Mathematical knowledge for teaching of a prospective teacher having a progressive incorporation perspective (PIP)

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This study examined mathematical knowledge for teaching of a prospective teacher having a PIP, Alin, and it revealed the relationship between PIP and mathematical knowledge for teaching. The case study design was used and the participant was selected by criterion sampling. Data were gathered from Alin’s lesson plans, transcripts of her practicum-teachings, interviews done before and after her teachings, and self-reflections on her teaching. Results showed that Alin having a PIP demonstrated all the codes in the Knowledge Quartet. Results also showed that knowing the perspective a prospective teacher has allows to draw on the basis of her mathematical knowledge for teaching. This indicates that the perspective a prospective teacher has might induce her mathematical knowledge for teaching. We suggest ways to promote the development of a PIP on the part of prospective teachers during methods courses.

Keywords: Progressive incorporation perspective, Knowledge quartet, Prospective teachers.

Introduction

The perspectives prospective mathematics teachers have on mathematics, mathematics learning and mathematics teaching have affordances and limitations on their teaching (Jin & Tzur, 2011). Mathematics teacher education is not an easy task because of the traditional views prospective mathematics teachers hold (Ball & Cohen, 1999; Lloyd & Wilson, 1998). Therefore, researchers suggested that to develop mathematical knowledge for teaching on the part of prospective teachers, their learning to teach should be highlighted instead of giving them certain skills during methods and practice-teaching courses (Ebby, 2000; Hiebert, Glad, & Morris, 2003; Shulman, 1986; Simon, 1995).

Based on a series of research with in-service teachers, some researchers have come to the conclusion that (prospective) mathematics teachers might have a continuum between traditional perspective, perception-based perspective (PBP) and conception-based perspective (CBP) on mathematics, mathematics learning and mathematics teaching (Simon, Tzur, Heinz, & Kinzel, 2000; Heinz, Kinzel, Simon, & Tzur, 2000; Tzur, Simon, Heinz, & Kinzel, 2001). Jin and Tzur (2011), in their work with a group of mathematics teachers in China, have placed an intermediate category in this continuum between the perception-based perspective and the conception-based perspective: the Progressive Incorporation Perspective (PIP) (see Table 1).
They used the term “perspectives” to refer to both the knowledge and beliefs teachers might hold regarding the nature of mathematics, mathematics learning and mathematics teaching and also the practices they might engage in based on such acknowledgment. Tzur and his colleagues proposed that teachers holding CBP would act accordingly with the views of radical constructivist epistemology. They proposed that PIP would be a more realistic target for the teacher education since “a PIP-rooted teacher’s practice can engender students’ learning processes envisioned by CBP without requiring the teacher’s explicit awareness of such view…”. They argued that providing ways for prospective teachers to develop a PIP during methods and practice teaching courses will contribute greatly to the field of mathematics education. In this respect, in the larger study, during the methods and practice-teaching courses, our purpose was to examine prospective secondary mathematics teachers’ development of PIP to teach mathematics effectively. In this paper, we report on the “mathematical knowledge for teaching (MKT)” of a prospective mathematics teacher who has been educated through these courses and has a PIP. Therefore, for this part of the study, investigating prospective teachers’ MKT once they have a PIP, we prepared a chart showing the list of characteristics a teacher would show in practice before, during and after the teaching. Also, when investigating MKT, we used the Knowledge Quartet (KQ) framework (see Table 2) (Rowland, Huckstep, & Thwaites, 2005).

<table>
<thead>
<tr>
<th>Perspectives</th>
<th>View of knowing</th>
<th>View of learning</th>
<th>View of teaching</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional Perspective (TP)</td>
<td>Independent of the knower, out there</td>
<td>Learning is passive reception</td>
<td>Transmission, lecturing instructor</td>
</tr>
<tr>
<td>Perception-Based Perspective (PB)</td>
<td>Independent of the knower, out there</td>
<td>Learning is discovery via active perception</td>
<td>Teachers as explainer (points out)</td>
</tr>
<tr>
<td>PIP (PIP)</td>
<td>Dialectically independent and dependent on the knower</td>
<td>Learning is active (mentally); focus on the known required as start, new is incorporated in to known</td>
<td>Teacher as guide and engineer of learning conducive conditions</td>
</tr>
<tr>
<td>Conception-based Perspective (CBP)</td>
<td>Dynamic; depends on the knower’s assimilatory schemes</td>
<td>Active construction of the new as transformation in the known (via reflection)</td>
<td>Engaging students in problem solving; Orienting reflection; facilitator</td>
</tr>
</tbody>
</table>

Table 1: Placing PIP within teacher perspectives (Jin & Tzur, 2011, p. 20)

The reasons we chose to use KQ in juxtaposition with the teacher perspectives framework was the following: First, we hypothesized that the framework corresponds to the three dimensions, nature of

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Contributory Codes</th>
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<tbody>
<tr>
<td>Foundation</td>
<td>(a) theoretical underpinning of pedagogy, (b) awareness of purpose, (c) identifying pupil errors, (d) overt display of subject knowledge, (e) use of mathematical terminology, (f) adherence to textbook, (g) concentration on procedures</td>
</tr>
<tr>
<td>Transformation</td>
<td>(a) teacher demonstration, (b) use of instructional materials (c) choice of representations, (d) choice of examples</td>
</tr>
<tr>
<td>Connection</td>
<td>(a) making connections between procedures, (b) making connections between concepts, (c) anticipation of complexity, (d) decisions about sequencing, (e) recognition of conceptual appropriateness</td>
</tr>
<tr>
<td>Contingency</td>
<td>(a) responding to students’ ideas, (b) deviation from lesson agenda, (c) teacher insight, (d) responding to the (un)availability of tools and resources</td>
</tr>
</tbody>
</table>

Table 2: The Knowledge Quartet: dimensions and contributory codes (Thwaites, Jared & Rowland, 2010, p. 86)
mathematics, mathematics learning and mathematics teaching, expressed in the perspectives. This is because as well the theoretical knowledge and beliefs related to mathematics and mathematics education are handled in KQ, the theoretical knowledge possessed by teachers is transformed into teaching through connections and the existence of contingency moments revealing students’ thoughts, mistakes and difficulties. Thus, student knowledge is also an important component of MKT. Secondly, we hypothesized that prospective teachers holding different perspectives regarding the nature of mathematics, mathematics learning and mathematics teaching might depict different MKT (Karagoz Akar, 2016). By the same token, even if prospective teachers depict the same MKT they might do so with having different reasons. For instance, prospective teachers with a PBP might not be able to anticipate students’ difficulties and respond to their ideas in spite of the fact that they might specifically know the ‘why’ behind the concepts since they are not able to think from their students’ point of view (e.g., Tzur et al., 2001). On the other hand, prospective teachers with a PIP might anticipate students’ difficulty and deviate from the agenda because they purposefully take every opportunity to expose and discuss students’ mistakes (Jin & Tzur, 2011). In this regard, scrutinizing the coherency between teacher perspectives and the domains in the Knowledge Quartet might help to uncover the reasoning behind (prospective) teachers’ MKT. In other words, we conjectured that once prospective teachers had the PIP perspective, their MKT would reveal itself during their teaching. We chose the Knowledge Quartet framework for evaluating MKT of prospective teachers since it allows to depict the MKT of teachers’ during their teaching qualitatively. Therefore with this study, we planned to make three main contributions to the field. First, how the practices of a prospective mathematics teacher with PIP before, during, and after teaching will be examined. Secondly, how such practices comprehensively revealing the relation between the characteristics of the PIP and the codes of KQ through empirical data, including before-during-after teachings and interviews, will be depicted. Third, and most importantly, how this correspondence presents evidences that a prospective teacher with a PIP is able to perform effective mathematics teaching will be explored. Diagnosing the reasons might provide teacher educators with particular steps to follow towards establishing more sophisticated perspectives and a full grasp of mathematical knowledge for teaching on part of prospective teachers. It is in this respect that the purpose of this study was to examine the MKT of a prospective mathematics teacher (Alin) who has a PIP and to reveal the relation between PIP and MKT. In light of our goal, the research questions were: “What are the indicators that Alin has the PIP?”; “How is Alin's MKT while examining the codes of KQ?”; “How is Alin's perspective reflected in MKT?

Methodology

Participants

In the larger study, the participants were seven prospective secondary mathematics teachers who were in their fifth year of study at one of the universities, in which the medium of language is English, in Turkey. For the report in this paper, we chose one prospective teacher, Alin, having the highest GPA (Grand Point Average) (3,44). Based on the first week of the classes during the methods course, we observed her as a verbal individual. Also, she volunteered to participate in the continuing set of interviews and the teaching sessions till the end of the study. We used the data
from her because the data were representative providing context that allowed us to examine the relationship between PIP and MKT.

Data Collection

For this study, Alin’s practice teachings were videotaped and transcribed. Also, we conducted interviews with Alin to talk about her lesson plans prior to the teachings, observed her teachings and conducted interviews upon completion of the teachings within the same week. In this regard, we used a modification of account of practice methodology (Simon & Tzur, 1999). Therefore, we conducted an interview prior to and after the practice teachings and we also observed the lessons. In addition, Alin wrote self-reflection papers after watching her videotaped lessons based on reflection tasks (Öner & Adadan, 2011). For instance, in the interviews, we asked the rationale behind Alin’s choice of the learning goal(s), the tasks and how she thought that the tasks she has chosen would allow students to learn meaningfully. We also asked what evidences of learning and difficulties of students she observed during teaching. In this paper, we report on one of Alin’s practice teaching. She taught an 80 minute lesson to the 10th grade students in a private high school.

Data Analysis

First, each researcher analyzed Alin’s perspective on mathematics, mathematics learning, and mathematics teaching and then analyzed her MKT through the Knowledge Quartet framework. We used coded analysis (Clement, 2000) for both frameworks. We analyzed the data to support theoretical hypotheses generated by the two frameworks and provided empirical data to show the coherence between these two frameworks that might yield to hypothesis generation, in the following way: First, each researcher read the lesson plans and transcripts from the practice teachings line by line, looking for Alin’s explanations regarding her perspective. Using the characteristics of teacher perspectives taking into consideration of previous research, we looked for her existing meanings. Once we spotted a line of explanation regarding her meanings in any of the data sources, we also checked her reflection papers that could possibly provide further evidence of such meaning. Based on the conjectures, we continued to examine the rest of the data. Then, we came together to have a consensus on the data set and our analyses and went back to the whole data set to challenge our conjectures. After determining Alin’s teaching perspective on mathematics, mathematics learning and mathematics teaching, secondly, using the codes from the Knowledge Quartet (see Table 2), we examined both this same data set and read further each of the data sources line by line to determine Alin’s MKT. Then we came together to have a consensus on the whole data set to challenge our conjectures. Finally, we wrote the narrative regarding the relationship between Alin’s perspective and her MKT.

Results

In this section, we share data showing Alin’s perspective on mathematics, mathematics learning, and mathematics teaching and her MKT in relation to her perspective. Alin modified a task for her students to make sense of the function \( f(x) = ax^2 + bx + c \) and the meaning of the coefficients \( a, b, \) and \( c \) on the graph of the function. During the pre-teaching interview, Alin stated that students could construct mathematical meanings by performing the mental operations required for the concept. Moreover, she said that her lesson plan would promote such learning.
Researcher: What do you want the students to learn as a concept?
Alin: ... we need to observe the change of “a” one by one, and keep “b” and “c” constant so that we can only be aware of the change in “a” ... Let me say the amount of change in y in terms of x, rather than amount of increase, because “a” can be negative too. It is necessary for students to observe how the amount of change in y is changing. When “a” changed and x changed as one unit, they can compare the amount of change corresponding to y, so that they can have an idea about the shape of the graph, I mean the arms (referring to the parts of the parabola)... Actually, what I am learning is to compare the amount of change in y with respect to change in x as one unit for different “a” values.

Researcher: Why do you think this [lesson] plan will promote your students’ learning?
Alin: ...I’m starting with the amount of change in y = x^2; therefore, students need to recognize the arms of the graph gets open and there is a decrease, I mean, there is a change in slope... Starting always with y = x^2, how this change is formed and how this change affects the graph, so thinking this point... I mean, my activity provides quantitative operations by playing with something existing in their mind that they know.

Researcher: You said playing with something they know, what do they know? Like could you explain one more time what is quantitative operations?
Alin: They know what y = x^2 is, what its roots are, how the change occurs in y = x^2, I mean, how the slope is changing and how it looks in the graph. However, they don’t have any idea about what happens to the graph when “a” changes, because they don’t observe ax^2 + bx + c for changing “a” values. Therefore, the quantitative operations formed in their mind when they changed “a”, I mean the thing they know in their mind, is like how the slope in y = x^2 is, how the amount of increase is, and drawing the graph.

Alin expressed that students need to compare the amount of change and examine how the change in the dependent variable occurs when the independent variable changes one unit, in order to construct the meaning of the coefficient “a” for a quadratic function. For Alin, students might create an idea about the graph and its structure by only performing these mental operations—comparing the amount of change. This suggests that for Alin mathematics is constructed depending on the learner's mind. Also, Alin planned her lesson hypothetically depending on her students’ mental operations and actions. We propose that this could be considered as an additional characteristic for the PIP.

When we consider the MKT, data showed that Alin had awareness of the purpose for her teaching, and concentrated on the ways to reach that purpose. She effectively analyzed which mental operations students need to perform to make sense of the meaning of “a”. Her analyses revealed that she has a strong subject knowledge and theoretical background about coefficient “a” for quadratic functions. Moreover, her expressions about mathematical structure of the concept and her use of mathematical terminology supported that she has a strong subject knowledge. Also, she anticipated the complexity of the concept: She planned her lesson in such a way that by keeping “b” and “c” constant, students’ examining the change of “a” would be more efficient. Moreover, she gave priority to students’ mental processes and planned her lesson depending on students’ ideas rather than adhering the textbook or planning randomly. Her consideration of graphical demonstrations while students were examining the coefficient “a” showed that Alin has knowledge about different representations and she could integrate these representations into her lesson with
relations to each other, too. Alin also stated that she planned to focus on students’ mental processes throughout the teaching so that students could create concepts and structures of quadratic functions relating with their prior knowledge. She leaned on her choice of examples to this.

**Researcher:** You said you are starting with \( y = x^2 \). Why do you start with this?

**Alin:** Because \( y = x^2 \) is easy for students to understand and play on it, I mean they can use it to examine others... They know it very well, also they won’t work away to find roots, but they will examine the change in “a” directly.

Data indicated that Alin chose \( y = x^2 \) as an example because students already knew this function. This suggested that she wanted to re-activated their prior knowledge: Alin believed that students learn new mathematical concepts by forming relationships between the new and old knowledge in the process. While planning her lesson, Alin’s consideration of possible students thoughts’ on examining the amount of change in the \( y = x^2 \) revealed that Alin took teaching process into consideration before she executed teaching. All these approaches Alin has taken in the process of before teaching displayed the first three dimensions of the knowledge quartet. In addition, data showed that Alin is aware that there might be unexpected events in the class:

**Researcher:** … What is the purpose of your questions? I mean why did you plan to ask these questions?

**Alin:** I will ask (questions) so that they can make relations. Some students could go faster but some could not. I can change my questions in a way to make them more understandable by the students. I can ask questions to fix their misunderstandings. I mean I will react regarding to students’ thinking.

She explained that she prepared questions in her lesson plan to enable students to form connections and she is willing to make differences in classroom context depending on students’ approaches because students could make connections in different ways. She also thinks that her plan could undergo some changes depending on the teaching process. Therefore, it can be interpreted that the contingency dimension is present in Alin’s actions even prior to teaching. This was also evident in her teaching: During teaching, she realized that students faced some difficulties. So, she deviated from her plan by presenting an example that was not in her lesson plan.

**Alin:** …how does y values change when \( y = x^2 \) as x increases one by one, so when you consider the change of \( y \)… When you consider the change of it from 0 to 1, how does my y values change? From 1 to 2, how does y values change? (Waiting for several seconds)…

Okay, let’s consider linear functions; like \( y = 2x \). (Drawing on the board) In the linear function like this, when \( x \) goes through 1 to 2. When \( x \) is 1; \( y \) is 2. and \( x \) is 2 which value of \( y \) do you get? When the students faced some difficulties to make sense of the change in \( y \) corresponding the one unit change in \( x \), Alin tried to overcome such difficulty by providing an example not given in her lesson plan. She gave a linear function example of \( y = 2x \) and wanted students to examine the amount of change in \( y \) given \( x \). This approach revealed the codes of responding to students’ ideas, deviation from agenda, and teacher insight during instruction presented in the contingency unit of the Knowledge Quartet. With this example and her questioning, Alin further allowed the students to overcome the difficulty and examine the amounts of change in the quadratic function. This showed
that Alin provided opportunities for her students to reflect on their prior knowledge and connect it to the new knowledge. Alin’s approach also showed that she possessed the PIP’s characteristic features: mathematics is both independent and dependent of the learner. That is, for her both linear and quadratic functions had qualities—the amount of change in the dependent variable given the change in the independent variable—out of the learner. Yet, the learners needed to make sense of such relationship on their own. She further explained her reasons about her choice of this example during the interview after teaching:

Alin: ….I did not give \( y = x \) as an example because it goes as one to one, so I gave \( 2x \) as an example…. We talked on the board to look at \( x^2 \). When we look at it, after the things between \( x \) values, when we draw the \( y \) values with arrows, students said that \( y \) values are increasing… In terms of their answers; whether they learned by heart or understood the rate of change. (I asked) Are you sure? Why? Re-express what your friend said… I was like pushing them like that...

Alin explained that she chose the example, \( y = 2x \) rather than \( y = x \) since the function \( y = 2x \) could have triggered them to focus on the amount of change. Her insight during her instruction was strong enough that she realized her students’ difficulties. She aimed at triggering their mental processes to form the concept by using the common features of linear and quadratic functions.

**Discussion and Implementation**

This study investigated mathematical knowledge for teaching of a prospective teacher having a PIP and revealed the relationship between PIP and mathematical knowledge for teaching. Results showed that the teaching of a prospective teacher having a PIP was effective because she planned her lesson with a view on mathematics as depending on learners’ minds and revised her hypothetical plan for her teaching by considering both the learners’ actions and reasoning. We might link the revisions of a hypothetical plan with contingency situations presented in the KQ, too. For instance, Alin considered her students’ thoughts and deviated from her plan when she determined that her students had some difficulties during teaching. This is important because hypothetically determining the teaching process envisioning students’ reasoning prior to the teaching is suggested to be an important part of MKT (Silverman & Thompson, 2008). This is also important in terms of noticing skills prospective teachers need to acquire (Jacobs, Lamb, & Philipp, 2010). Results of the study also suggest that if prospective teachers are trained to have a PIP, the strong MKT arises by itself. Particularly, the effects of Alin’s perspective are seen precisely in the deep examination of data done with respect to the KQ framework before, during and after teaching. For example, Alin was aware of her students’ having a different mathematics than hers. That is, she did not adhere to the textbooks, rather she focused on her students’ mental actions and prior knowledge while she was deciding and arranging the lesson activities. Also, she always took into consideration her students’ prior knowledge, even during teaching, and reactivated their prior knowledge to connect it to the newly established knowledge. So these were related with the connection, transformation and foundation dimensions in the KQ as well as her perspective on teaching. Based on these results, we suggest examining which codes of the KQ are activated before, during and after teaching and examining MKT of prospective teachers having different perspectives. Finally, we suggest to promote the development of a PIP on the part of prospective teachers during methods courses.

**Acknowledgement**
References


