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This paper reports the use of the Knowledge Quartet as a tool for analysing mathematical knowledge in teaching arising in interviews with four secondary mathematics teachers based around a pre-configured GeoGebra file involving circle theorems. This research was part of larger doctoral study, where the Knowledge Quartet was chosen as a means of providing a fine-grained analysis focusing on mathematical knowledge *in teaching. The sub-codes of the Knowledge Quartet were originally grounded in classroom observation and have been rarely exemplified through situations involving teachers use of technology. A tension is identified in applying the Knowledge Quartet to an interview situation involving technology centring around the sub-code 'adherence to textbook', with wider implications for how the framework is applied to situations involving teachers' use of (technological) tools.*

Keywords: Knowledge Quartet, teacher knowledge, technology.

Introduction

Using digital technologies, such as the dynamic software package GeoGebra, can place significant demands on teachers' mathematical knowledge (Laborde, 2001) and hence provide opportunities for making such knowledge visible and available for exploration. The research reported in this paper forms part of a larger doctoral study investigating mathematical knowledge in teaching using technology. The Knowledge Quartet (Rowland et al., 2005) is a widely used and influential framework for analysing mathematical knowledge in teaching, (see previous TWG20). The framework is composed of four supra-categories Foundation, Transformation, Connection and Contingency, generated by grouping 18 sub-codes representing prototypical classroom situations where mathematical knowledge for teaching is called upon, derived from analysis of classroom observations. The strengths of the Knowledge Quartet, in terms of this study, lie in the framework's focus on mathematical knowledge for teaching and the grounding of its codes in classroom observation, thereby maintaining strong face and content validity. For these reasons, the Knowledge Quartet was chosen as a means of providing a fine-grained analysis focusing on mathematical knowledge in teaching, as opposed to technology-focused frameworks such as the Technology Pedagogy and Content Knowledge (TPACK) framework (Mishra & Koehler, 2009). In addition, research on the importance of variation to structure sense-making (Marton & Booth, 1997; Watson & Mason, 2006) suggests the code *choice and use of examples* (Transformation) may be advantageous as a tool for analysing mathematical knowledge for teaching using technology in the context of the semi-structured GeoGebra interviews, since dynamic variation is central to such software (Leung & Lee, 2013).

The sub-codes of the Knowledge Quartet have rarely been exemplified through situations involving teachers use of technology. One such instance of a situation involving teachers' use of technology resulted in the addition of a new sub-code *responding to the (un)availability of tools and resources* to the Contingency category of the Knowledge Quartet (Rowland et al., 2015). Hence extending the use of the Knowledge Quartet to analyse situations beyond its original evidence-base may provoke further refinement of the framework. This paper seeks to address the following research question:

What tensions are exposed in using the Knowledge Quartet as a tool for analysing the mathematical knowledge in teaching arising in interviews based on a GeoGebra file involving circle theorems?

This paper also relates to a broader concern in TWG20 in CERME11 (e.g. Montes et al. (2019, February); Codes et al. (2019, February)) of investigating how tasks, in this case manipulating a GeoGebra file on circle theorems, can be used to explore, support and develop mathematics teachers' knowledge and what the implications for teacher education may be. The next section provides the theoretical background underpinning the study, by first setting out an overarching perspective of teachers' knowledge being distributed (Hutchins, 1995) e.g. across tools and then describing the Knowledge Quartet in more detail. In the following sections, analysis of interview data reveals a tension in applying the Knowledge Quartet, centring around the sub-code *adherence to textbook*, with wider implications for how the framework is applied to situations involving teachers' use of (technological) tools.

Background

Hutchins (1995) argues that conceptualising cognition as distributed assumes that cognition is not only a property of an individual person, but also occurs through human interaction with artefacts and other humans. In particular, he argues that cognition partially resides in tools – taken to mean any artefact appropriated for use by humans - since they incorporate in their construction the results of past cognitive efforts. Hutchins' (1995) view of distributed cognition provides a means of investigating how individual teachers' knowledge is involved in a participatory relationship with technology. In this study, the terms readerly and writerly response (Bowe, Ball, & Gold, 1992, drawing on the work of Barthes) are introduced to indicate how and to what extent knowledge is distributed (Hutchins, 1995) across teacher and technology. A readerly response indicates an uncritical acceptance by the teacher of a (technological) tool for mathematics teaching. By contrast, a writerly response entails recognition that the design of such a tool is open to critique and potential modification. Departing from their original meaning, that writerly texts invite the reader to participate in meaning-making and are therefore in a sense superior to readerly texts that make no such demands, the use of these terms in this study takes a less normative view. Instead, a readerly/writerly response indicates the role of individual teachers' knowledge in interacting with technology to produce mathematical knowledge made available in the classroom – or more pertinently, in the case of this study, in the interview. A readerly response suggests that the mathematical knowledge made available through this interaction is more dependent, i.e. more distributed, upon the technology, whereas a writerly response indicates that it is less so.

For the purposes of this study, the Knowledge Quartet provides a means of focusing on and analysing individual teachers' own knowledge in relation to using technology to teach mathematics i.e. within the participatory relationship with technology. The Knowledge Quartet emerged from research aimed at developing an empirically-based conceptual framework to guide lesson review discussions between teacher-mentor and student-teacher in the practicum placement of the

Postgraduate Certificate in Education course in the UK (Rowland et al., 2005). The purpose of developing such a framework was to focus these discussions on the mathematical content of the lesson under review. The Knowledge Quartet was initially developed from 24 lesson observations of student teachers, training to teach at primary level. These observations generated 18 codes relating to the student teachers' classroom actions that appeared significant in the sense that they were informed by the trainee's mathematical knowledge for teaching. The codes were then grouped into four super-ordinate categories, named Foundation, Transformation, Connection and Contingency. The foundation category consists of propositional knowledge of mathematical concepts and the relationships between them and of significant research findings regarding the teaching and learning of mathematics (Rowland et al., 2005). The second category of transformation refers to knowledge-in-action, concerning the ways that teachers make what they know accessible to learners: this category focuses in particular on their choice and use of representations and examples (Rowland et al., 2005). Connection also refers to knowledge-in-action, regarding the manner in which the teacher makes connections between different concepts, representations and procedures; and decisions about sequencing e.g. of topics. Contingency concerns the teacher's ability to 'think on one's feet', to provide an appropriate response to unanticipated pupil contributions, and also notable 'in-flight' teacher insights (Thwaites, Jared, & Rowland, 2011). While there are clearly no pupils to provoke contingencies in an interview context, technology can be a source of disruption to teachers' mathematical knowledge (Laborde, 2001). Indeed, in this study, contingent moments did arise through teachers' use of the GeoGebra file, however these are not the focus of this paper. The Knowledge Quartet has subsequently been examined in classrooms at secondary level (Thwaites et al., 2011) and in classrooms outside the UK, specifically in Ireland and Cyprus (Turner & Rowland, 2011), resulting in the addition of new codes and alteration of some of the original codes. Although Rowland et al (2005, p. 260) make use of an acquisition metaphor, implying individualist assumptions about knowledge by describing their foundation category as being about "knowledge possessed", Turner and Rowland (2011, footnote on p. 200) suggest that "this 'fount' of knowledge can also be envisaged and accommodated within more distributed accounts of knowledge resources". Hence the framework may be compatible with an account of teachers' knowledge as distributed e.g. across (technological) tools.

Methodology

Four teachers were selected from a group of English mathematics teachers who took part in a survey of secondary school mathematics teachers' use of ICT (n=183) and who further agreed to be contacted as case study teachers (Bretscher, 2011; 2014). The four case study teachers, Robert, Michael, Edward and Anne, were chosen along two dimensions of variation likely to be associated with mathematical knowledge for teaching using technology, based on their responses to survey items. Firstly, the case study teachers were chosen to be two of the most student-centred (Robert, Anne) and two of the most teacher-centred (Michael, Edward) in their approach to mathematics teaching in general (not limited to ICT use) of those who volunteered. Secondly, two teachers were chosen to be from schools with a high level of support for ICT (Robert, Michael) and two with a low level of ICT support (Anne, Michael). In addition, the four case study teachers had described themselves as being confident with ICT. As technology enthusiasts, the case study teachers were likely to display mathematical knowledge for teaching using technology; the variation in case selection aimed to highlight such knowledge – making it more 'visible'.

Semi-structured interviews based around a GeoGebra file on circle theorems provided a common situation across which the case study teachers' use of technology for teaching mathematics could be contrasted. The case study teachers were prompted to show and discuss how they would use a diagram presented in the GeoGebra file (see Figure 1) to demonstrate the angle at the centre theorem to their pupils. Circle theorems were chosen since it is a topic, in the English mathematics curriculum, which is commonly identified with the use of dynamic geometry software (Ruthven et al., 2008). It was therefore reasonable to assume that the case study teachers would be familiar with technological resources similar to the diagrams presented in the GeoGebra file and might even have previously used such resources in their own teaching. Thus they would be likely to have some mathematical knowledge for teaching circle theorems using the GeoGebra file, even if they were unfamiliar with the particular software. In addition, the topic of circle theorems is at the apex of geometry in the compulsory English mathematics curriculum, since it is typically where proof is introduced. Hence it provided a potentially challenging context even for experienced teachers who were both mathematically and technologically confident.

The GeoGebra file comprised three diagrams, all initially arranged in an 'arrowhead' configuration, relating to the circle theorem stating that angle at the centre of the circle, subtended by an arc, is double the angle at the circumference subtended by the same arc. The GeoGebra file also incorporated some text, setting the task of manipulating the diagrams in the pedagogical context of planning how to introduce pupils to this circle theorem based on a demonstration using these diagrams. In this paper, the analysis presented below focuses on the teachers' discussion of the first diagram (D1) only which was designed to be similar to resources found on a web-search. Thus the case study teachers were likely to have at least some familiarity with a dynamic diagram like D1 and possibly have even used something similar in their own lessons.

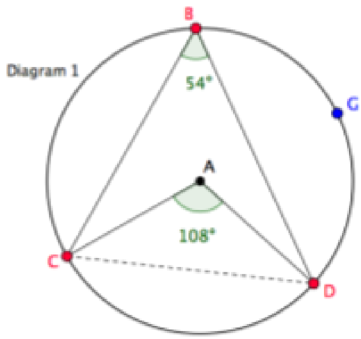

Figure 1 Diagram 1 (D1) in the GeoGebra interview file on circle theorems

Before opening the GeoGebra file on circle theorems, the case study teachers were asked to practise 'thinking-aloud' whilst manipulating a GeoGebra file contrasting two constructions of a square. The semi-structuring of the interview allowed the author some flexibility to respond to events during the interview, whilst maintaining an overall structure that would allow for and facilitate comparison. The GeoGebra interviews generally took place in a mathematics classroom at the case study teacher's school that was not being used for teaching at that time. The author's laptop with mouse attached was arranged on a desk so that both the author and the case study teacher could comfortably see the screen and use the mouse to manipulate D1, enabling collaboration on the task. Both the visual and audio aspects of the GeoGebra interviews were recorded on the author's laptop.

Results and analysis

Adhering to the starting configuration

Robert was the only case study teacher to consider modifying the starting configuration of D1 to suit his own pedagogical requirements. He suggested he might alter D1 so that the initial numerical example displayed when opening the GeoGebra file would be an almost implausibly 'nice' pair of numbers, setting the angle at the circumference to 60 degrees and the angle at the centre to 120 degrees as an example.

Robert: I'd probably I'd have it so when it [D1] came up I'd probably have it set up with I guess fairly nice numbers [drags D1 so angle CBD=60; CAD= 120] that they should be able to spot quite easily and I'd probably ask them what the relationship is. And then before dragging this point I'd probably you know I'd probably have it set up so that maybe it looks a bit well this is a kind of this is a nice symmetrical, that's horizontal, they almost look vertical you know. And so I'd probably ask them well what happens if I move this over here? Is it going to get bigger? Is it going to get smaller?

Interviewer, I: Can you show me?

Robert: So I probably would be, if this was an interactive whiteboard I'd be hovering over this and not actually touching it and saying I'm going to drag this this way. What's going to happen? And I'd probably try to lead them into, I probably wouldn't give them the option of it staying the same. I'd probably ask them is it going to get bigger or going to get smaller? To I guess when they see that it does stay the same to provide a bit of conflict there. And then I'd drag it and we'd drag it all the way around here and show that it never changes. [Rob-GGb-int, 13.6.2012]

His intention was to set up a situation that appeared 'too good to be true' so that pupils would assume no relationship was likely to exist and would therefore sustain cognitive conflict when the angle at the circumference remained invariant under drag, hopefully making the result more memorable. The other three case study teachers uncritically accepted the starting configuration, questioning neither the numerical example nor the geometric configuration. For example, Edward explained how he would begin using the diagram, without making reference to the starting position:

Edward: What I'd start with is look, just move B between C and D but don't cross it and move D just so it doesn't go further round than CD being a diameter. [Ed-GGb-int, 20.6.2012]

The geometric nature of the starting configuration, in particular, is important since it provides an implicit pedagogic structuring. For example, opening the GeoGebra file so that D1 initially displays an 'arrowhead' configuration implies a choice and use of examples and a decision about sequencing that alternative configurations will occur as a consequence of the arrowhead configuration, potentially reinforcing the impression of the arrowhead as the standard configuration of the angle at the centre theorem. An alternative would be to open the GeoGebra file so that D1 initially displays the convex quadrilateral configuration as a means of challenging this apparent orthodoxy. In addition, the starting configuration tends to impose decisions about sequencing, since some configurations are more difficult to obtain depending on whether they require dragging point B, C or D only or a combination of these points.

The case study teachers' adherence to or modification of the starting configuration appears to coincide with the meaning of the code *adherence to textbook*, in the Foundation category of the Knowledge Quartet, in the sense that it describes a situation involving mathematical knowledge for teaching where a teacher decides either to adhere to or to modify the pedagogic structuring of mathematics by a teaching resource. The teacher's decision, implicit or explicit, regarding the pedagogic structure of the teaching resource provides an indicator of foundational knowledge. An implicit (i.e. uncritical) adherence to the pedagogic structure of the teaching resource implies a negative reading of the code. Thus Anne, Edward and Michael's uncritical acceptance of the starting configuration suggests they lack foundational knowledge that the starting configuration of D1 might be (usefully or otherwise) critiqued in terms of the pedagogic structuring it provides. Hence they make a readerly response to D1 (Bowe et al., 1992). Nevertheless, a readerly response might apparently result in a positive choice and use of examples say, if the pedagogic structuring of the resource was sound. Thus, confusingly, a readerly response could also be interpreted as a positive example of the code adherence to textbook.

A writerly response (Bowe et al., 1992) to D1 would entail a recognition that the starting configuration of D1 might be critiqued in terms of the pedagogic structuring it provides, resulting in an explicit decision either to adhere to or to modify the pedagogic structuring of the teaching resource. This suggests a positive reading of the code *adherence to textbook*. Indeed, an explicit decision to adhere to the pedagogic structuring of the teaching resource would be a positive example of the code *adherence to textbook* if the pedagogic structuring of the resource were sound. On the other hand, such a decision could also be interpreted as a negative example of the code if the pedagogic structuring turned out to be flawed in some way. An explicit decision to modify the pedagogic structuring of the teaching resource which resulted in improvement, would again indicate a positive example of *adherence to textbook* – this latter is also dealt with by the new code *use of instructional materials* under the Transformation category introduced by Petrou and Goulding (2011). However, Robert's decision to modify the starting configuration could be interpreted as a deterioration in the pedagogic quality of the initial choice of example: it is geometrically too close to being symmetric and the numbers are exceptional. Thus Robert's modification could be interpreted both as a positive and negative instance of the code *adherence to textbook* under Foundation and a negative example of *use of instructional materials*. This analysis is rather cumbersome and symptomatic of the Knowledge Quartet's relative lack of focus on knowledge in relation to teaching resources in general.

Finally, there is a slight discordancy in using this code to describe the case study teachers' adherence to or modification of the starting configuration, since the code specifically refers to a textbook and not a digital resource such as the GeoGebra file on circle theorems. The specificity of the *adherence to textbook* code derives from the non-digital technology context in which it was grounded. The discordancy may be ameliorated by a minor alteration to the code, so that it refers to a more generic teaching resource as in *use of instructional materials* (Transformation) or *responding to the (un)availability of tools and resources* (Contingency).

Discussion

The analysis presented in the previous section shows that the Knowledge Quartet remained a useful tool for focusing analysis on mathematical knowledge despite the shift away from the classroom context in which the framework was originally developed and grounded. This finding is not entirely surprising since although the classroom is a particularly important context, it is not the only context in which teachers are likely to employ their mathematical knowledge for teaching.

In addition, the analysis of the GeoGebra interview data suggested some minor modifications to the Knowledge Quartet, in relation to technology context, that might prove useful when re-applied back to the original classroom setting or to other settings where teachers employ their mathematical knowledge for teaching. The cumbersome analysis of situations involving the code adherence to textbook is symptomatic of the Knowledge Quartet's lack of focus on knowledge in relation to (digital) technology. The difficulty with this code is that it categorises situations involving the application of foundational knowledge both in perceiving the technology as something requiring a pedagogic critique and in terms of the quality of the critique applied to the teaching resource. The former relates to the teachers' foundational knowledge in adopting a readerly or writerly approach to the resource (Bowe et al., 1992). The latter is also dealt with under the Transformation category, specifically the code use of instructional materials, which additionally reflects back onto the teachers' foundational knowledge indicated by the quality of the pedagogic critique applied to transform the resource for the purpose of teaching. There is no easy way to ameliorate this difficulty within the Knowledge Quartet, however, adherence to textbook could be modified to reflect a broader range of teaching resources rather than privileging this paper-based technology. Furthermore, as a result of new codes added by a range of researchers, the codes of the Knowledge Quartet use an impromptu variety of terms to refer to teaching resources including textbook, instructional materials, tools and resources. The variety of terms is not intended to make any productive distinctions, as far as I am aware, thus it might simplify and improve the coherence of the Knowledge Quartet to settle on a particular term or group of terms to refer to teaching technologies. Finally, the analysis above provides an exemplification of the Knowledge Quartet in relation to digital technologies, albeit not in a classroom context. This exemplification might be useful in helping teachers to use the Knowledge Quartet as a tool for professional development in relation to their use of technology, as described in Turner and Rowland (2011).

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