Adopting a discursive lens to examine functions learning and language use by bilingual undergraduate students

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This study examines the use of languages by bilingual undergraduate mathematics learners. It adopts a commognitive lens to examine whether language use by bilingual undergraduate mathematics learners impacts upon meta-level mathematical processes of learners, in particular in relation to functions. The findings suggest that there was a lack of code switching between languages when engaged in mathematical tasks and completing the mathematical tasks individually or in pairs impacted on the choice of language employed in the discourse. There was greater evidence of meta-level thinking through the medium of Irish than in English.

Keywords: Commognitive approach, language use, functions, bilingual students.

Introduction

An evaluation of the Irish and English languages demonstrates that differences exist in relation to syntax and semantics and that this may impact on the processing of mathematical text, and advantage those learning through the medium of Irish (Ní Ríordáin, 2013). Specifically, this paper examines two first year undergraduate students' (approx. age 18 years) thinking about functions and which language(s) (Irish and English) are used to facilitate their thinking. These students were participating in a bilingual first year Calculus module. In particular, we focus on concepts relating to functions, which are fundamental to mathematics learning across all levels of education, inclusive of university-level education (Gücler, 2016). We adapt Sfard's (2008) commognitive framework as an analytical lens to support our investigation of bilingual mathematics learners. This framework is adopted for a number of reasons including: to enable an examination of participants' discourses relating to functions; to facilitate a comparison of object-level and meta-level developments of functions in both English and Irish languages; and accordingly, to examine when participants are utilising their languages when engaged in mathematical thinking.

Therefore, considering object-level and meta-level level thinking and requirements in relation to functions, this research aims to make a contribution to the field by examining how languages (Irish and English) are utilised by bilingual undergraduate learners when engaged with functions discourse. Utilising Sfard's (2008) commognitive perspective, mathematical thinking is viewed as a form of communication, and that learning mathematics entails extending one's discourse from object-level to meta-level constructs. Adopting this perspective on thinking and considering that discourse is language dependent, the authors contend that discourses may differ between languages (Kim, Ferrini-Mundy & Sfard, 2012). In particular, the authors propose that there are variances between different linguistic forms of '"the same discourse"' (Kim et al., 2012, p.2) and accordingly it may impact on how bilingual students utilise their language(s) when engaged in mathematical thinking.

Planas and Setati (2009) argue that bilinguals switching between their languages is not just dependent on their proficiency in the languages but also due to the social situations and contexts. It is important that we study bilingual learners and their diverse contexts in order to gain a better understanding of the complexity of the issue and the influence of language(s) on mathematical thinking. It is well established that the 'choice' and use of language(s) may differ depending on the context (e.g., Planas & Setati, 2009). Research suggests that the type of mathematical tasks students encounter as well as the situated learning experiences facilitated, can influence language use and code switching (Moschkovich, 2007). Moschkovich asserts that some students may opt to use their first language when working on a task individually and code switch as necessary when engaged in interactive discourse concerning the same mathematical problem. Alternatively, if students associated mathematics with a particular language, it is reasonable that the students would then communicate their thinking in that language, i.e. their second language. Another alternative is that students might opt to code switch throughout the mathematical discursive processes. For example, Planas and Setati (2009) established differences in the ways that immigrant bilinguals utilised their languages whilst completing mathematical tasks. Switching between languages (Catalan and Spanish) occurred with changes in the complexity of the mathematical tasks required by the teacher. In general, they found that the students used their languages for different purposes and was dependent on the social context set up within the classroom (*ibid*). Ultimately, research suggests that that primary influencers of how students use their languages for learning mathematics are (1) the mathematical discursive competencies in a given language, (2) the social context, and (3) relative experience of mathematics (Moschkovich, 2007).

Theoretical frame

Gee's (1996) concept of discourse as encompassing, verbal, written, technical and physical modes of language use informed the close analysis of mathematical discourse in this study. The authors were cognizant of the social aspect of language use in learning and required a framework that allowed for the examination of discourse as a complex form of communicating while paying close attention to the cognitive processes of mathematical thinking. Therefore, the authors adopt Sfard's (2008) commognitive approach to analysing mathematical discourse in order to provide for the particular methodological considerations and central constructs of the study (mathematics discourse, bilingualism and language use). Sfard expresses thinking as comprising both personal and interpersonal communication and outlines four unique features of mathematical discourse, which this study explores as a process of socialisation into a mathematical community of practice. These discourse tenets are: 1) words use, 2) visual mediators, 3) routines, and 3) endorsed narratives. For the purpose of this paper the authors focus on word use and routines.

Word use is characterised by Sfard (2008) in terms of *passive-driven* (recognition or memorisation), *routine-driven* (using phrases or definitions), *phrase-driven* (task association) and *object-driven* (meaning-making) usage. Objectification is a feature of word use in mathematical discourse, which occurs through a process of reification (structural thinking) and alienation (impersonal presentation of phenomena). Therefore, this research was interested in whether participants engaged in objectified talk and in which language(s) this occurred. To this end, word use was categorised as above and considered in line with the routines feature of mathematics discourse. Sfard categorises routines as *deeds* (a change in the environment), *rituals* (cooperative acts of learning) and *explorations* (producing an endorsed narrative). Of particular significance to this study was the course of action employed by learners to solve mathematical problems (deeds) and the mathematical reasoning provided (explorations). A particular emphasis was placed on the language(s) in which each routine was discussed or employed. The practice of learners alternating between their languages is referred to as code switching in this paper.

The authors align with Gee's (1996) concept of discourse as a sense-making process, comprising modes of communication and participation in communities of practice. Comprised of both talk and non-talk forms of communication, Gee (1996) describes discourse as 'stretches of language that "hang together"' and are embedded in the situated and sociocultural practices of its use' (p.115). Further, the authors define meta-level developments in mathematical thinking as a process of endorsing the rules that govern the particular mathematics discourse, while object-level mathematical thinking refers to familiarisation with the objects of the discourse (Sfard, 2008). In terms of mathematics as discourse, the authors view mathematics as a complex register comprising subject content conveyed through the interchangeable use of everyday (colloquial) and mathematical (literate) discourses. As such, developing an advanced mathematics discourse culminates in a shift from a colloquial to a literate use of mathematical language, which signifies conceptual (meta-level) knowledge.

Methodology

At the National University of Ireland, Galway (NUI Galway), first year undergraduate mathematics students are afforded the opportunity to study honours mathematics through a bilingual approach. Modules offered bilingually are Calculus and Algebra. Four weekly lectures are provided through the medium of Irish, with all mathematics terminology offered bilingually (Irish and English). In addition, lecturers may opt to describe more complex concepts (such as the limit of a function) bilingually. The lectures are supplemented by the provision of a weekly tutorial in English in addition to an Irish-medium tutorial. Participants were comprised of two students, Val and Jean. Two Irish-medium education contexts exist – Irish-immersion education in English-speaking communities and schools located in Gaeltacht (Irish-speaking) regions. Val was from a Gaeltacht region and attended primary and secondary schools in that region, and therefore Irish was his dominant language. Jean attended immersion primary and secondary education, with English her dominant language. Accordingly, both Val and Jean learned Mathematics through Irish but in different contexts in terms of the dominance of the English and Irish languages. At university, the dominant language of informal communication for both participants was English.

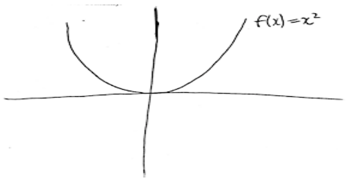
The study adopted a mixed methods approach to data collection. This study identified the plausible mathematical discourse trajectories in relation to functions for both the English and Irish languages. The purpose of these discourse trajectories is to examine object-level and meta-level developments in the both languages, and consequently identify which language(s) are used and how, as related to functions (Kim et al., 2012). Participants also completed a questionnaire. The purpose of the first part of the questionnaire was to gather participants' background data. The second part of the questionnaire engaged participants in functions discourse through the use of mathematical questions appertaining to graphing and limits of functions. All questions were presented bilingually, providing participants with the option of utilising English or Irish or both languages. Jean and Val were invited to think-aloud their thought processes as they recalled prior knowledge and experiences of mathematical discourses while answering the questions (Desimone & Carlson Le Floch, 2004). Finally, cognitive interview methods were employed in which both students were provided with the same mathematical problems as they had solved individually in the questionnaire (one week later). Students were invited to discuss their understandings, processes and thinking about how they approached the various questions and to negotiate their respective thinking towards substantiating the narrative of the particular mathematical constructs in question (functions). This method was employed to acquire comprehensive knowledge about what students' discursive patterns revealed about their comprehension of the specific mathematics functions concepts central to the study (Desimone & Carlson Le Floch, 2004).

Qualitative data analysis included a synthesis of thematic, framework, video and discourse analysis procedures. Sfard's (2008) Commognitive Framework for analysing mathematics discourse was employed to analyse students' meta-level mathematical thinking and related language use. The data was coded, recording both data-driven and concept-driven codes (Gibbs, 2007). Concept driven codes were the discourse tenets: word use, visual mediators; routines and endorsed narratives (Sfard, 2008). Meta-level discursive statements and linguistic patterns were gathered at this stage. Cross coding was employed to code a piece of data for more than one code. Three rounds of coding allowed for similar codes to be grouped and re-characterised and redundant codes were set aside. Parent and Child themes (Gibbs, 2007) were employed to further categorise the data and all codes, categories and themes were considered in terms of English, Irish or both languages, as well as object or meta –level learning.

Findings

A learning example is provided in Figure 1 which details students' mathematical problem solving routines and their language use. Both the individual explanations and the paired discussions are provided. Val and Jean mostly utilised Irish in the individual interview and English in the paired interview. In the example provided, the students' answers are provided in the language in which they spoke/wrote, with English translations provided for those instances where Irish was utilised.

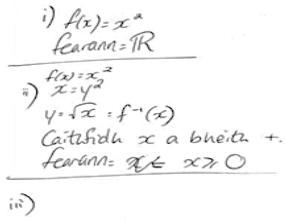
Questions: **(i)** What is the natural domain of a function $f(x) = x^2$? **(ii)** Explain to your friend how the domain for f must change if the inverse of f is to exist. **(iii)** Explain what the inverse function f^{-1} is when $f(x) = x^2$ (with suitable domain).
Val's Individual Answer: **Original text:** Anseo f(x), tá sé os cionn, mar síos anseo [pointing to the third quadrant] tá uimhreacha diúltacha ar fad, so abair lúide a haon, lúide a dó. Mar x cearnaithe [x^2] níl ann ach dá uimhir, [pauses] an uimhir x fá [meadaithe fá] é féin, so lúide a haon fá lúide a haon sin a haon. So chuile dá uimhir diúltach beith siad, em deimhneach. So, caithfidh gach páirt a bheith anseo [pointing to the region within the u-shape line drawn on the graph]. So sin é an graif. Maidir leis an inbhéarta ní, níor, like, ní cuimhin liom é a dhéanamh. Tá a fhios agam go ndearna muid é ach ní cuimhin liom and ábhar sin sa rang. ***English translation:*** *Here $f(x)$, it is above [the x axis], because down here [pointing to the third and fourth quadrants] the numbers are all negative. So let's say minus one, minus two. Because x^2, it is just a number [pauses], it is the number x multiplied by itself, so minus one multiplied by minus one is one $[-1 \times -1 = 1]$. So every second negative number would be positive. So, every part has to be here [pointing to the region within the u-shape line drawn on the graph]. So that's the graph. With regards to the inverse, I don't remember how to do it. I know we did it but I don't remember that topic from class.*



Jean's Individual Answer:

Original text: So an céad piosa ann, tá an fearann na réaduimhreacha go léir mar níl aon, níl aon fadhb leis. Mar shampla an ceann eile [referring to the previous question] bhí roinnt ar naid. Agus anseo, an inbhéarta, caithfidh an fearann a bheith x níos mó ná, nó cothrom le naid, mar nuair a déanainn tú amach an inbhéarta, an chaoi a déanann mise é, 'swapáil' mé an x is an y agus cuireann mé x ar y , like, leis féin. So faigheann tú préamh x , agus chun préamh x a ríomh, caithfidh x a bheith deimhneach, nó naid.

English translation: *So for the first part, the domain is all the real numbers because there is no problem with it. For example, the other one [referring to Q1] it was to divide by zero. And here, the inverse, the domain must be x more than or equal to zero, because when you do the inverse, how I do it is, I swap the x and the y and I put x on y, like, on it's own. So you get the root of x. And to calculate the root of x, x has to be positive or zero.* [does not complete (iii)]



Paired answer (original text was in English):

1. **Val**: [reads Q.2) Can that be Z?

2. **Jean**: Which one is Z again?

3. **Val**: ℝ, isn't ℝ like from 1 up and Z is like all the negative numbers and all the positive numbers?

4. **Jean**: Isn't N, N from 1 up, I think? I'm not sure, I was never really any good at this.

5. **Val**: And then, yeah, yeah, and ℝ then is N and Z, isn't is?

6. **Jean**: Is it? OK.

7. **Val**: That makes more sense, I think so. [reads Q.2 part (ii)].

8. **Jean**: I don't remember what I did here [in relation to Q.2(ii)]. [*Writes the answer on answer sheet*]

9. Oh yeah [*shows her answer sheet to Val*]. See like, the way in secondary school how I learned to get the inverse was change the x and the y

10. around, like say the original one is like that [writes $y = x^2$]. And then put y instead of x and x instead of y, and then get y on it's own to get 11. like, the inverse [*circles x on answer sheet*].

12. **Val**: Oh yeah, OK, OK.[*Reads part (iii) and says*:] I don't have a clue for that one.

13. **Jean**: Yeah, to be honest I didn't know either.

Val's written answer:

Jean's written answer

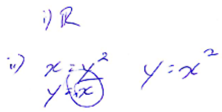


Figure 1: Learning example

Word use

In his explanation Val utilises Irish and employs a combination of deictic language with colloquial and some literate discourse. Deictic language, such as 'os cionn' (above), 'anseo' (here) and 'síos anseo' (down here) (in reference to the Cartesian plane), is combined with gestures (pointing to the associated quadrants on the graph) to highlight where the domain on the Cartesian plane is for function $f(x) = x^2$. There are elements of routine-driven words being employed since Val is describing the graph. There is little evidence of flexibility in Val's word use and objectified talk about the global features of the graph is not evidenced. In general, Val does not address the domain in this question and the mathematical explanation remains at an object-level of understanding. In contrast, Jean uses routine-driven mathematical words such as 'fearann' (domain), 'réaduimhreacha' (real numbers), 'inbhéarta' (inverse), and the phrase 'níos mó ná, nó cothrom le' (more than or equal to), which signifies familiarity with the subject content and related procedures and ease of use of the mathematical discourse in the Irish language. However, even when utilising Irish, Jean does not fully explain what R is or why in fact there is no problem with it in this instance, indicating that word use here is routine-driven and procedural. In their discussion of this same mathematics questions, Val opens this interaction in English and despite both students utilising Irish in their respective individual interviews, neither Val nor Jean negotiate their preferred language use for mathematics learning (which is Irish, as indicated in the questionnaire). When both students discuss in English the natural domain of function $f(x) = x^2$, they employ passive-driven words such as 'positive numbers' and 'negative numbers', and expressions such as 'from 1 up'. Their word use in English does not progress from passive-driven towards more object-driven usage as it tended to when the students utilised Irish.

Routines

In Figure 1, explaining the natural domain of a function $f(x) = x^2$, Jean explores the inverse of a function in the individual interview by first explaining (in Irish) that the domain must be x more than or equal to zero in order for the inverse of f to exit. Jean then clarifies that this reasoning is related to the routine of "doing" the inverse: 'an chaoi a déanann mise é, 'swapáil' mé an x is an y agus cuireann mé x ar y, like, leis féin. So faigheann tú préamh x , agus chun préamh x a ríomh, caithfidh x a bheith deimhneach, nó naid.' (I swap the x and the y and I put x on y, like, on it's own. So you get the root of x. And to get the root of x, x has to be positive or zero). There is an attempt at reification with the use of the word 'inbhearta' (inverse) as a verb. However, overall Jean's rituals (routines utilised for social rewards) define the repetitive discursive action or metarules for "doing the inverse" but there is little evidence of routine flexibility or explanations of why these rituals constitute an appropriate series of steps (meta-level thinking). Instead, there is a focus on pre-learned associations and repetitive patterns or routines (the domain of the function is R but does not explain what R is) and rote practices (learning how to swap the x and y).

Discussion and conclusion

There was a lack of language negotiation and code switching engaged in by both students throughout the learning process. Therefore, as Planas and Setati suggest (2009) the social situations and context of the learning scenario play a role in how the learner utilise their language for engaging in mathematics discourse. Both students employed Irish mostly when engaged individually in mathematics discourse but employed English when communicating with one another about the same mathematics topic. This is similar to Moschkovich (2007) who affirms that some students may opt to use their first language when working on a task individually. However, code switching was not evident in our study, even though the same mathematical problem was utilised in both the individual and paired tasks. Since there were two students only undertaking the bilingual stream of this undergraduate program, the whole group discussion was limited and group work was not facilitated. Hence, there were very few opportunities for social interactions within the learning context and as a result the opportunities for language choice and negotiations were also limited. Within the lectures, students conformed to the classroom linguistic norm of speaking Irish and carried this through to individual interviews. However, the language of paired communication was English, which is the dominant language within the Irish third level education setting. This study has found that the choice of language used by bilingual undergraduate students in this context was influenced by their views of language use in given situations (Caldas & Caron-Caldas, 2002). The impact of the social context is a key consideration for future studies relating to bilingual undergraduate mathematics learners.

Overall the learners demonstrated a combined use of colloquial and literate words and tended towards more colloquial word use when communicating together and through English. There are perceptible determinations from learners, especially Jean, to shift their discourse from colloquial to literate, particularly in Irish. Also, considering learners' word use in English remains mainly passive-driven, it appears that their word use tends towards literate and phrase-driven when communicating about functions individually through Irish. This discursive shift in word use is an indicator of meta-level thinking (Sfard, 2008; Moschkovich, 2002). However, although a shift from colloquial to literate discourse is evident, there is little evidence of objectified talk, even when Irish is utilised as the primary language of communication.

This study demonstrates that students utilised their languages in different ways when engaged in the same discourse (Kim et al., 2012), with greater evidence of meta-level thinking evident through the medium of Irish. Interestingly, both students show more competency and mathematical skills when using Irish in their individual explanations of the routines they employ to solve the problems and begin to shift towards a more literate and less colloquial discourse. In contrast, when engaged in conversation in English about the mathematics problems, they remained at a colloquial level of word use. In both instances, Irish was the preferred language utilised in the individual interview, whereas English was the language utilised during the paired interview. Although Jean displayed more dominance in mathematical discourse in the individual interview in Irish, this is not shown in the paired interaction. Jean does not use the same words nor employ the same visual mediators to explain the domain of function f. This might suggest a gap in Jean's discourse perhaps pertaining to meta-level understanding of the question, mathematical competency, or linguistic ability to discuss mathematics through English. Although learners displayed competence in terms of ritual-based routines from previous learning, either at university or school, reliance on such processual thinking resulted in occasional errors, or highlighted a lack of meta-level thinking when endeavouring to substantiate a particular narrative, in either language. This study demonstrates the importance of studying bilingual learners in diverse contexts in order to gain a better understanding of the intricacy and impact of their languages on mathematical thinking (Planas & Setati, 2009).

Overall, these students' mathematics discourse followed a basic language trajectory when engaged in functions discourse in English and collaboratively, whereas their mathematics discourse tended towards a more advanced mathematics discourse trajectory when engaged in functions discourse in Irish and individually. However, despite their mathematical language use progressing in Irish, this does not equate to progressing towards meta-level thinking, in either language. This insight raises additional questions relating to bilingual mathematics students' learning, in particular the role of the teacher in developing students' discourses and if there is value in explicitly teaching meta-level thinking/rules (Gücler, 2016) as related to language use by bilingual learners in undergraduate education. In additional, the social setting impacted on students' language choice in this study and raises the more general question of how the context of bilingual undergraduate mathematics learning influences choices relating to language use in bilingual mathematical discourse.

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