

The convective approximations

Raphaël Raynaud, Ludovic Petitdemange, Emmanuel Dormy

▶ To cite this version:

Raphaël Raynaud, Ludovic Petitdemange, Emmanuel Dormy. The convective approximations. European GdR Dynamo, Jul 2013, Ascona, Switzerland. hal-02428472

HAL Id: hal-02428472

https://hal.science/hal-02428472

Submitted on 6 Jan 2020

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers. L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

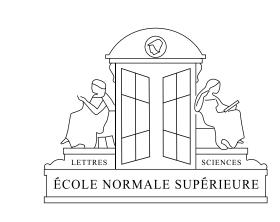


The Convective Approximations

Raphaël Raynaud, Ludovic Petitdemange, Emmanuel Dormy

raphael.raynaud@ens.fr

LRA, Département de Physique, École normale supérieure



INTRODUCTION

Convection can occur in a shallow layer of fluid with a small temperature contrast across the layer. Thus, it is natural to hope for a simpler description of buoyantly driven flows via an expansion of the fully compressible equations. The Boussinesq approximation performs well in modeling convection in laboratory experiments, but it is however unsatisfactory for large stratified systems, the lower part of which is compressed by the overlying material. More general approximations are then required to describe convection in natural systems like oceans, stars or planetary cores.

CONSERVATION LAWS

Mass conservation

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{V}) = 0$$

Momentum conservation

$$\rho D_t \mathbf{V} = -\nabla P + \rho \mathbf{g} + \mathbf{F}^{\nu}$$

with
$$\mathbf{g} = -\nabla U$$
 and $\mathbf{F}^{
u} = \nabla \cdot (au)$

and $\tau_{ij} = 2\mu \left[\frac{1}{2} \left(\partial_i v_j + \partial_j v_i \right) - \frac{1}{3} \partial_k v_k \delta_{ij} \right]$

where μ is the dynamic viscosity of the fluid

Energy conservation

$$\partial_t u^{tot} + \nabla \cdot (\mathbf{I}^{tot}) = 0$$

where

$$u^{tot} = \rho \left(E^k + E^I + U \right)$$
 (total energy)

$$\mathbf{I}^{tot} = \rho \mathbf{V} \left(E^k + E^H + U \right) + \mathbf{I}^q + \mathbf{I}^{\nu}$$
 (total energy flux)

with

$$E^H = E^I + P/\rho$$
 (specific enthalpy)

$$\mathbf{I}^q = -K\nabla T$$
 (heat flux)

$$I_i^{\nu} = -V_j \tau_{ji}$$
 (viscous energy flux)

Conservation laws and the first principle of thermodynamics

$$dE^{I} = -P d\rho^{-1} + T dS$$

lead to the heat transfert equation

$$\rho T D_t S + \nabla \cdot (\mathbf{I}^q) = Q^{\nu}$$

with the viscous heating $Q^{
u}= au_{ij}
abla_jV_i$

THE REFERENCE STATE

Decomposition of the thermodynamics variables into the sum of a steady variable corresponding to the reference atmosphere and a convective disturbance

$$f = f_{\mathsf{a}} + f_{\mathsf{c}}$$

The reference state must be in quasiequilibrium

mechanical quasiequilibrium: hydrostatic balance

$$-\nabla P_a + \rho_a \mathbf{g} = 0$$

• thermal quasiequilibrium: "well mixed" isentropic reference state

$$\nabla S_a = 0$$

This defines the adiabatic gradient

$$\nabla T_a = \left(\frac{\partial T}{\partial P}\right)_S \nabla P_a = \alpha^s \mathbf{g} = \frac{\alpha T_a}{C_p} \mathbf{g}$$

 $abla T_a = rac{\mathbf{g}}{C_n}$ (in the case of a perfect gas)

THE ANELASTIC APPROXIMATION

- non-dimensionalizing the equations: a dimensionless parameter must tend to zero when the temperature contrast goes to zero
- resulting set of equations

$$\nabla \cdot (\rho_a \mathbf{V}) = 0$$

$$D_t \mathbf{V} = -\nabla \left(\frac{P_c}{\rho_a}\right) - \alpha^S S_c \mathbf{g} + \mathbf{F}^{\nu}/\rho_a$$

$$D_t (T_a S_c) + \nabla \cdot (\mathbf{I}_a^q + \mathbf{I}_c^q) - \rho_a \alpha^S S_c \mathbf{V} \cdot \mathbf{g} = Q^{\nu}$$

THE BOUSSINESQ APPROXIMATION

- "thin layer approximation": a dimensionless parameter must tend to zero when the size of the convective region tends to zero
- resulting set of equations

$$\nabla \cdot (\mathbf{V}) = 0$$

$$D_t \mathbf{V} = -\nabla \Pi - \alpha T \mathbf{g} + \frac{\mu}{\rho_0} \nabla^2 \mathbf{V}$$

$$D_t T = \kappa \nabla^2 T$$

NUMERICAL IMPLEMENTATION

 mean field approach for the heat transfert equation split the variables into a mean and fluctuating part,

$$f = \langle f \rangle + f^t$$

so that

$$\langle \langle f \rangle \rangle = \langle f \rangle$$
 and $\langle f^t \rangle = 0$

This creates a turbulent entropy flux term

$$\mathbf{I}^{St} = \rho_a \langle S^t \mathbf{V}^t \rangle \propto \nabla Sc$$

Hypothesis: the turbulent diffusion of entropy dominates over the molecular diffusion of temperature in the heat transfert equation

- assume a polytropic reference state $(P \propto \rho^{\gamma})$ for a perfect gas Two additional non-Boussinesq control parameters:
 - the polytropic index $n = 1/(\gamma 1)$
 - the degree of stratification $N_{
 ho} = \ln \left(rac{
 ho_a ({
 m bottom})}{
 ho_a ({
 m top})}
 ight)$

ISSUES

- use of a non steady reference state
- consistency and domain of validity of the different variants of the anelastic approximation
- averaging the heat transfert equation

$$\rho T D_t S = \nabla \cdot (\rho T \kappa^t \nabla S) + \nabla \cdot (K \nabla T) + Q^{\nu}$$

CONCLUSIONS

- practical interest: filter out fast processes (acoustic/seismic processes) and retain slow processes (convection)
 - ⇒ low Mach approximation
 - ⇒ faster numerical integration
- method: from a thermodynamical point of view, consider the convective state as a small departure from a steady reference state

References:

- Braginsky & Roberts, GAFD, 1995
- The anelastic dynamo benchmark: Jones et al., Icarus, 2011
- Brown *et al.*, APJ, 2012