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Algebraic solutions of German out-of-field elementary school teachers

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In Germany out-of-field elementary school teachers are a frequent phenomenon but not regarded as unproblematic. In order to foster the pupils’ algebraic thinking, teachers themselves need to be aware of the structures. Additionally, tasks which foster process-related competences too can only be set when the teacher knows which discoveries can be made by using a specific task. In this article theoretical background is given to process-related competences, teacher training and out-of-field teaching at German primary schools. A teacher training course is presented and first insights into the algebraic solutions of the participating teachers are given.

Keywords: Professional development, teacher training, out-of-field teaching, algebraic thinking.

Introduction

In Germany, out-of-field teaching, which is to be understood as trained primary school teachers who have been trained in different subjects than mathematics, exists fairly frequently in primary schools due to terms and structures of the first and second phases of teacher education. As a result, a national study points out that nearly one third of all interviewed primary school teachers, according to their own statements, teach mathematics outside their area of expertise (Richter, Kuhl, Reimers, & Pant, 2012). Hence, teacher training courses are needed to support teachers in teaching their daily mathematics courses. Therefore, this paper presents a teacher training course for out-of-field primary school teachers with the focus on usage of task formats to foster process-related competences.

Firstly, process-related competences are described and why out-of-field teaching cannot be regarded as problem-free. In a second step, the design of the study is described. Thirdly, the teacher training design is presented to lead to the fourth part of giving insights into how out-of-field mathematics teachers themselves solve arithmetic tasks.

Theoretical background

There are several widespread theoretical topics relevant to fully acknowledge the key findings of the study which are briefly described in what follows below.

Process-related competences

In the German framework, agreements for elementary schools, math competences are divided into two different but intertwining kinds. On the one hand, there are content-focused competences, including numbers and operations; space and shape; data; frequency and probability; and sizes and measuring. On the other hand, there are so called process-related competences. They include problem-solving, arguing, communicating, presenting and modelling (KMK, 2004). By now, the latter are found in almost every curriculum in all German federal states. They are considered to be indispensable for a successful usage, acquisition and implementation of mathematics (KMK, 2005). The following example explains that in more depth:
Number-chains follow the rules of the Fibonacci sequences. The two first numbers are start numbers, which can be chosen freely from all natural numbers including 0 (they can also be identical or different). This task is one of those which “have the potential to address algebraic thinking” (Steinweg, Akinwunmi, & Lenz, 2018, p. 283). Not only (early) algebra can be fostered with number-chains but process-related processes as well. The children need to get adequate assignments and support. An assignment which fosters the competence of arguing is, for example, one which asks the students to (1) guess what happens when the first start number is increased by one, (2) calculate four examples and test their guess, (3) derive a rule from that and (4) give justification for the rule. It is important to describe and generalize the finding so that the children acquire knowledge about connections and relationships between numbers.

Eichholz (2018) showed in her study on out-of-field teachers in German elementary school that some of the participating teachers show little knowledge about these process-related competences. This indicates that there is a necessity to have a closer look at how teachers can be supported in learning and teaching general mathematics competences.

**Out-of-field teaching in German primary schools**

Due to the federal education system in Germany, study regulations vary widely in the 16 federal states. Although non-obligatory framework agreements state that studies at university to become a teacher comprise content knowledge and pedagogical content knowledge in both German and mathematics and an additional third subject (KMK, 2013), this is not the case in all states. For an overview on whether the education in the first and second phase of teacher education brings forth more generalists or specialist see Porsch, 2017. The principle of class teacher (Porsch, 2016) is dominant in German elementary school so that out-of-field teaching occurs frequently. However, there are enormous variations in percentages of teachers who teach mathematics without being fully educated from about 1.5% to almost 45% in the different federal states (Richter et al., 2012). Nevertheless, it is widely known that the competences of a teacher have considerable impact on the competences of their pupils. COACTIV, a German wide additional study to PISA, for example, showed that both content knowledge and pedagogical content knowledge of a teacher have an impact on students’ performances (Baumert & Kunter, 2011). There is not only a high correlation between content knowledge and pedagogical content knowledge, content knowledge also proved to be mandatory for pedagogical content knowledge (Krauss et al. 2008). Moreover, “[m]ore traditional beliefs were associated with more traditional practices” (Stipek, Givvin, Salmon, & Mac Gyvers, 2001, p. 221), such as emphasis on performance or speed, which contradicts the contemporary views on how a good mathematics class should look like. As Porsch (2015) pointed out, out-of-field teachers agree less often with constructivist views and therefore, according to self-disclosure, realize them less often.

These results imply that there is a need to focus research on out-of-field teaching in German primary schools and its consequences. However, there are quite contradictory results in German
research. While a nationwide investigation showed that elementary school students have poorer scores when being taught by an out-of-field teacher, which is especially true for the weakest 5% (Richter et al., 2012), reanalysis of the same data indicates opposing results. Ziegler and Richter (2017) state that other reasons are responsible for the comparably weak performances of these pupils. The fact that children with weaker cognitive skills and migrant background experience more out-of-field teaching could be an explanation for the first result. Nevertheless, it needs to be investigated why there are such differences in out-of-field teaching concerning that group of elementary school pupils.

(Pre-)Algebra in German primary schools

“Generalizations are the life-blood of mathematics” (Mason, Burton, & Stacey, 2010, p. 8). However, “German primary curricula and standards mention studies on algebra in a very limited way, if at all” (Steinweg, Akinwunmi, & Lenz, 2018, p. 284). Nonetheless, problems like little conceptual knowledge of variables are as present in Germany as they are in other countries (ibid.). This does not mean that there are no opportunities in German elementary school to foster algebraic thinking – but it stays implicit: “The objective is to encourage teachers to integrate algebraic thinking into their classroom. Moreover, the aim is to enable teachers to become aware of their already addressing algebraic thinking” (ibid., p. 286). The presented part of the study focuses on the algebraic thinking of teachers based on the assumption that teachers cannot set adequate tasks to foster algebraic thinking if they have not completely understood the mathematical structures themselves. This knowledge provides them with the opportunity to concentrate the pupils’ focus on interesting patterns and structures.

Teacher training

“Professional teachers require professional development” (Wilson, & Berne, 1999, p. 173). That there is a need for teacher training is commonly known, so various research results can be used to develop and legitimate the presented teacher training course. There are five core features which can be identified in most research literature: (1) duration, (2) content-focus, (3) active learning, (4) collective participation and (5) coherence (i.e., Desimone, 2009; Garet, Porter, Desimone, Biram, & Yoon, 2001). Derived from these core features of teacher training, the German National Center for Mathematics Teacher Training (Deutsches Zentrum für Lehrerbildung Mathematik, DZLM) has developed six design principles: participant orientation, competence orientation, diversity of teaching and learning, case study, stimulating cooperation and stimulating reflection (Barzel, & Selter, 2015). The following project is affiliated to the DZLM, however, elaborating on the six DZLM design principles is not possible in the scope of this paper (see Barzel, & Selter, 2015).

Design of the study

“Practice, at least in education, requires a cyclic alteration of research and development” (Freudenthal, 1991, p. 159). That is why this study was performed cyclically and was constantly revised. Prediger’s working group has developed a Design Research Cycle that is specifically adapted “for teachers with a focus on content-specific professionalization processes” (Prediger, Schnell, & Rösike, 2016, p. 97). The cycle is divided into four working areas which are “(a) specifying and structuring PD goals and contents in hypothetical intended professionalization
trajectories, (b) developing the specific PD design, (c) conducting and analyzing design experiments in PD settings, and (d) developing contributions to local theories on professionalization processes” (ibid., p. 98). The examined course was performed three times, with results from the first two courses being presented.

To gain insights into the level of knowledge and the beliefs of the participating teachers – about 25 in the first two circles – before and after the course, several research tools were used. At the beginning of the first lesson and in the last there were two different questionnaires. The first examines beliefs of how mathematics is learned, how it must be taught, the teachers’ self-concept regarding the process-related competences and other related features. These items are all from tested questionnaires which fulfil common quality criteria. The second questionnaire, which was designed by the author, is about how the participants themselves solve two different elementary school tasks. An operative change must be described in a first step. In a second step, the changes must be explained at elementary school level and, thirdly, the participants are asked to give an algebraic explanation. Each participant generates his or her personal code, so that the questionnaires stay anonymous but can be matched. Data evaluation is made mainly qualitatively by comparing pre- and posttest results and (possible) changes in beliefs. However, categories have been created, to allow statements about trends. Additionally, semi-standardized interviews with volunteers were conducted. The focus is on the questionnaire about the elementary school tasks to gain answers to the research question: How far do the algebraic solutions of out-of-field teachers primary school teachers, who participate in a training course with a focus on tasks (formats) to foster the process-related competences, change?

**Design of the teacher training course**

Based on the core features of teacher training and the DZLM design principles, the constructed course is presented briefly, before one exemplary period is described in more detail.

There are five appointments which last three hours. For each meeting one typical elementary school task format is combined with one process-related competence (except modelling), so that the participants gain knowledge in two different ways. On the one hand they are supposed to completely understand the task on a deeper mathematical level. On the other hand, they should be able to use their (newly gained) knowledge to set assignments which foster the process-related competences as well as the content-related ones (with help from Primakom, a platform for out-of-field teachers on which they can undertake training – regardless of point or period of time or place).

<table>
<thead>
<tr>
<th>Topic of the appointment</th>
<th>Primakom pages</th>
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<tbody>
<tr>
<td>1  Fostering process-related competences – documentation with the example of discovery calculation packs</td>
<td>• Mathematics – more than calculation</td>
</tr>
</tbody>
</table>
| 2  Recognizing and explaining patterns and structures, operative changes with the example of Fibonacci sequences | • Fibonacci sequence  
• Arguing                                      |
| 3  Discovering and communicative math classes – math conferences with the example of number pyramids | • Number pyramids  
• Communicating                               |
Combinatorics – problem solving strategies of pupils with the example of “building colorful towers”

Good geometric tasks – building up concepts of symmetry and space with the example of fold-and-cut tasks

<table>
<thead>
<tr>
<th>Topic</th>
<th>Content Focus</th>
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<tbody>
<tr>
<td>4</td>
<td>Building colorful towers Problem solving</td>
</tr>
<tr>
<td>5</td>
<td>Fold-and-cut-tasks Presenting</td>
</tr>
</tbody>
</table>

Table 1: Topics of the Appointment

Between two appointments there are always approximately four school weeks, distance phase, so that the participating teachers have the opportunity to try out what they have acquired in the presence phase beforehand. Reflecting on one’s own lessons can be enabled only when following this concept of duration, which is essential, according to Hippel (2001), when wanting to take the step from professional knowledge to professional action. In order to provide support during the distance phases, each topic is supplemented with one or two pages of an internet site (https://primakom.dzlm.de). The content focus is realized through the concentration on task formats and process-related competences. This covers the point of coherence, since these tasks can be found in almost every elementary school mathematics textbook allocated to different grades and are adaptable for all grades so that every participating teacher can use what is learned during the presence phases in their own class in the distance phases. To satisfy active learning, the participating teachers solve assignments with the focus on algebraic structures of the task. Additionally, they are asked to illustrate their findings using elementary school appropriate material, for example sachets, and variables.

First results
In the questionnaire about the teachers’ solutions of elementary school tasks, in this example number-chains, the teachers are asked to describe operative changes in number-chains, to give an explanation where these changes derive from what they could use in their elementary school classes and to give an algebraic explanation. It becomes clear that the participating teachers differ widely in their explaining and algebraic competences at the beginning of the course. Two examples, in each case of the same participants, of the pre- and post-tests for the following tasks are presented in what follows below: “Give an algebraic explanation (with variables) for the changes of the last number. Also give a reason for your algebraic explanation”. As shown in table 1, number-chains have been discussed during the course. In contrast, the other task in the questionnaire has not been talked about during the course. The first example of the pre-test illustrates that some conventions are known, for example, that variables are represented by small letters and often start with a. The explanation is, “When I add to a two numbers \( b + c \), I get the final number \( d \). If I increase both numbers by 1, the final number increases by two”. But the formation rules do not become clearer by the variables because they do not show the connections between the two starting numbers and the final number. That is why it cannot be counted as an explanation for the increase of the final number. But it has been recognized that the second starting number has increased by one, the third number as well and the final number by two. Nonetheless, the causal relation to the formation rules is not established. In the post-test, it can be seen that some other mathematical conventions have not been noted, such as that the notation of two variables without an operation sign stands for multiplication instead of addition. In the teacher’s explanation, however, it becomes clear that she knows it is addition: “In the addition of two variables \( b \) is taken twice, so that the result is increased by two times \( b \)”. This could be because, during the course, the elementary-school-appropriate representation to support the pupils’ generalizing competences are little sachets and no operation sign is needed in this case.

Teacher B tries to illustrate the formation rules similar to teacher A. She describes: “The first number-chain consists of (in the little chart) a: starting number, b: 2nd number, c: 3rd number, d: final number. The other numbers need to change”. This, again, is no illustration of the formation rules of the number-chains. But the changes are marked by the numbers in the circles, although this is not highlighting why these changes occur. In contrast to the pre-test, the post-test shows a different result for teacher B as well. She highlights the \( 2b \) by circling and underlining and explains “When \( b \) is increased by one, the final number is increased by...”
two because $b$ is two times in the final number”. The formation rule gets evident through the used variables, but she does not use numbers to illustrate the changes. Nonetheless, she explains that because of two $b$ in the final number, by whichever number $b$ is increased, it is doubled in the final number. This is, in contrast to the pre-test, a very good algebraic explanation.

Both examples show that there is a growth in algebraic competences for these two teachers. While in the beginning the formation rules cannot be expressed algebraically, the post-test shows that they can illustrate the changes and the reason for this case after the course. Their explanations change from being descriptive to being explanatory. This tendency emerges in a first analysis for the elementary school appropriate explanation as well as for the algebraic explanation. Before the course, the changes were described, but the answers of the participating teachers were not fully coherent and explanatory. This is only true for some teachers. A few participants were able to express the reasons for the operative changes in the pre-test as well. However, there is a very rare, in this case only one, questionnaire which is categorized as being fully explanatory in both questions (elementary school appropriate and algebraic). Whether the teachers were not able to see where the changes derive from or whether they could not express it properly, either with words or variables, cannot be answered.

**Conclusion and outlook**

This shows that there is a need for teacher training courses that foster the teachers’ algebraic competences. They are often able to name the changes and to describe them, but they are not able so explain why these changes occur and where these changes derive from. In order to foster students’ algebraic thinking and use the (full) potential of a task to foster content-focused and process-related competences at the same time, teachers need to be aware of the mathematical patterns and structures of such a task. Teachers need to be able to decide whether a student’s explanation is sufficient; teachers themselves need to know what is important to explain and not only to describe a process. In a next step, the other task of the questionnaire, which has not been discussed during the course, will be analyzed further. This may give insights into how far the participating teachers are able to transfer what is learned to other tasks. The solutions for both task formats will be categorized so that more than trends can be named. Additionally, the interviews will be considered to underline detailed analysis of some participants. Further investigations on design principles for teacher training courses and possibilities to support teachers need to be done.

**References**


