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This study investigates how principles of feedback to encourage students´ creative reasoning can be used by a mathematics teacher. An experienced teacher was introduced to principles, developed in pilot studies, and was instructed to plan a lesson based on four principles for feedback. During the lesson the teacher's interactions with students were recorded and the following analysis focused on the way feedback resonated with the principles. The result indicates that providing feedback which challenges students to reason creatively is difficult and complex. There are pitfalls that originate in established classroom norms for interaction, as well as beliefs about the object of teaching, and it appears that the principles, in order to become a powerful tool, require the teacher and students, to practice using them.

Keywords: Instructional design, teaching methods, feedback.

Introduction

Students, who are encouraged to construct their own solutions and create arguments when solving mathematical tasks, tend to, if they are successful, learn or remember more from such activities than students who are being guided by templates and prepared examples (Jonsson, Norqvist, Liljekvist, & Lithner, 2014). Despite the disadvantages, described in numerous research reports, of teaching mathematics by providing solution methods to tasks, such teaching is still prevalent in many classrooms, in Sweden, as well as around the world (Boesen, Lithner, & Palm, 2010; Hiebert & Grouws, 2007). Teaching where students create and justify their own solutions (i.e., engaging in creative reasoning) requires different teacher-student interactions than traditional teaching. Rather than explaining which method to use as well as how and why it works the teacher must encourage students, not only to construct own solutions, but also to challenge them to justify their choice of method (Hmelo-Silver, Duncan, & Chinn, 2007).

A teacher-student interaction aimed at supporting students' construction of solutions can be compared to feedback aimed at supporting the students' learning processes and thus relies on the active involvement of the student (Hattie & Timperley, 2007; Nicol & Macfarlane-Dick, 2006). Research on formative assessment and feedback often reports general guidelines on how to provide feedback in teaching but few studies present empirical results detailing how feedback can be prepared and designed in classroom situations (Hattie, 2012; Palm, Andersson, Boström, & Vingsle, 2017). Hence there is need for a deeper understanding of how formative assessment/feedback can be implemented at a classroom level (Hirsch & Lindberg, 2015). As a preparation for this study a series of four pilot studies, involving mathematical problem solving, was conducted. The interaction between students and their teacher was studied with the aim of identifying what characterizes the kind of feedback that leads students to reason creatively. The findings can be interpreted as four basic principles: (1) find out how the students are thinking, demand that students are specific, (2) encourage the students to formulate their thinking without interrupting or interpreting, (3) challenge the students to explain why their solution is working (or not working) – instead of confirming or disconfirming, and (4) encourage the students to find a way to test their solution. The present study investigates how the principles can be used by a teacher who is unfamiliar with them. Our aim is to refine the principles and to form guidelines for formative feedback that teachers can use to foster creative reasoning.

Background

Teaching that encourages students to construct and justify solutions to mathematical tasks entails a focus on reasoning. Furthermore, it is reasonable to assume that teachers' feedback may guide the character of students' reasoning. The following paragraphs will outline distinctions between different types of reasoning and feedback relevant for the didactic design addressed in this paper.

Imitative and creative reasoning

If the teacher explains a definition of a mathematical concept and then demonstrates how to solve tasks associated with this particular concept, it is possible for students to solve similar tasks by remembering the procedure without understanding the definition. Lithner (2008) found that students trying to apply memorized procedures often had difficulties when solving tasks for which there had been no recent teaching. For example, calculating $2^3 \times 2^4$ using a memorized process could mean mixing up whether the numbers should be added or multiplied. The reasoning associated with such an approach is defined as imitative (IR) (Lithner, 2008). A variant of IR is AR, algorithmic reasoning which is relevant for this paper. AR entails recalling a memorized, stepwise, procedure or following procedural instructions from a teacher or textbook, that are supposed to solve a task (Lithner, 2008). AR is algorithmic in the sense that it solves the associated task, but it does not require an understanding of the mathematics on which the procedure is based.

An alternative approach to the example above, $2^3 \times 2^4$, may be to consider what the mathematical meaning behind the expression is, i.e., 2^3 means $2 \times 2 \times 2$, and 2^4 means $2 \times 2 \times 2 \times 2$. After realizing this, the next step is to put the two together, $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$, which is 2^7. If the student can express mathematical arguments for the solution she is engaged in creative mathematical reasoning (CMR). CMR is characterized by the construction or reconstruction of a solution method and the expressing of arguments for the solution method and the solution (Lithner, 2008).

Feedback

It is possible for students to reach correct answers to tasks without understanding the mathematical concepts involved (Brousseau, 1997). If a student should fail in his or her attempts to solve a mathematical task, the most obvious feedback from the teacher may be an explanation regarding how to solve the task, not to explain the mathematics it is based on. Should the student, however, be responsible for the construction of the solution method, she is helped by understanding the mathematics required by the task. In such cases, it is appropriate for the teacher to inquire into the student's thinking. Inquiring into students' thinking can be seen as part of formative feedback at the process level whereas delivering or assessing a right or wrong answer is seen as task level feedback

(Hattie & Timperley, 2007). Research indicates that 90 % of feedback in classrooms is on task-level (Airasian, 1997). Feedback on process-level focuses on the underlying mathematical processes, which can support students' conceptual understanding as well as their autonomy (Kazemi & Stipek, 2001). This type of feedback creates interactions in which students have an opportunity to develop their ability to use mathematical ideas to formulate arguments and justifications for their choices (Michaels, O'Connor, & Resnick, 2008).

Development of principles for feedback aimed at supporting creative reasoning

The principles for feedback were developed iteratively during a year in four pilot studies in collaboration with a teacher (see Olsson & Teledahl, 2018). The starting point for the pilot studies was the design of appropriate tasks for CMR. Then possible solutions to the tasks, together with the potential difficulties that students will encounter, were considered. For each of these solutions and difficulties feedback aimed at supporting CMR was designed. The lessons were recorded and the interactions between the teacher and the students were analyzed with the aim of identifying the characteristics of the feedback that led students to reason creatively. The result of every analysis was used in the planning of the next lesson. After the fourth intervention, principles for feedback supporting CMR were formulated. These principles (see Introduction) were used in the present study to support the design of the intervention (to setup student activities where feedback for CMR is appropriate) and to prepare the teacher to formulate feedback for CMR.

Method

In order to test the previously developed principles for feedback a new study was set up in which a lesson was planned so that students would have the opportunity to engage in CMR. The principles were also used to prepare the teacher and help him support students' CMR through feedback. During the lessons the teacher's interactions with students were recorded. Compared to other possibilities (e.g., interviews) data was considered to be reasonably close to the thinking processes that create feedback and reasoning. Feedback is considered as both a response to students' actions and guiding their continued reasoning. The chain students' action – teacher's feedback – students' continued reasoning was the unit of analysis.

Sample

A teacher and students in 8th grade of a Swedish elementary school volunteered to participate. The teacher is experienced in teaching mathematics through problem solving and the students were used to problem-solving activities. The teacher did not participate in the pilot studies where the principles for feedback were developed. Instead, he was introduced, during a half-day seminar, to the idea of supporting students´ CMR through feedback according to the principles. Transcripts from the pilot studies were discussed and tasks suitable for the class participating in this study were considered. The teacher considered the class to be average in terms of achievement, and a mix of students with Swedish as their native or second language. With respect to feedback, in his everyday teaching, the teacher considered it his intention not to guide students to solutions, although he did not have explicit strategies for achieving this. Neither the lesson, nor the feedback, would thus be considered something out of the ordinary and according to the teacher, the interactions were typical for his style of teaching.

Lesson plan

The teacher was instructed to plan a lesson containing problem solving. The role of the teacher was to support students in constructing solutions rather than explaining how to solve the problems. It was assumed that successfully constructing solutions, without knowing a solution method in advance, would require students to engage in CMR. The tasks were designed in line with Lithner's (2017) principles: (1) creative challenge, no solution methods are available from the start and it must be reasonable for the students to construct the solution, (2) fair conceptual challenge to understand mathematical properties (e.g., representations and connections) and (3) justification challenge, is it reasonable for the particular student to justify the construction and implementation of a solution. The tasks were part of the curriculum and involved combinatorics. *Task 1a* asks the students to find all of the different ways to put three blocks of different colors in a row. The students were supplied with concrete materials. In *task 1b* another block, of yet another color, is added. The teacher was also instructed to prepare to give feedback according to the principles.

Procedure

During the lesson, the students worked in pairs. The tasks were presented in written text and the students were asked to present written solutions. The students were encouraged to collaborate and call the teacher if they did not understand the instructions, or if they got stuck. The teacher was wearing a recording device, which recorded his interactions with students. The recordings were transcribed into written text, with a focus on spoken language.

Analysis method

The analysis focused on the chain students' action – teacher's feedback – students' reasoning in connection teacher's feedback, and was performed through the following steps:

(I). Parts of transcripts where the teacher interacts with students, and where it is considered possible to provide feedback according to the principles, were identified. (II). Parts of transcripts, where the teacher's feedback resonates with the principles for feedback, were identified. (III). It was determined whether feedback, according to the principles, supported or did not support students' CMR. (IV). In interactions, where the principles were not implemented consistently or only partly implemented, possible trajectories leading to CMR were considered and the way the principles could have supported CMR was analyzed.

The analysis was a joint effort that involved the teacher.

Results

The lesson was conducted according to plan in the sense that students got engaged in problem solving where they did not have access to a solution method for the tasks. Interactions with the teacher could be observed when a solution to a task was reached or when students asked for help. We will now present two extracts from the transcripts which can be seen as examples of interaction where the teacher can be considered to have possibilities to provide feedback according to the principles.

In the first example, which is representative of a number of interactions during the session, the student had come to a solution to *task 1a*:

1. Teacher:	Explain the way you are thinking.... it looks like you have some system.
2. Student:	Well.... if you start with a block.... for instance, a green one.... you can always change the order of the other two blocks.... and in this case, there are three blocks.... so, you have plus two combinations.... so, in this case it can be green-white-yellow and green-yellow-white.... so, if we have three blocks it will be three times two.
3. Teacher:	OK.... you can go on to the next task

The teacher asks the student to explain her thinking (line 1) which is in line with the first principle. While the student did not know how to solve the task in advance it is reasonable to assume that the answer (line 2) represents, at least partly, CMR (constructing a solution). As a whole only the first principle is used. The student is not articulating arguments explicitly, which could have been encouraged by first inviting her to express her understanding (principle 2) and then challenge her to explain why her method results in correct answer (principle 3). The last utterance in line 2, when the student suggests a way to calculate the numbers of combinations, can be used to challenge her to justify their solution (principle 4). The teacher's comment on line 3 does not correct the student, which gives her an opportunity to find out that although the calculation is correct in connection to *task 1a*, it will not work in *task 1b*. This is an example of when the teacher's decision to refrain from explaining, gives the student an opportunity to discover the error in her conclusion. In the next task (1b) the teacher has the opportunity to challenge the student's reasoning using principles 2-4.

The second excerpt was chosen as an example where principle 3 could have been appropriate. It introduces a similar situation as example 1 and indicates the importance of the formulation of feedback. A pair of students have come to a solution on task 1a and now call for help with task 1b.

1. Teacher:	What did you conclude on task 1a?
2. Student:	We conclude that for every color.... if it is situated in one place.... there are two combinations for where the others could be situated.... that goes for every color situated in each place.... but if you add one block there will be four blocks and there will be three combinations for each block in each position.
3. Teacher:	Are you sure?
4. Student:	No.... there will be more then.
5. Teacher:	How many positions can this be in?
6. Student:	Then it would be.... these ones.... ok.
7. Teacher:	If there were three blocks they could be situated in....
8. Student:	If they were three they can.... yes.
9. Teacher:	This was the prior task [1a].... wasn't it?
10. Student:	Then there will be six ways for these.
11. Teacher:	Yes.... if the blue one is first.... but you don't have to put the blue one first.
12. Student:	No.
13. Teacher:	So, how many could it be.... how many ways is it if the yellow is first?

14. Student: If the yellow is first there will be six ways.
15. Teacher: OK, so how many is it all together?
16. Student: Twenty four.
17. Teacher: OK.... could it be more ways.... are there any others that could be first?
18. Student: No, these are the only ones you can have first.

In the planning for feedback it was considered that the reasoning connected to 1b could be continued from the solution of 1a. Therefore, on line 1 the teacher focuses the feedback to the task 1a. The question posed in line 1 may be associated to principle 1. The student´s reasoning on line 2 includes both how the solution is constructed and arguments which are components in CMR. The teacher's question on line 2 could have been in line with principle 3, but the student seems to perceive the question as an indication she is wrong. From here the teacher's feedback instead of following the principles is mainly on task-level, i.e., its purpose is to guide the students to a solution, not to support CMR. The student does not fulfil the CMR from line 2, instead, the reasoning turns into guided AR (imitating the teacher´s reasoning).

Summary of results

Mathematical reasoning is considered creative (CMR) if students themselves construct or reconstruct a solution method to a problem and express arguments to support this. The two examples above indicate that students, who are encouraged to explain their thinking, express CMR. Through their explanations it seems clear that they have constructed solutions to problems, for which they did not have access to a solution method in advance. Less clear is the way students formulate arguments for their solution. In neither of the examples are students encouraged to justify or articulate arguments. The principles for feedback in this study were developed to encourage students to construct their own solution methods and formulate mathematical support for their solutions. The results indicate that it may not be an easy task, even for experienced teachers, to implement these principles.

Discussion

Creative reasoning is a powerful tool in the learning of mathematics. In mathematical problem-solving creative reasoning, where conjecturing and justifying are viewed as important parts, leads students to construct their own solutions to mathematical tasks, something that previous studies have found beneficial for their learning (Jonsson et al., 2014; Olsson & Teledahl, 2019). How to design teaching, in particular feedback, which will encourage students to construct solutions and present support for why they are correct, is still something that we know too little about.

Teaching where students are constructing solutions while engaging in CMR does not entail a passive teacher (Hmelo-Silver et al., 2007). The teacher in this study planned the activities with the intention of creating situations where feedback, in line with the principles, would be appropriate. In the first excerpt, the students have managed to solve task 1a and principle 1 is used to challenge the students to articulate their reasoning explicitly. After the students have explained their thinking, the teacher without verifying the students' solution, encouraged them to move on to task 1b. By not confirming or disconfirming the solution to the first part of the task the teacher creates an opportunity for the students to discover that their method for solving in 1a will not work in 1b. He does not challenge them to explain anything, or ask them why they think it works, but rather leaves them to test their solution in the next stage. He is thus creating an opportunity for the students to continue their CMR, and for himself to challenge the students to formulate arguments for their solution, at a later stage when they, hopefully, have made useful discoveries about their original idea. This situation leads us to realize that the teacher not only has to consider the principles for *how* to challenge students to explain and provide argument but also *when* to do so. In this interaction giving students time and space might lead them to insights that helps them to argue for their final solution. It might be challenging to teachers to be consistent in constantly providing feedback, according to the principles, but also to retain a sensitivity to when it may be a good idea to leave the students alone.

Example 2 shows, what may be an even greater challenge, to avoid explaining how to solve a task when students ask for help. Again, principle 1 seems to work well. Students express their reasoning and leave space for feedback in line with principles 2, 3, and 4. Here, the teacher may have in mind not to explain the way to solve the task, but instead of asking the students to explain he uses questions that, step by step, guides the students towards a solution. On the surface, it might seem like he is sticking to the principles, given that he is using questions instead of statements and hints, but the questions can be seen as a way to implicitly provide the solution. The responsibility for the mathematical reasoning shifts to the teacher and the students miss an opportunity not only to develop their ability to provide mathematical arguments but also to retain their autonomy (Kazemi & Stipek, 2001; Michaels, O'Connor, & Resnick, 2008). It may be tempting for the teacher to explain or to guide the students to a solution, not only because it saves time when the teachers is stressed, but also because it produces a result, a solution. In this way the product, rather than the process, becomes the main object. A solution to a problem, even if it was obtained by imitating the teacher (AR according to Lithner, 2008) is considered more valuable than CMR, which may not even result in a complete solution. To focus feedback on the underlying thinking process that will solve the task is most likely difficult and it places high demands on both teachers and students.

Since the collaborative teacher in the pilot studies was involved in developing the principles for feedback she became gradually aware of how to use them and what they would lead to. In addition, while she was positive to the idea of students learning through CMR, she also practiced using the principles between the four interventions, which were planned in collaboration with the authors. The teacher, who participated in the current study, was also positive to teaching according to the principles. The difference was that he was introduced to the concept through a few hours of instructions. The two examples of interaction, shown above, highlight how important it is to carefully formulate your questions. In every classroom there is an established pattern for teacher-student interaction, and in order for such patterns to develop towards CMR teachers and students need, not only to practice using the guidelines, but also to reflect on their current patterns of interaction.

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