New Math at primary schools in West Germany – a theoretical framework for the description of educational reforms

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To describe the implementation of New Math in West German primary schools (known as "Mengenlehre"/"set theory" up to today) in the 1970s, a framework model is developed. The model shows an educational reform as a two-dimensional process, characterized by a chronological as well as an institutional dimension wherein for the latter the educational system is divided into four layers. The model is introduced and specified when used for the description of New Math. Suitability of the framework for this suggests that it might be well transferable to the historical description of further educational reforms.

Elementary school mathematics, elementary school curriculum, history of mathematics education, New Math.

Initial questions and aim

In West Germany the New Math movement had a massive impact on mathematics education at primary schools from 1972 on, especially regarding the curricular contents and classroom methods of elementary instruction. The most remarkable innovation was the introduction of concepts from naïve set theory to young children and the use of materials such as colored blocks (the most famous being the *Logic Blocks* credited to Z. P. Dienes) in the classroom. However, there is still a broad consensus within a considerable part of German society that the reform failed miserably. A subsequent valuation whether the reform, its concepts and implementations were "any good" cannot be accomplished nowadays as there exists hardly any empirical data from back then. A statement whether "*Mengenlehre*" must be considered a failed experiment from the historical point of view, though, would require a thorough definition of the concept of "failure", which – as far as the author of this paper is aware – does not exist in this context. An estimation of the historical significance of New Math within the development of primary mathematical education in Germany is yet to be given.

Hitherto, existing analyses of the reform in the Federal Republic are widely limited to works by contemporary witnesses, naturally lacking historical distance (Damerow, 1977; Zumpe, 1984; Keitel, 1980, 1996), and short summaries as part of general overviews of the historical development of primary education (e. g. Padberg, 2011). The need for a broader account that allows a historical grading of the "*Mengenlehre*" reform and a well-founded verdict on its historical impact and role within German mathematics education is obvious. Any judgement from a historical point of view requires a comparative integration of the event into a broader context, and this again requires a description of the historical event first. The challenges one meets when taking up the description of the introduction and implementation of New Math in West German primary schools must be considered typical, as they are inherent in any reform within a public educational system. The aim of this paper is to introduce a theoretical model, which provides a framework that has proven

suitable for describing the case of "*Mengenlehre*" reform. It will then be shown how the model has been used for this purpose.

A framework model for the description of educational reforms

A reform is never an isolated event, but a process bound in a multitude of influences (general cultural and societal conditions as well as current developments) and outcomes, and for describing it numerous aspects and directions – or dimensions, as will be the term used in this paper – must be taken into consideration and be structured in a sensible way. The obvious dimension is the chronological one as a reform does not happen in an instant but in a certain timeframe. Not only is there an initial situation as well as an aftermath, but the reform period itself takes time and in the course of events ideas, conditions and actions might be subjects to change. But especially in the case of governmental reforms there is another, non-chronological dimension, which might be called the institutional one. Numerous protagonists act on diverse institutional levels and layers and thereby take an active part in the course of reform. These institutional layers do not function independently, but they influence each other in a certain way. So altogether, the description of a public educational reform must consider multi-dimensionality of reforms. In order to allow describing the institutional dimension a model is needed, which identifies separate layers and their respective protagonists.

H. Fend (2008, pp. 30-33) has developed a model for the educational system consisting of four (more or less) separate layers that influence each other from top to bottom. In his model Fend assumes that the protagonists on each of the levels interpret the ideas, that are covered in products from the respective level above, from their very own point of view, in their own specific contexts, and that this re-interpretation always necessarily leads to substantial changes, shortenings and adjustments. He denotes these changes and adaptations as "recontextualizations" (p. 13). The four layers in Fend's model are the following:

1. the layer of **culture**: within each cultural sphere certain ideals of education exist and from these educational objectives are generated

2. the layer of **curriculum**: educational aims serve as starting point, from which subjects and contents are deduced that seem suitable to meet the aims; products on this layer are curricula as well as textbooks etc.

3. the layer of **practical schooling**: this is the layer where teachers plan and perform their lessons regarding the specific nature of their students

4. the layer of **reception**: pupils (and their parents) comprehend and assimilate contents in their specific, subjective way and make sense of it

Fend has generated his model in the context of his studies in educational sociology, it is therefore not fully applicable to the history of mathematics education. It serves as a suitable starting point but needs to be adjusted to the subject of educational reform.

As the model considers the educational system as one, the subjects play a minor role. Educational research institutions appear as part of the layer of curriculum where their function is limited to a descriptive one, whereas their normative function in designing educational concepts is not

mentioned. Looking upon the educational system from a subject didactical point of view the scientific discipline as well as didactical research and design need an evident place within the model, and regarding New Math this applies even more, as on the one hand the reform has been grounded on subject science to a high degree and on the other hand mathematics educationalists were widely responsible for designing practical concepts, courses and textbooks. Hence, it is obvious that (despite some overlappings) there is a considerable difference between the protagonists responsible for writing curricula (mainly personnel in politics and administration) and those writing textbooks. Due to their importance for classroom practice mathematics didactics is given a layer of their own.

Science in general is part of culture putting it on the layer of culture. However, it belongs to a global kind of culture, that is not limited by a community's borders. Undoubtedly, national culture and traditions in general are highly relevant for any educational reform within a nation's borders but they comprise such a vast number of aspects that they cannot be described in detail within the frame of a work like this and are therefore neglected. Mathematics as a theoretical discipline is different from didactical science regarding the latter's practical implementations and the former's function in providing a basis for the latter's work. That is why the two scientific branches must belong to different layers. Of course, the scientific subject is not the only discipline educationalists refer to when designing their concepts, educational sciences like pedagogy and the psychology of learning play an important role, as well.

As mentioned before, when it comes to New Math contemporary empirical data is scarce and gathering an amount of data to be able to describe the layer of reception in a satisfying way (e. g. by interviewing former pupils) is certainly a desideratum in due research, but as it cannot be part of this work either the layer is integrated in the layer of school practice in this case.

From all these specifications in consideration of an educational reform derive the following adapted layers of the educational system:

1. the layer of **theoretical scientific discipline**: institutions and protagonists on this layer are scientists from subject and educational science; their products are subject-specific foundations, scientific theories for example on the development of knowledge etc.

2. the layer of **curriculum**: institutions and protagonists are personnel from politics (e. g. from the ministry of education) and administration, sometimes educationalists as far as they contribute to decisions concerning curricula; their products are curricula, syllabi, decrees etc.

3. the layer of **classroom concept**: institutions and protagonists are subject didactics, textbook authors (these can be teachers, as well), editors and publishers of schoolbooks etc.; their products are textbooks, workbooks, teacher's handbooks, manipulatives, suggestions for instruction etc.

4. the layer of **practical schooling**: institutions and protagonists are schools, teachers in classroom, pupils, parents; their products are lessons, learning progress, knowledge as well as tests, written tasks, exercise books, portfolios, reports and so on

As for personal overlaps between the layers (i. e. someone acts on more than one level) it is assumed that the actor's function is a different one on each layer and that they must recontextualize their own ideas in another context, as well.

On each of these layers the chronological dimension additionally comes to effect. If we stick to the top-down model developed by Fend, the adapted model can be visualized as follows, the arrows meaning "recontextualized by".

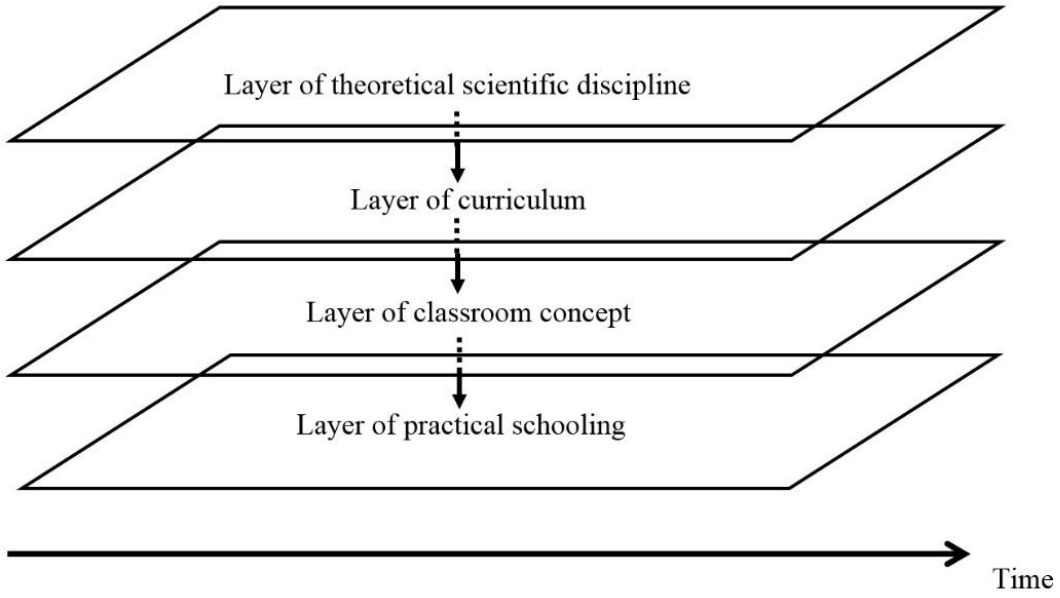

Figure 1: Adjusted layer model

This framework model now allows concretization dependent on the specific reform that is about to be described, the sources that have been selected and the focus chosen. Guiding questions that are implicitly suggested by the model are:

1. What central aims and/or ideas (on content, didactical principles, methods, curriculum concept…) can be deduced from the products of each layer?

2. How can one characterize the process(es) of recontextualization between the different layers?

3. How did ideas and concepts develop and change over time?

New Math in West German primary schools – sources, focus and findings

Choice of sources and focus

We are already dealing with a model naturally underlying diverse reductions. Nevertheless, further reductions become mandatory when one chooses exemplary sources for each layer and decides on a focus.

It has been mentioned before that sources for pupils' (and parents') reception of "*Mengenlehre*" courses are scarce, the same is true for the whole layer of practical schooling, therefore this part of the educational system will not be the focus. From a subject didactical point of view the layers on

which scientific personnel is most active – that are the layer of theoretical scientific discipline and, at least in the case of New Math, the layer of classroom concepts – seem the most relevant, that is why the emphasis will be laid on these. By this, the layer of curriculum gets assigned the role of an intermediary, which is relevant, if one wishes to describe the recontextualization that has been taking place between the other two layers, so that it must be looked at, as well.

In this case however, analysis of curricular documents leads to a specific difficulty, based on the German federal system, which involves that issue of curricular documents belongs to the federal states' field of responsibility. Hence, the number of syllabi that has been generated during the time of reform cannot be surveyed at once, and there is no work yet giving the due overview of primary school curricula in the way Damerow (1977) has provided of secondary school curricula. There are few nationwide documents, though, which must be taken into consideration. Explicitly concerning mathematics as a school subject, there are two directives on behalf of the *Kultusministerkonferenz* (the common conference of the federal states' ministers of education, referred to in figure 2 as KMK), the first one from 1968, which at the same time is seen as the starting point of reform in Germany, and the second one from 1976. In order to get an impression of at least one example of federal state curricula (*Rahmenrichtlinien*, referred to in figure 2 as RRL), those of Lower Saxony were chosen. The latest of these syllabi shows a massive decrease of the concept of "*Mengenlehre*" and therefore, its year of release, 1984, marks the terminal of the time axis.

Of course, the sources for classroom concepts are numerous, as well. Due to the facts that during the New Math reform primary school textbooks were written by mathematics educationalists and that they were widely accompanied by teachers' handbooks giving explicit insight into aims, concepts, didactical foundations and methods intended, samples from textbook series provide adequate sources for the layer of classroom concept. Here, the sample that has been chosen for thorough analysis of conceptual elements comprises 1st grade materials from three textbook series.

The textbook series are *alef* by H. Bauersfeld et. al. (1970), *Wir lernen Mathematik* ("We learn mathematics", referred to in figure 2 as N & S) by W. Neunzig and P. Sorger (1968) and *Mathematik in der Grundschule* ("Mathematics in primary school", referred to in figure 2 as F & B) by A. Fricke and H. Besuden (1972). *Alef* is a textbook series that derived from the only large-scale empirical project in the history of New Math in Germany, namely the *Frankfurter Projekt*, in the course of which children were educated with specially designed all new materials throughout their elementary school years. *Wir lernen Mathematik* was the earliest textbook containing "*Mengenlehre*" to be released in West Germany, even earlier than the first *Kultusministerkonferenz* directive, and thus it holds a special role in the process of the reform. *Mathematik in der Grundschule* has been chosen because it initially appeared in pre-reform times and has then been adapted to the reform standards. Of all three textbook series several editions exist, which are compared and thus contribute to the question how concepts developed and changed along the time axis.

As for the layer of theoretical scientific discipline, pedagogy has usually not been a point of reference for "*Mengenlehre*" concepts, so there is no need to consider the subject of pedagogy any further. Scientific mathematics, especially the Bourbaki work, is said to have had a big impact on

the global New Math movement but going into the scientific subject any further would lead astray. A major impulse to the start of reform activities within Europe is attributed to the seminar, which was organized by the OEEC (precursor of the OECD) and took place in Royaumont, France, in 1959. The seminar was attended by high school teachers and university mathematicians, who put a strong focus on how modern scientific mathematics could find its way into school, turning them into protagonists from the layer of theoretical scientific discipline. So, the report from the Royaumont seminar (OEEC, 1961) serves as source for the influence of scientific mathematics and according to the impact of the event, the date of the seminar serves as starting point of the time axis.

Another discipline the authors of textbooks have constantly referred to is the psychology of learning. Especially the landmark findings and theories by J. Piaget (e. g. Piaget & Szeminska, 1965) were most prominent. J. S. Bruner on the other hand has had a big influence even on modern day German curricula and his theories, which emerged in pre-"*Mengenlehre*" time, as well, were well-known. One of the biggest influences on primary school math certainly was Z. P. Dienes. Dienes is not easily allocated to one of the layers as he provided a vast amount of concrete classroom examples. As he never created a complete course, though, but rather provided theoretical foundations for German courses which partly arose from his own psychological research, Dienes is assigned to the scientific layer, as well.

From the above the general model can now be specified as it is filled with names of the documents that serve as sources for the description of New Math in West German primary schools. The arrow indicates the focus on the process of recontextualization from basic ideas and concepts as they can be derived from the layer of theoretical science to their concretization in textbooks on the layer of classroom concepts.

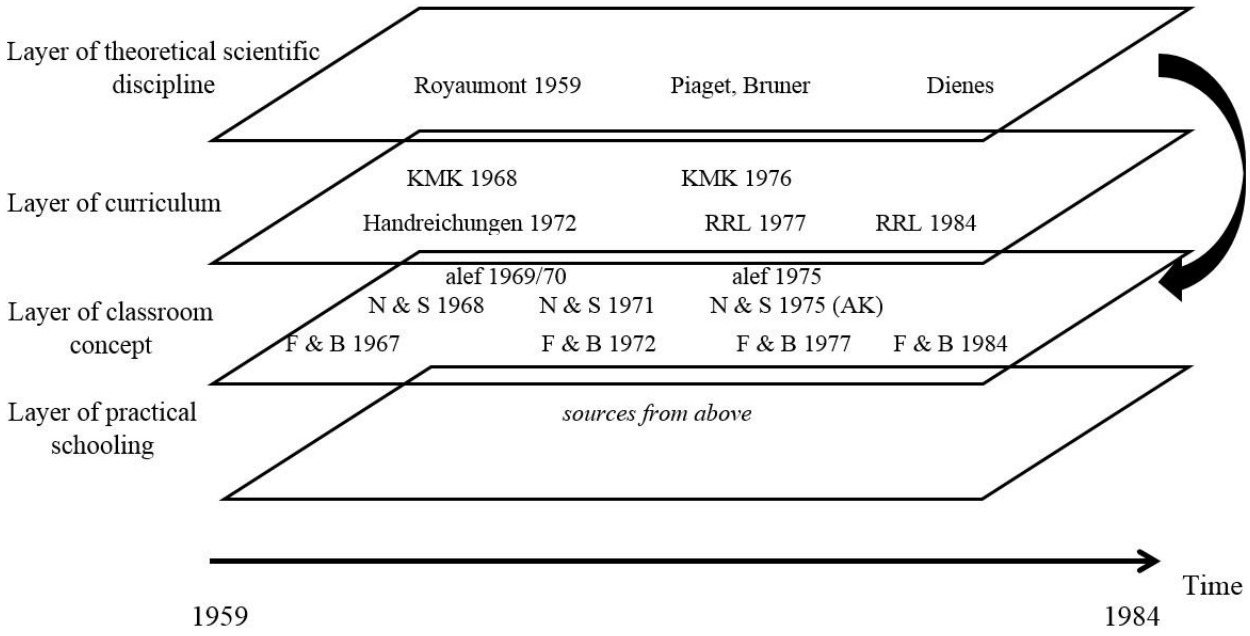

Figure 2: Layer model filled with sources used for the description of "*Mengenlehre*" in West Germany

Findings[1]

Regarding the focal question for recontextualization of the scientific roots of reform it becomes obvious that the extent to which original ideas have been implemented differs. Altogether they have not been fully transferred to textbooks, but the authors of each course have selected a certain part and by this, shortened the original concept towards their own respective purpose. Bauersfeld comes closest to the ideas from the scientific layer but in this case, he was forced by decisions coming out of the curriculum layer to apply changes to his concept. The shortcomings that are caused by the incomplete transfer of the original concept altogether lead to diverse inconsistencies in the implementation of "*Mengenlehre*". Independently of what happened on the layer of practical schooling, this finding even allows us to state that New Math at West German primary schools has indeed been a failure, and as educational reforms never go without recontextualization processes there is a high probability for extensive innovations in the classroom to fail according to this definition of failure.

Another finding concerns the influence of the layer of practical schooling. The sources suggest that teachers' customs, beliefs and attitudes, along with an education that did not meet the demands presented by New Math, proved as an obstacle in classroom. Many parents are said to have had reservations towards reform, anyway, and these most probably increased as soon as things were not implemented according to plan. Presumably, one political reaction to this was a change in curricula, which in turn led to a change in textbooks and therefore, regarding recontextualization processes along the time axis, one finds that the process of reform cannot be described using a top-down-model. Instead, within the framework model, mutual influences rather must be pictured the following way:

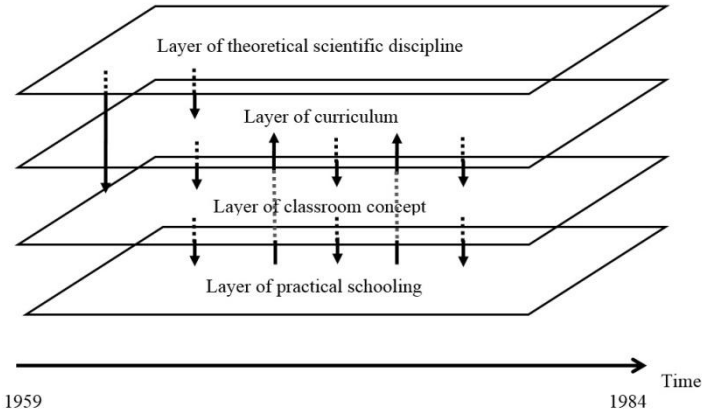

Figure 3: Layer model showing the process of reform

In this case, even though the original model did not prove fully suitable for a description of New Math in West German primary schools, it still served as a useful device for illustrating the process of reform.

[1] This paper presents an extract from a bigger research project. Due to the focus that has been set here findings can only be displayed in a strongly abridged way. For details and a complete account of the sources see Hamann, 2018.

Conclusion

It was the aim of this paper to introduce a framework model for description of complex educational reforms from a historical point of view. As for the example of New Math in West German primary schools, the model proved suitable for providing a restricting frame, a helpful device for structuring the work and a starting point that could be adjusted according to the process of the reform. It therefore allowed relevant findings which may function as a ground on further research on the topic. All of this suggests that the model of recontextualization might be suitable for historical studies on other educational reforms, as well.

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