'Practicing place value': How children interpret and use virtual representations and features

Axel Schulz, Daniel Walter

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Axel Schulz[1] and Daniel Walter[2]

[1]University of Bielefeld, Germany; axel.schulz@uni-bielefeld.de

[2]University of Münster, Germany; daniel.walter@uni-muenster.de

The tablet-app 'Stellenwerte üben' (Practicing place value)[1] was developed to provide a software for practicing place value concept. There are two goals associated with this project: 1) Developing the app and 2) Investigating how children interpret the given non-symbolic representations and how they use the implemented features. This article presents the underlying theoretical background, a short description of selected features, and a sketch of initial empirical findings on the interpretation and use by primary school children in their second and third school years. First findings suggest that previous experience with concrete physical material overlaps and influences the intended use of the implemented virtual features.

Keywords: Place value, digital media, representational change, primary mathematics education

Place value understanding

The development of the understanding of place value and therefore of the composition of multi-digit numbers is one of the most important goals in mathematics education in primary school (Wittmann, 1994). In addition to that, lack of understanding of the concept of place value is a main indicator for learning difficulties in arithmetic (Scherer et al., 2016, p. 636). Understanding of place value can be assumed when someone 1) knows that the place value system is based on continually grouping by tens (powers of ten), 2) knows about the meaning of place value: The quantity of the value is determined by its position represented by the digits – the values of the positions increase in powers of ten from right to left, 3) knows about the meaning of face value: The digits have to be multiplied by the power of ten assigned to its position, 4) knows that the quantity of the whole numeral is the sum of the values represented by the individual digits and 5) knows the rules and structure of number word formation corresponding to the written number (e. g. Fuson et al., 1997).

In the literature, we can find numerous appropriate activities to foster the development of students' place value understanding and to strengthen it in practice phases. Some of these activities are the following: Bundle and unbundle unstructured sets of objects according to the power ten (van de Walle, 2004; Hiebert & Wearne, 1992), arrange the bundles according to the place value to introduce and practice the place value system and name the bundles to compile and practice the rules of number word formation (van de Walle, 2004; Gaidoschik, 2003). In addition to these activities there are several manipulatives and representations that seem to be appropriate to work with: Unstructured objects such as linking cubes and tiles, pre-structured objects, for example base-ten blocks (Dienes-Blocks), 'place value mats' for sorting or trading base-ten material (e. g. van de

[1] The German version of the tablet-app is provided on Google Play. An English version is in progress.

Walle 2004, pp. 165–169, Gerster & Walter, 1973) and place value charts with tiles or digits for a standardized and conventional representation.

(Virtual) representations and their potentials

Theoretical analyses and empirical findings suggest that there are several features *of virtual representations* that have the potential to support mathematical learning processes. Some examples for these features are the following: Multitouch-Technology (e.g. Sinclair & Heyd-Metzuyanim, 2014), support in structuring an unsorted set of objects (Walter, 2018), reducing an extraneous cognitive load by the means of computational offloading (Rogers, 2012) and informative feedback (e.g. Harrass, 2007). For the present study, we focus on the following two features (see below): *Fitting between virtual representations and mathematical ideas* (e.g. Peltenburg, van den Heuvel-Panhuizen & Doig, 2009) and *synchronously linked representations* (e.g. Ainsworth, 2006). However, it must be pointed out that the existence of the features mentioned above does not automatically lead to an intended use (e. g. Walter, 2018).

Fitting between representations and mathematical ideas

Mathematical objects can not be empirically observed (like for example astronomical or biological objects) and therefore mathematical knowledge can not be developed through this observation. "The only way to have access to […] [mathematical objects] and deal with them is using signs and […] representations" (Duval, 2006, p. 107). These representations can be symbolic (e.g. written or spoken numbers "30", "three tens") or non-symbolic such as iconic or concrete representations (e.g. three ten-rods of the base-ten blocks; thirty apples in a picture), or something in between (e.g. three tiles in the tens place in a place value chart). In addition to that, these representations can be 'material/external' or 'mental' – for example three dots in a place value chart or *thinking* of three dots in a place value chart (Duval, 2006, p. 105). In this context, there are (among other things) two aspects that have to be taken into account: 1) The intended way from external representations to mathematical ideas 'is not straight, easy or clear' (Söbbeke, 2015, p. 1499) and these representations are not self-explanatory (Dreher & Kuntze, 2015, p. 91). 2) Mathematical ideas are abstract concepts and the representations must not be confused with the mathematical idea or object (Duval, 2006, p. 107).

Nevertheless, the structure of a given non-symbolic representation (no matter if these are concrete, virtual or mental) should 'fit' with the structure of a given mathematical idea (e.g. Sarama & Clements, 2016). On the one hand this fitting should be provided by an 'intended structure' built in by the manufacturer (Söbbeke, 2005, p. 135; Sarama & Clements, 2016, p. 82), on the other hand this fitting is the result of an epistemological process (Steinbring, 2015, p. 288). These aspects and conditions are taken into account through developing the tablet-app and analyzing its interpretation and use (see section "Design research and study design" below).

Synchronously linked representations

"Comprehension in mathematics assumes the coordination of at least two registers of […] representations" (Duval, 2006, p. 115). On the one hand this coordination is a learning goal in mathematics education. On the other hand, it is a challenge because representations can be misinterpreted and the representations of different registers do not have to be congruent and, therefore, a translation between registers is not simple, obvious or self-evident (Duval, 2006, pp. 115–120). An appropriate method to foster the ability to coordinate and translate between different registers and to realize what is mathematically relevant is to analyze *representation variations in at least two registers* (Duval, 2006, p. 125). In contrast to non-virtual representations, virtual representations can automatically be linked so that changes to one representative have a direct effect on another representation. This feature has the potential to support children in overcoming difficulties in switching between different representations (e.g. Ainsworth, 1999; Goodwin & Highfield, 2013).

Design research and study design

There are two goals associated with the presented project: 1) Developing the app considering the described theoretical background and 2) Investigating how children interpret the given non-symbolic representations and how they use the implemented features. To give an insight into the process of this design research project, we give a short description of the implemented features, a short description of the study design and in the next section a sketch of initial empirical findings on the interpretation and use by primary school children.

Implemented features

There are numerous apps focusing on place value (e. g. Burris, 2013; Ladel & Kortenkamp, 2016; Utah State University, n. d.). Though, none of these focus on the practicing phase of the learning, which provide self-generating tasks and informative feedback after entering a result. In addition to that, the implemented features of the app 'Stellenwerte üben' as described below can also be found in these apps. However, there is no app that combines all of these features.

Bundle and unbundle: The module 'Bundle' requests the bundling of *exactly* ten objects (ten ones or ten tens) to create a bundle of the next higher value (ten ones → one ten; ten tens → one hundred). To this end, one has to 'activate' exactly ten objects (e.g. by using the lasso-function) and to push the bundle-button. Then the ten objects assemble automatically into the next higher bundle and this slides to the respective column of the place value sorting map (see Fig. 1). An essential aspect of this assembling is the transparent value preserving: Even after bundling ten objects, these objects are still apparent (a ten is made of ten ones, a hundred is made of ten tens). In reverse, unbundling is possible, for example, by sliding one hundred in the tens (or ones) column. After doing this the hundreds automatically split into ten tens (or hundred ones). This feature is implemented in all modules with virtual representation of the base-ten material. Sarama and Clements point out that these virtual bundling activities are "more in line with the *mental actions* that we want students to learn" (Sarama & Clements, 2016, p. 85, emphasis in original) than activities with concrete-physical base-ten blocks.

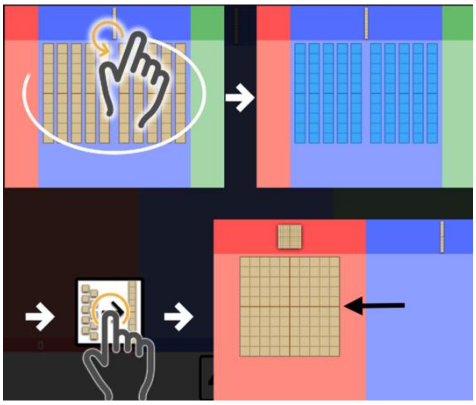

Figure 1: Bundling ten ten-rods

Sorting bundles: The module 'Sort' requests the sorting of an unorganized set of ones, tens and hundreds in the place value sorting map. By sliding the objects in the respective columns, the students practice the assignment of a given bundle to the appropriate place value column. In this context, the sorting is aligned to the visible quantity of a given bundle. This visual assignment is supposed to strengthen the integration of the 'quantity' value aspect and the 'column' value aspect of place value understanding (Sayers & Barber, 2014, pp. 24–25).

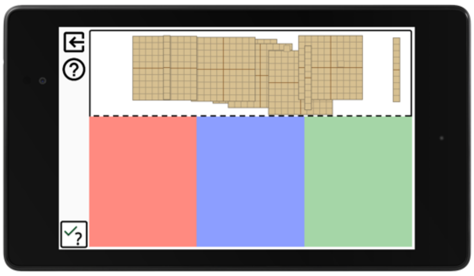

Figure 2: Sorting objects

It has to be taken into account, that these practicing activities (virtual bundling and sorting) *do not only by themselves lead* to a sustainable place value understanding, but there have to be several prior activities and accompanying instruction (e. g. van de Walle, 2004). In addition to this, it has to be taken into account that the described activities are based on representations that are visually clearly distinguishable by their quantity. These activities can be interpreted as didactic stages on the way to an abstract understanding of place value. A rigid adherence to these stages and an unreflected use of this representation could lead to common errors (as described by Ladel & Kortenkamp, 2016, p. 297).

The research interests arising from these considerations are the following:

- Research question 1 (RQ1): In which ways do students interpret and use the intended fitting between representations and mathematical ideas?
- Research question 2 (RQ2): How do students interpret the non-symbolic representation particularly with regard to the hundred-squares, ten-rods and unit-squares in the place value sorting mat?

Synchronized linked representations: Changes in the iconic register (e.g. unbundle a ten rod) lead to changes in the nonverbal-symbolic register at the bottom of the surface (0+10+2 changes into 0+0+12, see Fig. 3).

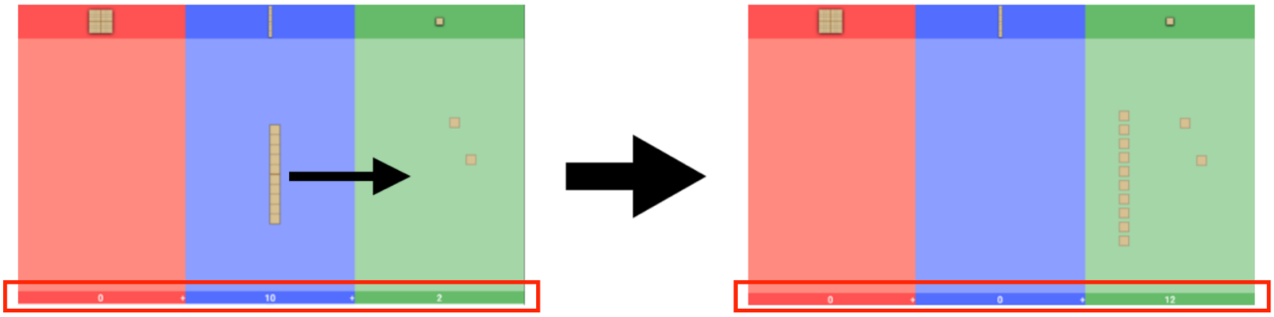

Figure 3: Synchronously linked representations in 'Stellenwerte üben'

- This leads to research question 3 (RQ3): Do children notice the synchronously linked representations and in what ways do they use this feature?

Study design

One focus of our research is to analyze students' *methods of usage and interpretation* while working with the tablet-app 'Stellenwerte üben' (see the research questions developed above). To this aim individual clinical interviews with n=29 German second and third graders have been conducted. All children knew the physical base-ten material from classroom. Main tasks have been, among other things: 1) the creation of non-symbolic representations: "Please show me the number two-hundred-fourty-five using the tablet-app." (RQ1), 2) the explanation of a created non-symbolic representation: "How do you know this is the number you are looking for?" (RQ1 and 3), 3) an interpretation of 2 ten-rods in the tens place: "Another child has told me this is a representation of twenty tens. What do you think?" (RQ2), and 4) taking away two tens from two bundled hundred-squares "Can you please remove two tens." (RQ1). All interviews have been transcribed (obtaining both speech and actions) and analyzed using a qualitative content analysis (Mayring, 2015).

Initial empirical findings

It was examined whether and how children *make use* of digital media's features implemented in the developed tablet-app. Furthermore, we wanted to evaluate how children *interpret* the given non-symbolic representation. These findings might lead to implications for further development, research and for lessons using the described app.

Using the fitting between representations and mathematical ideas

As expected, we found different types of usage. On the one hand, there are children who use the implemented features (here bundling and unbundling by sliding and lassoing) independently and are aware of what they are doing (for an example see the following transcript).

Interviewer:	What would you have to do to remove twenty?
Student:	I know, I know. Like this. *(Unbundles a hundreds-square by sliding it to the tens-column. Swipes away two ten-rods.)*
Interviewer:	Ok, and what is your result?
Student:	Hundred-and-eighty.

On the other hand, there are children who imitate the action with concrete base-ten blocks (namely trading). These children erase a given hundred-square, create ten ten-rods instead and take away two of these ten-rods. This finding suggests that previous experience with concrete physical material overlaps and influences the intended use of the implemented feature. This assumed interdependence between virtual and concrete actions is worth a closer lock in further investigation.

Interpretation of the non-symbolic representation

The non-symbolic representation of the tablet-app was developed to integrate both the quantity value aspect and the column value aspect of place value understanding (Sayers & Barber, 2014, pp. 24–25). To examine children's interpretation of this representation (two ten-rods in the tens-column) we asked explicitly for this interpretation ("Another child told us… What do you think?", see above). By doing this we found two different types of explanation: 1) Children who are aware of the fact that there are exactly two tens. Some of these children referred explicitly to the intended integration of quantity value and column value. 2) Children who mix up the naming of the given representation (“There are twenty tens”). Through further analyses and interviews we want to find out if these answers have their roots in an inaccurate use of mathematical vocabulary or if they are indicators for a conceptual misinterpretation of the intended structure.

Using the synchronously linked representations

As expected, our findings indicate that not all children consider the linked representations *independently* in their methods of use. Most children only focus on and refer to the non-symbolic representation. There are at least two possible explanations for this finding: 1) in the surface’s graphical proportioning there is much more room for the non-symbolic representation than for the symbolic representation and 2) the main tasks requested actions with the non-symbolic representation. Nonetheless, it can be stated that there are both low- and high-achieving students who do independently use the linked representations, and all students use this feature when asked for an explanation. In these cases, the linked representations are used to support children’s argumentation and as self-monitoring.

Interviewer: Please show me the number two-hundred-fourty-five.

Student: (*Creates two hundreds in the hundreds-column, four tens in the tens-column, five ones in the ones-column.)*

Interviewer: Ok, and how did you know, how many hundreds, how many tens and how many ones you needed?

Student: One square is hundred ones (*points at the hundred-square*) and there are two, thus, two-hundred. You can read this here, too (*points at the symbolic representation*).

Conclusion

The described study investigates children’s *usage* and individual *interpretation* concerning a virtual practicing material. The tablet-app was developed to foster and strengthen children’s place value understanding and it provides several features which are supposed to support mathematical learning processes. The implemented features are based on mathematical didactic theories and findings. In the following, we summarize the main results and draw some conclusions for teaching and further research.

The described findings validate the previous assumptions (Walter, 2018) that the existence of the described mathematical didactic features of virtual representations does not automatically lead to an intended use. Especially instead of using the synchronously linked representations children mainly focused on the non-symbolic representation, which is quite explainable. Nevertheless, we found that some children (both low- and high-achieving) used the linked representations *independently* and others used them for argumentation when asked for an explanation. This knowledge gives teachers the opportunity to discuss the connection between the different representations in the context of ‘math conferences’ with all students. In these settings students may become aware of the possibility to change between and use different representations, which they do not yet do independently.

Another feature which was not used as intended by some children is the fitting between the virtual actions and the intended mental actions (Sarama & Clements, 2016, p. 85). Some students traded a hundred for ten tens and vice versa, instead of bundling or unbundling. These different types of usage (trading vs bundling and unbundling) could be used for constructive discussions in class: Do these different actions lead to the same results? What aspects of these actions are the same, what are different? This discussion about actions, representations and the underlying mathematical ideas might lead to a better understanding of some basic aspects of the place value concept.

As mentioned above, some of the reported findings lead to further research interests like the investigation of an interdependence between using concrete and virtual representations. Also of interest is the question if the different interpretations of the non-symbolic representation ground merely on inaccurate use of mathematical vocabulary or if they are indicators for a conceptual misinterpretation of the intended structure. Since the tablet-app was developed for practice, further investigations also might focus more on autonomous usage (without an interviewer) and ways of practicing.

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