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Procedure for the Determination of Local Gravimetric-Geometric Geoid Model

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ABSTRACT

As the surface adopted for geodetic computation is a mathematical surface which is different from the physical surface, the geoid adopted as a reference for the vertical coordinate system, the ellipsoidal heights obtained from GPS observation are transformed to practical heights known as orthometric heights. The transformation of the ellipsoidal heights to orthometric heights requires the knowledge of the geoid-ellipsoid separation at the point of observation. Since the geometric method requires the computation of geoid heights of points from GPS observation and geodetic leveling carried out over long distances which are labor intensive and prone to human errors, the accurate geoid heights of the points should be obtained from gravity measurement and a geometric geoid surface fitted to the gravimetric geoid heights. This paper presents detailed procedures for determining local gravimetric-geometric geoid model of an area or a region. The detailed procedures which consist of selection of suitable/evenly distributed points, DGPS and gravity observations of selected points, processing of DGPS and gravity observations, computation of gravimetric geoid heights of the points, fitting of geometric geoid surface to the computed gravimetric geoid heights and computation of accuracy of the geoid model are presented in sequential order.

Keywords: *Gravimetric-geometric, Geometric geoid surface, Gravimetric geoid height, Stokes integral.*

1. INTRODUCTION

The advent of Global Positioning System, GPS which determined accurately the three dimensional coordinates of points on the earth surface has led to the determination of local geoid models in various parts of the world. This is because the third dimension which is the ellipsoidal heights is determined on a mathematical surface as well as the ellipsoid. The ellipsoid is different from the surface adopted as a reference for vertical coordinate system. The surface adopted as a reference for vertical coordinate system is known as the geoid. The geoid is the surface which coincides with that surface to which the oceans would conform over the entire earth, if free to adjust to the combined effects of the earth's mass attraction (gravitation) and the centrifugal force of the Earth's rotation. Specifically, it is an equipotential surface, meaning that it is a surface on which the gravitational potential energy has the same value everywhere with respect to gravity (Olaleye et al, 2013). The transformation of the GPS ellipsoidal height to more meaningful or practical height known as the orthometric height requires the knowledge of the geoid-ellipsoid separation as well as the geoid height at the point of observation. The relationship among the ellipsoidal, orthometric and geoid heights according to Komarov (2007) is

$$N = h - H \tag{1}$$

where, N is the geoid height, h is the ellipsoidal height and H is the orthometric height. Figure 1 also shows the relationship among the ellipsoidal, orthometric and geoid heights with respect to the reference surfaces.

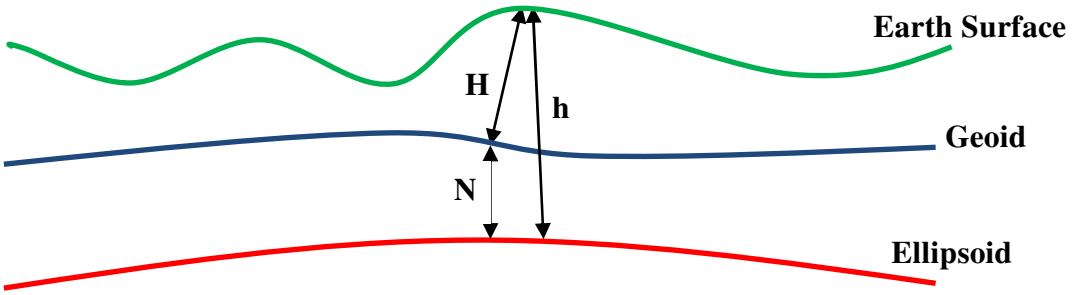


Figure 1: Relationship among Orthometric, Geoid and Ellipsoidal Heights.
Source: Isioye and Youngu (2009)

With the height difference determined using the orthometric heights, suitable gradient can be chosen and water can flow from one end to the other.

The geoid can be determined using various methods such as the gravimetric, geometric, astro-geodetic and transformation methods amongst others.

The gravimetric method can be carried out by the well-known Stokes-integral, equation (2) and the use of accurately determined absolute gravity data (Heiskanen and Moritz, 1967, Eteje, 2015 and Eteje et al, 2018).

$$N = \frac{R}{4\pi\gamma}\iint_{\sigma}\Delta g S(\psi)d\sigma \qquad (2)$$

Where, N is geoid undulation, Δg is gravity anomaly, $S(\psi)$ is stokes function, γ is normal gravity on the reference ellipsoid and R is mean radius of the earth. The geometric method is to use the known "geoid heights" at some points, which are derived from co-located GPS-determined heights and levelled heights to interpolate the geoid heights at other points (Chen, and Luo, 2004). The interpolation of the geoid heights at any other point involves the use of interpolation models such as bicubic model and other models like multiquadratic model, etc. In astro-geodetic method of geoid determination, the geoid heights of points are determined with reference to a geodetic (reference) station whose geoid height is known. The geoid heights differences between points are determined using the components of deflection of the gravity vector which can be obtained by carrying out astronomical and geodetic observations. The astronomical observation is carried out to determine the astronomical coordinates (astronomical latitude, Φ and astronomical longitude, Λ) by observing stars. The geodetic observation is used to determine the geodetic latitude, φ, geodetic longitude, λ and ellipsoidal heights, h as well as the azimuths, α and geodetic distances, ℓ between network points. Using the astronomical and geodetic coordinates, the components of deflection of the gravity vector can be computed. The transformation method involves the use of the well-known Euclidean similarity transformation model which is used to convert Cartesian coordinates between two geodetic reference frames that generally differ in terms of three translation parameters (tx, ty, tz), three orientation parameters (εx, εy, εz) and a factor of uniform spatial scale change (δs).

Using the geometric method, the geoid heights of points are obtained from GPS observations and geodetic levelling. The levelling of these points is carried out with respect to the Mean Sea Level, MSL as well as the geoid over long distances as these points are selected to cover the entire study area. Ideally, these points are selected along different roads so that their orthometric heights can be determined using closed loops differential levelling. Based on the fact that the points are chosen along various roads for accessibility and intervisibility reasons during levelling, these points are not evenly distributed over the entire study area. Also, the geodetic/differential levelling is labour intensive and prone to human errors over long distances. With the gravimetric method, the points can be chosen such that they are evenly distributed to cover the entire study area or region. This is because gravity observation does not require line of sight which in turn require intervisibility of points. Carrying out gravity observation for geoid heights computation is not labour intensive. Although, it is expensive but the utmost concern here is the accuracy with which the geoid heights are determined. Using the grid as well as the summation method of the modified Stokes integral in which the study area is divided into different compartments, the gravimetric geoid heights of each of the compartments are computed and averaged. The average geoid height is the geoid model of the study area. Taking the mean geoid height as the determined geoid model, it is certain that there will be residuals if the differences between the mean geoid and the geoid heights of the points are computed. This is because geoid heights vary with no constant value.

Therefore, to determine accurate local geoid model of an area, the gravimetric and the geometric methods should be combined such that the geoid heights of the selected points are obtained from gravity observation and a geometric interpolation surface is fitted to the gravimetric geoid heights. Fitting an interpolation surface to the gravimetric geoid heights enables geoid heights of

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new points to be obtained with the determined geoid model using variables such as geographic and rectangular coordinates. The fitting of the geometric geoid surface to the gravimetric geoid heights requires the model parameters to be computed. The computation is carried out with least squares adjustment method.

This paper presents detailed procedures for determining local gravimetric-geometric geoid model of an area or a region.

2. GRAVIMETRIC GEOID HEIGHTS COMPUTATION USING THE MODIFIED STOKES INTEGRAL

Using the modified Stokes integral given in equation (1), the geoid height of points can be computed if their gravity anomalies and geographic coordinates are known. Featherstone and Olliver (1997) gave the integration of equation (1) as well as the Stokes formula as

$$N = \frac{r\Delta g}{8\gamma}\left(\begin{array}{l} -6\sin^2\psi_o \ln\left\{\sin\left(\frac{\psi_o}{2}\right)+\sin^2\left(\frac{\psi_o}{2}\right)\right\}+16\sin\left(\frac{\psi_o}{2}\right)+12\sin^2\left(\frac{\psi_o}{2}\right) \\ -24\sin^3\left(\frac{\psi_o}{2}\right)-12\sin^4\left(\frac{\psi_o}{2}\right)-4\cos\psi_o+5\cos 2\psi_o-1 \end{array}\right) \quad (3)$$

Where, N is the geoidal height of individual point, ψ_o is the surface spherical radius, γ is the theoretical as well as normal gravity, Δg is the gravity anomaly and $r = R$ is the mean radius of the earth.

2.1 Computation of Surface Spherical Radius, ψ_o

The surface spherical radius, ψ_o is computed as (Shrivastava et al, 2015)

$$\cos\psi = \sin\varphi\sin\varphi^1 + \cos\varphi\cos\varphi^1\cos(\lambda^1 - \lambda) \quad (4)$$

Where,

$\varphi =$ Mean latitude of the points

$\varphi^1 =$ Latitude of individual point

$\lambda =$ Mean longitude of the points

$\lambda^1 =$ Longitude of individual point

2.2 Computation of Normal Gravity, γ

The normal as well as the latitude gravity is computed on a specified ellipsoid. According to Hinze et al (2005), the normal gravity is computed using:

$$g_T = \frac{g_e(1+k\sin^2\phi)}{(1-e^2\sin^2\phi)^{1/2}} \quad (5)$$

Where, e^2 is the first numerical squared eccentricity, g_e is normal gravity at the equator and k is a derived constant = 0.001931851353. Based on equation (5), Aziz et al (2010) gave the formula for the computation of the normal or theoretical gravity as:

$$g_T = 9.7803267714\left(\frac{1+0.0019318513\,8639\sin^2\phi}{(1-0.0066943799\,9013\sin^2\phi)^{1/2}}\right) \quad (6)$$

2.3 Computation of Gravity Anomaly, Δg

The gravity anomaly is the difference between the points gravity reduced to the geoid and the latitude as well as the normal gravity computed on a specified ellipsoid and corrected for free air and effect of rock. The points gravity anomalies are grouped into two: the free air gravity anomalies and the bourgue gravity anomalies. The free air gravity anomaly, Δg_{fa} is computed as (Aziz et al, 2010):

$$\Delta g_{fa} = g_{Obs} - g_{fa} - g_o \quad (7)$$

Where, Δg_{fa} is free-air anomaly, g_{Obs} is observed gravity and g_o is theoretical gravity and $g_{fa} = 0.3086[\text{mGal/m}]H$.

The bourgue gravity anomaly, Δg_B is given by Vermeer (2016) as

$$\Delta g_B = \Delta g_{fa} - 0.1119H \tag{8}$$

The orthometric heights of the points can be obtained from the Digital Terrain Model of the area.

2.4 Computation of Mean Radius of the Earth, $r = R$

The mean radius of the earth, R is computed using:

$$R = \sqrt{MN} \tag{9}$$

Where, M is the radius of curvature in meridian section and N is the radius of curvature in prime vertical.The formula for computation of the radius of curvature in prime vertical, N is given as (Olaleye et al, 2013)

$$N = \frac{a}{(1 - e^2 \sin^2 \varphi)^{1/2}} \tag{10}$$

while that for computation of the radius of curvature in meridian section, M is given as (Kotsakis, 2008)

$$M = \frac{a(1 - e^2)}{(1 - e^2 \sin^2 \varphi)^{3/2}} \tag{11}$$

Where,

$a =$ semi-major axis

$\varphi =$ latitude of observation point

$e^2 = 2f - f^2 =$ eccentricity squared

$f = \frac{a-b}{a} =$ flattening

$b =$ semi-minor axis

2.5 Computation of Combined Topographic Effect

To obtain a precise geoid height of a point, the combined topographic effect is calculated and applied to the computed geoid height of the point. The formula for the computation of the combined topographic effect, δN_{Comb}^{Topo} is given as (Sjöberg, 2000 and Kuczynska-Siehien et al, 2016):

$$\delta N_{Comb}^{Topo} = -\frac{2\pi G\rho}{\gamma}\left[H^2 + \frac{2}{3R}H^2\right] \tag{12}$$

Were, G is the earth gravitational constant, ρ is density, R is the mean radius of the earth and H is the orthometric height of observation point which can be obtained from the DTM of the area.

3. GEOMETRIC GEOID SURFACES

Geometric geoid surfaces are mathematical interpolation surfaces fitted to geoid heights to enable geoid heights of new points to be determined using variable such as geographic or rectangular coordinates of the points. These surfaces include: plane surface, bi-linear surface, second degree surface, third degree polynomial and fifth degree polynomial. The surface to be adopted as well as the degree and order of the polynomial depends on the size of the study area and the variation of the geoid heights. For small area, the plane surface is used, for relatively large area, bi-linear surface is applied while for large area either the third or the fifth order polynomial surface is used.

3.1 Plane Surface

According to Alevizakou and Lambrou (2011), when the area of interest is small and the geoid has a normal variation, then it is possible to approach it by using a plane with a mean slope. The equation of such plan as given by Alevizakou and Lambrou (2011) is

$$h_i - H_i = N_{f_i}(\varphi_i, \lambda_i) = a_o + a_1.(\varphi_i - \varphi_o) + a_2.(\lambda_i - \lambda_o) \tag{13}$$

where, φ_i, λ_i = geodetic coordinates, latitude and longitude, of point *i.*

φ_o, λ_o = geodetic coordinates, latitude and longitude, of a central point in the region of interest.

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N_{f_i} = undulation of the geoid in the point *i*.

h_i, H_i = geometric and orthometric heights in point *i*

a_o, a_1, a_2 = unknown parameters of the plane.

3.2 Bi-Linear and Second Degree Surfaces

The bi-linear and the second degree surfaces are used where the study area show a more prominent surface. Alevizakou and Lambrou (2011) respectively gave the bi-linear and the second degree surfaces as

$$h_i - H_i = N_{f_i}(\varphi_i, \lambda_i) = a_o + a_1.(\varphi_i - \varphi_o) + a_2.(\lambda_i - \lambda_o) + a_3.(\varphi_i - \varphi_o).(\lambda_i - \lambda_o) \quad (14)$$

$$h_i - H_i = N_{f_i}(\varphi_i, \lambda_i) = a_o + a_1.(\varphi_i - \varphi_o) + a_2.(\lambda_i - \lambda_o) + a_3.(\varphi_i - \varphi_o)^2 + + a_4.(\lambda_i - \lambda_o)^2 + a_5.(\varphi_i - \varphi_o).(\lambda_i - \lambda_o) \quad (15)$$

3.3 Polynomial Surface

The polynomial surface used when determining geoid is given as (Kirici and Sisman, 2017)

$$N_{(x,y)} = \sum_{i=0}^{m} \sum_{\substack{j=k-i \\ i=0}}^{k} a_{ij} x^i y^j \quad (16)$$

Where,

a_{ij} = polynomial coefficients

m = degree of polynomial

x, y = plan coordinates of point

In applying the polynomial, the degree should be chosen and the polynomial should be formed for the chosen degree.Kirici and Sisman (2017) gave the 3rd degree polynomial as

$$N = a_{00} + a_{10}X + a_{01}Y + a_{20}X^2 + a_{11}XY + a_{02}Y^2 + a_{30}X^3 + a_{21}X^2Y + a_{12}XY^2 + a_{03}Y^3 \quad (17)$$

Where,

$Y = ABS(y - y_o)$

$X = ABS(x - x_o)$

y = Northing coordinate of observed station

x = Easting coordinate of observed station

y_o = Northing coordinate of the origin (average of the northing coordinates)

x_o = Easting coordinate of the origin (average of the easting coordinates)

The fifth degree polynomial which was applied in an area of $50 \times 45\,\mathrm{km}^2$ in Turkey by Erol and Çelik (2004) is given as

$$N(X,Y) = A_0 + A_{01}X + A_{10}Y + A_{02}X^2 + A_{11}XY + A_{20}Y^2 + A_{03}X^3 + A_{12}X^2Y + A_{21}XY^2 + A_{30}Y^3 + A_{04}X^4 + A_{13}X^3Y + A_{22}X^2Y^2 + A_{31}XY^3 + A_{40}Y^4 + A_{05}X^5 + A_{14}X^4Y + A_{23}X^3Y^2 + A_{32}X^2Y^3 + A_{41}XY^4 + A_{50}Y^5 \quad (18)$$

Where,

$X = k \times (\varphi - \varphi_o)$

$Y = k \times (\lambda - \lambda_o)$

$k = 100 / \rho^o)$ = scaling and unit adjustment factor

φ = latitude of observation point

λ = longitude of observation point

φ_o = latitude of the origin (average of the latitudes)

λ_o = longitude of the origin (average of the longitudes)

3.4 Observation Equation Method of Least Squares Adjustment

The fitting of geometric geoid surface to a set of geoid heights required the model parameters to be computed. The computation of these parameters is done using observation equation method of least squares adjustment. The functional relationship between adjusted observations and the adjusted parameters as given by Ono et al (2014) is:

$$L_a = F(X_a) \tag{19}$$

Where, L_a = adjusted observations and X_a = adjusted parameters. Equation (19) is linear function and the general observation equation model was obtained. The system of observation equations is presented by matrix notation as (Mishima and Endo, 2002 and Ono et al, 2018):

$$V = AX - L \tag{20}$$

But the residual matrix is not necessary when applying least squares adjustment technique for determination of local geometric geoid model parameters. Thus the general matrix notation becomes

$$L = AX \tag{21}$$

where, A = Design Matrix, X = Vector of Unknowns, L = Observation Matrix.

$$X = (A^T A)^{-1} A^T L \tag{22}$$

The step by step procedures for determining geometric geoid model parameters are detailed in Eteje and Oduyebo (2018).

3.5 Evaluation of the surface fitting

In choosing a geometric geoid surface to be fitted to a particular region, the fitting of the surface has to be evaluated to determine if the chosen surface is actually fitting to the region. According to Alevizakou and Lambrou (2011), the unknown parameters must be tested in order to assess whether they are statistically important for a certain confidence level (for example 95%). Each of the unknown parameters a_i must conform to the following formula (Alevizakou and Lambrou, 2011):

$$\sigma_{a_i} . Z_{95\%} \leq a_i \tag{23}$$

Where, σ_{a_i} = standard deviation of each unknown parameter a_i

$Z_{95\%}$ = coefficient of the normal distribution for one dimension arrays for confidence level 95%.

The parameters a_i will be considered statistically important only if equation (23) is valid. The standard deviation of the unknown parameters, σ_{a_i} is obtained from the variance-covariance matrix generated during least squares computation of the model parameters.

3.6 Accuracy/Reliability of Gravimetric-Geometric Geoid Model

The accuracy of the determined local gravimetric-geometric geoid model is obtained using the Root Mean Square Error, RMSE index. To evaluate the determined local gravimetric-geometric geoid model accuracy, the local geoid model is used to determine the geoidal heights of points whose gravimetric geoid heights are known. The gravimetric-geometric geoid model geoidal undulations are compared with the known gravimetric geoidal undulations of the points to obtain the residuals. The Root Mean Square Error, RMSE index for the computation of gravimetric-geometric geoid model accuracy as given by Kao et al (2017) and Eteje and Oduyebo (2018) is

$$RMSE = \pm\sqrt{\frac{V^T V}{n}} \tag{24}$$

Where,

$V = N_{GGH} - N_{G-GMGH}$ (Residual)

N_{GGH} = Gravimetric Geoid Height of Point

N_{G-GMGH} = Gravimetric-Geometric Model Geoid Height of Point

n = Number of Points

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4. PROCEDURES FOR LOCAL GRAVIMETRIC-GEOMETRIC GEOID MODEL DETERMINATION

The local gravimetric-geometric geoid model determination involves the following procedures: selection of suitable/evenly distributed points, DGPS and gravity observations of selected points, processing of DGPS and gravity observations, computation of gravimetric geoid heights of the points, fitting of geometric geoid surface to the computed gravimetric geoid heights and computation of accuracy of the geoid model. Figure 1 shows the local gravimetric-geometric geoid model determination procedures flow chart.

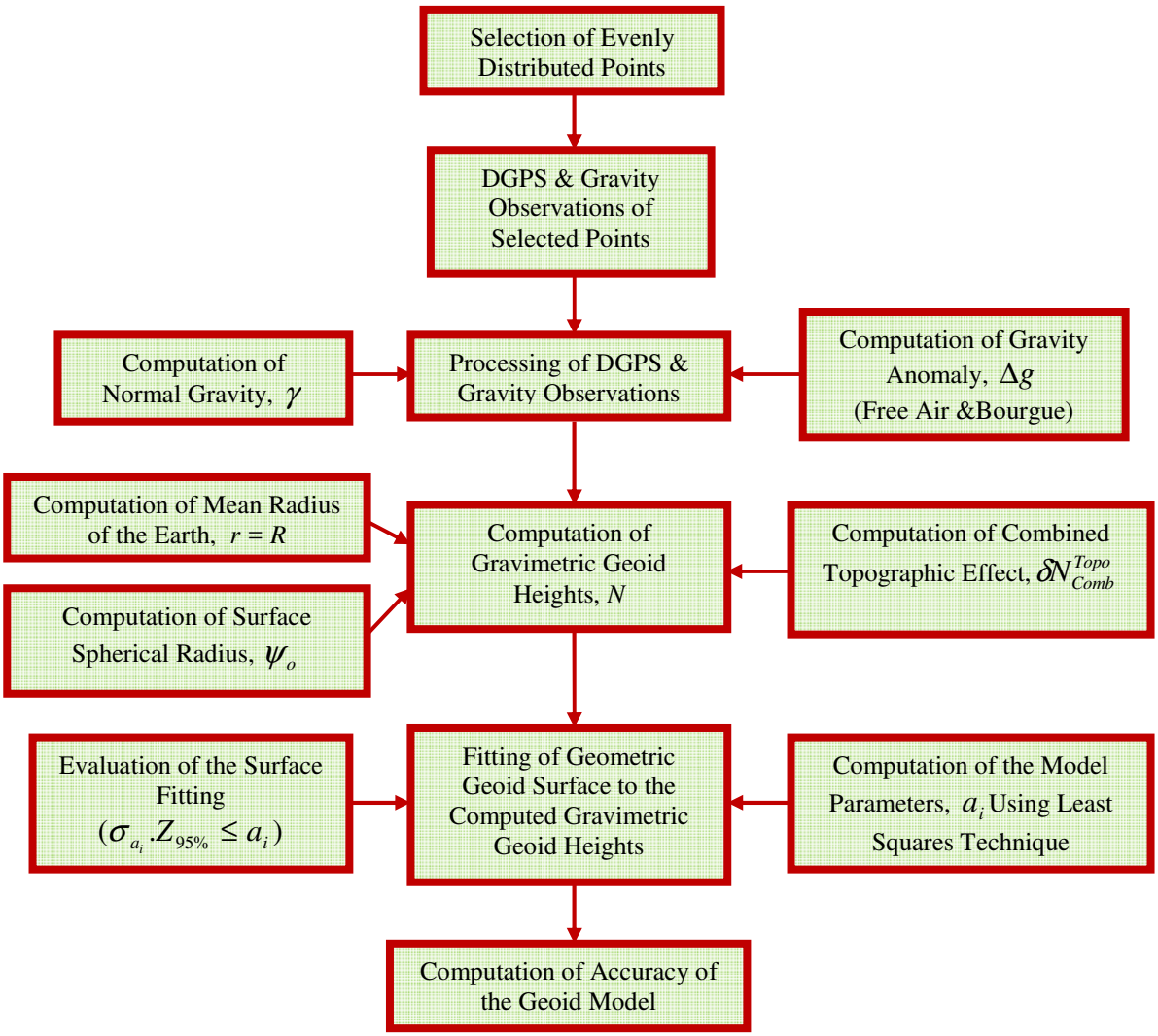


Figure 2: Local Gravimetric-Geometric Geoid Model Determination Procedures Flow Chart

4.1 Selection of Suitable/Evenly Distributed Points

Select suitable points within the study area such that the points are evenly distributed to cover the entire area. This will enable the gravimetric geoid heights of the study area to be evenly determined to cover the whole area.

4.2 DGPS and Gravity Observations of Selected Points

Carry out DGPS and gravity observations to respectively obtain the geographic coordinates and the gravity of the selected points. The geographic coordinates of the points are used during the processing of the gravity observations. The gravity observation of the points should be carried out with a gravimeter.

4.3 Processing of DGPS and Gravity Observations

The acquired DGPS and gravity observations should be processed on the local ellipsoid adopted for geodetic computation in the study area. The normal gravity, γ should be computed on the local ellipsoid using equation (5). The free air and bourgue gravity anomalies, Δg of the points should be respectively computed using equations (7) and (8).

4.4 Computation of Gravimetric Geoid Heights of the Points

Having computed the gravity anomalies of the points, the next step is to compute the gravimetric geoid heights of the points using equation (3). To apply equation (3) for computation of the geoid heights of the points, the mean radius of the earth, r and the spherical radius, ψ_o have to be respectively computed using equations (9) and (4). The computation of the mean radius of the earth requires the computation of the radius of curvature in prime vertical, N and radius of curvature in meridian section, M using equations (10) and (11) respectively. To obtain precise geoid heights of the points, the combined topographic effect should be calculated with equation (12) and applied to the computed geoid heights of the points.

4.5 Fitting of Geometric Geoid Surface to the Computed Gravimetric Geoid Heights

Having computed the precise gravimetric geoid heights of the points, a geometric geoid surface is fitted to the geoid heights to enable geoid heights of new points to be determined within the study area using their rectangular or geographic coordinates. Any of the geometric geoid surfaces given in equations (13) to (18) can be used depends on the size of the study area and the variation of the gravimetric geoid heights of the points. It is advisable to fit two or more geometric geoid surfaces to the geoid heights so as to determine the surface that is most suitable for application in the study area. To fit a geometric geoid surface to the gravimetric geoid heights, the model parameters, a_i have to be computed using least squares technique as well as equation (22). To assess whether each of the computed model parameters is statistically important for a certain confidence level, equation (23) is applied.

4.6 Computation of Accuracy of the Geoid Model

The determined gravimetric-geometric geoid model has to be validated to determine its reliability. The determination of the reliability of the geoid model requires the computation of the accuracy of the model using the Root Mean Square Error, RMSE index as well as equation (24). To do this, the differences between the gravimetric geoid heights of randomly selected points for validation and their corresponding model geoid heights are computed to obtain the residual matrix, V in equation (24).

5 CONCLUSION

The accurate transformation of geometric heights obtained from GPS observation to practical as well as orthometric heights has resulted in the determination of local geoid models in different parts of the world. Local geoid models are determined using various methods such as gravimetric, geometric, astro-geodetic, astro-gravimetric, transformation and hybrid methods. The geometric method requires the computation of geoid heights of selectedpoints from GPS observation and geodetic as well as differential levelling carried out over long distances which is labour intensive and prone to human errors.Therefore, the accurate geoid heights of thepointsshould be obtained from gravity measurement and a geometric geoid surface fitted to the gravimetric geoid heights. Consequently, the this paper has in simple terms presented detailed procedures of fitting a geometric geoid surface to gravimetric geoid heights. Thus, local gravimetric-geometric geoid model determination.

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