A simple formula giving the speed of light

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Abstract

The contemporary physical theories are based on several fundamental physical constants. No theoretical framework provides their experimental values. Here, we might have found two formulas yielding the speed of light and the Planck constant according to the vacuum properties and fundamental constants. In particular, they highly suggest that gravitational and electromagnetic waves must actually propagate in matter. Because there is no dense matter between high-mass bodies in the universe, the relations would mean that the structure of the universe is at least pentadimensional (four spatial and one temporal dimensions). We then discuss the implications of this hypothetical feature on contemporary physics.

Keywords: universe; vacuum; fundamental constants; speed of light; gravitation; theory

1. Introduction

In the mid-17th century, Isaac Newton published his law of gravitation in *Philosophiae naturalis principia mathematica*. Classical mechanics remained the mainstay of science until the end of the nineteenth century. The electromagnetism theory suggested that light had to propagate in a medium named luminiferous aether by scientists. In the 1880's, while most of them were confident in its existence, A. Michelson and E. Morley [1] carried out several experiments that challenged this hypothesis. In particular, they did not measure any "aether wind". Classical mechanics failed to explain this result and more and more scientists gave up the aether concept. It was the case of A. Einstein who contributed to the special and general theory of relativity with M. Grossmann, D. Hilbert, H. Lorentz and H. Poincaré (for the most famous ones). These theories did not need a tangible material such as the aether medium and made possible the observation of new physical phenomena. Based on redshift, the discovery of the expansion of the universe is one example [2,3]. Lemaître [4] and then Hubble [5] independently formulated a law that states that galaxies move away from each other at a speed proportional to their distance. At the same time, the development of quantum mechanics turned the perception of particles upside down.

Many experimenters carried out tests to challenge these theories. For now, special relativity and quantum mechanics passed these tests. General relativity could also be in accordance with observations. The perihelion precession of Mercury [6], the bending of light by the Sun [7-9], gravitational redshift [10] and gravitational waves [11,12] are successes of general relativity. However, this theory needs dark matter [13] to agree with the galaxy rotation curves for instance [14]. Up to now, this hypothetical matter has never been observed. The accelerating expansion of the universe is another issue which is explained by dark energy [15]. This can be embedded in the equations of general relativity but its origin remains unknown. Linking all the physical processes, from the infinitely small (quantum mechanics) to the infinitely large (general relativity), remains an unsolved problem as well. Finally, the value of the fundamental physical constants on which contemporary physics is based, is not explained yet.

Some efforts are being conducted to understand these physical problems. Scientists are trying to detect dark matter where it is supposed to be. Others are focusing on Modified Newtonian dynamics (MOND [16]) which corresponds to a theory where Newton's laws are modified to fit with the properties of galaxies. Theoretical physicists are also developing ideas to couple gravitation and the three other fundamental interactions (electromagnetic, weak and strong interactions). Such theories are named "theories of everything". The two most famous ones correspond to string theory and loop quantum gravity. Like in string theory, extra dimensions are thoroughly considered in many other theories. The first attempt consisted in Kaluza-Klein theory which adds a fifth dimension to spacetime [17,18]. Similarly, a fifth dimension is considered in Randall-Sundrum models [19,20].

In this paper, we first present two relations which link some fundamental physical constants with vacuum properties of the universe. These formula, which derive from a dimensional analysis, highly suggest that gravitational and light waves propagate in matter, out of our field of view. Then, under the assumption that both relations do not result from two coincidences, we discuss about their consequences. In that respect, gravitation would actually correspond to the deformation of matter. A fourth spatial dimension would be necessary to explain the existence of this material. We then focus on the interpretation of some contemporary physical phenomena in the framework of the developed theory. Finally, we touch upon the variation of some fundamental constants.

2. Results

The purpose of this section is to find a relation giving the speed of light. Special and general relativity theories assert that gravitational and light waves propagate in vacuum. However, common sense could make us think, like nineteenth-century scientists, that waves need a material to propagate. Despite the apparent demonstration that it is false (see introduction), we will still trust in this common sense and assume that waves do not travel in vacuum but in some hypothetical continuous matter.

One can observe that the speed of material waves is often related to the pressure and density of the medium by $c_{waves} \propto \sqrt{\frac{P}{\rho}}$. Perfect gases, polytropes, incompressible liquids or elastic solids obey this relation (the pressure term is replaced by Young's modulus for a solid). If the continuum hypothesis holds for the universe, the velocity of gravitational and light waves could be given by a similar relation. For the calculation, we can use the critical density (vacuum density) $\rho_c = \frac{3H^2}{8\pi G} \sim 9.2 \times 10^{-27}\ kg.m^{-3}$ (with H the Hubble constant and G the gravitational constant). Considering that many measurements do not agree on the value of the Hubble constant, we assumed $H \sim 70\ km.s^{-1}.MPc^{-1}$ because this corresponds to an average of the observations [21-24]. No known parameter can directly be selected for the pressure term P. We will thus try to find a parameter with the dimension of a pressure from the vacuum properties and physical fundamental constants.

Table 1 itemizes some fundamental constants and vacuum properties that could, *a priori*, be involved in the calculation.

Temperature (Cosmic Microwave Background)	$T \sim 2.73 \quad K$
Density	$\rho_c \sim 9.2 \times 10^{-27} \quad kg.m^{-3}$
Boltzmann constant	$k_B \sim 1.381 \times 10^{-23} \quad kg.m^2.s^{-2}.K^{-1}$
Planck constant	$h \sim 6.626 \times 10^{-34} \quad kg.m^2.s^{-1}$
Vacuum permittivity	$\varepsilon_0 \sim 8.854 \times 10^{-12} \quad m^{-3}.kg^{-1}.s^4.A^2$
Elementary charge	$e \sim 1.602 \times 10^{-19} \quad A.s$
Gravitational constant	$G \sim 6.674 \times 10^{-11} \quad m^3.kg^{-1}.s^{-2}$
Speed of light	$c_{light} \sim 2.998 \times 10^8 \quad m.s^{-1}$

Tab. 1 Fundamental physical constants and vacuum properties [25].

The temperature and the density should be selected since the calculation of a pressure often depends on both variables (perfect gas, liquid, ...). The vacuum permittivity is, by definition, a vacuum property, so it should participate. In the dimensional analysis, the electric charge of the vacuum permittivity can only be balanced by the elementary charge. Necessarily, both parameters will be associated such as $\varepsilon_0 e^{-2}$. The same argument is applied to the temperature. So, the temperature and the Boltzmann constant are embedded in one variable $k_B T$. The Planck constant takes part to the study because pressure depends on microscopic effects.

We can now manage the dimensional analysis with the following parameters: P, ρ_c, $\varepsilon_0 e^{-2}$, $k_B T$ and h. There are five physical quantities and three physical dimensions (length, time and mass). According to the Buckingham π theorem, two dimensionless numbers can be constructed. The method consists in the arbitrary separation of the quantities into two groups. We consider both dimensionless numbers $\Pi_1 = \rho_c^{\alpha} \left(\varepsilon_0 e^{-2}\right)^{\beta} \left(k_B T\right)^{\gamma} P$ and $\Pi_2 = \rho_c^{\delta} \left(\varepsilon_0 e^{-2}\right)^{\zeta} \left(k_B T\right)^{\eta} h$. The resolution of the linear systems yields

$$\Pi_1 = \frac{P}{\left(\varepsilon_0 e^{-2}\right)^3 \left(k_B T\right)^4} \tag{1}$$

$$\Pi_2 = \frac{\left(\varepsilon_0 e^{-2}\right)^{5/2} \left(k_B T\right)^2 h}{\sqrt{\rho_c}} \tag{2}$$

Relation (1) allows us to calculate $\sqrt{\frac{P}{\rho_c}}$ and compare it with the speed of light. With $\Pi_1 = 10$,

$$c = \sqrt{\frac{P}{\rho}} = \sqrt{10 \frac{\left(\varepsilon_0 e^{-2}\right)^3 \left(k_B T\right)^4}{\rho_c}} \sim 3.0 \times 10^8 \quad m.s^{-1} \tag{3}$$

The inaccuracy on the Hubble constant prevents certifying that Π_1 is exactly equal to ten which corresponds to $H \sim 70.1\ km.s^{-1}.MPc^{-1}$ but its value should be between 8.5 and 11.5. The calculation of the second dimensionless number yields $\Pi_2 \sim 21.7$. Again, the uncertainties on the Hubble constant only allow us to know the order of magnitude of this number.

One can wonder whether pure chance could lead to the good order of magnitude for both relations. Given the great discrepancy between all the involved constants and properties (Tab. 1), one can check that any other dimensionless combination of these variables always leads to huge or tiny orders of magnitude. Then, the probability that two such relations between independent physical variables exist must be close to zero. In the following, we thus assume that these relations are not two coincidences and discuss their possible implications.

3. Discussion

3.1 A tangible fourth spatial dimension

Before proceeding, we make some remarks about the current state of the art of gravitation. General relativity is based on the curvature of spacetime due to the presence of matter. Since human beings cannot depict the deformation of a three-dimensional space, literature often describes it through an analogy with the deformation of a two-dimensional space into a three-dimensional one (Fig. 1). But this only corresponds to an illustration of the three-dimensional space warp in a four-dimensional one.

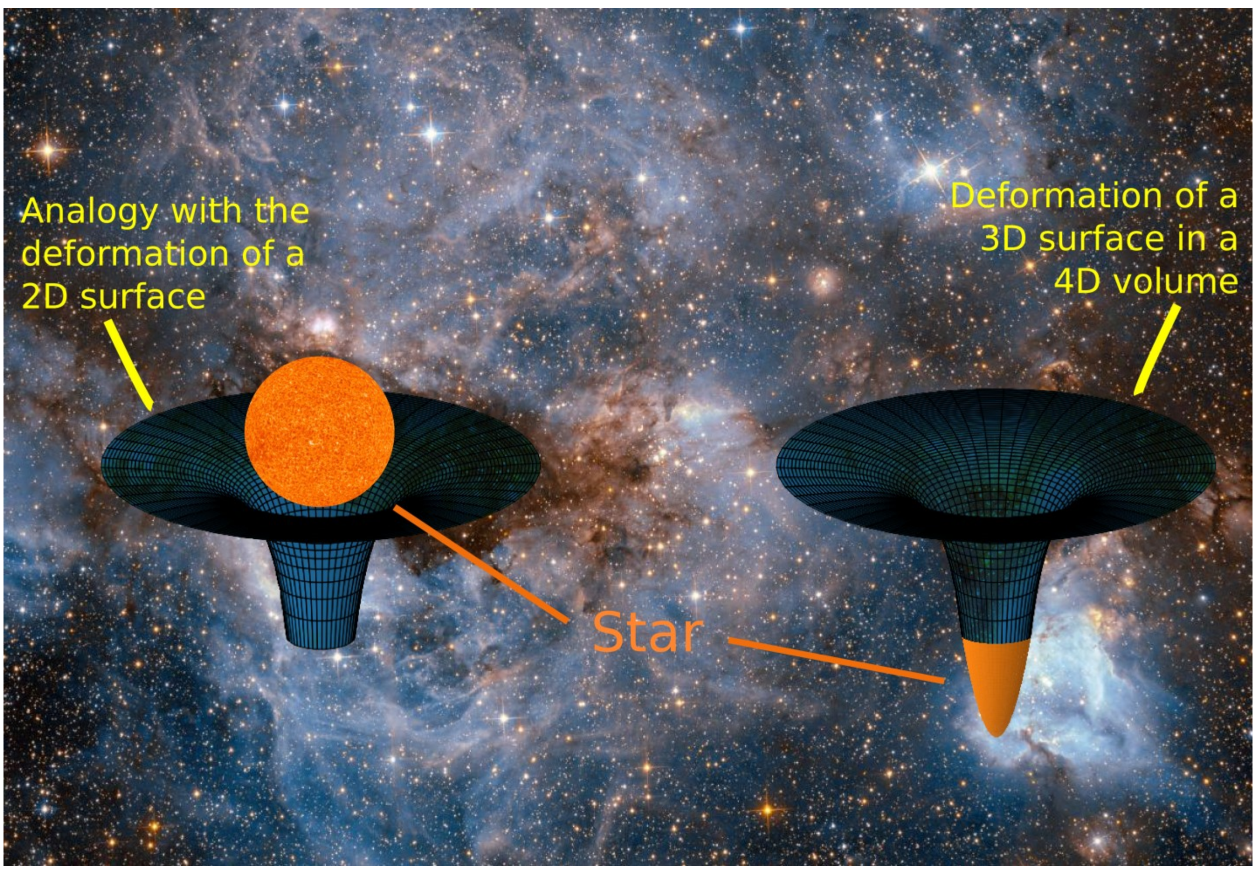

Fig. 1 Schematic representations of space deformation. Left: With a two-dimensional analogy for space but not for the star (general relativity). Right: With the analogy for both (more consistent).

Besides, the universe's expansion shows that the observable universe does not contain any point that could be considered as the big-bang origin. Indeed, the Hubble-Lemaître law states that all points move away from Earth and there is no reason for our planet to be the center of the universe. To understand this process, scientists again use an analogy with two-dimensional spaces. For instance, if the curvature of the universe is positive, our living space can be understood as the surface of an inflating ball. All the points of the surface of an inflating ball move away from each other while the center does not belong to this surface. By extending this analogy to three dimensions, our living space would be a 3-sphere (sphere in three dimensions) embedded into a four-dimensional space. This was already Einstein's feeling after the publication of general relativity. He considered that the universe was a static 3-sphere [26].

The relation $c_{waves} \propto \sqrt{\frac{P}{\rho}}$ derives from continuum mechanics. However, the critical density is too low to assume that vacuum is continuous so this relation should provide an aberrant result. Yet, relation (3) gives the value of the speed of light with a density of a few atoms per cubic meter. How is it possible? One way of overcoming this argument is to add at least one spatial dimension to the universe. If we assume that our world seems to be in three dimensions but presents a fourth one, then the pressure P should be given in $N.m^{-3}$ and the density in $kg.m^{-4}$. The density in $kg.m^{-4}$ would then be given by $\frac{\rho_c}{L}$ with L the thickness of our "three-dimensional" home (in the direction of the fourth dimension). A very weak value of L could lead to a great four-dimensional density (same argument for the pressure) and make continuum mechanics applicable. This would not change the proportional relation giving the speed of waves since only the ratio of both variables intervenes in this relation. Therefore, relation (3) would entail that the spatial dimension of the universe is at least four. The existence of a fourth spatial dimension could involve the reality of the analogy presented at the beginning of this section. Like Einstein's feeling, our living space would be the surface of a 4-ball which corresponds to a 3-sphere but the 4-ball would contain matter. The propagation of waves in vacuum would be a four-dimensional phenomenon. The surface would actually be a very thin volume (in four dimension) so that we underestimate the number of dimensions.

3.2 The nature of gravitation

According to general relativity, gravitation corresponds to the deformation of spacetime induced by masses. Einsteins's theory does not concern the physical process of this deformation. But the presence of matter in the fourth dimension would seem to require the deformation to result from a physical phenomenon. According to the strong equivalence principle, gravitation corresponds to acceleration. In adhering to this principle, the force applied on high-mass bodies, which produces the deformation of the surface, must derive from an acceleration. The observations show that the expansion of the universe is accelerating. Gravitation might then be explained by the following process: due to the acceleration of the expansion of the universe, the high-mass bodies in the 3-sphere would be maintained against some matter that is located in the fourth dimension, out of our field of view. Then, bodies would locally deform the three-dimensional surface because of the acceleration. This deformation would thus stem from the same process as the deformation of a material supporting a mass on Earth (or in an accelerated frame of reference). The motion of a body in a gravitational field would correspond to the motion of a body, which experiences a vertical acceleration, on a deformed surface. In this theory, it is worth noting that gravitation would only exist in the 3-sphere and the four-dimensional material would not be affected by gravitation. Figure 2 displays this vision of the universe.

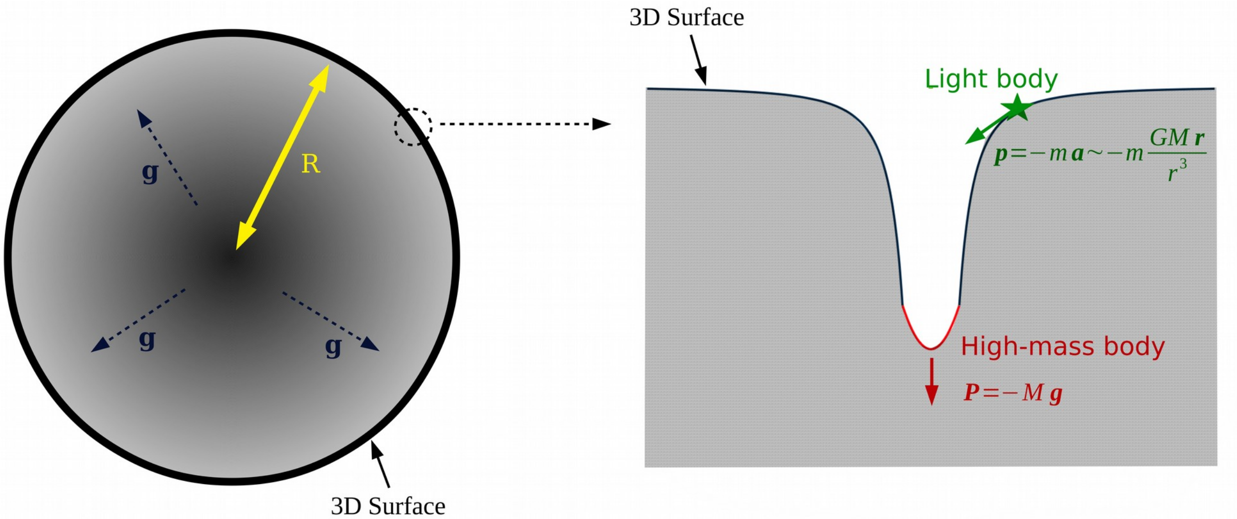

Fig. 2 Pentadimensional structure of the universe (four spatial and one temporal dimensions). It is assumed that the space curvature is positive. On the left: The universe would be a four-dimensional ball so its expansion would correspond to a radial acceleration g. On the right: Physical deformation of the surface due to the radial acceleration. A light body would only experience a part of it, approximately $a=\frac{GM}{r^2}$ according to Newton's law of universal gravitation.

3.3 Interpretation of some physical phenomena

This way of understanding the universe might explain several physical matters. First, the universe's expansion would be the result of some processes inside the four-dimensional ball. For instance, the interaction between particles could generate this expansion. This energy could correspond to the so-called dark energy. Its location, out of our three-dimensional world, could explain its mysterious origin. The asymmetry of matter and antimatter might also originate from this extra dimension.

Second, a physical explanation of gravitation is provided. With this definition, gravitation would not be a fundamental interaction. It would result from the acceleration of the universe's expansion and the other interactions. In particular, the bodies at the surface would interact with the particles in the fourth dimension. At macroscopic scale, this process would correspond to a support reaction. Gravitation would thus be intimately linked with electromagnetism. In addition, this definition removes the singularities from Newton's law of universal gravitation and general relativity. The gravitation law in $\frac{1}{r^2}$ would not be reliable until the center of bodies since continuum mechanics does not present such a uniform deformation profile. An inflection point would exist at the radius of bodies and another law would take place inside.

Third, some effects predicted by general relativity could also be interpreted in the framework of this theory. The loss of energy due to gravitational waves could be the consequence of friction with the matter in the fourth dimension. It is worth noting that friction, to a certain extent, also exists in the current scientific models. The observations show that spacetime is expanded because all bodies are moving away from each other. Without friction between bodies and spacetime, the former would slide on the latter and would not follow its expansion. The Lense-Thirring effect could stem from the dragging of four-dimensional matter due to the rotation of high-mass bodies and friction.

3.4 Incompatibility with relativity?

Contrary to aether theories, we do not consider additional matter between us and stars radiating light. The propagation of waves is not achieved in a three-dimensional material but in a four-dimensional one that has the same density as the apparent 3D “vacuum”. However, like these theories, there is an absolute frame of reference. This contradicts Einstein’s feeling about the isotropy of the speed of light. Yet, in addition to relation (3), it is well-known that vacuum presents some continuous properties similar to materials such as a non-zero permittivity or a stiffness to some extent (according to general relativity). Besides, Einstein admitted that general relativity required the existence of a particular form of aether without mechanical properties [27]. Then, is it safe to conclude that a four-dimensional absolute frame of reference cannot exist?

Based on the isotropy of the speed of light, special relativity explains time dilation and length contraction. Can we ensure that it is not the other way around? Could time dilation and length contraction make the speed of light isotropic from our point of view? The isotropy would just stem from the nature of spacetime. Spacetime would be intimately linked to the properties of the four-dimensional material: speed of material waves (special relativity) and topology (general relativity). Then, from the absolute frame of reference, the speed of light would not be a fundamental constant (like material waves), but from the three-dimensional surface, time dilation and length contraction would always make us measure the same speed. The relativity theories would then only be applicable in our three-dimensional visible world, where there is no absolute frame of reference (3-sphere), but not outside it. This hypothetical interpretation authorizes light to propagate in matter without disagreeing with the isotropy of the speed of light on which relativity theories are based.

Thus, special and general relativity would be the good mathematical framework to describe spacetime, but would not allow us to understand all the involved physical processes. We highlight that the propagation of waves in vacuum could be incompatible with physics.

3.5 Quantum mechanics

A fourth dimension containing matter could lead to another interpretation of quantum mechanics. If gravitation results from a support reaction, this means that we are always subject to interactions with particles from the fourth dimension. Therefore, the apparent chance, on which quantum mechanics is based, could correspond to these interactions (which we cannot observe). Moreover, quantum fluctuations might correspond to the motion of particles from the three-dimensional surface to the fourth dimension and *vice versa*. In this sense, energy would be conserved in the four-dimensional space and not necessarily in our 3D world. This travel, with a certain probability, might be possible for particles, but not for too big bodies that dense four-dimensional matter would block at the surface. Thus, from the three-dimensional surface point of view, a probabilistic theory would be needed to describe particles but not big enough bodies. Relation (2) could substantiate this point of view because it might be understood as

$$h=\frac{\Pi_2\sqrt{\rho_c}}{\left(\varepsilon_0 e^{-2}\right)^{5/2}\left(k_B T\right)^2} \tag{4}$$

In this case, the Planck constant (and quantum mechanics) would depend on the vacuum properties.

A fourth dimension containing matter where the propagation of waves is possible might also give an explanation of wave-particle duality. Could the wave nature of particles stem from the propagation of waves in the four-dimensional material? At this scale, particles could be affected by such waves and behave like them whereas they are only particles. They would present a wave motion because of the existence of waves impacting them. It is worth noting that Bell's theorem does not avoid such a non-local hidden variable theory. This is similar to De Broglie's idea and the De Broglie-Bohm theory [28], [29]. This pilot wave theory is based on the Madelung equations [30], [31]. Madelung noted the similarity between the Schrödinger equation and the Navier-Stokes equations. In particular, the quantum equation can be understood as a continuity equation for the probability density.

3.6 A minimization of the size of the universe

It is possible to minimize the size of the assumed structure of the universe (Fig. 2). The Hubble-Lemaître law states that the recessional velocity v is proportional to the proper distance D: $v=HD$ with H the Hubble constant. In the framework of the theory, the Hubble constant would not be truly a constant but we will take the actual Hubble constant for the minimization. Therefore, the recessional acceleration is given by $a=H^2 D$. We note R the radius of the universe and g the radial acceleration (Fig. 2). The same relation holds for both variables: $g=H^2 R$. The radius of the universe can then be deduced from an estimation of the radial acceleration. This acceleration must be greater than all the gravitational accelerations of the bodies in the universe. The Earth's acceleration is about 10 $m.s^{-2}$. This value, which should be far lower than the greatest acceleration, yields $R\sim 2\times 10^{20}$ light-years. This corresponds to 4.5 billion times more than the observable universe. This agrees with the apparent flatness of the universe from our point of view.

3.7 Variation of some fundamental constants?

The previous interpretations raise the question of the variation of some fundamental constants. The speed of light and the Planck constant could vary against time if relations (1) and (2) really describe physical phenomena. In this case, the definition of gravitation (Fig. 2) would also involve the variation of the gravitational constant. The latter would indeed depend on both the acceleration of the expansion and the pressure term P. For instance, the deformation of a solid depends on the applied forces and Young's modulus. Since both quantities should vary according to time, the gravitational constant would also vary.

The fine-structure constant is defined as $\alpha = \frac{e^2}{2\varepsilon_0 h c_{light}}$. This dimensionless number has been subject to discussion about its value and its possible variation for decades. Indeed, its order of magnitude indicates that an underlying phenomenon must take place. Like A. Eddington, many scientists have tried to find a simple formula giving its experimental value without success.

Thanks to relations (3) and (4), we can substitute the speed of light and the Planck constant in the definition of the fine-structure constant. This yields

$$\alpha^{-1} = 2\sqrt{\Pi_1}\Pi_2 \sim 137.035 \qquad (5)$$

Contrary to the Planck constant, the speed of light and the gravitational constant, the fine-structure constant would truly be a constant. Despite this new form of α, only doubtful complex formulas giving Π_2 were found.

4. Conclusion

From a dimensional analysis, we have found two relations linking the Planck constant and the speed of light with fundamental constants and properties of vacuum. These relations either genuinely describe physical phenomena or are just two coincidences. Given that the relation about the speed of light is based on a well-known continuum mechanics formula, this is rather probable that they are not a matter of pure chance. This is corroborated by the order of magnitude (10^1) of both dimensionless numbers (1) and (2) in spite of the great disparity between the involved fundamental constants and vacuum properties.

In this framework, these relations could involve the existence of at least one another spatial dimension. Based on Einstein’s theories, the universe would be a 4-ball containing matter while our living world would correspond to its surface, a 3-sphere. The developed theory could explain the so-called propagation of waves in “vacuum”, some astonishing aspects of quantum mechanics, dark energy and the nature of gravitation. The apparent contradiction with relativity can be overridden with another interpretation of Einstein’s theories. In particular, the isotropy of the speed of light would derive from time dilation and length contraction which would be linked to the properties of the four-dimensional material. The speed of light would not be a constant from the hypothetical absolute frame of reference, but would appear as such from our point of view. It could also account for the potential variation of some fundamental physical constants. Finally, the two relations could lead to another interpretation of the fine-structure constant. In particular, they could provide clues on the underlying phenomenon inherent to this constant that scientists have tried to discover for decades.

In this study, the mathematical part of this theory is left to be constructed. Regarding quantum mechanics, we do not question the Schrödinger equation, only its interpretation. In particular, relation (4) is more consistent with pilot wave theories. Even if this study does not question the equations stemming from general relativity neither, we can however wonder whether four-dimensional mechanical deformations can lead to Newton's law of universal gravitation as a first approximation (without relativistic effects).

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