Camilla Gilmore, Silke M. Göbel & Matthew Inglis. An Introduction to Mathematical Cognition. Routledge, New York, USA, £ 31.99, pp .248, ISBN 978-1-138-92395-9.

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**Understanding mathematical thinking – the key for improving mathematical performance and a test case for exploring the human mind**

How well you do in mathematics is often taken as a direct index for general intelligence. People who struggle with mathematics fear being intellectually insufficient. Even worse, understanding of mathematics is also perceived as a stable and persistent trait - irremediable. While it is true that deficiencies in mathematical understanding do have a negative impact on career success, medical decision making and even personal health, it only represents one cognitive domain that does not represent an index of general intelligence. Also, various intervention studies have demonstrated that mathematical understanding is malleable. The key for changing these (and other) misbeliefs lies in a better understanding of the cognitive mechanisms that underlie mathematical cognition.

The increasing awareness of the fact that many individuals struggle with mathematics was the motivation for writing *An Introduction to Mathematical Cognition*, the new book by Gilmore, Göbel and Inglis. The book starts out with a definition of what mathematical cognition is: an interdisciplinary endeavour that sets out to explain how mathematical understanding develops and what factors define individual differences in performance.

The second chapter provides a brief and well-structured overview of the perception of non-symbolic numerical information. Understanding the number of items in a set (i.e., its numerosity), is a capacity that provides many species with an evolutionary and vital advantage. The authors empirically motivate the differentiation between core mechanisms that serve perceiving smaller (< 5) numerosities on one hand and larger numerosities on the other. They describe the developmental trajectories of both mechanisms as well as the existing theoretical models and the underlying neural correlates. The chapter finishes with a critical evaluation of the cognitive system for perceiving larger numerosities and features alternative explanation for a multitude of related findings.

In the third chapter, the authors focus on symbols that represent numerical magnitudes (e.g. Arabic digits, number words, etc.). Since nothing relates these symbols to the quantity they describe (Why do "three" and "3" describe the number of lines depicted hereafter? ||| ), the authors discuss how children acquire a conceptual understanding of this relation (the symbol grounding problem). Two theoretical alternatives are introduced that explain this process either as a refinement of an innate system that allows for approximate numerical estimates or a bootstrapping process in which an initially void placeholder system (i.e., number words) is successively filled with meaning, starting from the mapping between small number words (one – four) and capacity limited object representations. The chapter then delineates the development of digit writing and multi-digit number processing before ending on a description of the relationship between symbolic number knowledge and mathematical performance.

Chapters 4 to 6 cover the development of arithmetic skill, the understanding of arithmetic concepts and mathematical difficulties. The authors describe that different problem representations can have significant consequences for children's arithmetic performance and what strategies can be differentiated. Arithmetic concepts (additive composition, commutativity, etc.) are explained by using well-chosen examples. The authors also present different ideas on how procedural and conceptual development may be intertwined during development. By introducing a hypothetical case study of an 8-year-old girl who struggles with mathematics in school, the authors introduce developmental deficits (developmental dyscalculia) and emotional dysfunctions (math anxiety) that impede performance.

Chapter 7 introduces number systems beyond natural numbers (i.e., positive integers), including rational numbers (e.g., 2/3), irrational numbers (e.g., $\sqrt[2]{2}$ ), and complex numbers (e.g., for solving equations like $x^2 = -5$). They discuss how inconsistencies between natural and rational numbers (e.g., understanding that larger numbers as denominators lead to smaller magnitudes) lead to biases in the understanding of fractions. For example, many participants erroneously respond that ½ is smaller than ¼ (because 2 is smaller than 4). The understanding of rational numbers hence calls for a conceptual change.

Chapter 8 takes it yet another step forward and introduces the reader to the cognitive basis of algebra. The authors describe the difficulties students experience when being introduced to the concept of unknowns in equations (e.g. 14 + n = 43) and discuss the merit of introducing algebra and arithmetic at the same time. A crucial concept that often poses problems is how students understand equivalence. Authors show that a relational understanding of equivalence that hinges on the notion that the equal sign indicates that two sides of an equation are equivalent is advantageous in comparison to an operational understanding where the equal sign merely indicates an instruction to carry out an arithmetic operation.

Chapter 9 introduces the reader to mathematical proofs. On the basis of an introductory example, the authors demonstrate the different purposes of a proof, the difficulties students encounter with proofs (their construction; their validation; their comprehension), and why students have these difficulties (epistemic; strategic; logical). This chapter is particularly interesting because it sheds light on the largely overlooked difficulties with a central concept in mathematics from a cognitive point of view.

Chapter 10 describes how findings from the realm of reasoning and decision making (e.g. logic and conditional reasoning) relate to the investigation of mathematical cognition. Starting from the well-known Wason selection task, the chapter nicely works out the logical meaning of conditional statements (if -> then clauses) and the way these statements are used in real life. Since mathematical proofs often take the same form, knowing about the fallacies in conditional reasoning is important for understanding them. The authors conclude by showing that logical reasoning is predictive of mathematical success and vice versa – representing a rare example of knowledge transfer to skills outside the trained domain.

As becomes evident from the brief description of the chapters' content, the book has a clear structure, going from basics of numerical understanding to higher-level mathematical concepts and mathematical reasoning. Each chapter provides a succinct description of the major theoretical concepts. One of the strengths of the book is that it identifies the most important theoretical strands and delineates their differential explanations of a given question. This is a very useful guide that can help readers who come to the field in order to better classify a given empirical finding and better apprehend its theoretical implications. The book provides instructive examples that demonstrate how mathematical cognition is an integral part of every-day life. Where available, the book describes the underlying neural correlates, mainly focusing on results from functional magnetic resonance imaging studies. Another strength of the book is the integration of developmental aspects of mathematical cognition.

It should be noted that the book does not make extensive use of specialized jargon, which makes it easily accessible to readers without a background in developmental psychology or cognitive neuroscience. At the end of each chapter, recommendations are provided for readers who want to expand on their knowledge of a given topic. Central concepts are explained in separated text boxes, which simplifies reading.

There are few things that this book misses (e.g., in-depth description of computational models) but, nevertheless, the authors' selection of topics is excellent in targeting psychology students and researchers in cognate disciplines.

In sum, this book is an excellent introduction to the field of mathematical cognition that provides a succinct and clear overview of the most important concepts and findings. Its structure makes it an ideal candidate textbook accompanying an introductory class on mathematical cognition. Finally, the book shows how studying the cognitive mechanisms of mathematical cognition can help developing mathematical skills, which is – in light of the pessimistic lay views I started this review with – an optimistic message.