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Solving problems collaboratively in multi-age classes – a possibility for learning?

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From an interactionist perspective, interaction is seen as a foundational constituent of mathematical learning, as it creates the possibility for students to negotiate meaning, which then can lead to the construction and/or modulation of meanings which outlast the situation - framings. According to Krummheuer (1992), this is seen as learning. In multi-age classes, students have even more diverse framings then in single-age classes and when they are given the opportunity to learn collaboratively, an increased need to coordinate their framings might lead to an optimised possibility for learning. This research project has the objective of identifying how students participate in processes of collective negotiation of meaning and therefore how possibilities for learning mathematics emerge. First analyses show that the possibilities for learning emerging in the interaction vary depending on whether differences in framings can be coordinated and a taken-as-shared meaning is brought forth.

Keywords: Multi-age education, collaborative learning, interactional analysis.

Multi-age education

No matter the school, students in a classroom are never homogeneous, also in relation to their learning preconditions. Each individual is unique and therefore has differing abilities and experiences. Using age as the distinguishing factor for placing students in specific classes is the norm, because it is seen as a possibility to make classes as homogeneous as possible. But even in learning groups which are homogeneous in age, the differences in students' preconditions for learning can be up to five years at the time they enter into school (Lorenz, 2000, p. 22). One concept with the aim to purposefully value and meet this existing diversity is merging students of different ages and grades in one class deliberately (Prengel, 2007, p. 69), which can be referred to as *multi-age classes*. The benefit in making learning groups heterogeneous in age is e.g. seen in that, because the diversity of learning preconditions increases, the necessity for differentiation increases also. However, mathematics is often presumed to be a subject built up rectilinear whose content can only be learned following a specific order (Lorenz, 2000, p. 19). Nührenbörger and Pust (2006) found that because of this common assumption students in multi-aged classes are rarely given the opportunity to learn from and with others in mathematics classrooms. Instead, this often causes extremely individualised and separated learning to take place as teachers either feel they need to separate the learners back into the different age groups or they need to let each student learn individually at their own pace in order to meet the increased diversity of the students. However, both of these waste the opportunity seen in collaborative learning (Nührenbörger & Pust, 2006, p. 17), especially from an interactionist perspective on mathematics learning.

Besides overarching concepts, which have been developed for collaborative learning within multi-age mathematics teaching (e.g., Nührenbörger & Pust, 3006), interactions between learners who are heterogeneous in age have been examined within multi-age mathematics education research. These studies predominantly focus on dialogues between two students from different grade levels and concentrate either on how knowledge is constructed in the interaction (Nührenbörger & Steinbring, 2007) or on when interactions are beneficial for learning in these settings (Gysin, 2018). Our focus is also on possible benefits for students' learning conditions when children with diverse preconditions interact with each other while solving problems collaboratively. However, we base our study on the interaction theory of learning, which will be described below, and include analyses of group settings of three students working together.

Interaction theory of (mathematical) learning

From an interactionist perspective, the individual process of learning is constituted in the social process of the negotiation of meaning. The social interaction, therefore, holds the potential to generate meanings new to the individual. Especially for young learners, interaction is seen as a foundational constituent of learning. In this context, Miller (1986) creates a sociological theory of learning of at least two individuals – the theory of collective learning processes – in contrast to psychological theories of learning which predominantly focus on isolated individuals. Hereby, he does not, on principle, question that learning is also an individual process, but rather that, when learning fundamentally in the early stages of a child's development, an interactive collective process precedes or determines this process of learning something new (Jung & Schütte, 2018). In this sociological theory of learning, the concept of argumentation is essential and Miller (1986, p. 37) differentiates it from "communicative action." Argumentation, and hereinafter more specifically collective argumentation, is mainly characterized by the aspect of rationality, whereas communicative action is based on something uncontroversial. Mathematical learning, therefore, takes place when two or more people negotiate what is viewed from a mathematical perspective as rational. Krummheuer and Brandt (2001) draw on this aspect of learning in collective argumentation and create an interaction theory of mathematical learning. For them, it is of great importance that students participate in collective argumentation within classroom interaction in which meaning is collectively negotiated (Krummheuer & Brandt, 2001, p. 20). This, at best, will lead to learning to occur which is seen as the construction and/or modulation of meanings which outlast the situation – also called framings (see below) (Krummheuer, 1992; Jung & Schütte, 2018, p. 1091) and can be reconstructed in the increasingly autonomous participation in these collective argumentations (Krummheuer & Brandt, 2001). Based on this understanding of learning and the significance of interaction in mathematics learning, diverse theories with rich empirical content have been developed in recent years from a foundation of "sensitizing concepts" (Blumer, 1954, p. 7) – a kind of theoretical skeleton for the interaction theory of learning, which will be described below.

At the beginning of an interaction, the participants develop preliminary interpretations of the situation on the basis of their individual experiences and knowledge – called *definitions of the situation* (Krummheuer, 1992). Furthermore, participants in the interaction attempt to attune their definitions of the situation to each other, ideally leading to the production of *taken-as-shared*

meaning (Voigt, 1995, p. 172), a working consensus. The participants' interpretations of a situation can never be exactly the same, however, by creating an understanding between them concerning the objects, ideas and rules of the interaction, the participants are enabled to work together. The result of the participants' negotiation only is something "temporarily" which serves as a basis for further processes of negotiation but might also be rejected or transformed as the interaction unfolds (Bauersfeld, Krummheuer, & Voigt, 1988; Voigt, 1995). The taken-as-shared meaning – *the working consensus* (Goffmann, 1959, p. 9) – is on the one hand socially constituted and, on the other, potentially novel for the individual if it pushes systematically beyond his or her interpretive capacities. The working consensus represents the 'stimulation potential' of individual cognitive restructuring processes. When taken-as-shared meanings are brought forth repeatedly in the interaction, the participants' individual definitions of situations can become standardized and routinized and can therefore be produced by the individual in similar situations. Drawing on Goffmann (1974), Bauersfeld et al. (1988) call these definitions *framings*. Thus, ideally, mathematical ascriptions of meaning which sustain beyond the situation (framings) are constructed, or the working consensuses, which are repeatedly collectively negotiated, 'converge' in individual mathematical framings.

However, it often happens that the framings produced by young learners either among themselves or in interaction with individuals with advanced skills are not in alignment with each other (Krummheuer, 1992). Students, for example, each interpret situations based on framings from the environments they have experienced outside of school and in their previous school career. In order to maintain the progressive mutual negotiation of content relating to a particular theme, the *differences in framing* between the participants need to be coordinated. This coordination should be done by an individual who is advanced in the subject-specific interaction. While these differences in framing can make it more difficult for the participating individuals to adjust their definitions of the situations to fit each other, they also provide the "'motor' of learning" (Schütte, 2014, p. 927) since, on the interactional level, they generate a certain necessity for negotiation.

Analysing students' participation in processes of negotiation of meaning

One of the unique characteristics of multi-age education is that learners most likely have even more diverse framings then in single-age classes. We believe that this can provide and optimized opportunity for learners when working collaboratively, as there is an increased need to coordinate their differences in framing by negotiating meaning. In order to coordinate the differences in framings, the students have to bring forth a working consensus collectively. This can then lead to the construction and/or modulation of framings which is seen as learning. Based on this interaction theory of learning, the overall question this research seeks to address is: How do students participate in processes of collective negotiation of meaning within multi-age mathematics education and do possibilities for learning mathematics emerge.

Interactionist approaches of classroom research

Methodologically, this work can be located within qualitative methods of social research which follow a reconstructive-interpretative qualitative approach and have the aim to 'understand' the actions of the individuals participating in class and develop local theories (Schütte, 2011, p. 776).

More specifically, this work is located within interactionist approaches of mathematics educational classroom research (e.g., Krummheuer, 2011). For this study, the classes are filmed several times over a period of one to two years in order to reconstruct possible age-specific changes concerning the participation in collective argumentation of the children. The interactions of the students are then transcribed and analysed using the interactional analysis, which was first deployed in the area of interpretive classroom research in mathematics education in studies by Bauersfeld et al. (1988). The interactional analysis allows to reconstruct the ways in which negotiations of mathematical meaning are interactively constituted by individuals. Furthermore, it can help to reconstruct patterns and structures of verbal actions of the teacher and the students. Overall, this research seeks to describe how mathematical learning takes place collaboratively in multi-age education and will use the sensitizing concepts in order to describe these processes.

Collective argumentation of students in multi-age education

The group works analysed in this paper are from a mathematics lesson in a multi-age class including 24 children from first, second and third grade. At the beginning of the lesson, the students are divided into groups of three which mostly contain one student of every grade level. The students are given this task: "Tina and her family want to decorate their Christmas tree with hand painted Christmas ball ornaments[1]. They want every ornament to look different. To paint the ornaments, they have the colours red, green and blue. With these colours, they can paint dots or stripes. For each ornament they want to use two colours and one pattern at the most. Which possibilities do they have when painting their ornaments?" To solve this task, the students have material on their tables which includes circles, stripes and dots of each colour to try out different combinations without painting them. The interactions of two groups of three students, including a student from every grade level, working on this task were analysed and the summaries of these analyses will be presented below. Afterwards, the analyses will be compared to each other and a conclusion will be drawn.

Analysis of the first group work

The first group includes Isabella (1st grade), Hans (2nd grade) and Elias (3rd grade). During the first part of this group work, the students start by creating two green circles, one with red stripes and one with blue dots. Then the following interaction emerges.

Transcript 1: Excerpt of the first group work

1		Isabella	I wouldn't just use green now but (if we) also the other two colors *[points to the blue and red circle]*
2		Elias	on red wait *[takes a big red circle from the pile]* on red we could *[picks up a small blue dot]*
3		Hans	no now now *[touches red dots]* now green dots *[picks up two small green dots]*
4		Elias	no we already have green dots . twice *[first points to a big green circle and then with two fingers to the two green circles with patterns on them]*
5	<	Elias	now we have to

[1] In Germany, Christmas trees are often decorated with different kinds of ornaments (e.g. ball ornaments) which can be in one or more colors. Therefore, children in Germany are usually familiar with ball ornaments.

6	<	Hans	but green dots *[places his two green dots on the red circle in front of Elias]*
7		Elias	yes okay . that that yes

In this excerpt, Isabella argues that they should not only use green circles but also the two other colours because they should not only use the same colour <1>. Elias then picks up this idea and starts suggesting to put blue dots on a red circle <2>. Hans also follows up on Isabella's idea and suggests to put green dots on a red circle <3>. Elias seems to misunderstand Hans' utterance and reiterates Isabella's idea <4>. Then Hans repeats his idea to clarify it <6> and Hans and Elias agree on creating a red circle with green dots <7>. Hans and Elias then briefly disagree on the number of dots they should use but both seem to implicitly agree that for the differentiation of the possibilities the number of dots is not relevant. After settling for three, they also disagree on which background colour they should continue with. Elias now suggests to use a blue circle but Hans disagrees and suggests to create a second red one using the other colour left for the pattern (blue). They then agree on using a red circle and add two blue stripes. Isabella says that she has found another possibility and points to a blue circle with one green stripe. Contrary to Elias who then says he wants to create another red circle with a pattern, Hans explains that they have created two green and two red ones so far and suggests to now create blue circles. All three quickly agree that they will first use a blue circle with red dots.

In this part and in the continuation of this group work, one can see that all three children participate in the creation of the possibilities. All of them contribute suggestions but also disagree with each other directly. However, before they continue working on the next possibility, they always come to an agreement concerning a possibility. In doing so, they collectively negotiate their suggestions. In the arguments, which are brought forth in the processes of reaching agreements on the procedure and correctness of the specific possibilities, various strategies of the children can be reconstructed (even usage of all colours; maximum difference in the materials used; alternating colours and patterns at the same time; aesthetic reasons such as using the same or deliberately different number of dots and stripes; conservative usage of materials to avoid wasting materials). The two strategies which prevail – even usage of all colours, as well as the variation of the number of dots and stripes for aesthetic reasons – are both introduced by Isabella first and then picked up by Elias and Hans. It is noticeable that Isabella contributes a lot of ideas but does not implement them directly but rather formulates them in the subjunctive. Because she mainly relies on verbal suggestions without using the material, whereas Hans and Elias often directly use the material while bringing forth a suggestion (e.g. <3, 4>), her suggestions and arguments are often very explicit. Elias, in particular, takes up Isabella's suggestions over and over again and implements these as well as his own suggestions. Hans primarily takes up Elias' suggestions but often also contradicts them and starts acting directly on his own. However, in these contradictions - at the beginning especially between Elias and Hans – no competition develops for who prevails, instead the contradictions are used for finding the best possible strategies and methods for solving the given task. Elias, in particular, involves everyone in the process of solving the task by asking for their approval. Therefore, a working consensus seems to establish that all three participants should agree on a suggestion (or at least no longer disagree on a suggestion) before moving to the next possibility. This leads to the group bringing forth collective argumentations in order to solve the emerging differences in

framing. Throughout this entire task, they all continue to work collaboratively because, even though they briefly start working alongside each other once in a while, they always agree collectively on a possibility before moving forward.

In this group work, the differences in framings seem to be coordinated as the children bring forth a working consensus through collective argumentations. Therefore the possibility for learning can be reconstructed as this working consensus can lead to the students constructing new or modulating existing framings. In further analyses, different roles of the individual students within these negotiations could be reconstructed more specifically. Elias, for example, seems to act as one who coordinates the differences in framings and therefore seems to help them come to a working consensus as a group.

Analysis of the second group work

The second group includes Kim (1st grade), Erich (2nd grade) and Hannes (3rd grade). In this group, we can also see that all three children participate in the creation of the possibilities by making suggestions, responding to the suggestions of others, but also by acting immediately on their own suggestions. As a group, however, they only agree on a possibility briefly before revising it again. At the beginning, they start by working on one possibility collectively, however Erich's proposal is rejected by Kim and Hannes without giving clear reasons. Kim only says „no I believe I don't know if that will work with red dots I don't believe [*takes the red dots off the blue circle*]" <78> and after Erich moves the blue circle Hannes says *„no Erich red needs to go [takes the blue circle which Erich moved and puts it in front of himself*] <82>. From then on Erich withdraws from working collaboratively. The disagreements between the children are often solved by exercising authority and rarely by giving arguments and if an argument is given, this is not taken up by the others. This is also reflected in the fact that from the start Hannes repeatedly contradicts suggestions by referring to the task – only two colours may be used – but then does not comply with it himself. Therefore, a pattern is established in which all three children work side by side. Kim and Hannes seem to interpret this situation as aimlessly trying out various possibilities, as Kim, for example, repeatedly rejects or reverses possibilities and never leaves a possibility permanently. Erich, on the other hand, seems to be looking for possibilities in a more structured way. His structured approach can also be reconstructed by him letting his suggestions remain visible and not taking them apart again like the other two. At different times during the group work, Kim looks at other tables and then introduces new suggestions for strategies for solving the task (using the sheet of paper; gluing the patterns onto the circles; using a more structured approach) but her suggestions are not taken up or even rejected by Hannes and Erich. All in all, it can be seen that all three act very autonomously right from the start but do not find a working consensus about how to solve the task. They then start working side by side, presenting their own suggestions to each other but when contradictions arise they neither bring forth arguments (other than aesthetics), nor agree on possibilities and secure their results as a group.

In this second group work, the few attempts of the participating children to coordinate the differences in framing, which emerge, fail and therefore no working consensus seems to emerge. This may be due to the fact that they bring forth separate arguments but no collective argumentation

is brought forth. From an interactionist perspective, the possibility for learning in this group work is therefore limited as without bringing forth a taken-as-shared meaning the possibility for constructing new or modifying existing framings is impeded.

Conclusion

When comparing the interactions of the two groups, even though all children participate in solving the task, from an interactionist perspective, only in the first group an optimized possibility for learning can be reconstructed. Therefore, solving problems collaboratively in multi-age classes can vary in the way it enables possibilities for learning to emerge. In further analyses, collective argumentations seems to be initiated e.g. when learners explicitly verbalize disagreements or ask questions about the correctness of a solution as this seems to put the other learner(s) – often the more advanced ones - into an interactive obligation to bring forth arguments. In the process of the students then bringing forth the argumentation collectively, both the advanced and the less advanced student seem to have the possibility for learning even though these may be different (cf. Friesen, forthcoming). However, as seen in the two group works presented in this paper, this interactive obligation does not always lead to a collective argumentation in which a taken-as shared meaning is produced. In both groups, there are arguments that are brought forth but only in the first group these are brought forth collectively as can be seen in the excerpt. In the second group, the students do sometimes give an argument when a disagreement is verbalized, however, these are not taken up by the others and therefore no argumentation is brought forth collectively. In the future, these analyses will be compared with analyses of further group work in order to more clearly describe the patterns that enable children to bring forth and participate in collective argumentation.

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