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Comparative water pricing analysis:
Duality formal-informal in a CGE model for Senegal

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Abstract:
In this paper we build a static CGE model for Senegal that allows for comparative analyses of different water pricing schemes. The model defines five sectors (drinking water production, drinking water distribution, the agricultural sector, the industry and services). The originality of this model consists on defining the duality formal–informal in the drinking water distribution sector. The main reasons are firstly that the current water prices are different between both segments and secondly, that we focus on substitution possibilities of households regarding their choice of water distributor. The objective is to estimate the production and employment impacts of water policy pricing on the development of both formal and informal water distribution segments. The aim is to determine the policy which makes water service affordable for all households and to analyse the effects on the income distribution for three consumers categories (rural, Dakar and other urban areas).

JEL classification: C68, O17, Q25

Keywords: Water Pricing, Computable General Equilibrium Model, Informal, Africa

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Introduction

The World Bank does not cease promoting the participation of the sector deprived in particular in the water sector. However the analysis of the current situation of the developing countries, and in particular of sub-Saharan Africa, shows that many households (especially with low income) of the urban or rural areas still do not have access to drinking water in spite of the modernization of the sector. Indeed the large distributive firms of water have evil to ensure the service in the small centers and secondary districts of the large cities, probably because of their structure and policy badly adapted to these marginal or not very profitable zones. However these zones shelter a significant part and in full growth of the population. This is why, in the absence of a more effective modern water service, there is an "alternative" service, ensured by small private operators who generally belong to the "informal" sector. Surveys of ground (Hydroconseil, 2000) carried out in several countries of sub-Saharan Africa revealed their great dynamism and especially, their dominating economic weight in the sector, in terms of employment, a number and sales turnover of served families.

Any water policy in the countries of sub-Saharan Africa should integrate the existence of these actors ignored too a long time. This is why, the article proposes the construction of a coherent theoretical framework making it possible to describe and include/understand, the transmission channels of the shocks of water tariff policy in Senegal. To treat this question, the recourse to a Computable General Equilibrium Model (MEGC) is a particularly suitable methodology since it makes it possible to model the behavior of all the sectors and agents intervening in the described economy, as well as the relations which bind them.

The MEGC concentrates on the sectors users of the water resource: drinking water production, drinking water distribution and a agricultural sector. More still, taking into account the significant number informal operators intervening in the drinking water distribution, the accent is related to the possibilities of substitution and arbitration between the formal and informal segments of this market. The treated question is that of the effects of changes of water tariff policy in Senegal on the respective evolution of the two segments in terms of drinking water distribution and employment. If a tariff policy makes it possible to increase the access to the formal network for all, which will be to become to it informal sector of water? Which will be the impacts in term of efficacy and equity, like on the total wellbeing? The first part of the article dealt with the specificity (duality) of the drinking water market in sub-Saharan Africa to show its impact in term of heterogeneity on the prices of water supported by the households. The second part presents the government policy, changes in water tariffs in Senegal (1996-2002) and the inequities to the poor. The third part describes the possibilities of water tariffing which will be then considered like scenarii in the MEGC. The fourth part presents the MEGC methodology and the last part relates to the justifications of the modeling choices of the various productive sectors and agents of the described economy.
1. Duality formal-informal in the drinking water distribution in sub-Saharan Africa and its impact in term of the water prices heterogeneity

Empirical work (Hydroconseil, 1998) on the drinking water distribution in sub-Saharan Africa highlights a certain number of common characteristics which we quickly present in this section.

First of all, the service offered by the informal operators must be regarded as a true complement of the public company, which answers badly at the request of the populations with low income. These private operators develop in the "interstices" of the public utility, i.e. they supplement the gaps of the service offered by the distributive firm, while answering a request atomized in badly parcelled out districts or of recent installation. The largest consumers (in a number) of water *via* the informal segment of distribution are located in the called peripheral districts secondary centers or in difficult access points. All these areas with the margin of the remainder of the city complicate the supply water by the traditional networks of distribution.

This informal water sector has as a dominant function the balance of the shortage. Within the total circuit, the purchase with the abstract operators fulfills a regulating function of adaptation of the water service to the economic risks. This offer makes it possible many households to be able to have each day a quantity of minimal drinking water, by splitting their expenditure according to their daily and irregular incomes.

The macroeconomic weight of these operators in the water sector is considerable. It is significant in term of added value since according to a report/ratio:2 the private operators of the drinking water sector realize between 21% and 84% of the added value of the die although they are located for the majority in the informal sector. Moreover, the proportion of jobs created by these informal activities is even larger (3 to 13 times more) that of the distributive firms, even if it is often about precarious or transitory employment. The personnel thus employed would account for 2 to 4% of the cities population, that is to say 2 to 3% of the stable use of the cities.

1.2 Impact of this "atypical" market structure on the drinking water tariffing

1.2.1 The heterogeneity of the water prices on the “dual” drinking water market in Senegal

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2 Synthesis of the various terrain surveys led to Burkina-Faso, in Cape Verde, in Haiti, in Mali, in Mauritania and in Senegal (Hydroconseil, 1998).

2.1 New water tariffs

In past (1996) at the time reform (water privatization) was initiated, water tariffs in Senegal were at levels that covered operations and maintenance, but they did not come close to meeting the investment needs of the sector. Given the precarious situation of the central government’s finances at the time, it was imperative that the water sector be self-financing if the badly needed investments in network renewal and extension were to be made. The government has thus looked to consumers to finance these investments through tariffs, while trying to buffer the effects of this policy on the poor.

Senegal uses an “increasing block tariff” structure, with a subsidized “social tariff” for levels of consumption below 20 m³ in a 60-day period, a regular tariff for consumption over this, and a “dissuasive” tariff for consumption above 100 m³ per 60 days, which is designed to be a disincentive for excessive water use. The idea behind an increasing block tariff is that a cross-subsidy will be in effect: that consumption in the higher blocks will generate enough surplus to finance the subsidy delivered to customers consuming water in the lowest “lifeline” block, who are, in theory, the poor (however, this is not always the case).

The tariff consists of several components: the rate the operator charges for operation and maintenance of the system, a component to cover the costs of SONES and ONAS (the state organization responsible for sanitation), and various taxes.

The average tariff has risen steadily. In 1996, the average tariff faced by consumers for both water and sanitation, including all taxes and surcharges, was 380.42 FCFA/m³, and in 2002 it was 464.84 FCFA/m³, an overall increase of just over 22% (SONES).

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3 Under the recently introduced new tariff structure, the second tariff block ends at 40 m³.
4 Société Nationale des Eaux du Sénégal (state asset-holding company in Senegal)
5 Office Nationale de l’Assainissement du Sénégal (National Sanitation Office)
In most years (1996-2002), the tariff increase was applied more or less evenly across the domestic consumer categories (the “social tariff”, “full tariff”, and “dissuasive tariff”).

More recently, however, there have been large variations in the revisions to the various tariff categories, partly due to an overall change in the tariff structure introduced in 2003. This new tariff structure (which actually took effect in November 2002) was introduced by SONES because of the need to streamline the system and to raise extra revenue for ONAS. This structure preserved the increasing block tariff for domestic connections, but reduced the size of the second block (the “full tariff”) from 100 m³ to 40 m³, so that the “dissuasive” tariff was applied for any consumption over 40 m³. A single tariff block was introduced for all non domestic-customers, including industry and government clients. In addition, the price of water purchased by market gardeners above a certain quota was substantially increased. In fact, market gardeners had benefited from very low prices for water. Below the quota, the price they pay is still the lowest price within the tariff structure. However, this change was extremely controversial and the new tariff for market gardeners was almost immediately suspended.

Under the 2003 tariff the price for water in the “full” tariff block was decreased for the first time. The social tariff went up, but this merely reversed a decrease in the social tariff that had occurred in 2002. The overall increase in the social tariff has been 16% over the seven years of the reform.

The reform has provided a considerable increase in the price of water for standposts in 2002. This appears to have occurred in part due to the fact that, for the first time, a sanitation surcharge was added to the tariff category under which standposts are billed. In towns with sanitation systems, this resulted in a 9.7% increase in the prices charged to standpost operators. It can be assumed that this price increase was passed on to their customers. Overall, the price of water charged to standpost operators has increased by 35% over the seven years of the reform, compared to about 20% for other customers: What is the poverty impact?

In terms of the changes faced by consumers with connections, between 1995 (before the reform started) and 2003, consumers in all domestic categories would have seen their bills (for both water and sanitation combined) increase by over 20%. A family with low consumption of 10 m³ per month, falling in the social block of the tariff, would have paid a monthly amount of 1 567 FCFA in 1995, and 1 863 FCFA (about 3.40 $US) in 2003 (an increase of 19%). A family with higher consumption of 30 m³ per month would have paid 12 257 FCFA per month in 1995, and 15 689 FCFA (about 28.50$US) in 2003 (an increase of 28%).

2.2 Water service to the poor

A substantial proportion of the population of Senegal is poor. It is estimated that 54% of the population of 9.2 million lived below the poverty line in 2002\(^6\). Many of the poor live in rural areas, but urban areas also have high rates of poverty: in 1995 it was estimated that 16% of Dakar’s population was poor\(^7\), a percentage that is probably higher today.

These poverty statistics are reflected in water supply coverage, which WHO estimated in 2000 to be 65% for water in rural areas. SONES estimates in 1996 were that urban coverage overall was 72.5%, of which about 22% was through public standposts. If approximately 46% of the total population of Senegal lives in urban areas, this translated at the time to over one million urban dwellers who were completely unserved, and over 850,000 who were provided water through public standposts only, (Trémolet, 2002).

Improvements to water supply for the poor were an important part of the reform process, and were identified as a priority by the government, bilateral donors, and the World Bank. Increasing accessibility to safe drinking water for the urban poor was identified as an important feature of the World Bank-funded Water Sector Project, and funds for connections for low-income households and public standposts amounting to 8.7 million $US were allocated in the budget. It was predicted that these funds would be sufficient to install 34,000 social connections and 400 public standposts over the project. Using an estimate of 10 people served per social connection and 300 people per standpost, this means 460,000 urban dwellers would be served: 57.5% of the estimated urban uncovered population.

2.3 Government water policy to the poor

The policy of the Senegalese government is to provide water service to all households through formal mechanisms, such as private connections and official, licensed vendors at standposts. The standposts themselves are seen as a temporary method of supply, and the goal is eventually to provide each household with a private connection.

The government has a policy of providing small diameter (15 mm) private connections to poor households at subsidized rates: these are referred to as “social connections” (“branchements sociaux”). There is an established set of criteria for determining which households are eligible. The government of Senegal also has a policy of subsidizing water consumption up to a certain volume. This is currently done through the “social” block in the tariff and this subsidy is, in theory, aimed at the poor.

The subsidies targeted at the poor in the water sector thus take three forms:

- Subsidized connections through the social connection program, financed by government funds (some of which are provided through the World Bank project).
- Construction of standposts in areas where there are people without private connections, financed by the government with funds from the World Bank project, and supply of water to these standposts at low rates (the standposts are managed by private operators recruited by SDE in consultation with the local community).
- Subsidized consumption at low levels of consumption, financed through a cross subsidy between customer categories and delivered through an increasing block tariff with a “social tariff” for household consumption consumption under 10 m³ per month.

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10 Consumption under 20 m³ in 60 days is billed at a low rate (about 30% of the price of water in the next block of the tariff). The underlying assumption is that poor families have low water consumption and will not exceed 20 m³ whereas wealthier households will (there are, however, no data in Senegal to confirm this assumption, and research in other countries has found that water consumption does not tend to be correlated with income.
11 Sénégalaise des Eaux (Senegalese water utility)
The government thus had a clear commitment to making service both accessible and affordable to the poor, and proved to have the political will to act upon this commitment.

2.4 Inequities in service to the poor

Despite the fact that government policy was oriented towards improving service to the poor, and that this was backed with funds for both consumption and connection subsidies, there are certain flaws in the way the government targets and delivers these subsidies.

High effective tariffs at public standposts:

Public standposts have the advantage of providing service quickly to those living in areas where the piped network does no reach, but they do not necessarily provide the lowest-cost water to the poorest. For example, under the 2003 tariff, introduced in late 2002, the water at the standposts was sold to the licensed vendors who run the standposts for 315.09 FCFA/m³, while the “social tariff” block was priced at 186.32 FCFA/m³. Over the seven years of the reform (1996-2002), the standpost tariff has systematically been higher than the social tariff.

Consumers using water from the standposts have to pay the overhead of the licensed vendor, or re-seller, who manages the standpost. An example of the high prices standpost users thus pay was observed in November 2002 at a standpost: users purchase water in 40 liter plastic basins, at 25 FCFA per basin. The volumetric cost of this water is thus 625 FCFA per m³. The official tariff category for public standposts is 315.09 FCFA. This means that users are paying a 98% “overhead” on this water. Users at this standpost are therefore paying about 350% of the current social tariff and about 2% more than the regular, unsubsidized tariff for private connections and commercial institutions.

Many of the poor are excluded from social connections:

The standposts are seen as a temporary solution. The government wants to reach all the poor with private connections through the policy of providing subsidized “social connections”. However, this policy suffers from a major flaw: the criteria that make a household eligible for the subsidy more or less guarantee that it is not poor. “The poorest households are precluded from having subsidized connections because the social connection programs are intended for stable neighborhoods where the residents have established themselves”, (D.T. Lauria, O.S. Hopkins, 2003). In order to obtain a social connection, an applicant must have title to the land, and an existing house must be located on it.

3 Water pricing scenarii in the MEGC

3.1 Marginal Cost Pricing: the “first best optimum”

Theoretically, in a partial equilibrium state, the Pareto optimal prices would be equal to the marginal cost of producing water.

The marginal cost pricing approach implies that the marginal cost of producing water equals the marginal benefits of consumption water. In the model, we have several demands for

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primary water and several demands for drinking water. The equation defining the marginal pricing approach for each demand is the following:

\[ P_{wau} = Cm_{wau} \]

### 3.2 Ramsey-Boiteux Pricing (RBP): the “second best optimum”

However, given the cost structure of producing water (natural monopoly with important fixed costs), this type of pricing can lead to surpluses. But more generally, it leads to deficits.

So, authorities have to finance these deficits through taxes. The taxation methods are almost all distorted, therefore a second best optimum should be considered.

Ramsey (1927) and Boiteux (1956) proposed a taxation formula that was further formalized by Baumol and Bradford (1970). The method is known as Boiteux-Ramsey pricing (BRP). In the model of Baumol and Bradford (1970), the pricing method consists of setting quasi-optimal prices for each market such that they deviate from the marginal cost inversely proportionate to the demand price elasticity in the given market.

Consequently in the BRP method, the government’s objective is to maximize consumer surpluses while imposing a budgetary equilibrium on water authorities.

Decaluwé et al (1999) have tested this water pricing scenario in a CGE model applied to Moroccan. The authors demonstrate that the Boiteux-Ramsey water price for each market is given by:

\[ P_{BR, water,j}^{water} = \frac{Cm_{water}}{1 - \frac{\xi_{water}}{\varepsilon_{d, water,j}}} \]

\[ \xi_{water} : \text{The Ramsey weighted number} \]

\[ \varepsilon_{d, water,j} : \text{The price elasticity of demand for each water consumer j} \]

The procedure consists of imposing a budgetary equilibrium on water authorities and generating RB prices from the base year equilibrium. “The results will permit to measure the impact on water users and the general equilibrium effect on other micro and macroeconomic variables. With the RBP, we obtain several prices for water according to consumers. The discrimination is based on the elasticity of demand of each consumer as stated previously and not on socio-politico criteria” (Decaluwé, 1999).

### 3.3 “Increasing block tariff” structure with a subsidized “social tariff”

Senegal uses an “increasing block tariff” structure, with a subsidized “social tariff” for levels of consumption below 20 m³ in a 60-day period, a regular tariff for consumption over this, and a “dissuasive” tariff for consumption above 100 m³ per 60 days, which is designed to be a disincentive for excessive water use.

But, “increasing block tariffs” do not target consumption subsidies well. Indeed, there are many criticisms of increasing block tariffs as subsidy delivery mechanisms. In fact many poor people in Senegal are not benefitting from the subsidized block of the tariff at all, simply
because they do not have connection. In low-income areas where connections are few, many families may share a single connection, and thus consume most of their water in a higher block of the tariff. “These people would then cross-subsidizing small families who used less than the 10 m³ per month of the social tariff, some of which might be wealthy. As 10 m³ is a fairly generous monthly amount, there may be many relatively well-off households in urban areas of Senegal who only consume subsidized water, thus benefiting from assistance intended for the poor. Subsidies delivered this way are thus not necessarily related to need; household water consumption is a notoriously poor proxy for poverty status” (J. Boland, D. Whittington, 2000)\textsuperscript{13}.

So, the question is who is eligible for social connections and how the tariffs are structured. The very poorest are disadvantaged by the fact that they are not eligible for social connections and pay more for water at the standposts than customers with private connections pay for their water. We want to test different social tariff and the impact on equity.

We propose this tariff structure to model “increasing block tariff”:

![Tariff Diagram](image)

Levels of consumption below 20 m³ in a 60-day

With:

- (AB): Drinking water “poor” demand
- (CD): Drinking water “no-poor” demand
- Ps: social tariff
- Pm: price equal marginal cost

4 Applied General Equilibrium Models and Water Policy in Senegal

Although we can obtain some important insights from partial equilibrium analysis, this framework presents limits for the analysis of the efficiency of water allocation. In fact, water is used in almost all production activity, being an essential input in many of them. Also, water value is highly dependent on time and location. Any change in water policy will probably have consequences on the sectoral composition of the economic product, on employment, on costs and prices, and on the income distribution between the rural and urban sectors. A Computable General Equilibrium Model provides a framework able to capture all the relevant economic effects of a changed structure of water prices.

4.1 Review of Some Literature on Applied general equilibrium models for water managment

Applied general equilibrium models for water managment are well suited to compare alternative policy scenarii such as in Berck, Robinson and Goldman (1991) who use a CGE which studies the reduction of water use in San Joaquim Valley as an efficient alternative to solve drainage problems. The works of Dixon (1990), Horridge et al. (1993), Decaluwé et al. (1999) and Thabet (2003) analyse the impact and efficiency of water prices. Nevertheless the use of CGE to analyse the reallocation of water rights between users is less common. Seung et al. (1998) studies the welfare gains associated to the transfers of water uses from agricultural to recreative uses in the Walker River Bassin (California). Seung et al. (2000), combines a dynamic CGE model with a recreative demand model to analyse the temporal effects of water reallocation in Churchill County (Nevada). Diao and Roe (2000) provide a CGE model to analyse the consequences of a protectionist agricultural policy in Morocco and show how the liberalization of agricultural markets ceates the necessary conditions for the implementation of efficient water pricing (particulary through the possibility of a market for water in the rural sector). Goodman (2000), by using an applied CGE, shows how temporary water transfers provide a lower cost option than the building up of new dams of the enlargement of the existing water storage facilities.

So the analysis of water allocation then requires a comprehensive view of the economy, and applied CGE methodology gives a potential framework to assess and compare policy options.

4.2 Methodology : the Social Accounting Matrix

The central theme of the study is to examine the impact of water policies on income distribution in Senegal with specific focus on water distribution sector policies.

The starting point for the CGE model is the construction of the Social Accounting Matrix (SAM) of the Senegalese economy. This is because the SAM provides a useful framework for the construction of the CGE model by describing the structure of the economy and illustrating the interdependencies in the economy. It also serves as a consistent database to be inputted into the model.

The SAM is a square matrix, which organises information from different sectors and different agents in the economy. SAM is a synthesis of two main economic concepts in economics, which are the input-output table and the national income accounting. The input-output table portrays the economy as a system of inter-industry linkages. This is generalised in SAM, as one sector’s purchase is another sector’s sale. As in the national income accounting, income
must equal expenditure. As a result, the information in the SAM is organised in such a way that each transaction or account has its own row and column. The payments are listed in the column and the receipts in rows. Since each account must balance, the corresponding row and column total must be equal.

The Senegalese’s SAM must be structured to capture the pronounced Dakar/Other urban areas/Rural disparity in economic structure and other socio-economic characteristics of the different households according income quintile. The SAM must be used to examine the income, efficiency and equity effects of alternative water pricing.

5 The model structure

5.1 Basic assumptions

We consider basic assumptions: economic rationality, perfect information, perfect competition, neutral currency. The economy uses six production factors: primary water, capital, formal labor, informal labor, irrigated land and non-irrigated land.

We build a static theoretical model which tries to capture both the economic structure and the drinking water distribution problems of Senegal. For this purpose, we distinguish five economic sectors: drinking water production, drinking water distribution (formal/informal), agricultural production (two segments are distinguished between irrigated/no-irrigated), industrial production and services. Any sector produces a particular good or service.

There are four agents in the economy: firms, consumers, government and the Rest of the world. There is a representative firm in any economic sector. Household groups are distinguished on the basis of income quintile according to the geographic area Dakar/Other urban areas/Rural. The only activity of the government consists of selling primary water, collecting the tax water, tax revenues and distributing them to other agents (consumers, firms and the rest of the world) as lump-sum income transfers. Senegal is assumed to be a small open economy (price-takers) and, consequently, import demand and export supply of any good or service.

In the model, water resource is considered as a primary production factor in the drinking water production sector and in the agricultural sector. This factor is called “primary water” which quantity is assumed to be fixed, totally held and sold by government, and bought by the drinking water production sector and the agricultural sector.

5.2 Key sectors

5.2.1 Drinking water production

The MEGC developed by the economic literature to treat management of water differ on several levels. First of all, the model of Goldin and Roland-Holst (1995), suppose the existence of a perfectly elastic offer of water, being able to answer any request. Löfgren (1996) made the assumption of a total quantity of fixed water in the economy while

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Decaluwé et al. (1998, 1999) explicitly model the water production with various technologies according to whether water is extracted from the stoppings or the underground tablecloths. On the other hand, the model doesn’t comprise a drinking water distribution. This means whereas water coming from the stoppings or the tablecloths can directly be used for human consumption, without particular treatment. This assumption (of the MEGC of Decaluwé) seems to us strong and it appears relevant to us to build at the same time a sector of water production (with treatment) and a sector of drinking water distribution. Work of Thabet (2003) is for the moment the only ones to have to integrate in a MEGC a drinking water production/distribution sector, but this one is aggregate since no distinction is made between production and distribution.

Concerning the water demand, certain MEGC consider only the demand for water of irrigation by the agricultural sector (Löfgren, 1996; Goldin and Roland Host (1995)) while others (Decaluwé, 1998, 1999) distinguish a domestic request, agricultural and industrial. We integrate also these three distinct requests for water.

To define the drinking water production sector, one leaves the following assumptions:

It is supposed that the sector of drinking water production is represented by only one producer: the distributive firm of water with mission of public utility. It is charged to transform primary water by combining it with other factors of production (capital and labor) and with others intrants (such as energy, chemicals which make it possible to treat water…) to produce a "drinking water" of quality higher than primary water. This "drinking water" thus produced will be required as intrant by the sector of drinking water distribution, as an intermediate consumption by the industrial sector.

One defines the productive structure of following drinking water:

With the level low of the structure, the capital factor is combined with the labor factor by a function CES to form a called composite factor "capital-labor". Then, the composite factor is combined by a function CES with primary water to form the added value of the sector of drinking water production. Finally with the most level of the tree structure, the added value is combined with the total intermediate consumption of the sector by a Leontief function in order to form the total production of drinking water.

5.2.2 Drinking water distribution

The economic literature does not provide any example of modeling of a drinking water distribution sector since to our knowledge, no MEGC integrated such a sector. Taking into account our problems of research, we consider it very important to represent this sector in order to integrate the existence of a strong duality in the countries of sub-Saharan Africa between the formal segment of distribution and the informal segment (informal private drinking water retailers). This representation is essential to analyze thereafter, the potential effects of new water tariff policies in terms of substitutability between the two modes of provisioning of drinking water of the households (formal/informal), and the impacts on informal employment.

One postulates the assumption according to which the total offer of drinking water distribution emanates on the one hand from a formal segment and on the other hand, of an informal segment. The two segments are characterized by the fact that only one uses capital. Indeed, it is considered that the formal segment is equipped in capital factor (infrastructures
related to the public fountains for example) and in factor “formal labor” (dealers deprived in contract with the distributive firm or the State). While the informal segment is very intensive in factor “informal labor” (although the workers are generally accustomed to cumulating two types of employment: a formal use of farmer and, an informal employment related to the private water resale per bearing: case of the carters). From the informal statute of these last activities, it appears impossible to circumvent to regard this factor labor as having an informal statute.

Taking into account the empirical facts on the operation of the market of distribution of water, and of the followed problems, we propose the following productive structure:

In a conventional way, one poses on the 1st level of the tree structure a relation of imperfect substitution between the factors capital and “formal labor” what thus justifies the combination of both by a function CES to form a formal “capital-informal labor” composite factor.

On the 2nd level, one distinguishes two pennies levels. With the 1st under-level, intrant it drinking water of the formal segment combines with the formal “capital-informal labor” composite factor by a function CES describing imperfect substitutability between the two, to form the added value of the formal segment of drinking water distribution. With the 2nd under-level, intrant it drinking water of the informal segment combines only with the factor “informal labor” by a function CES translating imperfect substitutability between the two, to form the added value of the informal segment of drinking water distribution.

On the 3rd level, one distinguishes two more under-levels. With the 1st under-level, the added value of the formal segment is combined with the aggregate intermediate consumption of the formal segment by an additive function (Léontief), thus forming the formal drinking water distribution. With the 2nd under-level, the added value of the informal segment is combined with the aggregate intermediate consumption of the informal segment by a Léontief function, thus forming the informal drinking water distribution.

Lastly, with the most level of the tree structure (4th level), the productions of the segments formal and informal are combined by a Cobb-Douglas function to incorporate these two productions in a total offer of drinking water distribution.

The choice of the recourse to a Cobb-Douglas function is explained by the fact why the actions of the government as regards tariffing of drinking water and support for the domiciliary connection, should make the two segments perfectly substitutable to the long-term. Although terrain surveys show that currently the two services are differentiable according to the segment which operates, the distribution of water should be a homogeneous service. Ideally (taking into account the objective of the government of generalization of the access to drinking water for all), the informal network of water resale as a palliative of the too ineffective, under-dimensionalized and defective formal network current, should disappear under the effect of more effective policy. This disappearance would be positive in the direction where it would mean greater effectiveness of the formal network. A contrario, the impact in terms of disappearance of informal employment could be negative if employment is not created more in the formal segment. It is this reading of the facts which inspires the choice of the Cobb-Douglas function.

5.2.3 Agricultural sector

The agricultural sector, by the recourse to the irrigation, is a sector largely consuming water. This is why, we grant also a reflexion particular to the choice of the specifications of
production technologies of this third key sector of the economy. Those give significant information on the relations of substitution between the ground, fertilizers, water, capital and labor, like with the intermediate consumptions.

To build the agricultural tree of production, we took as a starting point the economic literature (denser concerning this sector), by establishing an assessment of the advantages and disadvantages of the various structures suggested by the agricultural MEGC integrating the water resource.

The regional MEGC of Berck, Robinson and Goldman (1991) aimed at evaluating the effects of a strong reduction of the water quantity allocated with the agricultural sector on the GDP, the sectoral production, the use of the land in the area of the valley San Joackim in California. Their function of agricultural production comprised four primary factors: water, land, labor and capital. In this model, the authors differentiate the land according to quality in three categories according to a hierarchical structure: the land of good quality can be used for a lower use but not the reverse. The water stock available is in fixed aggregate quantity. This resource is used only by the agricultural sector, which is not the case in our model. The authors distinguish two alternatives from the sector. An alternative with strong elasticity of substitution of the offer in which the added value (agricultural) is represented by a Cobb-Douglas function connecting the land, labor and capital. An alternative with low elasticity of substitution of the offer in which capital and land are used in fixed proportions and work is combined with this aggregate by a Cobb-Douglas function. However the use of such a specification of agricultural production limits the possibilities of substitution between the primary factors on the one hand, and the others intrants intermediate on the other hand. In particular, in the alternative with low elasticity "water, land and capital are used in fixed proportions" and there exists thus, "no relation of substitution between land and intermediate consumptions such as manures and the pesticides". In the alternative with raised elasticity, "substitutions between land, labor and capital are the same ones in all the agricultural activities and are reduced to the unit" (Thabet, 2003).

The MEGC proposed by Decaluwé, Patry and Savard (1999), called Aquam model, aimed at analyzing the impact of a irrigation water tariffing at the marginal cost then at the average cost on the Moroccan economy. The model represents two areas: abundant water North and South slightly equipped out of water. The authors take into account the various uses of the resource (domestic, agricultural and industrial). The structure of production agricultural is represented by a function CES encased on several levels. This one is based on the argument of Just (1991) according to which the introduction of possibilities of substitution between intrants into the agricultural function of production is of primary importance for a suitable analysis of the water question. In particular, it appears essential to represent relations of substitution between water, the factors of production and certain intermediate consumptions. Thus, one CES encased makes it possible to postulate a substitutability between the various factors of production and the intermediate consumptions, and this, at several stages of the production.

Taking into account the dense literature, we propose the following specifications of agricultural production:

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On the most level of the structure (5th stage), the agricultural production breaks up by an additive function (Léontief) between the added value of the agricultural sector and the total intermediate consumption (aggregate) of the agricultural sector.

On the 4th level, the added value of the agricultural sector breaks up by a function CES between two large substitutable composites: composite "Land-Fertilizer-Water" and composite "Capital-Labor" of the agricultural sector. It appears with the sight of the economic literature, that land, fertilizers and water can form a composite by a function CES. In the Aquam model, the composite comprises also the capital factor since this last is connected to the land to form under composite, which itself is connected to one 2nd under-composite (formed by fertilizers and water). We choose here not to integrate the capital factor in the formation of the composite, since it appears to us "unrealistic" to consider it substitutable by same elasticity with fertilizers, water, and land. However, to consider the same elasticity of substitution between fertilizers and water initially, then another elasticity of substitution between the composite "Fertilizer-Water" and land, appears more reasonable to us.

On the 3rd level, each of the two large composites disaggregate by a function CES. Composite “Land-Fertilizer-water” disaggregates in two composites: composite “Land-Fertilizer-Water” of the irrigation land (TFEIA) and composite “Land-Fertilizer-Water” of the no-irrigation land (TFESA). Indeed, in accordance with an assumption of the model of Thabet, we choose to distinguish land and thus, the composite which includes land, according to their utilization factor of the irrigation water. This assumption seems significant to us if we plan to distinguish the price from the irrigation water according to the intensity of the consumption of this rare resource (small-scale irrigation /large-scale irrigation). In the same way, according to the shock of irrigation water tariff policy, it seems probable that substitutions will be not only between water, fertilizers, and land within each segment (irrigation land/no-irrigation land), but also between the recourse to grounds of irrigation or dry. Composite “Capital-Labor” disaggregates (CES) in capital and labor of the agricultural sector.

On the 2nd level, each composite TFEIA and TFESA is disaggregated by a function CES. Composite TFEIA disaggregates in a factor irrigation land of the agricultural sector and in a composite "Fertilizer-Water" used by irrigation land. Composite TFESA disaggregates in a factor no-irrigation land and a composite "Fertilizer-Water" used by the dry grounds. It seems desirable to preserve here the composite "Fertilizer-Water" to substitute it for land, rather than to disaggregate more "brutally" out of land, fertilizers and water because the degree of substitutability between the three is surely not identical. Moreover, the literature showed the crucial relation of substitutability between the fertilizing intrants and water, considered as stronger than that between the factors of production (Hertel, Ball, Huang and Tsigas, 1989)\(^\text{16}\). On the level low of the structure, each composite FEIA and FEASA are to disaggregate by a function CES between the intrants Fertilizers and Water according to the respective ground of use (irrigation or dry) (Hexem and Heady, 1978)\(^\text{17}\).

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\(^{16}\) T.W. Hertel, V.E. Ball, K.S. Huang and M.E. Tsigas (1989), "General Computing Equilibrium Farm Level Demand Elasticities for Agricultural Commodities", Agricultural Experiment Station Research Bulletin no 988, Purdue University, West Lafayette (AZ).

5.3 Consumption, Government and foreign trade

5.3.1 Households consumption

The MEGC comprises three categories of households (three representative consumers): Dakar, Other urban areas and the Rural areas. The modeling of the structure of their preferences is similar, which does not prevent from preferably considering distinct parameters of behavior in term of consumption according to the category.

On the most level of the structure, the households decide total amount allocated with the consumption of goods and services, and that of the saving (Cobb-Douglas). Then, they determine the composition of their basket of consumer goods (made up of four aggregate products: drinking water, the agricultural good, the industrial good and service). These four goods constituting the basket of household consumption are incorporated by a linear system of expenditure (LES). The choice of this functional form is not arbitrary. It makes it possible to formulate modes of consumption which seem to us appropriate to the case of the developing countries since it is centered on the concept of "standard of living". Indeed, this system distinguishes within volume consumed two types of consumption: the "incompressible" consumption which is the volume of the product that must consume the representative consumer if he wants to maintain a standard of living minimal (exogenic), and, the "discrétionnaire" consumption which depends in an endogenous way, the price changes and the income available.

For the incorporated drinking water consumption, the households arbitrate, through function CES, between the water consumption via the individual connection offered by the formal segment of distribution, and the water consumption via other modes of formal and informal provisioning (no private connection).

Finally the demand for drinking water via other modes of provisioning (no private connection), disaggregates in a request for drinking water via the public fountains (standposts) of the formal segment, and a request for drinking water via the informal operators (carters: deliverymen in residence). A function CES described imperfect substitution between these the last two modes of provisioning.

However, the arbitration of the households between the water consumption via the public fountains of the formal segment and the informal operators (carters), is in more complex African reality. The concept of time "wasted" with the water collection is comparable to a measurable opportunity cost by the wages. Indeed, the water delivery in residence by the carters (informal) makes it possible the households (especially "poor") to allocate time with remunerative productive activities.

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18 The recourse to the form to describe the behavior of household consumption, in particular in the developing countries is recommended by literature (Decaluwé et al., 2001). The origin of the SLDS (linear system of expenditure of Stone) is due to work of Klein and Rubin (1947-1948), then of those developed by Stone (1954). Function of utility, of which the SLDS can be deduced, was formulated by Samuelson (1947-1948) and Geary (1950-1951), the SLDS being frequently called Stone-Geary system besides.
Households income:

For the households of the category $h$, the resources are consisted of the whole of the incomes of the primary production factors held to which the transfers of the government (assistances and social security benefits) are added, the received private transfers of the other categories of households and the RDM.

$$R^h = RES^h (1 - \tau^h) - tr_{h,G} - tr_{h,P} - tr_{h,RDM}$$
avec : $$RES^h = p_k \overline{K} + p_l \overline{L} + p_{Ih} \overline{I} + p_r \overline{E} + tr_{G,h} + tr_{P,h} + tr_{RDM,h}$$

5.3.2 Government

The only activity of the government consists in selling water, collecting tax and distributing them (transfers) to other agents of the economy. For government, revenue is generated from water sends, direct taxes on households income, indirect taxes on domestic goods, taxes levied on imports and transfers from other agents. Government spend it on subsidies and transfers.

The disposable revenue of government is:

$$R_G = RES_G - Sub_C - Sub_P - tr_{G,h} - tr_{G,RDM}$$
with:

$$RES_G = p_{ep} E_{pe} + p_{E_a} E_{Ea} + p_{E_s} E_{Es} + p_{CI} E_{CI} + DD + IINS + TEXP + \sum_{h} IR + tr_{RDM,G} + tr_{h,G}$$

5.3.3 Foreign trade

One poses the assumption of "small country" for all the products of the economy. This means that all the producers and consuming Senegalese are "price-takers". They are without influence on the world prices (neither with export, nor with the importation).

The majority of the MEGC model the foreign trade in an identical way. The assumption of "differentiation of the products according to their origin" as well with supply with is often posed. It is true that intuitively, it is understood that the industrial products and agroalimentary can be distinguished according to their geographical origin (preference different according to the origin from the imported products). This assumption would be also valid for the agricultural food products "rough" according to several work: over rice (S. Ito, 1990)\(^9\), on the corn (Johnson, 1979)\(^{20}\) or the fruit and vegetables (Sparks and Ward, 1992)\(^{21}\).


\(^{20}\) P. Johnson, T Grennes and Mr. Thursby (1979), "Trade models with differentiated products", American Newspaper of Agricultural Economics ", flight 61, n° 1, 120-127.

These studies recommend the recourse to the assumption of Armington\textsuperscript{22} to describe the behavior as regards choice between the domestic products and the foreign products. Moreover, the recent study of Hertel (1999)\textsuperscript{23} show that the recourse to this assumption is all the more appropriate to the agricultural produce to study the exchanges, taking into account climatic and agronomic considerations which can temporally limit the productions of certain agricultural produce which requires the recourse to the importation then: climatic, environmental constraints and on the water resource. Countries of sub-Saharan Africa whose Senegal, are particularly vulnerable with respect to these constraints.

Exports: it is considered that the domestic production $Q_i$ is made up of a part intended for the export ($X_i$) and of another intended for the provisioning of the domestic market ($QD_i$). In a very conventional way, one chooses to represent the behavior of the producers as regards transformation of the national production into $X_i$ and $QD_i$ by the function CET (Constant Elasticity of Transformation). It describes the structure of the preferences of the local producers in term of market aimed for the sales of their products.

Imports: it is supposed that the consumer of product $i$ sees himself offering a total volume of product $i$ ($CD_i$) which it buys in variable proportion with the RDM ($M_i$) and on the domestic market ($QD_i$). Its choice between the two sources of provisioning is expressed by a function of commercial substitution to elasticity of constant and finished commercial substitution.

**Conclusion**

Few resources have a as significant impact on the human life as water. This one is of primary importance with the maintenance of the human conditions and medical minimal. Taking into account the scarcity of this resource, economic science can unquestionably bring elements of reflexion on the instruments ensuring the conditions of its rational use. The problems of the tariffing of water cannot validly be studied that within a framework EGC which holds account on the one hand, of the bonds which link the various components of the economy and on the other hand, of the feedback effects of the economic agents decisions on the structure of the production and the request.

The immediate stages of work are as follows:

1) Specification of the arbitration of the households between the demand for drinking water distributed via the standposts of the formal segment and that distributed to residence by the informal operators (carters). We will define a micro-function of utility which will take into account the opportunity cost supported by the households (with low income) represented by the time allocated with the water collection, time which could be reallocated with a remunerative productive activity.

2) The determination of balances of market: goods and services and factors of production. The stress must be laid on that of work (formal and informal) and the mobility conditions.


\textsuperscript{23} T.W. Hertel (1999), "General Applied Equilibrium analysis of agricultural and resource policies", Staff Paper 99-2, department of Agricultural Economics Purdue University, march.
3) The construction of the Matrix of Social accounting (MCS) of Senegal, starting from the at the same time macroeconomic and sectoral data, will make it possible to carry out the calibration of the model. Elasticities will be borrowed from the economic literature.

4) We will be able to carry out simulations. The three modes of tariffing suggested will be the subject of the scenarii MEGC. The interpretation of the results will make it possible to deduce from them the impacts in economic term of effectiveness, equity, access to drinking water, as well as the possible substitution effects between the various factors of production. We will determine also the effects in term of poverty and total wellbeing.

Appendix

1. Arborescent structures

Figure 1: Technology of the water production

Figure 2: Technology of the drinking water dual distribution sector
Figure 3: Technology of the agricultural sector

Q_A
  \text{Leontief}
  \quad \text{VES} \\
V_{A_A} \quad C_{I_A} \quad 5
  \quad \text{CES}
T_{E_A} \quad K_{L_A} \quad 4
  \quad \text{CES}
T_{F_E_{IA}} \quad T_{F_E_{SA}} \quad K_A \quad L_A \quad 3
  \quad \text{CES}
T_{I_A} \quad F_{E_{IA}} \quad T_{S_A} \quad F_{S_A} \quad F_{E_{SA}} \quad 2
  \quad \text{CES}
F_{I_A} \quad E_{I_A} \quad F_{S_A} \quad E_{S_A} \quad 1

Figure 4: Households consumption

U
  \text{Cobb-Douglas}
  \quad \text{S}
C \quad 4
  \quad \text{ES}
C_{D_E} \quad C_A \quad C_I \quad C_S \quad 3
  \quad \text{CES}
C_{D_E_{FB}} \quad C_{D_E_{FI}} \quad 2
  \quad \text{CES}
C_{D_E_{FB}} \quad C_{D_E_{I}} \quad 1
2. Equations

2.1 Technology of the water production sector

\[ K_L^E = \left[ \alpha \cdot K_E \cdot \frac{1}{\sigma} \right brack \left( 1 - \alpha \right) L_E \cdot \frac{1}{\sigma} \]

\[ L_E = K_L^E \left( \frac{1}{p_k} \right) \]

\[ K_E = K_L^E \left( \frac{1}{p_k} \right) \]

\[ V_A^E = \left[ \alpha \cdot E_{p \cdot p} \cdot \frac{1}{\sigma} \right \rbrack \left( 1 - \alpha \right) K_L^E \cdot \frac{1}{\sigma} \]

\[ K_L^E = V_A^E \left( \frac{1}{p_k} \right) \]

\[ E_{p \cdot p} = V_A^E \left( \frac{1}{p_k} \right) \]

\[ Q_e = V_A^E + Q_e \sum a_{\text{DE}} \quad \text{and} \quad C_{\text{IE}} = Q_e \sum a_{\text{DE}} \quad \text{i = 1 à 5 (5 sectors)} \quad \text{i = E, DE, A, I, S} \]

2.2 Technology of the drinking water distribution sector

\[ V_A^{DEI} = \left[ \alpha \cdot E_{DE} \cdot \frac{1}{\sigma} \right \rbrack \left( 1 - \alpha \right) L_{DEI} \cdot \frac{1}{\sigma} \]

\[ L_{DEI} = V_A^{DEI} \left( \frac{1}{p_k} \right) \]

\[ E_{DEI} = V_A^{DEI} \left( \frac{1}{p_k} \right) \]

\[ K_L^{DEF} = \left[ \alpha \cdot K_{DF} \cdot \frac{1}{\sigma} \right \rbrack \left( 1 - \alpha \right) L_{DF} \cdot \frac{1}{\sigma} \]

\[ K_{DEF} = K_L^{DEF} \left( \frac{1}{p_k} \right) \]

\[ L_{DF} = K_L^{DEF} \left( \frac{1}{p_k} \right) \]

\[ V_A^{DEF} = \left[ \alpha \cdot E_{DF} \cdot \frac{1}{\sigma} \right \rbrack \left( 1 - \alpha \right) K_L^{DF} \cdot \frac{1}{\sigma} \]
\[ KL_{\text{DEF}} = VA_{\text{DEF}} \left( \frac{p'_{\text{V luxe}} (1 - \alpha_2)}{p'_{\text{K luxe}}} \right)^{\sigma_2} \]

\[ E_{\text{DEF}} = VA_{\text{DEF}} \left( \frac{p'_{\text{V luxe}} \alpha_2}{p_{\text{E luxe}}} \right)^{\sigma_2} \]

\[ Q_{\text{DEI}} = VA_{\text{DEI}} + C_{\text{DEI}} \quad \text{and} \quad C_{\text{DEI}} = Q_{\text{DEI}} \sum a_{\text{DEI}} \]

\[ Q_{\text{DEF}} = VA_{\text{DEF}} + C_{\text{DEF}} \quad \text{and} \quad C_{\text{DEF}} = Q_{\text{DEF}} \sum a_{\text{DEF}} \]

\[ Q_{\text{DE}} = A_{\text{DE}} Q_{\text{DEF}} a^1 - a \]

\[ Q_{\text{DEI}} = \left( \frac{p'_{\text{DEI}}}{p_{\text{DEI}}} \right) \left( \frac{(1 - \alpha) Q_{\text{DE}}}{\alpha} \right) \]

\[ Q_{\text{DEF}} = \left( \frac{a p'_{\text{DEI}}}{p_{\text{DEI}}} \right) Q_{\text{DE}} \]

2.3 Technology of the agricultural sector

\[ FE_{\text{IA}} = \left[ \alpha_6 F_{\text{IA}}^{1 - \frac{1}{\sigma_6}} + (1 - \alpha_6) E_{\text{IA}}^{1 - \frac{1}{\sigma_6}} \right]^{\left(1 - \frac{1}{\sigma_6} \right)} \]

\[ E_{\text{IA}} = FE_{\text{IA}} \left[ \frac{p'_{\text{FIA}} (1 - \alpha_6)}{p_{\text{FIA}}} \right]^{\sigma_6} \]

\[ F_{\text{IA}} = FE_{\text{IA}} \left[ \frac{p'_{\text{FIA}} \alpha_6}{p_{\text{FIA}}} \right]^{\sigma_6} \]

\[ FE_{\text{SA}} = \left[ \alpha_7 F_{\text{SA}}^{1 - \frac{1}{\sigma_7}} + (1 - \alpha_7) E_{\text{SA}}^{1 - \frac{1}{\sigma_7}} \right]^{\left(1 - \frac{1}{\sigma_7} \right)} \]

\[ E_{\text{SA}} = FE_{\text{SA}} \left[ \frac{p'_{\text{FSA}} (1 - \alpha_7)}{p_{\text{FSA}}} \right]^{\sigma_7} \]

\[ F_{\text{SA}} = FE_{\text{SA}} \left[ \frac{p'_{\text{FSA}} \alpha_7}{p_{\text{FSA}}} \right]^{\sigma_7} \]

\[ TFE_{\text{IA}} = \left[ \alpha_8 T_{\text{IA}}^{1 - \frac{1}{\sigma_8}} + (1 - \alpha_8) FE_{\text{IA}}^{1 - \frac{1}{\sigma_8}} \right]^{\left(1 - \frac{1}{\sigma_8} \right)} \]

\[ FE_{\text{IA}} = TFE_{\text{IA}} \left[ \frac{p'_{\text{TFEA}} (1 - \alpha_8)}{p'_{\text{FIA}}} \right]^{\sigma_8} \]
\[
T_{I4} = TFE_{I4} \left[ \frac{p_{TFE_{I4}} \alpha_{I4}}{p_{T_{I4}}} \right]^{\sigma_{I4}}
\]

\[
TFE_{S4} = \left[ \alpha_{I4} T_{S4} \sigma_{I4} + (1 - \alpha_{I4}) FFE_{S4} \sigma_{I4} \right]^{1\left(1 - \frac{1}{\sigma_{I4}}\right)}
\]

\[
FE_{S4} = TFE_{S4} \left[ \frac{p_{TFE_{S4}} (1 - \alpha_{I4})}{p_{T_{S4}}} \right]^{\sigma_{S4}}
\]

\[
T_{S4} = TFE_{S4} \left[ \frac{p_{TFE_{S4}} \alpha_{I4}}{p_{T_{S4}}} \right]^{\sigma_{S4}}
\]

\[
TFE_{A} = \left[ \alpha_{I4} TFE_{I4} \sigma_{I4} + (1 - \alpha_{I4}) TFE_{S4} \sigma_{I4} \right]^{1\left(1 - \frac{1}{\sigma_{I4}}\right)}
\]

\[
TFE_{S4} = TFE_{A} \left[ \frac{p_{TFE_{A}} (1 - \alpha_{I4})}{p_{T_{S4}}} \right]^{\sigma_{I4}}
\]

\[
TFE_{I4} = TFE_{A} \left[ \frac{p_{TFE_{A}} \alpha_{I4}}{p_{T_{S4}}} \right]^{\sigma_{I4}}
\]

\[
KL_{A} = \alpha_{11 K} A, \quad (1 - \alpha_{11} L_{A}) \quad \frac{1}{\sigma_{I1}} \quad \left[1 - \left(1 - \frac{1}{\sigma_{I1}}\right)\right]
\]

\[
L_{A} = KL_{A} \left[ \frac{p_{KL_{A}} (1 - \alpha_{I1})}{p_{L_{A}}} \right]^{\sigma_{I1}}
\]

\[
K_{A} = KL_{A} \left[ \frac{p_{KL_{A}} \alpha_{I1}}{p_{K_{A}}} \right]^{\sigma_{I1}}
\]

\[
VA_{A} = \alpha_{I2 TFE_{A}} \sigma_{I2} + (1 - \alpha_{I2}) KL_{A} \sigma_{I2} \quad \left[1 - \left(1 - \frac{1}{\sigma_{I2}}\right)\right]
\]

\[
KL_{A} = VA_{A} \left[ \frac{p_{VA_{A}} (1 - \alpha_{I2})}{p_{KL_{A}}} \right]^{\sigma_{I2}}
\]

\[
TFE_{A} = VA_{A} \left[ \frac{p_{VA_{A}} \alpha_{I2}}{p_{TFE_{A}}} \right]^{\sigma_{I2}}
\]

\[
Q_{A} = VA_{A} + CI_{A} \quad \text{and} \quad CI_{A} = Q_{A} \sum a_{iA}
\]

2.4 Technology of the industrial sector

\[
CI_{AE_{t}} = \left[ \alpha_{I0} CI_{A_{t}} \frac{1}{\sigma_{I0}} + (1 - \alpha_{I0}) CI_{E_{t}} \frac{1}{\sigma_{I0}} \right]^{1\left(1 - \frac{1}{\sigma_{I0}}\right)}
\]

\[
CI_{E_{t}} = CI_{AE_{t}} \left[ \frac{p_{CI_{E_{t}}} (1 - \alpha_{I0})}{p_{CI_{E_{t}}} \sigma_{I0}} \right]^{\sigma_{I0}}
\]
\[CI_{AE} = CI_{AE_i} \left[ \frac{p'_{Cl,AE_i} \alpha_{10}}{p'_{Cl_i}} \right] \]

\[CI_{IT} = \left[ \frac{1}{\sigma_{11}} \alpha_{11} CI_{AE_i} \left( 1 - \alpha_{11} \right) CI_{IT_i} \right]^{t_1} \left( 1 - \frac{1}{\sigma_{11}} \right) \]

\[CI_{IT_i} = CI_{IT} \left[ \frac{p'_{Cl_i} \left( 1 - \alpha_{11} \right)}{p'_{Cl_i}} \right]^{t_1} \]

\[CI_{AE_i} = CI_{IT} \left[ \frac{p'_{Cl,AE_i} \alpha_{11}}{p'_{Cl_i}} \right]^{t_1} \]

\[VA_i = \alpha_{12} K_i \left( 1 - \alpha_{12} \right) L_i \left( 1 - \alpha_{12} \right) \left( 1 - \frac{1}{\sigma_{12}} \right) \]

\[L_i = VA_i \left[ \frac{p'_{Va_i} \left( 1 - \alpha_{12} \right)}{p_{i}} \right]^{t_1} \]

\[K_i = VA_i \left[ \frac{p'_{Va_i} \alpha_{12}}{p_{i}} \right]^{t_1} \]

\[Q_i = VA_i + CI_{IT} \]

2.5 Production technology of services

\[VA_S = \left[ \frac{1}{\sigma_{13}} \alpha_{13} K_S \left( 1 - \alpha_{13} \right) L_S \left( 1 - \alpha_{13} \right) \right]^{t_1} \left( 1 - \frac{1}{\sigma_{13}} \right) \]

\[L_S = VA_S \left[ \frac{p'_{Va_S} \left( 1 - \alpha_{13} \right)}{p_{S}} \right]^{t_1} \]

\[K_S = VA_S \left[ \frac{p'_{Va_S} \alpha_{13}}{p_{S}} \right]^{t_1} \]

\[Q_S = VA_S + CI_S \quad \text{and} \quad CI_S = Q_S \sum a_{S} \]

2.6 Households consumption

\[U^h_{DE} = \left[ a_h^3 C^h_{DEB} \left( 1 - 1/\sigma_{13} \right) + a_h^4 C^h_{DEI} \left( 1 - 1/\sigma_{13} \right) \right]^{t_1} \left( 1 - \frac{1}{\sigma_{13}} \right) \]

\[U^h_{DE} = \left[ a_h^3 C^h_{DEB} \left( 1 - 1/\sigma_{13} \right) + a_h^4 C^h_{DEI} \left( 1 - 1/\sigma_{13} \right) \right]^{t_1} \left( 1 - \frac{1}{\sigma_{13}} \right) \]

\[U^h_A = \left[ \sum_{i=1}^{l} b^h_{AI} \left( 1/\sigma_{13} \right) C^h_{AI} \left( 1 - 1/\sigma_{13} \right) \right]^{t_1} \left( 1 - \frac{1}{\sigma_{13}} \right) \]

\[U^h_I = \left[ \sum_{i=1}^{m} d^h_{II} \left( 1/\sigma_{13} \right) C^h_{II} \left( 1 - 1/\sigma_{13} \right) \right]^{t_1} \left( 1 - \frac{1}{\sigma_{13}} \right) \]
\[ U_S^h = \left[ \sum_{i=1}^{n} e^{h} S_i \left( C_{S_i}^{h} \right) \right]^{\frac{1}{1 - \sigma_{en}}} \]

\[ U_c^h = \alpha^c_1 \log(c_{DE}^h - \bar{c}_{DE}^h) + \alpha^c_2 \log(c_{A}^h - \bar{c}_{A}^h) + \alpha^c_3 \log(c_{I}^h - \bar{c}_{I}^h) + \alpha^c_4 \log(c_{S}^h - \bar{c}_{S}^h) \]

\[ U^h = A^h \cdot C_{S_i}^{h} \cdot S_i^{1-\rho_s} \]

2.6 Income households

\[ RD^h = R^h - S^h \quad \text{and} \quad S^h = s^h R^h \]

\[ RES^h = p_h \tilde{K} + p_l \tilde{L} + p_{l_i} \tilde{T_i} + p_{E_i} E + tr_{G,h} + tr_{P,h} + tr_{RDM,h} e \]

\[ R^h = RES^h (1 - \tau_h) - tr_{h,G} - tr_{h,p} - tr_{h,RDM} e \]

2.7 Income government

\[ RES_G = p_{E_i} E_{m_i} + p_{E_s} \sum_i E_{i,s} + p_{E_a} \sum_i E_{i,a} + p_{C_i} \sum_i CI_{i} + DD_{i} + II_{INS} + TEXP + \sum_{h=1}^{n} IR + \]

\[ tr_{RDM,G} e + tr_{h,G} \]

\[ DEP_G = Sub_{h} + Sub_{p} + tr_{G,h} + tr_{G,RDM} \]

\[ R_G = RES_G - Sub_C - Sub_P - tr_{G,h} - tr_{G,RDM} e \]

2.8 Foreign Trade

\[ Q_i = A^q_i \left( \psi_{j} X^{-\rho_j} + (1 - \psi_j) QD^{-\rho_j} \right) \frac{1}{\rho_j} \]

\[ \frac{QD_j}{X_i} = \left[ \frac{1 - \psi_j}{\psi_j} \right]^{\sigma_q} \left[ \frac{p_{X_i}}{p_{QD_j}} \right] \]

\[ \frac{CD_j}{QD_j} = A^m_i \left[ \alpha^m_i M^{-\rho_i} + (1 - \alpha^m_i) QD^{-\rho_i} \right] \frac{1}{\rho_i} \]

\[ \frac{M_i}{QD_j} = \left[ \frac{\alpha^m_i}{1 - \alpha^m_i} \right] \left[ \frac{p_{QD_j}}{p_{M_i}} \right] \]

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