Introduction to the papers of TWG14: University Mathematics Education

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Introduction

The TWG14, *University Mathematics Education* (UME), was launched in CERME7 (Nardi, González-Martín, Gueudet, Iannone, & Winsløw, 2011) showcasing the fast growth of research in UME and also the specificity of the research context. In particular, the abstract, formal nature of a significant portion of the mathematical content; the absence of national curriculum guidelines, and therefore the great variations in organization and practices across institutions; the general lack of professional development or of systematic preparation for teaching; and, the volume of content to learn in a short period of time, and the degree of autonomy expected from the students and faculty. The consolidation of this research area, both outside and inside CERME, was recognized by the ERME community in inviting the first, three-time leader of this TWG to present a summary of UME research as a CERME10 plenary lecture (Nardi, 2017). Moreover, an ERME Topic Conference, *International Network for Didactic Research in University Mathematics* (INDRUM) was launched in 2015 with two successful conferences (INDRUM2016 and INDRUM2018) since then, with the first one leading to a special issue in the *International Journal of Research in Undergraduate Mathematics Education*, the new journal in the UME field. Future developments include a forthcoming book on *Research and development in University Mathematics Education* (Durand-Guerrier, Hochmuth, Nardi, & Winsløw, in preparation).

The number of papers submitted to TWG14 has been increasing since its inception. This year, we received 52 paper and 13 poster submissions, with 35 papers and 15 posters presented at the conference and published in the proceedings. Having received the largest number of papers in CERME10 and the second largest in CERME11, the number of papers led to the decision to split TWG14 into two isomorphic groups, TWG14A (18 accepted papers) and TWG14B (17 accepted papers) that ran in parallel; we held some common sessions during the conference. Each of the groups covered the same range of areas of research pertaining to UME, therefore the isomorphic label. The decision was prompted because splitting thematically would have resulted in losing perspective on the complexity of the issues that participants wanted to address. Furthermore, because the call for papers was open to a range of themes it would be unfair to assign contributions to a new group that might not align with the authors' original aim. This introductory paper summarizes the works presented in both groups organized according to the discussed themes, as well as the common discussions.

Some themes continue work from CERME10. For instance, we received a large number of papers focusing on students' learning of specific topics (10 papers), such as calculus, reasoning, and proof. We accepted seven papers proposing interventions, whereas in CERME10 we received five papers; however, relative to CERME10, the number of papers about teachers and their practices and about mathematics for non-specialists decreased from ten to four and from seven to two, respectively. We had two new themes: one on curriculum and resources and one on transition. In the next section, we briefly present the seven themes used to organize the work this year, with examples from paper contributions. It is important to note that many papers could fit in more than just one theme; this classification helped us to structure the presentations and the discussions during the sessions.

Themes and paper contributions

Students' learning of specific topics

Ten papers were classified under this theme: Aaten, Roorda, Deprez, & Goedhard; Gabel & Dreyfus; Herrera Alva, Figueroa, & Aguirre de la Luz; Kondratieva; Kontorovich; Lankeit & Biehler; Malaspina & Torres; Bašić & Milin Šipuš; Mkhatshwa; and, Pinto & Cooper.

Calculus was prominent in several papers (Aaten et al.; Herrera Alva et al.; Lankeit & Biehler; Malaspina & Torres; Mkhatshwa), with an increasing interest in two-variable calculus (Malaspina & Torres; Mkhatshwa) and analysis (Kondratieva). Under-researched topics were also present: differential geometry and its connections with two-variable calculus (Bašić & Milin Šipuš) and the use of conventions (Kontorovich). We also received papers addressing reasoning and proof (Gabel & Dreyfus; Pinto & Cooper).

Some of the papers discussed in this theme also concerned questions related with teachers and teaching. For instance, an intervention to improve in-service teachers' comprehension of discontinuity in one or two variables was also presented (Malaspina & Torres), as well as studies about the challenges in providing feedback that is actionable by students when revising a proof (Pinto & Cooper) or when teaching proofs (Gabel & Dreyfus). These last two studies raised questions in relation to the teaching of, and communication about, proof relevant to other themes we discuss below.

Resources and curriculum

Five papers were discussed in this theme: Bosch, Hausberger, Hochmuth, & Winsløw; Howard, Meehan, & Parnell; Pepin & Kock; Sabra; and, Viirman & Jacobsson.

From a student perspective, but also in relation to lecturers' intentions, Pepin and Kock explored what type of resources first year engineering students use and how they use/orchestrate these resources for their study of mathematics. They found that students' type and use of resources varied in relation to the type of course and the lecturer's expressed expectations. In relation to assessment, Howard et al. discussed student engagement with resources and continuous assessment in a mathematics course for business students.

Some of the papers studied the relation of university mathematics instructors with resources. For instance, Viirman and Jacobsson put forward the role of the mathematical content in the didactical choices made by university mathematics lecturers in course design; they highlighted how differences in the epistemological character of the topics to be taught may have affected the course design. Similarly, Sabra showed how different teachers with different training may focus on different aspects of the content to teach using some resources.

Finally, Bosch et al. presented a study about the didactic transposition processes that lead to the creation of undergraduate mathematics programmes, which fills a gap on the rationale behind the creation of such programmes. Overall, the papers discussed in this theme show how the use of resources is related to learning and teaching practices, a question that was discussed in the other themes as well.

Transition

Four papers were discussed in this theme: Bampili, Zachariades, & Sakonidis; Bergsten & Jablonka; Doukhan & Gueudet; and, Pinkernell.

Different aspects of transition were present in these papers. For instance, Doukhan and Gueudet studied praxeologies in pre-university and university courses regarding the notion of random variable; they showed that the content learned in secondary school may not prepare students for tertiary courses. Regarding institutional support, Bampili et al. showed how institutional support (or the lack of it) at university can influence the trajectories of novice students. Furthermore, Pinkernell reported on a frame (WiGORA), founded on five characteristics of understanding mathematics, for supporting students' transition towards university mathematics.

Transition towards and within university courses has been investigated in the last years through several lenses. Bergsten and Jablonka presented a review of literature concerning the secondary-tertiary transition, referring to five theories. Their review evidenced that, while there is a move towards socio-cultural theorizing, some dimensions pertaining to social issues still appear under-researched.

Literature on transition addresses, in many cases, students' challenges. This calls for intervention, and some specific examples were discussed in the following theme.

Interventions

Seven papers were discussed in this theme: Feudel & Dietz; Florensa, Barquero, Bosch, & Gascón; Ghedamsi & Lecorre; Hochmuth, Schaub, Seifert, Bruder, & Biehler; Kuklinski et al.; Thomas, Jaworski, Hewitt, Vlaseros, & Anastasakis; and, Uysal & Clark.

A relatively recent approach to constructing interventions—framed by the Anthropological Theory of the Didactic (ATD)—is the study and research paths (SRPs); SRP is a specific form of inquiry-based intervention. Florensa et al. discussed ten years of SRPs implemented at university level, showing their utility as a methodological tool for the systematic design of interventions. Other interventions included the implementation of a coaching program to support conceptual learning in a first-year mathematics course (Feudel & Dietz) or innovative lectures that foster more social interactions and student engagement than traditional ones (Kuklinski et al.). These studies investigated the impact of the interventions on student engagement. Finally, Uysal and Clark reported on the use of history in learning mathematics and its potential to support mathematical and emotional aspects of students' transition from high school to university.

On a more theoretical level, Ghedamsi and Lecorre compared and contrasted the Theory of Didactical Situations (TSD) and ATD with the aim of combining them in the design of a teaching intervention on real numbers and sequences. Many papers presented in this theme were related to other themes. Hochmuth et al. presented the structure of a diagnostic and evaluative test that helps students decide about what bridging courses are more appropriate for them; this paper has connections to the transition theme as well. Thomas et al. presented a project involving a bridging module, and showed that student-partners in task design furthered their own understanding of the topics (complex numbers) and developed didactical insights when designing mathematical tasks for others; this paper has connections to the transversal issue of collaborations in UME.

Students' identity and experience

Three papers were discussed in this theme: Thoma & Nardi; Toor, Mgombelo, & Buteau; and, Voigt, Rasmussen, & Martínez.

Students' experiences were seen by Thoma and Nardi in relation to their participation in the mathematical discourse. They used the theory of commognition to study students' substantiation of claims in exam scripts, in a first-year course called "Sets, Numbers and Theory." They demonstrated that students' participation in mathematical discourse, at least in the exam, was mostly ritualized. On the other hand, Toor et al. studied the experiences of post-secondary mathematics students learning to use programming as a computational thinking instrument for mathematics. They argued that identity is essential to the development of productive dispositions in learning to program for mathematics investigation and modeling. The notion of identity was central in the work of Voigt et al. as well. Adopting a sociocultural perspective and the notion of *figured world*, they showed how three different variations in a calculus course can have an impact on a student's trajectory and relationship with mathematics.

Teachers and teaching

Four papers were discussed in this theme: Costa Neto, Giraldo, & Nardi; Mali, Cawley, Duranczyk, Mesa, Ström, & Watkins; Mesa, Duranczyk, Watkins, & AI@CC Research group; and, Stewart, Epstein, Troup, & McKnight.

The involvement of and the tension between different communities in relation to the teaching of mathematics was addressed by Costa Neto et al. Using a re-storying methodology, their study examined disputes between mathematicians and mathematics educators in Brazil over the curriculum of a pre-service teacher education program. Teaching at the post-secondary level is the focus of Mesa et al. who describe the development of an instrument for the evaluation of community college algebra instruction, Evaluating the Quality of Instruction in Postsecondary Mathematics (EQIPM), through the analysis of video recorded data that include codes for student-content, instructor-content and instructor-student interactions. Specifically, Mali et al presented two of the twelve characteristics of instruction included in EQIPM: *Instructors Making Sense of Procedures*, and *Student Mathematical Reasoning and Sense Making*. Although these characteristics can contribute to the quality of instruction, the authors showed that exemplary instances of instruction with these characteristics were relatively rare. Finally, Stewart et al. addressed the work of a mathematician preparing his course on eigenvalues and eigenvectors, showing how the use of some didactic constructs may be useful for the mathematician to organize content and to reflect on his practice.

Use of mathematics by non-mathematicians

Two papers were discussed in this theme: González-Martín & Hernandes-Gomes and Tetaj & Viirman.

Artigue, Batanero, and Kent (2007) stated ten years ago that the study of issues related to mathematics as a service course would certainly need the use of sociocultural approaches. This is the case of the two papers discussed in this theme. For instance, González-Martín and Hernandes-Gomes studied the teaching of mathematics to future engineers with an institutional perspective based on ATD. Using textbook analysis, they compared how the integral is taught in a calculus course and in a mechanics of solids course for engineers. The authors identified a rupture in the study of a similar task (the sketching of the graph of an antiderivative), likely to prevent students from using their calculus knowledge in professional courses. Tetaj and Viirman used the commognitive approach to analyze the mathematical discourse of undergraduate biology students. Their results showed students' tendency to develop a ritualized approach, as well as their difficulties to mathematize a biological phenomenon; their lack of familiarity with relevant construction routines may prevent them from developing abilities to deal with certain problems.

Transversal issues addressed in plenary discussions

Resources in university mathematics education

Research on resources and their use in UME is growing, with works investigating the use of textbooks (e.g., González-Martín, Nardi, & Biza, 2018; Mesa & Griffiths, 2012) and technology (e.g., Lavicza, 2010). Some authors consider a general concept of resource, as anything likely to resource the activity of a teacher (e.g., Gueudet, 2017) or a student (e.g., Gueudet & Pepin, 2018). Since CERME10, TWG22 has been dedicated to studies related to the use of resources in mathematics education, but, in TWG14, some papers explicitly or implicitly addressed resource use and analysis of resources. For instance, the design of interventions is linked to the design of specific resources and the teaching of mathematics for non-specialists uses particular resources. The discussions on this topic in TWG14 raised several questions. Choosing an appropriate definition of resource is a first complex step. Should this definition encompass mathematical knowledge, professional knowledge (in the case of teachers), or affect? Different definitions may be needed depending on the question studied. The question of the use of resources for the research activities of the lecturers, as well as lecturers' interactions (or absence of interactions) with resources for teaching also requires further study.

University mathematics: teachers and teaching

Relative to school settings, research on teachers and teaching is less common in UME (e.g., Speer, Smith III, & Horvath, 2010). In contrast, CERME has three groups dedicated to teachers and teaching (TWG18, TWG19, and TWG20), four if one counts assessment (TWG21). The growing interest of UME in this area is evidenced in our TWG14, which discussed some of the challenges of working at the intersection of university mathematics teaching and research in mathematics education (RME). To begin with, the conception of what it means to know mathematics at university may be quite different from conceptions that are common in pre-tertiary RME. As a result, the models of university-level teaching may not be consonant with theoretical models and frameworks that typically guide RME. This state of affairs can be particularly challenging for researchers who set out to investigate their own mathematics lecturing. Another type of challenge has to do with relationships between the two communities involved: university lecturers do not always value RME as a scientific field, while educational researchers are often critical of lecturers' approach to teaching. Thus, to support research-based professional development for university lecturers there is a need for teacher educators with expertise in both content and teaching, credentials that may be scarce. These challenges and others were addressed in TWG14 contributions, as summarized in the *Teachers and Teaching* section. Finally, many research works involving the study of practices (e.g., teaching, resource use, planning, and assessment) of mathematics lecturers and also mathematicians call for the development of fruitful collaborations.

Collaboration with mathematics lecturers, mathematicians, and other professionals in research and development in UME

Collaboration and relationships between mathematics lecturers[1] (ML) and mathematics education researchers have been addressed in UME research studies (e.g., Nardi, 2008). This collaboration can be seen from (at least) two different perspectives: (1) through the roles of each group in such collaboration (e.g., UME researchers researching the practice of ML; ML researching their own

[1] We use "mathematics lecturer" or "mathematics teacher" to refer to a person who teaches mathematics at university. This person may be active in research in mathematics or not.

practice in collaboration with UME researchers; or UME researchers and ML collaborating in UME research and development); and (2) through the contribution of research in UME to developmental projects in industry and practice (e.g., what is the ‘utility’ of research to practice?) and to mathematics teaching and curriculum development (e.g., how a research finding is transformed in order to be used as a curricular ‘product’?). Collaborations and tensions in such collaborations were addressed in contributions to this conference as we have exemplified in the themes above. The discussion on this topic had two directions: (a) theoretical and methodological tools that will help to study such collaborations and make them fruitful; and (b) the role and impact of theory in such collaborations, from the perspective of both UME researchers and ML. Indicative points raised in the discussion concerned the observation that collaborations engage communities with different practices, as a result we should work on the: need of theoretical tools to study collaborations; need for tools to study boundary crossing, even though the boundaries between the communities are not clear (e.g., ML may also be UME researchers); need of ways to identify the goals and expectations of different professionals (e.g., mathematicians, chemists, biologists, engineers, etc.) who are involved in such collaborations; and, a need for better understanding of institutional factors (e.g., institutions may or may not facilitate such collaborations).

Reflection and ways forward

We initiated the discussion in the concluding session of our work at the conference summarizing some of the main points of the chapter *Research on university mathematics education* (Winsløw, Gueudet, Hochmuth, & Nardi, 2018), published in the book celebrating twenty years of CERME. Winsløw et al. (2018) synthetize the main contributions of CERME to research in UME in terms of: (a) *what is it?*, namely research into current practices of UME (with no direct intervention), such as: mathematical content; methods and resources; transition phenomena; student experiences; and, teaching non-mathematics specialists; and (b) *what could it be?*, namely developmental or experimental research, that includes an intervention design as part of the research project (e.g., research on, and for innovation in UME; i.e., interventions in specific courses or programmes) and professionalization of UME practice (preparation of ML).

In our discussion in the concluding session we reflected on these areas as well as on the studies we discussed at the sessions and we suggested the following ways forward: 1) because many areas and topics of UME require the establishment of varied collaborations (with ML, mathematicians, engineers, school mathematics researchers) we need to develop theoretical tools to study these collaborations; 2) some papers presented an articulation or networking of different perspectives which could be a way to study complex phenomena (there were papers using “external” theories, e.g., sociology); 3) large-scale studies are needed to consolidate some results; 4) innovative methodologies are also being developed, such as those to study the use of digital textbooks; 5) most of the current studies in UME are local, and therefore, we need to develop studies and tools that help to transfer findings to other educational contexts; and 6) new topics appeared in this conference: the use of history of mathematics, curriculum design and its origin, the role of course coordinators. We predict more new topics will appear in the years to come. We anticipate that the coming CERME conferences will allow us to pursue research on the areas and questions discussed this year and to bring research on some of these areas forward.

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