Cross-linking maths: Using keynotes to structure a curriculum for future teachers

Tanja Hamann[1] and Barbara Schmidt-Thieme[2]

[12]University of Hildesheim, Institut für Mathematik und Angewandte Informatik, Germany; bst@imai.uni-hildesheim.de

Keywords: Teacher education curriculum, history of mathematics, language.

Initial conditions and requirements

Subject-specific content knowledge is an essential component of professional teacher knowledge (for the concept of professional knowledge see for example Schwarz, 2013). It forms a basis for the ability to judge a specific topic's significance for an entire subject and thus forms an important requisite for didactical reflections (Dreher et al., 2018). Considering this, mathematics teachers' content knowledge must comprise not only advanced insight into specific topics but also a netlike overview of mathematics as one, consisting of notions and relations between them as well as methods and basic principles. A survey in a second-year lecture on arithmetic showed that the students were not able to connect what they heard about equivalence relations, functions or groups to corresponding contents they had learned in previous lectures (e. g. congruence and the group of congruence mappings in geometry, isomorphisms and linear functions in linear algebra).

To achieve that students are able to see these parallels, a university curriculum for future teachers demands cross-links between the single courses. Basic notions (such as sets, functions, algorithms, relations…) that are found in diverse mathematical subdomains seem suitable for serving as cross-links between scientific lectures, both, in a horizontal and a vertical way. This would transfer Bruner's (1969) concept of fundamental ideas and a spiral curriculum onto teacher education, also suggested in Dreher et al. (2018, pp. 330 f.).

Basic ideas and fundamental notions can be found when one looks into the history of a subject as they should appear throughout time (Schweiger, 1982), though maybe in different disguises. Apart from that aspect, including the history of mathematics into teacher education has been suggested long-since, presumably bearing a whole lot of further benefits (Schubring, 2000; Katz, 2000; Jankvist et. al., 2016), for example experiencing mathematics as a process conducted by human beings rather than as a mere product and thus influencing future teachers' beliefs. In addition, through history of mathematics one learns about the synthesis of notions and terms. This is of special importance for future teachers as it surely facilitates due decompression of scientific content into school content, which requires thorough analysis. (Dreher et al., 2018, 331)

Generation, clarification and precision of mathematical concepts and notions require the use of language (e.g. Morgan et al., 2014). Without language, definitions and propositions could not be worded and not be taught either, so language forms another essential part of content as well as pedagogic content knowledge. Due to increasing heterogeneity in pupils' language abilities, it is necessary that future teachers learn to illustrate subject matters on differing language levels and that they become able to vary by those levels.

The Hildesheim concept of "Learning along Keynotes" (LaK)

We suggest that integrating the three keynotes – basic notions, history and language of mathematics – into a teachers' curriculum and thus cross-linking the courses supports the construction of a professional teacher content knowledge as required. Therefore, the keynotes should become a constitutional part of all scientific lectures throughout studies. Our poster presents a concept of how the three keynotes might constitute a scaffolding for a spiral curriculum, where historical and language elements serve as enrichment as well as embedding of basic contentual and methodical concepts. We also present an exemplary concretization for the basic notion "algorithm".

LaK started 2018 in a first-year (linear algebra) and a second-year (arithmetic) lecture at the university of Hildesheim. Historical and linguistic tasks that are mostly integrated into exercises associated to the lectures comprise transferring formulas into common language or working on original historical texts.

To emphasize the netlike structure students are encouraged to design concept maps for a selection of basic notions (see poster for complete list) and augment them in the course of their studies, starting from first year and going on through all lectures. The next step will be to design appropriate tasks for all scientific lectures as well as a research frame for evaluating the concept in the form of a long term study.

References

Bruner, J. S. (1969). *The process of education*. Cambridge, MA: Harvard University Press.

Dreher, A., Lindmeier, A., Heinze, A., & Niemand, C. (2018). What kind of content knowledge do secondary mathematics teachers need? A conceptualization taking into account academic and school mathematics. *JMD, 39,* 319-341.

Jankvist, U. T., Mosvold, R., & Clark, K. (2016). *Mathematical knowledge for teaching teachers: The case of history in mathematics education*. Paper presented at History and Pedagogy of Mathematics 2016, Montpellier, France.

Katz, V. J. (2000). *Using history to teach mathematics: An international perspective*. Washington, DC: Mathematical Association of America.

Morgan, C., Craig, T., Schuette, M., & Wagner, D. (2014). Language and communication in mathematics education: an overview of research in the field. *ZDM, 46,* 843-853.

Schubring, G. (2000). History of mathematics for trainee teachers. In Fauvel, J., & van Maanen, J. (Eds.), *History in mathematics education: The ICMI Study* (91-142). Dordrecht, The Netherlands: Springer Science+Business Media.

Schwarz, B. (2013). *Professionelle Kompetenz von Mathematiklehramts-studierenden: Eine Analyse der strukturellen Zusammenhänge*. Wiesbaden, Germany: Springer Spektrum.

Schweiger, F. (1982). "Fundamentale Ideen" der Analysis und handlungsorientierter Unterricht. *Beiträge zum Mathematikunterricht 1982* (103-111).