The development of Thales theorem throughout history

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Thales theorem may have different functionalities when using distances, algebraic measurements, or vectors. In addition to that, the utilization of a figure formed of secant lines and parallels or a figure relating to two similar triangles.

The aim of this work is to categorize different formulations of Thales theorem and explain why in teaching we must know the appropriate mathematical environment related to each Thales Theorem statement. The analysis of many geometry books in history makes it possible to distinguish two points of view according to different forms, demonstrations and applications of this concept.

The Euclidean point of view

The general statement of Thales theorem shows us the idea to move from one triangle to another, moreover, the link with similar triangles (immediately following it and generally with similar figures) is a characteristic of this point of view.

Proposal 2 of Book VI states that:

"If a straight line be drawn parallel to one on the sides of a triangle, it will cut the sides of the triangle proportionally; and, if the sides of a triangle be cut proportionally, the line joining the points of section will be parallel to the remaining side of the triangle". (Heath, 1956).

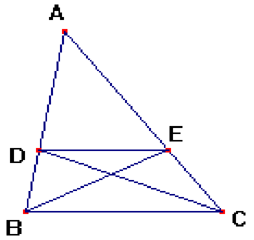

Figure 1 : figure of Euclid

The demonstration is based on the surfaces method; using equalities of triangles cases and making cuts and re-compositions in order to compare surfaces.

The point of view of transformations

This point of view is characterized by the disappearance of the link between Thales theorem and similar triangles, and, a passage from one line to another by projection appears in a figure of type: "parallels and secants".

In the seventeenth century, Euclidean treaty did not seem to satisfy some researchers. In fact, the latter prefer not to prove a result on lines using surfaces. In Thales theorem proof, Arnold rejects the detour by the surfaces made by Euclid.

For Arnold (1667), the statement of Thales theorem uses several parallels cut by secants:

"If several lines, being differently inclined in the same parallel space, are all cut by parallel lines to this space, they are cut proportionally"

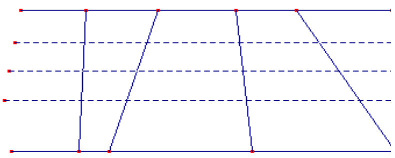

Figure 2 : Figure of Arnold

When applying the Thales theorem, we find essentially the search of 4th proportional, and the division of a segment with a given ratio.

This type of statement is also found in Hadamard (1928) where the study of figures marks the appearance of transformations that coexist with traditional objects of geometry.

The homothety, introduced with distances, consolidates for Hadamard dynamic aspect of geometric figures. It also provides the possibility to use similarity of polygons by decomposing them into triangles.

Note that starting from the 20th century, new conceptions of geometry appeared and benefit from linear algebra. The traditional methods of Euclid and his successors were eventually set aside.

With Choquet (1964), Dieudonné (1964) and other renowned researchers, geometry is evolving in each century. For Choquet, Thales theorem allows proving the relation $\alpha(u+v)=\alpha u+\alpha v$, in order to study the linearity of an oblique projection.

Conclusion

In Thales theorem proof, Euclid uses surfaces method to avoid problems of irrational numbers. In majority of perused publications, Thales theorem proofs often use commensurable segments, thus, the transition to immeasurable segments is admitted.

In teaching, it is important for instructors to conceive the real line without holes. We also think that the two points of view of Thales theorem have their efficacy and both should have enough time to reach the students. We recommend set each one of them in the appropriate mathematical environment that can result in a better understanding of geometric themes introduced in class.

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