Exploration of patterns in different contexts

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As part of a research project focusing on investigative approaches to mathematics, a teacher in a Sámi school created an interdisciplinary teaching unit about patterns: skiing- mathematics- Sámi language. The 1–2 grade students painted their own patterns and they created patterns using their skis and poles in the snow. The analysis shows that the students developed their rules for what constitutes a pattern when the context changed from classroom work to skis on snow. One more aspect of pattern is focused: Sámi languages do not use any overarching term for pattern. At least three words are needed to cover the meaning of pattern in the mathematics curriculum.

Keywords: Pattern, hearva, language, Sámi upbringing, skiing.

Introduction

This paper presents an analysis of an investigative teaching unit, which focuses on *patterns* related mainly to the students' local culture and some physical activities. Regarding *pattern* as mathematical concept(s), two aspects of language are highlighted, a) relations between Indigenous (Sámi) language and majority (English and Norwegian) language and b) how students express their conceptions of *hearvvat* (patterns) in paintings and through their physical activities. Unjárgga oahppogáldu/Nesseby oppvekstsenter/Unjárgga centre of education is a mixed age school where the students have either Sámi or Norwegian as their first language.[1] Sámi is the first language of the teacher and the students in this paper. Sámi schools, like Unjárgga oahppogáldu, have to provide an education that is based on Sámi culture, language and social life (Sámediggi, 2017). As part of the research project SUM (Coherence through inquiry based mathematics teaching), the school develops investigative teaching units in mathematics from kindergarten to grade 10. Outdoor schooling is on the school's regular program. During a SUM meeting with teachers and the researcher, Anne, Lisbet got the idea of an open task where the students investigate skiing patterns they create in snow. She argued, "… I could work with patterns with skis. They could investigate patterns, because that is an open task…" (Transcript from conversation, 18.06.2018). In January 2018, students in grades one and two carried out an interdisciplinary teaching unit skiing - mathematics - Sámi. Lisbeth teaches mathematics and Sámi language for grades 1–2, so she often works interdisciplinarily with these subjects.

Trinick, Meaney and Fairhall (2016) point out that revitalization and maintenance of Indigenous language, including the teaching of mathematics, is insufficient unless cultural knowledge is also revitalized and maintained. The teaching unit considers their point, and thus mathematics, language

[1] Sámi and Norwegian languages are equal in the Norwegian municipalities governed by the Sámi language act.

and culture are intertwined in the teaching unit. According to Vorren (1995), there are several findings of ancient Sámi skis from the Unjárga area. One of these, the Mortensnes ski, is two thousand years old, so skiing has strong roots in the students' local culture.

Zazkis and Liljedahl (2002) point out, that patterns are the heart and soul of mathematics, but the exploration of patterns does not always stand on its own as a curricular topic or activity. According to the national mathematics curriculum's competence aim for grade 2, the students shall be able to "make and explore geometric patterns... and describe them orally" (Ministry of Education and Research, 2013). The teaching unit in this paper is about students' understanding of geometrical patterns. The teacher and the researcher communicate in Norwegian, while the teaching unit's language is North Sámi. Sámi culture and languages has a variety of pattern concepts, related to meaning and context. There is no overarching concept of pattern. Germanic languages, on the other hand, like English and Norwegian, use *pattern* and *mønster*, respectively, as overarching concepts.

Students' exploration of patterns/*hearvvat* in snow is the teaching unit's main mission. *Hearvvat* is a new concept for many of the students and the teaching unit opens with *hearvvat* in contexts related to clothes and fabrics. One goal is to provide the students with experiences of investigative approaches to mathematics, related to local culture. An additional goal is for students to develop their understanding of *hearvvat* through oral reasoning and reflections. The research question is; how can students' investigations of patterns in different contexts contribute to their conception of pattern? The data is a) an audio recording of teacher Lisbet's narrative about the teaching unit and b) posters with the students' work. The teaching unit includes five steps in total; two preparatory lessons that focus on patterns in fabrics, on clothes and in students' paintings, two outdoor skiing lessons and one reflection lesson in the classroom. The preparatory lessons intend to provide the teacher with an overview of the students' previous knowledge and to create a common idea of *hearvvat*.

Sámi languages and Sámi upbringing

Fishman, Gertner, Lowy and Milán (1985) point out that language itself is a part of culture and every language becomes symbolic to the culture with which it is intimately associated. The Sámi is an Indigenous people who live in northern Scandinavia and in the Kola Peninsula of Russia. There are ten different Sámi languages and Sámi culture can be described as diverse. This paper refers to North Sámi language. Sámi handicraft, *duodji*, is a powerful identity marker and generator of many traditions, such as language, social relationships, customary rules etcetera (Helander-Renvall & Markkula, 2017). The traditional Sámi livelihood of hunting, fishing, trapping and reindeer husbandry are important for sustaining Sámi culture and language (Keskitalo & Määttä, 2011). Skiing is an old Sámi tradition and the art of skiing originates from the ancient Sámi hunting, fishing and trapping culture (Birkely, 1994; Vorren, 1995). Skis are among the eldest relics of culture from Sámi settlements, and variations in landscapes and snow conditions caused the development of a variety of skis and sleds, for which the Sámi languages developed a wide nomenclature of expressions (Vorren, 1995). Due to the Norwegianization process (Heidemann, 2007) many of these words are not in daily use any more. In Unjárga today, Norwegian terms for

skiing are commonly used in the students' Sámi language. The teaching unit introduces students to Sámi skiing concepts.

The main goal of traditional Sámi child rearing is to develop independent individuals who can survive in a given environment; and to give the children self-esteem and zest for life and joy (Balto, 2005). The focus is on the learning process and less on teaching, so evidently experience-oriented learning is favoured. Trial and error is important for the learning process. As for skiing, knowing how to ski uphill and downhill is learned through trial and error and from the support of grown-ups and the older children. Keskitalo and Määttä (2011) developed a framework for Sámi pedagogy, where Balto's (2005) work has the central position. Upbringing is based on story-telling, connection with nature, and independence. Sámi pedagogy is based on a holistic and constructivist idea of learning. Strengthening the Sámi language is a core factor in the framework of Sámi pedagogy (Keskitalo & Määttä, 2011). All students should have the possibility to develop as language users at their own pace.

According to Balto (2005) as well as Keskitalo and Määttä (2011), autonomous students is a central theme in Sámi upbringing; the role of the teacher is to be advising, guiding, and trusting, while the students' role is to be active, flexible, and autonomous. Children should be provided with opportunities to find information and learn in nature and other places outside the classroom. Skovsmose (2001) underlines the importance of leaving the traditional mathematics textbook-based lessons when the teacher aims to support the students' autonomy. A move towards more investigative approaches can cause the students to be acting subjects in their learning process. The teaching unit allows the students to learn at their own pace and to develop as autonomous individuals.

Mathematisation of hearvvat or patterns

To mathematise phenomena from a non-mathematical or insufficiently mathematical matter, like skiing, means learning to organize it into a structure that is accessible to mathematical refinements (Freudenthal, 1991). Mathematising patterns in cloth, gloves, and skiing involves representing phenomena from these contexts by mathematics, for instance as drawings and geometrical figures, or representations by more or less mathematical concepts, expressed in different languages. However, topics related to Sámi culture exist because they are intertwined with cultural context; they cannot just be mathematised as if they exist independently from context. This is an important issue to consider for researchers and teachers with non-Indigenous backgrounds. Lakoff and Núñez (2000) point out, that mathematical symbols are just symbols and not ideas. The intellectual content of mathematics lies in its ideas, not in the symbols themselves. Symbols are representations of ideas that often can be expressed in more than one way. In this paper, we present students' descriptions of mathematical ideas; students' own *hearvvat*, patterns, with a focus on their drawings and their words.

Context plays an important role when translating the English word *pattern* into Sámi. Repeating patterns in woven bands and around gloves' wrists, like in Figure 1, are called *hearvvat* in Sámi. According to Nielsen's (1932/1979) dictionary, *hearva* means finery, adornment, and ornamentation. He presents two more Sámi words for pattern; *minsttar*, which means pattern, model

or template, and *girje*, which has two meanings. *Girje* means a spot of another colour (on an animal). In plural form, it means coloured ornamental patterns, like *girjefáhccat;* gloves that are patterned all over. However, the word *girje*'s second meaning is *book.* According to a more recent dictionary (Kåven, Jernsletten, Nordahl, Eira & Solbakk, 1995), *hearva* means decoration, trimming, or embroidery, while *minsttar* means pattern or formula, and *girji* means book, letter, or spot. So, in total there are three different North Sámi words that can mean *pattern* in English. The North Sámi version of the mathematics curriculum (Ministry of Education and Research, 2013) consequently uses *minsttar* for pattern (Norwegian: mønster). *Minsttar* may work for number patterns, but not for the geometrical patterns in the teaching unit. Lisbet uses the word *hearva* in the teaching unit. She discussed the question of using *minsttar* or *hearva* with Harald Gaski who is a professor in Sámi culture and literature. He too found *hearva* to be more appropriate for *pattern* in the teaching unit's contexts. Gaski (1998) describes the lyrics in the traditional Sámi song, *luohti,* as something that follows a pattern. In that context, he uses the Sámi word *girji* for pattern. *Pattern* is a central concept in mathematics, but the literature has not problematized the Sámi translations of it.

Figure 1. Example of *hearvafáhcca:* The patterned Unjárgga/Nesseby glove (photo: Lisbet Hansen)

Sámi languages do not use overarching terms to the same extent as Norwegian and English do, they have chosen different ways of expression (Fyhn, Eira, Hætta, Juuso, Nordkild & Skum, 2018). This should be an important issue to consider for mathematics education in Scandinavia. Bishop (1990) warns that it is very difficult for anyone conditioned to the western way of naming and classification to imagine that there exists other ways of conceptualizing and using language. McMurchy-Pilkington, Trinick and Meaney (2013) point at the importance of debating standardization of terms and the place of dialectical differences, in the development of Indigenous mathematics curricula. It could be that *hearva* and *minsttar* are used differently in eastern and western parts of the North Sámi area.

The teaching unit's preparatory steps

The first step of the teaching unit was based on a collection of patterns that the teacher brought to her classroom; Unjárgga gloves as shown in Figure 1, pieces of fabric like the examples in Figure 2a and 2b, an apron, homemade woollen socks, and a *holbi* from an Unjárgga *gákti* (the bottom part of the local Sámi dress) as shown in Figure 2c. Stars and candles in Figure 2a were the first items the

Figure 2a. Stars and candles

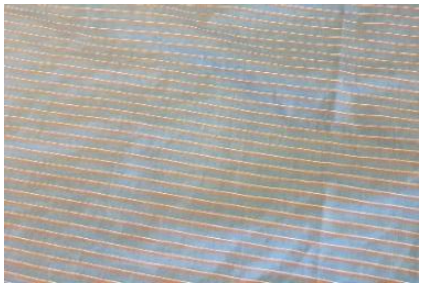

Figure 2b. Stripes

Figure 2c. *Holbi* from Unjárga

students managed to recognize and name here. The students' discussions lead to a common rule for what counts as a *hearva;* it has to be a system of colours. When asked to search for repetitions, everyone found something; for example red-green-red-green …, square-rectangle-square-rectangle … and star-candle-leaf-star-candle-leaf… After a while, one student pointed at two fabrics, claiming that one has *hearva* while the other has not. This student distinguished between what is *hearva* and what is not by a counterexample; that something is not *hearva.*

In Step 2, most of the students' work was autonomous. This is in accordance with Keskitalo and Määttä's (2011) description of Sámi pedagogy and with Balto's (2005) description of traditional Sámi child rearing. The students' task was to create and paint their own *hearvvat* and the teacher had no control of their choices. Every student created multiple paintings. The paintings in Figure 3 show how three students represent their ideas of *hearva*, pattern. Each painting shows a system of colours. Each painting also shows repetitions of something. The three paintings show repeating dots. The leftmost painting shows repetitions of blue dots; some dots are only blue and some dots have a brown circumference. The two other paintings also show zigzag patterns, this could be related to the zigzag pattern in Figure 2c, which is named *njunnesuorránat*. Considering that the Sámi word *hearva* means *finery, adornment* and *ornamentation* in English, it is reasonable to expect that the students include colours as a property of *hearva*. The other paintings also showed systems of colours and repeating items.

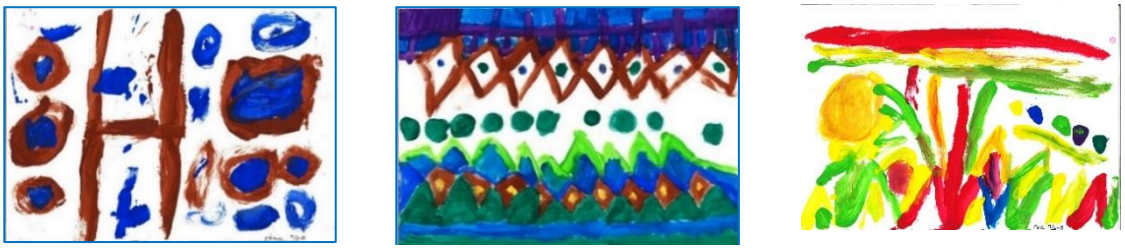

Figure 3. Three painted *hearvvat*

Work with *hearvvat,* patterns, in snow

Step 3 took place outdoor. When the teacher asked if the students could make any *hearvvat,* patterns, in the snow with their skis and poles, the answer was no. Then Lisbet skied around the schoolyard with the students following her trail. Pointing at the fresh ski trail, she asked if there

were any *hearvvat*. This is interpreted to be a leading question, a support for those who had ideas they did not dare to present the first time they were asked. Several students answered no and no one objected. This is interpreted to mean that the students' conceptions of *hearvvat* did not include patterns in snow. Most likely it was limited to coloured patterns on clothes, fabric and paper. The students' rules for *hearva* included a system of colours. The white snow is just white; there is no system of colours. The teacher's aim was that the students developed their ideas of what constitutes a *hearva* to something independent of colours.

The teacher offered the students time for thinking. After a while, one student suggested that maybe the poles create *hearvvat* in the snow. Lisbet then asked whether the skis make any *hearva*. One student replied that the skis make lines, like the stripes in the fabric in Figure 2b. When you go skiing, you usually move forwards while the patterns and marks from your skis and poles appear behind you. One student expressed this as "Ovddes guvlui go čuigen šadde sárgát" [when you skied forwards, there appeared some lines] (Lisbet, transcript from conversation, 18.06.2018). In order to see what you have created, you have to look behind you. Children at school age are familiar with what ski trails look like. Students' investigations of skiing patterns also include investigations of how they move their entire bodies; they learn at their own pace, "In one way they investigate the *hearva* as well as the skiing technique… they have to try things out to see what happens", (Lisbet, transcript from conversation, 18.06.2018). "They have to try things out", means that the students are acting subjects in their learning process, as Skovsmose (2001) underlines. This is in accordance with Sámi traditional upbringing (Balto, 2005) and Sámi pedagogy (Keskitalo & Määttä, 2011).

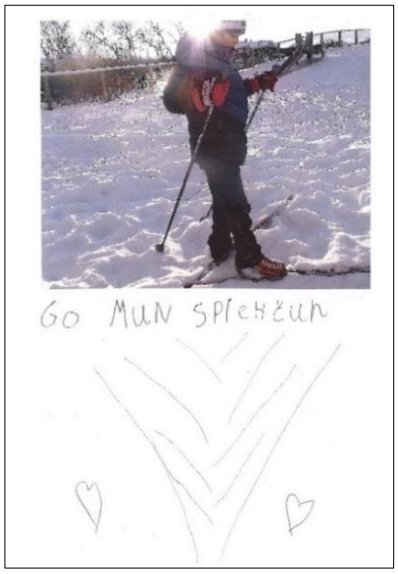

Figure 4a. ***Spiehčču t***

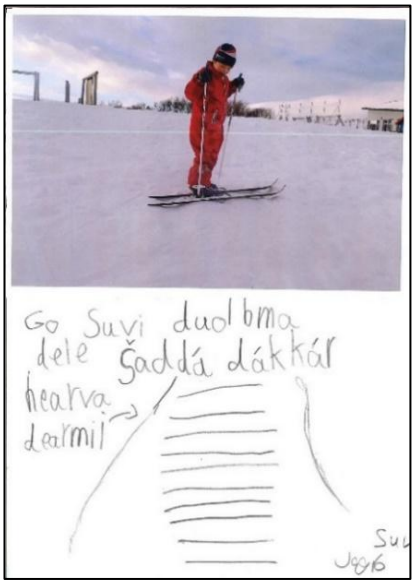

Figure 4b. ***Duolbmá***

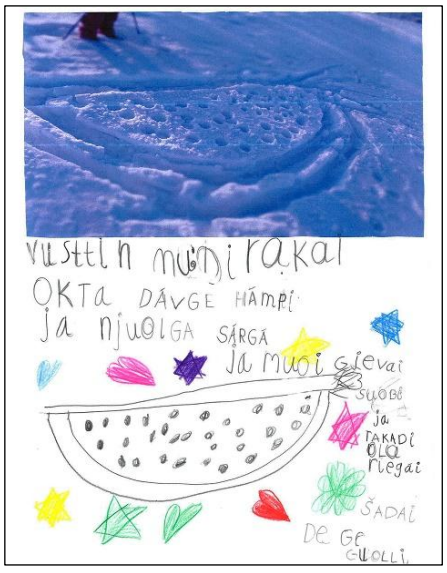

Figure 4c. ***Guolli***

Step 4 took place at the school's soccer field. The students were asked to create their own *hearva* in the snow. They explained their *hearvvat* to the others, and Lisbet captured photos of their work. Usually, the students use Norwegian skiing concepts when they speak Sámi, but now they have started using Sámi concepts like *spiehčču t* (herringbone skiing) as shown in Figure 4a, and *duolbmat* (sideways uphill tramping) as shown in Figure 4b. When you perform skiing techniques like in Figure 4a and 4b, the outcomes are patterns in the snow. These patterns are named by the

technique; by the way you move your body. So, the students' ideas of *hearvvat* were represented by how they moved and by the marks made by their skis and poles. At the end of the lesson, the teacher and the students looked at all the patterns. The students explained in their own words. The two students in Figure 4c first made a *dávgehápmi* (curved shape) and *njuolga sárggá* (straight line) and then *rieggát* (rings). The result was *guolli* (a fish). One student said, "When I skied forwards, the result was straight lines." In order to bridge the students' daily Sámi language and their mathematical language, Lisbet supplied their choice of words with mathematical terms like *circle*.

The drawings and texts in Figures 4a–c present outcomes from Step 5; a reflection lesson that took place in the classroom. The students were presented to A4 photos of their snow *hearvvat*, and they recognized the photos and discussed their *hearvvat*. Each student chose one photo, made a drawing of the *hearva* and wrote a text for that drawing. The teacher had no control of the students' choices. Some of them needed assistance in writing down their words related to the photo; they asked how to spell some of the words and the teacher then showed them before they wrote on their own. The students were active and autonomous, while the teacher's role was supporting and trusting; this lesson was in line with Sámi pedagogy. The first graders needed more assistance in writing than the elder ones.

Closing words

The conversation between the teacher and the researcher revealed that the teaching unit provided the students multiple experiences with investigative approaches to mathematics related to their local culture. Students were acting subjects in their learning process by creating their own *hearvvat*, patterns in different contexts. Their development of rules for what is a pattern was guided by the teacher. The students' first rules for what is *hearva*, pattern, was related to clothes, fabrics and paintings; it had to be a system of colours and it had to be repetitions of something. When the context changed to skiing, the students at first found no *hearva*. Maybe that is because there were no contrasting colours or maybe that is just because the context changed. The teaching unit caused a development of the students' concept of *hearva* such that it does not have to include different colours. The teaching unit highlights the challenge students meet when they move from one context to another; students do not automatically transfer knowledge into a new context. The analysis also revealed a need for discussing the mathematics curriculum's choice of Sámi terms for *pattern.* It turned out that the three different Sámi words *hearva*, *girji* and *minsttar* all can be translated to pattern. These words are related to different cultural contexts. This may have influenced the students' ideas when they made up their rules for what a *hearva* is. More research is needed on the meaning of these words in order to prevent misunderstandings between teachers, students, and parents, not to mention those who write textbooks and curriculum texts. The teaching unit also appreciates and strengthens local Sámi language; the students used Sámi concepts for skiing techniques that they usually refer to with Norwegian concepts in their daily Sámi language. An interesting follow-up study would be to interview Lisbet's students one year later in order to see what words they use, when describing what is meant by *hearva*, and what is meant by *minsttar*, the curriculum's word for pattern.

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