q \\ \dot{r} &= q.\end{aligned}\tag{9}$$

Notice that in this system, τ acts as a parameter, given that t and τ are independent variables. The above system can be written in vectorial form $\dot{\underline{z}} = \underline{f}(\underline{z})$, with $\underline{z} = (z_1, z_2) = (q, r)$, is autonomous and its stability can be studied using the classical tools of Lyapunov stability theory (see Brauer and Nohel (1969)). This system has an equilibrium point (fixed point) at $\underline{z}^* = (q^*, r^*)$ satisfying:

$$\begin{aligned}-a(r^*, q^*) + c(\tau - r^*) &= 0 \\ q^* &= 0.\end{aligned}\tag{10}$$

The above fixed point shows that the system has zero velocity in terms of the fast time variable t , i.e. $q^* = \frac{\partial y_0^*(t, \tau)}{\partial t} = 0$, leading to $y_0 = y_0(\tau)$, which is a function of the slow time variable, only. It should be emphasized that the velocity in terms of the slow time variable τ is not zero.

The eigenvalues, λ , of the Jacobian of $\underline{f}$ satisfy the characteristic polynomial:

$$\lambda^2 + \lambda \left(\left. \frac{\partial a}{\partial z_1} \right|_{\underline{z}^*} + d \right) + \left(\left. \frac{\partial a}{\partial z_2} \right|_{\underline{z}^*} + c \right) = 0.\tag{11}$$

The system is unstable when a positive eigenvalue exists and stable when all the eigenvalues are negative. Therefore, if for any τ , exists $\underline{z}^* = (0, r^*)$, such that $\left. \frac{\partial a}{\partial z_2} \right|_{\underline{z}^*} + c < 0$ or $\left. \frac{\partial a}{\partial z_1} \right|_{\underline{z}^*} + d < 0$, the system is unstable. The system is asymptotically stable when $\left. \frac{\partial a}{\partial z_2} \right|_{\underline{z}^*} + c > 0$ and $\left. \frac{\partial a}{\partial z_1} \right|_{\underline{z}^*} + d > 0$. The limiting case $\left. \frac{\partial a}{\partial z_2} \right|_{\underline{z}^*} + c = 0$ or $\left. \frac{\partial a}{\partial z_1} \right|_{\underline{z}^*} + d = 0$ is of no particular interest for the physical problem at hand and it is not studied herein. However, we can show that in this particular case the system is stable.

The time evolution of the stable solution of the system is $y_0 = y_0^* = y_0^*(\tau)$. We call here this state *steady-state*, given that $\frac{\partial y_0^*}{\partial t} = 0$. Eq.(8).2 becomes:

$$\frac{\partial^2 y_1}{\partial t^2} = -\alpha y_1 - \beta \frac{\partial y_1}{\partial t} - \beta \frac{\partial y_0^*}{\partial \tau} + d,\tag{12}$$

where $\alpha = \left. \frac{\partial a}{\partial y} \right|_{(y_0^*, 0)} + c$, $\beta = \left. \frac{\partial a}{\partial y} \right|_{(y_0^*, 0)} + d$ and $\frac{\partial y_0^*}{\partial \tau}$ can be determined by differentiating Eq.(10.1) in terms of τ :

$$\frac{\partial y_0^*}{\partial \tau} = \frac{c}{\left. \frac{\partial a}{\partial y} \right|_{(y_0^*, 0)} + c} = \frac{c}{\alpha}.\tag{13}$$

The general solution of (12) is:

$$y_1^*(t, \tau) = \frac{\alpha d - \beta c}{\alpha^2} + C_1 e^{\frac{1}{2}(-\beta + \sqrt{\Delta})t} + C_2 e^{\frac{1}{2}(-\beta - \sqrt{\Delta})t}, \quad (14)$$

where C_1 and C_2 are constants determined by the initial conditions of the ε^1 problem and $\Delta = \beta^2 - 4\alpha$.

The asymptotic approximation of the steady-state solution of the problem, y^* , is therefore:

$$y^*(t) = y_0^*(\varepsilon t) + \varepsilon y_1^*(t, \varepsilon t) + O(\varepsilon^2). \quad (15)$$

The steady state velocity, $v^*(t)$, is approximated by the series (see Eq.(5)):

$$v^*(t) = \varepsilon \left[\frac{c}{\alpha} + C_1 \frac{1}{2} \left(-\beta + \sqrt{\Delta} \right) e^{\frac{1}{2}(-\beta + \sqrt{\Delta})t} + C_2 \frac{1}{2} \left(-\beta - \sqrt{\Delta} \right) e^{\frac{1}{2}(-\beta - \sqrt{\Delta})t} \right] + O(\varepsilon^2). \quad (16)$$

Notice that if $\beta > 0$, $v^*(t)$ approximates asymptotically the solution $v^*(t) = \varepsilon \frac{c}{\alpha} + O(\varepsilon^2)$. Positive β is usually the case, due to the high viscosity of the surrounding to the fault rocks.

2.3 Conditions for steady-state slip of the spring-slider analog

Considering a monotonous motion ($\dot{\delta} \geq 0$) of the spring-slider system, the friction can be considered as a non-linear function of slip δ and can be expanded in power series as done in Eq.(7). Under these assumptions, the multiscale asymptotic approach presented above can be used for the spring-slider system. More specifically, according to Eq.(16), the slip-rate at steady-state of the spring-slider model is given by:

$$v_{ss} = \frac{v_\infty}{1 + \frac{1}{k} \frac{\partial F}{\partial \delta}}, \quad (17)$$

where $\frac{\partial F}{\partial \delta}$ is calculated at a given $\delta(t) = \delta_0$ and at zero slip-rate $\dot{\delta} = 0$. The above relation can give a useful global estimation of velocity, if the frictional properties of the system are known. Inversely, if the time evolution of the block velocity is known (e.g. measured), $\frac{\partial F}{\partial \delta}$ can be determined:

$$\frac{\partial F}{\partial \delta} = k \left(\frac{v_\infty}{v_{ss}} - 1 \right). \quad (18)$$

According to the previous paragraph, the steady-state slip is *unstable*, if and only if for any δ_0 :

$$\left. \frac{\partial F}{\partial \delta} \right|_{(\delta_0, 0)} < -k \quad \text{or} \quad \left. \frac{\partial F}{\partial \dot{\delta}} \right|_{(\delta_0, 0)} < -\eta. \quad (19)$$

The first condition refers to slip-weakening and coincides with the one described in Dieterich (1978) (see also Goodman (1989); Scholz (2002)). The so-called *nucleation length* (Andrews, 1976) can be retrieved setting $k = A \frac{2}{\pi} \frac{\lambda + \mu}{\lambda + 2\mu} \frac{\mu}{L}$, where λ and $\mu = G$ are the Lamé constants (μ not to be confused with the friction coefficient in the next paragraphs) and A the slip area. The second condition shows that the system is unstable when the friction is slip-rate weakening (velocity-weakening). However, for common parameters of frictional velocity weakening (Reches & Lockner, 2010) and viscosity of rocks (Vutukuri & Katsuyama, 1994) this instability condition is not critical.

We assume Coulomb friction, $F = \mu N'$, where N' is the effective normal force applied on the block, as shown in Figure 1, and μ is the friction coefficient. The coefficient of friction varies from an initial value $\mu_{\max}$ (static friction coefficient), to a residual one μ_{res} (kinetic friction coefficient). Figure 2a shows schematically the transition between static and kinetic friction. This transition is made in a characteristic distance D_c and depends on the frictional properties of the fault system. In Figure 2b we show the in-

Figure 2. Schematic representation of the evolution with slip of (a) the friction coefficient and (b) of the Coulomb friction force with and without fluid pressure, P_f . The system becomes more ductile (lower slope) when the fluid pressure increases.

fluence of N' on the friction force. Due to fluid injection, $N' = N - P_f$, where P_f is the force exerted to the block by the fluid due to fluid pressure and N the total force which is a fraction of the overburden load depending on the tectonic setting. Forces can be expressed in terms of stresses by dividing by the slip area A . It is worth noticing that the slope of the softening branch $(N - P_f) \frac{\Delta\mu}{D_c}$ on P_f , i.e. $(N - P_f)$ multiplies μ . Under constant P_f , unstable, seismic slip happens when $(N - P_f) \frac{\Delta\mu}{D_c} < -k$, provided that the

applied destabilizing force from the spring is high enough. If the force exerted from the spring to the block, $F_s = k(v_\infty - v_{ss})$ is not high enough ($F_s < F$), then no sliding takes place and the system is stable. Therefore, the necessary condition for instability is $F_s = F$. In this case we say that an (existing) fault is (re)activated. However, it is worth emphasizing that this condition is not a sufficient one for seismic slip (unstable behavior). If the aforementioned instability conditions (19) are not satisfied, we have slip, but this a slow, aseismic, creep-like slip with velocity given by Eq.(18).

From the physical point of view, a dynamic instability takes place if the slip weakening is higher than the negative slope of the effective elastic response of the surrounding to the fault rocks or, in other words, when the elastic unloading of the surrounding rocks cannot be counterbalanced by fault friction. The same happens also when the friction shows velocity weakening that cannot be counterbalanced by the viscosity of the surrounding rock mass. However, the situation changes when the fluid pressure is not constant with time $P_f = P_f(t)$. The question addressed in the next section is exactly how one should control $P_f(t)$ in order to assure stable slip.

3 Controlling instabilities

3.1 Control system configuration

We assume a general negative-feedback control system as depicted in Figure 3. $\Sigma(P)$ is the system to be controlled, the spring-slider in our case, $\Sigma(C)$ the controller we need to design, $y(t)$, the output of the controlled system $\Sigma(P, C)$, i.e. the displacement of the block $\delta(t)$, $u(t)$ the input of $\Sigma(P)$, $u_c(t)$ the input of the controller $\Sigma(C)$ and $y_c(t)$ its output. $u_1(t)$ and $u_2(t)$ are inputs to the system (e.g fluid pressure, long and short range perturbations, applied slip velocity etc.). We seek the controller $\Sigma(C)$ that can stabilize the spring-slider model by modifying (controlling) the applied fluid pressure $P_f(t)$ (input). The problem is challenging due to friction and the consequent non-linearities it introduces. Additionally, the pore pressure multiplies the friction coefficient and does not allow us to write the mathematical system in canonical forms frequently used in the mathematical theory of control (Vardoulakis, 1991, 2012). However, the target of the present work is to stabilize the system and stay in the vicinity of an evolving steady-state. This justifies the linearization of the equations in terms of slip, slip-rate and fluid pressure.

Figure 3. Negative feedback control system $\Sigma(P, C)$. $\Sigma(P)$ is the system to be controlled with the controller $\Sigma(C)$.

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3.2 Linearization

We want to stabilize the fast dynamics of the system described by Eq.(2). Therefore, we need to assure that the system is stable in the fast time scale described by the order ε^0 in the cascade system of equations (8). Setting $y_0 = \tilde{y} + y_0^*$ in Eq.(8).1 and the same equation expressed at the reference state y_0^* , we obtain:

$$\ddot{\tilde{y}} = -a(\tilde{y} + y_0^*, \dot{\tilde{y}}, \phi) + a(y_0^*, 0, \phi_0^*) - c\tilde{y} - d\dot{\tilde{y}}, \quad (20)$$

where $\tilde{y}$ represents a perturbation of the reference steady-state solution, $\phi = \phi(t)$ the dependence of the $a(y, \dot{y}, \phi)$ function in terms of an input (here the fluid pressure), $\phi(t)$, which takes the reference value ϕ_0^* at steady state. Expanding $a(\tilde{y} + y_0^*, \dot{\tilde{y}}, \phi)$ in power series around $\tilde{y} = 0$, $\dot{\tilde{y}} = 0$ and $\phi = \phi_0^*$ (steady-state) we obtain:

$$\ddot{\tilde{y}} + a_1\dot{\tilde{y}} + a_0\tilde{y} = b_0u, \quad (21)$$

264 where $u(t) = \phi(t) - \phi_0^*$, $a_0 = \alpha = \frac{\partial a}{\partial y} \Big|_{(y_0^*, 0, \phi_0^*)} + c$, $a_1 = \beta = \frac{\partial a}{\partial \dot{y}} \Big|_{(y_0^*, 0, \phi_0^*)} + d$ and $b_0 =$
 265 $-\frac{\partial \alpha}{\partial \phi} \Big|_{(y_0^*, 0, \phi_0^*)}$. In the following, the tilde over y is dropped for simplicity in notation.

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3.3 Stabilizability

The above equation describes the behavior of the system for small perturbations from its steady-state. It is linear both in terms of y and the input u and its characteristic polynomial is:

$$D(s) = s^2 + a_1s + a_0, \quad (22)$$

while for the input (right-hand-side) is:

$$N(s) = b_0. \quad (23)$$

The transfer function of the system is:

$$P(s) = \frac{N(s)}{D(s)} = \frac{b_0}{s^2 + a_1s + a_0}. \quad (24)$$

The poles of the system are at s_0 satisfying $D(s_0) = 0$. As expected, the poles have non-negative real part when α or β are negative (instability). Since the system has no decoupling zeros in the closed right half complex plane, i.e. in $\overline{\mathbb{C}}^+ = \{s \in \mathbb{C}, \text{Re}(s) \geq 0\}$, the system is *stabilizable*. Decoupling zeros are called the common roots of $N(s)$ and $D(s)$ that are not roots of its transfer function $P(s)$. Consequently, a stabilizing compensator (stabilizing controller) can be designed. This is a major result for the application at hand as it shows that earthquakes can be controlled, at least from the mathematical point of view.

3.4 Proper stabilizing controller

According to Vardoulakis (1991) the system $\Sigma(C)$, with transfer function $C(s) = \frac{Y(s)}{X(s)} \in \mathbb{R}_{pr}(s)$, is a stabilizing compensator, if and only if, the characteristic polynomial of the closed system $\Sigma(P, C)$:

$$D_c(s) = X(s)D(s) + Y(s)N(s), \quad (25)$$

has all its roots in $\mathbb{C}^- = \{s \in \mathbb{C}, \text{Re}(s) < 0\}$. Following the procedure described in Vardoulakis (1991, 2012) it is possible to determine $Y(s)$ and $X(s)$ and, therefore, design the desired stabilizing controller. Let $\Lambda_5 = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5\}$ the set of roots of $D_c(s)$ such that $\lambda_i = \lambda \in \mathbb{C}^-$. The polynomials $X(s)$ and $Y(s)$ are determined by solving the following linear system:

$$\underline{\omega}^T \underline{M}_4 = \underline{d}^T, \quad (26)$$

where $\underline{M}_4$ is the Wolovich resultant of rank 6 (Antoniou & Vardoulakis, 2005):

$$M_4 = \begin{bmatrix} a_0 & a_1 & 1 & 0 & 0 & 0 \\ b_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_0 & a_1 & 1 & 0 & 0 \\ 0 & b_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_0 & a_1 & 1 & 0 \\ 0 & 0 & b_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_0 & a_1 & 1 \\ 0 & 0 & 0 & b_0 & 0 & 0 \end{bmatrix} \quad (27)$$

and

$$\underline{d}^T = \begin{bmatrix} -\lambda^5 & 5\lambda^4 & -10\lambda^3 & 10\lambda^2 & -5\lambda & 1 \end{bmatrix} \quad (28)$$

ω^T contains the coefficients of $X(s)$ and $Y(s)$:

$$\omega^T = \left[\begin{bmatrix} \chi_0 & \psi_0 \end{bmatrix} \begin{bmatrix} \chi_1 & \psi_1 \end{bmatrix} \begin{bmatrix} \chi_2 & \psi_2 \end{bmatrix} \begin{bmatrix} \chi_3 & \psi_3 \end{bmatrix} \right] \quad (29)$$

such that:

$$X(s) = \chi_3 s^3 + \chi_2 s^2 + \chi_1 s + \chi_0 \quad \text{and} \quad Y(s) = \psi_3 s^3 + \psi_2 s^2 + \psi_1 s + \psi_0. \quad (30)$$

The transfer function of the closed system $\Sigma(P, C)$ is then:

$$H_{cl}(s) = \begin{bmatrix} \frac{P(s)}{1+C(s)P(s)} & \frac{P(s)C(s)}{1+C(s)P(s)} \\ -\frac{C(s)P(s)}{1+C(s)P(s)} & \frac{C(s)}{1+C(s)P(s)} \end{bmatrix} \quad (31)$$

and therefore the Laplace transform of the output of the closed system is:

$$\begin{bmatrix} Y(s) \\ Y_c(s) \end{bmatrix} = H_{cl}(s) \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}, \quad (32)$$

where $U_1(s)$ and $U_2(s)$ are the Laplace transforms of the inputs $u_1(t)$ and $u_2(t)$, respectively (see Figure 3).

3.5 Asymptotic tracking

We want to control the response of the system $y(t)$ in order to asymptotically track a reference input $u_2(t)$ as $t \rightarrow \infty$. In other words, we want the error function $u_c(t) = u_2(t) - y(t)$ tend to zero for $t \rightarrow \infty$. Consider the transfer function between the error function $u_c(t)$ and the reference input $u_2(t)$:

$$S(s) = \frac{U_c(s)}{U_2(s)} = \frac{1}{1 + C(s)P(s)} = \frac{X(s)D(s)}{X(s)D(s) + Y(s)N(s)}, \quad (33)$$

where $U_c(s)$ and $U_2(s)$ are respectively the Laplace transforms of $u_c(t)$ and $u_2(t)$. Let $u_2(t)$ be the step function, such that $u_2(t) = K$ for $t \geq 0$ and zero for $t < 0$ or $u_2(t) = Kt$ for $t \geq 0$ and zero for $t < 0$. Then according to Vardoulakis (2012), if $\Sigma(P, C)$ is asymptotically stable, then $\lim_{t \rightarrow \infty} u_c(t) = 0$, if and only if, $C(s)$ has a double pole at $s = 0$ or equivalently if and only if $\chi_0 = 0$ and $\chi_1 = 0$. Solving the linear system (26) under these constraints we determine the stabilizing controller, which has the following coefficients:

$$\begin{aligned} \chi_0 &= 0, \chi_1 = 0, \chi_2 = -(a_1 + 5\lambda), \chi_3 = 1 \\ \psi_0 &= -\frac{\lambda^5}{b_0}, \psi_1 = \frac{5\lambda^4}{b_0}, \psi_2 = \frac{5a_0\lambda + a_0a_1 - 10\lambda^3}{b_0}, \psi_3 = \frac{5a_1\lambda + a_1^2 - a_0 + 10\lambda^2}{b_0} \end{aligned} \quad (34)$$

and therefore:

$$C(s) = \frac{Y(s)}{X(s)} = \frac{\psi_3 s^3 + \psi_2 s^2 + \psi_1 s + \psi_0}{(\chi_3 s + \chi_2) s^2}. \quad (35)$$

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3.6 Approximate frictional parameters

Knowing a priori the frictional parameters of a fault system is practically impossible. Various geophysical methods can give only approximate estimations. Given a controller designed as described in the previous paragraphs, we will investigate here the tolerance in the frictional parameters that guarantee stabilization. Let $\Sigma(P)$ be the system for which a stabilizing controller, $\Sigma(C)$, was designed and $\Sigma(P')$ the real system that having different frictional parameters than $\Sigma(P)$. The characteristic polynomial of the closed system $\Sigma(P', C)$ is:

$$D'_c(s) = X(s)D'(s) + Y(s)N'(s), \quad (36)$$

Let also the frictional parameters of $\Sigma(P')$ be $a'_0 = a_0 + \Delta a_0 = \left. \frac{\partial a'}{\partial y} \right|_{(y_0^*, 0, \phi_0^*)} + c$, $a'_1 = a_1 + \Delta a_1 = \left. \frac{\partial a'}{\partial y} \right|_{(y_0^*, 0, \phi_0^*)} + d$ and $b'_0 = b_0 - \Delta b_0 = -\left. \frac{\partial \alpha'}{\partial \phi} \right|_{(y_0^*, 0, \phi_0^*)}$. Then Eq.(36) becomes:

$$D'_c(s) = D_c(s) + X(s)\Delta D(s) + Y(s)\Delta N(s), \quad (37)$$

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where $\Delta D(s) = \Delta a_1 s + \Delta a_0$ and $\Delta N(s) = \Delta b_0$. This system is stable when D'_c is a stable polynomial, i.e. when it has all its roots in $\mathbb{C}^-$. The stability of the polynomial can be explored using the Hurwitz matrix or approximately using asymptotic methods, which gives the required tolerances for Δa_1 , Δa_0 and Δb_0 in function of λ or numerically for a given system.

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4 Numerical examples

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4.1 Geological setting and scaling laws

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We consider a fault system at 5km depth. This is a common depth for many energy-related human activities in the earth's crust and in the range of modern drilling technology. At 5km the normal to the fault stress is about $\sigma_n = 100\text{MPa}$ ($\sigma_n = \frac{W}{A}$) and the fluid pressure $p_f = 50\text{MPa}$ ($p_f = \frac{P_f}{A}$), leading to an effective normal stress of $\sigma'_n = \sigma_n - p_f = 50\text{MPa}$. Notice that these values vary considerably with the tectonic configuration, i.e. they depend on whether the fault system is in extensional, compressional or strike-slip setting. The density of the rock is taken equal to $\rho = 2500\text{kg/m}^3$, its apparent shear modulus equal to $G = 30\text{GPa}$ and its apparent viscosity $C = 10^5\text{MPa s}$.

295 Viscosity is not a well constrained quantity. The chosen value is several orders of mag-
 296 nitude lower than the viscosity of rocks at ambient temperature, in order to account for
 297 the earth's crust temperature gradient and the high deviatoric stresses during (pre-)seismic
 298 slip. We refer to Vutukuri and Katsuyama (1994) for experimental results on viscosity
 299 and its dependence on temperature and high stresses.

The shear stress drop caused by an earthquake varies considerably over the rup-
 ture area. This is due to material heterogeneities, fault roughness, geometrical factors,
 multiphysical couplings, locking etc.. Nevertheless, interesting conclusions can be drawn
 if one considers the average shear stress drop over the whole rupture area, $\Delta\tau$. Accord-
 ing to seismological inversions of actual earthquakes, $\Delta\tau$ varies between 0.1 and 10 MPa.
 Here, we take $\Delta\tau = 5\text{MPa}$. It is worth emphasizing that this is a spatial average of the
 shear stress drop over the whole area of the fault and that stress drop can be much higher
 locally. According to Kanamori and Brodsky (2004) the expected (average) slip is:

$$D = \theta^{-1} G^{-1} L \Delta\tau, \quad (38)$$

where θ is a geometric constant of order unity. The seismic moment is defined as $M_0 =$
 GDA , where A is the rupture area, which here is assumed circular, $A \approx L^2$. Consequently:

$$M_0 = \theta^{-1} L^3 \Delta\tau. \quad (39)$$

The seismic magnitude M_w is defined as follows:

$$M_w = \frac{2}{3} \log_{10} M_0 - 6.07 \quad (M_0 \text{ in Nm}) \quad (40)$$

300 From the above scaling equations it is clear that the magnitude of an earthquake is log-
 301 arithmically related to the length of the fault L , or in other words to the size of the fault's
 302 rupture area.

Assuming $\mu_{max} = 0.6$ and $\mu_{res} = 0.5$ such that $\Delta\tau = \sigma'_n \Delta\mu = 5\text{MPa}$ the spring-
 slider model provides useful insights regarding earthquake instability. It models earth-
 quake nucleation and seismic slip in terms of average quantities. However, it ignores the
 spatial rupture process and propagation, which will be studied in details in a future work.
 Based on field measurements, we take $D_c = 10\text{mm}$. Notice that this value is much higher
 than the D_c measured in the laboratory (Kanamori & Brodsky, 2004). Expressing Eq.(1)
 in the form of Eq.(3) we obtain:

$$a = \frac{\sigma'_n}{\rho L} \mu(\delta) \frac{T^2}{D}, \quad c = \frac{G}{\rho L^2} T^2, \quad d = \frac{C}{\rho L^2} T, \quad (41)$$

where T and D are, respectively, any reference/characteristic time and length scales leading to Eq.(2) such that $\varepsilon = \frac{v_\infty}{v_{ref}} \ll 1$, with $v_{ref} = \frac{D}{T}$, and $y = \frac{\delta}{D}$. The denominators ρL and ρL^2 are derived by considering that the mobilized mass, m , is equal to ρL^3 . Regarding friction evolution with slip, i.e. $\mu(\delta)$, two cases are explored here. The first one is a piece-wise linear function of δ and the second the exponential expression $\mu(\delta) = \mu_{res} \left(1 - \frac{\Delta\mu}{\mu_{res}} e^{-\frac{\delta}{D_c}}\right)$ (Figure 2). We neglect velocity weakening as at low slip velocities it is several orders of magnitude lower (Reches & Lockner, 2010) than the apparent viscosity of the surrounding rock and, therefore, it does not influence instability (Eq.(12)). Finally, we assume a far field tectonic movement of $v_{inf} = 1\text{cm/year}$.

In Figure 4 we present the response of the spring-slider for $L = 5\text{km}$. The equations of the system were integrated using *Wolfram Mathematica 11.2*. According to Eq.(38) the seismic slip is 0.83m and, as expected, it coincides with the final displacement obtained from the spring-slider model. The seismic moment is $M_0 = 6.25 \times 10^{17}\text{Nm}$, which corresponds to an earthquake of magnitude $M_w = 5.8$.

Figure 4. Evolution of slip and slip velocity in function of time for a fault length equal to 5km.

For smaller fault length $L=500\text{m}$, the seismic moment is $M_0 = 6.25 \times 10^{14}\text{Nm}$ and the magnitude $M_w = 3.8$. Figure 5 shows the evolution of slip and slip-rate for this fault length as obtained by the spring-slider model. Piece-wise linear or exponential evolution of μ has minor influence on the response of the system.

4.2 Scenario #1: Controlling induced seismicity

This scenario concerns a fluid injection project in the earth's crust. Examples of this scenario are deep geothermal projects, deep wastewater disposal, CO_2 sequestra-

Figure 5. Evolution of slip and slip velocity in function of time for a fault length equal to 500m.

tion, unconventional oil and gas production etc. (see Rubinstein and Mahani (2015)). Of course, in all these projects, avoiding seismic events is of paramount importance. We assume that the system is not on the verge of instability and that the actual shear stress due to the far field tectonic movement is lower than the critical one. Critical we call the average shear stress required to render the system unstable and provoke the earthquake event of $M_w = 3.8$ described in the previous paragraph (Figure 5). Knowing the exact state of stress in a fault system is not trivial. In this scenario we assume that the real (average) shear stress state along the fault system of $L=500\text{m}$ is at 90% of the critical shear stress, i.e. $\tau_{real} = 0.9\tau_{crit}$. At this stress state the system becomes unstable when the fluid pressure increases for $\Delta p_{f,crit} = 5 \text{ MPa}$. However, due to inaccurate measurements and other uncertainties, the project coordinators have considered an average shear stress level lower than the real one. Based on this wrong estimation, they have decided to inject fluid leading to a maximum fluid pressure increase of $\Delta p_{f,applied} = 10 \text{ MPa}$, which is double than $\Delta p_{f,crit}$ and will cause a considerable seismic event without any control system. The injection program will last one week.

To mitigate this risk the fluid pressure increase is continuously regulated by the stabilizing compensator designed in section 3. We choose $\lambda = -0.1$. The connectivity of the controller assures negative-feedback to the fault system as shown in Figure 3. We use the friction parameters at $\delta = 0$ for setting the controller (Eq.(29)). More specifically, $\mu = 0.6$, $\frac{d\mu}{d\delta} = -0.01$ and $a_0 = -3.5 \times 10^{-4}$, $a_1 = 1.6 \times 10^{-1}$, $b_0 = 4.8 \times 10^{-4}$ for $L = 500 \text{ m}$ (see Eq.(41)).

Before the reactivation of the fault no slip is observed (locked). Nevertheless, when the critical fluid pressure is reached the system becomes unstable. The controller suc-

ceeds in stabilizing the system assuring zero slip. No seismicity is observed and the $M_w = 3.8$ earthquake is prevented.

Figure 6 depicts the total pressure change, which is automatically adjusted by the controller. Immediately after fault reactivation, the controller inhibits any further increase of the applied pressure, which according to the injection program should reach 10 MPa. Then it slowly reduces the pressure in order to keep the system stable. This reduction is barely perceptible, but it is necessary for the stabilization of the system. Notice that the far field tectonic velocity is always acting and it has a destabilizing effect as we are at the verge of instability. If the controller is deactivated the system will loose stability and an earthquake of $M_w \approx 3.8$ will take place. In this case we would like to drive safely the system from its unstable state to a stable one. This the objective of the next paragraph.

Figure 6. Evolution of fluid pressure change. The total injection operation time was set to 1 week. On the left we present the fluid pressure change and on the right the fluid pressure change normalized by the effective stress at that depth. Immediately after the fault reactivation at $\Delta p_f = 5\text{MPa}$, the controller cancels out the applied pressure and progressively reduces the fluid pressure (suction/pumping) in order to stabilize the system, which is now unstable.

4.3 Scenario #2: Controlling earthquakes

In this paragraph we present an example of active fluid pressure control for avoiding earthquakes, like the ones presented in the previous paragraphs. In particular we focus on avoiding the $M_w = 5.8$ earthquake described in paragraph 4.1 (Figure 4).

Contrary to the previous scenario, in this scenario we don't want to avoid slip, but to induce it in a controllable way. Our target is to assure slow, aseismic slip such as to

move the system from its initial unstable state to a stable one. In this way we will dissipate in a controllable manner the energy surplus and we will avoid the sudden energy release that leads to earthquakes.

The earthquake control operation is performed in a time window of ten minutes. The technological feasibility of such an intervention is not discussed herein. We assume the worst case scenario, i.e. the system is on the verge of instability. However, it is constantly controlled by the stabilizing compensator we designed in Section 3. In this scenario, we choose to make the system slip for 0.83 m, which coincides with the seismic slip distance of the uncontrolled system (see Figure 4). Consequently, the target velocity that we want to apply is $v_c \simeq 8.3\text{cm/min}$.

As far it concerns the controller, we choose $\lambda = -0.1$ and we calculate its parameters using the friction parameters at $\delta = 0$ (Eq.(29)). More specifically, $\mu = 0.6$, $\frac{d\mu}{d\delta} = -0.01$ and $a_0 = -4.0 \times 10^{-5}$, $a_1 = 1.6 \times 10^{-3}$, $b_0 = 4.8 \times 10^{-5}$ for $L=5\text{km}$ (see Eq.(41)).

In Figure 7 we show the response of the controlled system by integrating the differential equations and connecting the controller as shown in Figure 3. We observe that no abrupt sliding takes place and that the system is successfully controlled (cf. Figure 4). The seismic event is avoided.

Figure 8 shows how the controller alters the fluid pressure from its initial hydrostatic value in order to drive the system from its initial unstable state to a stable one in an aseismic way. We observe that the controller reduces the pressure (pumping, suction), but in a non-monotonous way. In the beginning of sliding, the fluid pressure is decreased rapidly reaching a minimum value of approximately $\Delta p_f = -9$ MPa. This phase corresponds to the unstable phase of the system (critical distance, D_c). Then the fluid pressure is progressively increased and when we reach the target distance of the operation it recovers its initial hydrostatic value, i.e. $\Delta p_f = 0$ MPa. Notice that at this stage, the fault system is stable even in the absence of the controller ($\frac{d\mu}{d\delta} = 0$, see Eq.(19)).

In Figure 9 we compare the kinetic energy of the controlled system with the uncontrolled one. The kinetic energy in the controlled system is four orders of magnitude lower than the one developed during the seismic event. Moreover, it is practically constant during the earthquake control operation.

Figure 10 shows the energy dissipation at the fault. The controlled system dissipates energy under almost constant rate, which depends on the chosen operation time. Of course, this is not the case for the uncontrolled system where high dissipation rates are reported due to fast, seismic slip.

Finally, Figure 11 illustrates the drop of the elastic energy of the system. Again, the controlled system manages to reduce its potential energy smoothly (linearly in this example) and avoid its sudden release as in the case of the uncontrolled system.

Figure 7. Evolution of slip (left) and slip velocity (right) in function of time normalized with the control operation time (here 10 minutes). The earthquake event is avoided and the system is successfully controlled.

Figure 8. Fluid pressure change, automatically adjusted by the stabilizing controller. On the left we present the fluid pressure change and the on the right the fluid pressure change normalized by the effective stress at that depth. The fluid pressure change is not monotonous and it is calculated in real time by the stabilizing controller.

The above simulations were performed with a predetermined friction law that follows the exponential relation that was given in paragraph 4.1. However, it has to be emphasized that the parameters of the controller were kept constant during the simulation and they were not updated by taking into account the exact evolution of the frictional

Figure 9. Comparison of the kinetic energy of the controlled system (left) with the uncontrolled one (right). The designed controller manages to reduce four orders of magnitude the kinetic energy and keep it practically constant during the earthquake control operation.

Figure 10. Comparison of energy dissipation due to friction between the controlled system (left) and the uncontrolled one (right). In the controlled system the energy is dissipated progressively, while in the uncontrolled system the energy is dissipated abruptly (earthquake).

properties. This shows that the system is controlled even if the frictional properties of the system are not exactly known (see paragraph 3.6). In order to explore further the robustness of our approach we keep the same parameters as before for the controller and we add a sinusoidal perturbation to the friction coefficient as shown in Figure 12. Our stabilizing controller manages to control the system and avoid the earthquake instability despite the fluctuations of the frictional properties. In Figure 13 we show the fluid pressure change that the controller automatically adjusts to assure stability. The system is driven again from its unstable state to a stable one. The perturbation in the friction coefficient is reflected in the fluctuations of the fluid pressure.

5 Conclusions

The current paper presents a theoretical work focusing on exploring the possibility of preventing earthquakes by controlling fluid injection pressure. Our analysis is based on the classical spring-slider model (frictional slider), which we actively stabilize. We adopt

Figure 11. Comparison of the elastic energy drop between the controlled (left) and uncontrolled (right) systems. The designed controller avoids the sudden energy release that happens during an earthquake.

Figure 12. Sinusoidal perturbation of the frictional coefficient.

Figure 13. Fluid pressure evolution as calculated by the stabilizing controller for the perturbed friction coefficient (Figure 12). The controller stabilizes the system by automatically adjusting fluid pressure (fluctuations).

a general frictional law that accounts for slip and slip rate for the fault behavior. The fault is considered fully saturated and ideally oriented for slip in the ambient stress-field. The rocks that surround the fault are considered as a Kelvin-Voigt material.

We describe the dynamics of the system using two time scales, i.e. a slow and a fast one. This scale separation technique allows to asymptotically approximate the solutions of the system as power series of a small parameter, ε , that expresses the ratio between the fast and the slow time scales. We define as steady-state, the motion that does

not involve inertia effects related to the fast time scale at dominant order, ε^0 (Eqs. (8)). This state represents the slow, creep-like, aseismic motion of the system and has a simple mathematical expression. More precisely, it is proportional to the far field tectonic velocity and inversely proportional to a term that involves the first derivative of the apparent friction with respect to slip and the apparent elastic stiffness of the surrounding the fault rocks (see Eqs.(17) and (18)). The stability of this slow-slip motion is then investigated using Lyapunov's first method of stability and the conditions for which steady-state motion is stable are determined. Notice, that due to the far field tectonic velocity and the general rheology considered for friction, the system is non-autonomous. With this double-scale methodology, we manage to alleviate the explicit dependence of the system on the fast time scale, rendering it time-invariant at the dominant order.

Based on these results and the abovementioned time-invariance, the application of well established tools of the classical mathematical theory of control is possible. More specifically, control theory is used for stabilizing the system and make it remain in the vicinity of an evolving steady-state. This justifies the linearization of the equations in terms of slip, slip-rate and fluid pressure.

We show mathematically that the system is stabilizable by controlling fluid pressure. In other words, it is possible to stabilize and control the system when it is unstable, by appropriately adjusting the fluid pressure. This is a major result. The contrary would mean that earthquake control is impossible. Moreover, the opposite conclusion would imply that ongoing, large-scale industrial applications, involving injection of large quantities of fluids in the earth's crust, have high degree of risk, which cannot be mitigated.

Assuming a general negative-feedback control system, we designed a proper stabilizing controller. By monitoring slip, the designed controller succeeds in adjusting the fluid pressure and assures stable, aseismic slip, even in the absence of complete knowledge of the exact frictional properties of the system (robustness). Moreover, it succeeds in automatically controlling the pore fluid pressure and impose a prescribed slip or slip velocity (asymptotic tracking). These features of the controller allow a) to mitigate the seismic risk related to induced seismicity and b) to drive the system from an unstable state of high energy to a stable one of lower energy with constant slip velocity.

These features are illustrated through two scenarios of earthquake control. In these scenarios, fluids are injected under controlled pressure at 5km depth. More specifically, the first scenario refers to a fictitious injection project, where injection under constant pressure rate was planned. It is shown that the controller automatically stabilizes the system and avoids the anticipated earthquake event of $M_w = 3.8$. In particular, the controller inhibits any further fluid pressure increase when the system enters to the unstable regime. This is done automatically by the controller by simply monitoring slip evolution. In this way the earthquake event is avoided. As the time increases, the controller progressively reduces the fluid pressure by pumping, in order to prevent seismic slip. Fluid removal reminds us the hypothetical scheme for EQ control proposed by Raleigh et al. (1976) and the related experiment at Rangely, Colorado in USA. However, with our approach, fluid pressure is automatically regulated in real-time, based on a designed negative-feedback control system. Consequently, more complex situations can be treated and stability can be actively assured without prescribing the fluid pressure history in advance.

The next scenario concerns the mitigation of a $M_w = 5.8$ event by imposing constant slip velocity. For simplicity we assume that we are at the verge of unstable, seismic slip. The controller reduces the fluid pressure by pumping, but this is done in a non-monotonous way. In the beginning of (imposed, desired) sliding, the fluid pressure is decreased rapidly reaching a minimum value of approximately $\Delta p_f = -9$ MPa. This phase corresponds to the unstable phase of the system (slip smaller than the critical distance, D_c). Then, the fluid pressure is progressively increased by the controller and, finally, it recovers its initial hydrostatic value. At this stage, the fault system is stable. During the controlled injection, the kinetic energy is kept constant and it is four orders of magnitude lower than the one that would develop abruptly, if the system was not controlled (seismic event). Regarding energy dissipation, the controller assures an almost constant dissipation rate, which depends on the chosen operation time. The same happens with the elastic energy, which is decreased smoothly over the duration of the EQ control operation. Of course, this is not the case for the uncontrolled system, where high dissipation rates and fast elastic energy drop are observed due to fast, sudden, seismic slip. Notice, that in the case of faults that are not at the verge of instability, the controller would increase the pore fluid pressure to enhance slip and then, once the system enters to the unstable regime, it would automatically reduce the pore-fluid pressure in order to stabilize the system and drive it to a stable state of lower energy, as mentioned above.

The above two scenarios show the ability of the designed controller to prevent earthquake events. Of course, these are academic examples and intensive research is needed before eventual real-scale applications. Notice that both physics and geometry of the fault system are kept as simple as possible. The role of heterogeneities, of pore fluid diffusivity and special hydrological conditions are not considered as well. Moreover, fault rupture and slip propagation is considered only on average over the whole fault's length. The investigation of the above mentioned limitations, as well as the observability of the real system and other techno-economical aspects of EQ control, exceed the scope of the present article and they are explored in the frame of the ongoing ERC project "Controlling earthquakes - CoQuake" (<http://coquake.com>).

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