Referring and proffering: An unusual take on what school mathematics is about

Jean-François Maheux, Jérôme Proulx, Rox-Anne L'Italien-Bruneau, Marie-Line Lavallée-Lamarche

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Jean-François Maheux[1], Jérôme Proulx[2], Rox-Anne L'Italien-Bruneau[3] and Marie-Line Lavallée-Lamarche[4]

Laboratoire Épistémologie et Activité Mathématique, Université du Québec à Montréal, Canada; [1]maheux.jean-francois@uqam.ca; [2]proulx.jerome@uqam.ca; [3]l'italien-bruneau.rox-anne@courrier.uqam.ca; [4]lavallee-lamarche.marie-line@courrier.uqam.ca

When it comes to appreciating school mathematics, there is a strong tendency to turn to the work of professional mathematicians as a reference. Although we see good reasons to do so, in this paper we question that reference to professional mathematicians as a "standard" for school mathematics and pay more attention to what the practice of mathematics in school itself has to offer. This raises significant political and cultural issues for school and research practices. The concept of doing|mathematics is discussed for this purpose, along with that of proffering.

Keywords: School mathematics, mathematicians' practices, reference, doing|mathematics.

Introduction

Historically and culturally, learning mathematics has often, if not always, been closely associated with the work of professional mathematicians. Evidence of this, for example, can be found in ancient Babylonian and Egyptian traditions, where training seems to mingle formal education and apprenticeship (Karp & Schubring, 2014). Closer to us, when the French decided to establish widespread public education after the 1789 Revolution, they naturally turned to mathematicians like Monge, Laplace, Lagrange, and Carnot to design and teach mathematical curricula to young people. One also thinks of Klein, who supervised the publication in Germany of many volumes on teaching mathematics at all levels. This turn toward mathematicians appears quite natural, because, after all, professional mathematicians are said to be the experts of the discipline: They spend their days doing mathematics and so they probably know better than anybody what mathematics is really about and what one needs to do in order to know about mathematics and practice it (Hersh, 1997).

Hence, we could say we have good reasons to refer to what mathematicians do to think about what is going on in schools. For one, paying attention to how mathematicians work helps us see the difference between doing mathematics and learning about mathematics (Papert, 1972). We also realize how much asking questions and trying out ideas is central to this practice (e.g., Brown & Walter, 2005; Lockhart, 2009). Also, we understand why problem-solving or mathematical modeling cannot be reduced to linear 4- or 5-steps procedures (Schoenfeld, 1985). Looking at what it means to do mathematics for a professional mathematician thus provides us with good grounds to challenge the simplistic idea of direct instruction and the possible overemphasis on the procedural part of school mathematics. Attending actual professional mathematicians' work is also important to humanize the discipline: mathematics is often seen as cold, uncreative, abstracted from real-life, homogeneous, and indubitable: all aspects that a close account of professional struggles and enjoyment of mathematicians can help transform (Burton, 2004).

This being said, we are, however, also familiar with the disastrous political upshot of the “new math” movement started in the 1960s, when governments tried to include “up to date” topics such as modular arithmetic, matrices, symbolic logic, and Boolean and abstract algebra in elementary and secondary school. This was also the case in the vivid discussions we know as the 1990 “math wars” in the US and elsewhere. The political role that mathematicians played in school mathematics was then talked about in relation to so-called traditional and reform mathematics philosophy and curricula; these ideas were often criticized as being conceived in an isolated way and disconnected with the everyday reality of the classroom. Nevertheless, it is still common today to hear mathematicians comment on how school mathematics “should” be: One thinks of Wolfram (2010) in the US, of Liu (2000) in Canada, and of Villani and Torossian (2018) in France.

Mathematicians undoubtedly have something to contribute to school systems, but the nature of that contribution might be interesting to discuss and not left unquestioned. Thus, in this paper, we raise a number of these questions, and in turn raise cultural and political issues, by reflecting on the issue of *referring* to mathematicians’ practices. Through drawing on the concept of *doing|mathematics* (Maheux & Proulx, 2015, 2018), we consider what the practice of mathematics in schools offers, and even *proffers*, as we call it, to mathematics itself as a discipline.

Referring to mathematicians’ practices

One way of examining how mathematicians’ work relates to school mathematics is to think about how we use this work to talk about what is, or needs to be, happening in schools. As mentioned above, it is quite common to refer to mathematics as practiced by professional mathematicians to express or orient what students ought to be doing in a mathematics classroom. Schoenfeld (1994), for example, describes true mathematics as the science of patterns and thus suggests that curricula be organized in the form of mathematical inquiry resembling what mathematicians do. The idea of being “authentic” to what mathematics is really about is also often used to critique didactical approaches to teaching mathematics in schools in order to promote a variety of practices that are said to be better aligned with what mathematicians really do: for example, problem solving (Borasi, 1992; Lampert, 1990), modeling-driven curricula (English & Gainsburg, 2016; Lesh & Zawojewsky, 2007), and classroom culture (Bauersfeld, 1998; Papert, 1996).

This notion of referent, when considered in relation to mathematicians’ practices for school practice, is multifaceted. One of the facets consists of referring to mathematicians’ practice as what ought to be happening in the classrooms. This would entail tailoring students’ mathematical experiences to what doing mathematics represents for a mathematician and try to have them experience this. So, if much of what mathematicians do is read papers, ask questions, try to find answers to these, and eventually change the questions based on the answers they find, this is also what students ought to be doing in classrooms. Of course, we do not see this often, but *some* parts of it are easily selected from what mathematicians do as we decide what to ask of students.

A second facet is referring to mathematicians’ practices as considering them as an objective to attain, that is, as the finality of school mathematics work, and the end goal to achieve. In that sense, students are not expected to reproduce the mathematicians’ activity, but to prepare themselves to perform it. From this perspective, one could say that although students might need to start learning how to ask good mathematical questions, they might mostly need to “learn the basis” (or the basics) so they can later engage in mathematicians’ practices. This is a common view, especially in undergraduate mathematics. However, there is no easy agreement on what exactly students most need as preparation to become mathematicians. For some, the answer to this question might be exactly what is suggested in the first facet.

A third facet is related to referring to mathematicians’ practices as a means of devising classroom practices. In analyzing what mathematicians do, we can identify key elements around which educational practices or activities can exist. For example, one could draw on mathematicians’ ways of discussing during conferences to organize classroom debates or boil down peer-review processes to students through checking one another’s work in writing, or again transform mathematicians’ lab interactions as small-group talk. Hence the activities considered are not necessarily direct examples of mathematicians’ practices, nor are they done to prepare students to be mathematicians, but are mostly inspirations from which to guide the design of specific activities. That is, the starting point could even be any given school practice (like testing, lecturing, note-taking, homework) into which some of the “essence” of mathematicians’ practices has been injected.

A fourth way of referring, more at a meta-level, is for mathematicians’ practices to be used, referred to, as a source of justification of practices attempted in schools. For example, the idea of changing a mathematics classroom’s ethos to make it more engaging for girls because we do not see enough of them in the professional community would be one example. Arguing in favor of introducing the history of mathematics because of its importance to the discipline itself could be another. Here, it is about mathematics as a practice, but more as an authority for justifying actions. Obviously, the three facets mentioned above can also be seen as a way of doing just that.

Although all these make sense for taking the mathematicians’ practice as a referent, one can wonder if they are legitimate for thinking of school mathematics. Hence, aside from the habits of doing things like this culturally, aside from political agendas related to curriculum aimed at reproducing societies’ current goals, one can wonder: is this reference to mathematicians’ practices, in all their possible facets well aligned with school mathematics’ practices?

Questioning professional mathematics as a referent

If only to make more transparent the choices we make, the reference to mathematicians’ practices is something to be examined. Indeed, a number of possible questions could be raised in relation to this reference. For example, who are the mathematicians to whom we are referring? Squalli (2010) answers this question, first by wishing to include all people who have a university mathematical background, but then arguing that we should be more inclusive and consider any person who uses or produces mathematics. This would include not only engineers, economists and many artists, for example, but also teachers of mathematics and even students who do mathematics every day. For Squalli, all these persons can be considered mathematicians because they are all engaged in activities that produce[1] mathematics at various levels, for diverse uses and

[1] In the sense of making mathematics happen, causing it to take place, to come into existence. It thus includes both the invention of some “new” mathematics and realization of any mathematical work, since both are based on the

needs. From this angle, the concept of mathematicians' practices as a referent no longer makes much sense: If everybody engaged with mathematics is a mathematician, then everybody acts as its own referent! Albeit lightly, this raises questions about the relevance of mathematicians' practices for school purposes. And many other questions can be raised on the matter, each leading to different viewpoints on the use of professional mathematics as a referent for the classroom. We offer here a sample of these questions, as a range of reflections frequently heard of when discussing mathematics education matters. This list is obviously not exhaustive, nor does it raise all possible matters on the issue.

- It is said that mathematics is everywhere around us, that we use it all the time. Would it not be more relevant to use everyday life mathematics skills as a referent?
- Mathematics is an evolving domain, as are mathematicians' practices. How can we use mathematicians' practices as a referent for school if they are not fixed and established?
- Most teachers have not been trained as mathematicians. Is it reasonable to expect teachers to have mathematicians' practice as a referent if they have not experienced it themselves?
- For many, school mathematics should prepare for citizenship, for example, by reproducing values that society deems relevant. How is mathematicians' practice of any help for this, especially if mathematicians' work is seen as lodged in a space where relevance to society is rarely desired, or even cumbersome?
- We often assert that mathematicians have a tendency to conceive mathematics as a platonist/absolutist domain. Is this a reference aimed at for school mathematics?
- Mathematicians are not homogeneous and are widely oriented (topologists, algebraist, logicians, etc.) by diverse (kinds of) problems, expectations, goals, and so forth. When we refer to mathematicians' practices, to whom are we referring?
- Mathematicians engage in the solving and posing of problems never solved or encountered before. How is this a valid referent for school when most problems, if not all, already have the answers at the end of the textbook?
- Mathematicians do not gather in a room to solve, in a given time frame, problems imposed by someone else. How close can school be to professional mathematicians' context?
- We might generously estimate the number of mathematicians on earth at 100 000. This contrasts with millions of kids doing mathematics in schools. Is it fair to impose the view of a few on the many?
- Most (previous and current) mathematicians are white occidental males living in privileged countries. How equitable is it to aim at reproducing their practices in schools?
- We know that pure and applied mathematicians, as well as engineers and other professionals, do not use the same mathematics and do not do mathematics in the same way. How is this to be taken into account as a referent for school practices?

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occurrence of some mathematical activity, or, again, as one mathematician in Burton's (2004) study argues, re/creation for oneself.

- Almost all mathematicians learned their mathematics in school and use that knowledge to create new mathematics. Would not professional mathematics then be considered as a form of applied school mathematics? Which would be the referent of which?
- Studies have shown that even in the university, classes taught by mathematicians do not align with mathematicians' practices. Hence, how is this "project" doable for schools if it cannot even be achieved in the university by the mathematicians themselves?

Each one of these questions could also be examined closely as a way to clarify why and how we think or not about mathematicians' practices as a reference for schools. These questions highlight how using mathematicians' practices as a referent for school mathematics is not trivial and hides diverse interpretations or outcomes, as well as agendas. But, while we see how the reference to mathematicians can be questioned, the notion of "referent" in itself can also be scrutinized.

Doing away with the referent? From referring to proffering

Desiring a referent for school mathematics comes with a prescriptive attitude. Why do we wish to look at the classroom mathematical activity from the outside, if not to "judge" it, one way or another, in relation to something else? A question we might ask is: what does it mean to *refer* to something? The verb *refer* comes from the Latin *referre* meaning "carry back", from *re-* "back" + *ferre* "bring". We refer to professional mathematicians' work when we turn back to it in order to appreciate what is going on in school, to describe either what it is, what it is not, or what it can be.

Reference is a relation in which one object designates another. As such, it suggests the defined, rather static existence of these objects in themselves. Searle (1983) explains that descriptive content of the sentence "Aristotle was a philosopher" does not define the name (what is "Aristotle") but establish the name's reference. There is something called "a philosopher", there was someone called "Aristotle", and by saying "Aristotle was a philosopher" we do not at all fix or change who Aristotle was and what a philosopher is: we simply state an assignation relationship. This view on language is challenged by other linguists (e.g., Bakhtin, Wittgenstein) for that very reason: thinking that a word can actually represent something is too restricting in regard to how words live in actual languaging (where their signification changes with the intonation, the context, the intention, the response, and so on), and the continually evolving nature of meanings (across time or cultures, for example). "Carrying back" school mathematics to professional mathematicians' work can certainly be unsatisfactory in this sense. What we mean by being a mathematician or doing mathematics is something open, fluid, and so is school mathematics. The designation relationship is problematic once we accept the dynamic, irreducible complexity *evoked* by the words "school mathematics" or "professional mathematician's practices".

One way to go around this tension is to think in terms of self-reference: an approach developed in many fields of studies, such as language, biology, philosophy and others. In mathematics, for example, self-reference is fundamental to the notion of fractals: mathematical objects exhibiting similar patterns at various scales. The specific nature of a given fractal is expressed in how it recursively reproduces its structure. Looking at coherence and similarity at various scales of mathematical activity (in and out of school) could be one way to further examine and develop it without having to make continual comparison with some externally posited entity. We offered elsewhere such conceptualization of mathematical activity (Roth & Maheux, 2015), contrasting the idea of defining mathematics "in itself" (i.e., as a fixed, independent, objective thing) with a dynamical approach in which mathematics is a way of making difference "in its own terms". Coherently, these terms are themselves observer-dependent and evolve under their own movement.

We have explicitly discussed this idea through the use of the dialectical expression *doing|mathematics* (Maheux & Proulx, 2015, 2018). *Doing|mathematics* is both *doing* something (some thing) recognizable as *mathematics*, but also producing *mathematics* as this thing that we are *doing* when what we do is mathematics. The Sheffer stroke between "doing" and "mathematics" serves here to emphasize the dialectical relationship between the two terms. *Doing|mathematics*, as an activity that produces mathematics in its production, represents as much the activity of mathematics than the mathematics produced: it is an act of meaning-making, where meaning is made through it. Thus, in this view, it is *doing|mathematics*, its activity and its product, which constitutes the landscape that we call Mathematics. That landscape is a "reference" for any mathematical idea or activity, but it is also something to which all contribute by nourishing, complexifying, defining and developing this very landscape of "reference". The two processes are inherently tied, dynamically: As we contribute to the mathematics landscape, this landscape influences us, and as it influences us, we contribute back to it. This also means that students, university professors, statisticians, teachers, and so on, in their ways and time, all indirectly but inescapably influence one another through this evolving landscape.

What is suggested here is *also* to consider school mathematics more like a *response*, a dialogue with mathematicians' professional activity, and not merely something that exists *in reference* to it. In this sense, school mathematics as an instance of *doing|mathematics*, is an act of *proffering*: offering itself as an answer to what it means to do mathematics. If mathematicians are seen as producers of mathematics, it is precisely because they engage in an activity that we identify as mathematics. They do mathematics in order to produce what we recognize as mathematical, while that very product of their activity also affects how we conceive of mathematics. It is the same for mathematics in schools, or in any other places where mathematics happens: workplace, street, institutes, etc. Mathematics in schools affects itself and contributes to itself while being influenced by the mathematical landscape with/in which it occurs and evolves. From such perspective, school mathematics legitimately has a life of its own (something often called for, see Hart & Johnson, 1984; Watson, 2008). But like just any other living entity, it is in constant interaction with other life forms (as non-living material).

Focusing on school, we could say that classroom mathematics dialogues with professional mathematician's practices; not to see what ought to be, but in terms of possibilities that they both embody. Thinking about what might happen is very different from setting something as an objective. Enabling students to further engage in mathematics can mean many things. Similarly, if school mathematics is seen as contributing in its own terms to the landscape, that is, to understandings of what it means to do mathematics, thinking of how students might experience things similar or different to what mathematician do is not merely justifying or reducing the later to the former, but exploring the very nature of those experiences. Considering how similar or different mathematical experiences might occur outside school or professional practices simply adds to these possibilities and enriches their very nature and essence. To some extent, this is at the core of ethnomathematical research (e.g., Powell & Frankenstein 1997), leading to wonder what really are "mainstream" mathematics practices. These are, of course, only very general orientations: something to be discussed, worked on, and engaged with more deeply.

Coda: On mathematics education research

The term *proffering* is provocative. It suggests turning the tables and thinking about what school mathematics actually offers to our understandings of mathematics as a practice. The idea that professional mathematicians could learn from schoolchildren does have a revolutionary twist. Taken more generally, the proposition is less aggressive: we can learn about what is mathematics and mathematical activity from both lay and professional mathematicians. Following such an assertion, one might ask: what is the role of mathematics education research in relation to this?

Evolutionary epistemology (Campbell, 1974) teaches us that by paying attention and studying a concept (or phenomenon) in order to understand it, that very concept (or phenomenon) is transformed. Studying professionals' or schools' mathematical practices also has this effect. By reifying, deconstructing and explicating them, we affect those practices. Analyzing a phenomenon renders some elements (more) salient; it imposes an interpretation that changes how we view things (regardless of how we acted on them or not). Observing always disturbs the observed, and discourse always shape how we do things. From this perspective, we can see mathematics education research as also contributing to the development of mathematics in a broad sense. If research investigates mathematical practices in schools and elsewhere, it not only makes these mathematical practices available to one another, but it also contributes in its own ways to them, affecting them in the process, and thereby also contributing to the mathematical landscape through it. So, coherently, whatever comes out of mathematics education research is not to be taken as *the* reference for mathematical activity. Proffering is also something mathematics education research does.

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