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► To cite this version:

C. Escolle, Vincent Michau, Marc Ferrari, Thierry Fusco, E. Hugot. Misalignment estimation for active telescopes. CEAS Space Journal, 2019, 11 (4), pp.553-559. 10.1007/s12567-019-00272-w. hal-02414025

HAL Id: hal-02414025

<https://hal.science/hal-02414025>

Submitted on 4 Nov 2021

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Misalignment estimation for active telescopes

C. Escolle^{1,2} · V. Michau¹ · M. Ferrari² · T. Fusco^{1,2} · E. Hugot²

Abstract

The estimation of the misalignment of an active telescope, from aberrations measured in its focal plane, is a tomographic issue. MMSE-based estimation is successfully used in optical tomography issues, like multiconjugate adaptive optics, where the problem is linear, and the phase perturbation spectrum rapidly decreasing. The implementation of MMSE developed in the case of multiconjugate adaptive optics has been applied to the alignment of active telescopes, although the forward problem is not linear, and the aberration spectrum different. Its performance is characterized by numerical modeling in the case of a TMA type large-field imaging space telescope, and compared to the classical Least-Square approach. The MMSE estimation brings a significant gain in terms of robustness and accuracy.

Keywords Active optics · Telescope · MMSE · Inverse problem · Space

1 Introduction

Due to volume and weight constraints, the next generation of large telescopes will combine compact optical concepts with lightweight primary mirrors and structures [6]. The static load, as well as the harsh satellite environment, will induce mechanical deformations all the more critical as compact telescopes are sensitive to misalignments [4]. In this context, active optics is envisaged to guarantee a constant image quality.

Active optics have first been studied in the 1980s for ground-based telescopes with thin primary mirrors [32]. After a first demonstration of feasibility, this technique has been applied to the 10 m-class telescopes [9, 24, 26, 30]. In most of these systems, Focus and Coma are corrected by positioning actively the secondary mirror, the higher order aberrations being removed by the active support of the primary mirror. The correction is achieved from a wavefront measurement performed in a single direction. In 10 m-class ground telescopes, as in existing active space telescopes, the estimation of short term misalignments does not deal with tomographic separation between primary and secondary

mirrors, either the alignment being sufficiently stable [18] or the primary mirror being considered as rigid [14].

In the future 30 m-class ground telescopes, the deformations of the secondary and tertiary mirrors should be significant, thus inducing aberrations evolving in the field of view. The estimation of the misalignment and deformation of each mirror will be mandatory [3, 5, 21]. In the same way, the new generation of wide-field space telescopes will include several active surfaces [8]. A tomographic approach seems appealing to estimate the corrections to perform [2].

This article deals with the estimation of the misalignment of an active telescope from wavefront measurements in the field of view. Note that the issue of the actuators control is not addressed. Although aberrations are not linear functions of the misalignment, the inverse problem is derived from a model based on an approximation of these functions by their first-order expansion. The tomographic estimation is deduced from an approach proposed by Fusco in the context of multi-conjugated adaptive optics [7]. A wide-field telescope, representative of space telescopes, is chosen for testing this estimation in a practical context. Its performance is characterized with the help of a numerical modeling.

Section 2 provides an overview of the active telescope chosen for the validation. Section 3 presents the estimation approach, as well as its implementation with the active telescope. The limit of the linear approximation for the telescope is studied in Sect. 4. Finally, the gain provided by

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MMSE estimation, as evaluated by numerical modeling, is illustrated in Sect. 5.

2 Active telescope characteristics

The telescope is a three-mirror anastigmat (TMA) [11] with:

- a 1.5 m parabolic primary mirror, M_1 , defining the telescope aperture. The optical deformation of M_1 is assumed to be significant, typically 150 nm rms.
- a 0.36 m elliptic secondary mirror, M_2 . Its misalignment aberrations of the order of 2 μm rms are predominant before alignment. M_2 is supposed to be rigid.
- a 0.5 m elliptic third mirror, M_3 , assumed to be perfectly aligned.

The field of view is linear ($\pm 0.6^\circ$ radius on sky). The schematic diagram of the telescope is presented in Fig. 1. Besides these three mirrors, the system also includes three folding mirrors and a deformable mirror, M_d . The folding mirrors are assumed to be perfectly plane. M_1 and M_d are optically conjugated.

The optical performance of the telescope is simulated by the ray tracing model implemented in Zemax. The wavefronts obtained with Zemax are processed with in-house algorithms. The residual aberrations of the perfectly aligned telescope, as estimated in that way, vary between 11 and 16 nm rms in the field (see Fig. 6).

The misalignment of M_2 has three main consequences: a shift of the field of view (0.2° for 0.5 mrad tilt or 0.5 mm decenter), a shift of the aperture image (0.1% for a 0.5 mrad tilt), and third-order misalignment aberrations [17, 25, 27–29]. The slope of the linear astigmatism is 360 nm rms per degree,

considering a 1 mrad tilt of the secondary. A typical despace of 250 μm induces a defocus of 6.2 μm rms.

In general, in case of moderate misalignments, the impact of the field and aperture shift on the misalignment aberrations is weak. Hence, the measure of aberrations in the field may be used to align the telescope, as already done since the beginnings of the 1990s, with ground telescopes such as 1.2 m Whipple observatory [12] or 8 m VLT telescopes [18]. With the chosen TMA, the same assumption may be made.

3 Estimation of misalignment and deformation of mirrors

3.1 Linear direct model

The proposed estimation is based on a linear direct model. Although misalignment aberrations are clearly non-linear functions of mirror displacements, a first-order approximation may be made around their perfect positions. Similarly, the mirror surface deformations are assumed to be small enough that the deviations of the optical rays are neglected. Note ψ_k the perturbations of the mirror M_k , i.e., the deformation of its surface and its displacement (decenters, tip/tilt and despace), and $\psi = (\psi_k)$. At first order, in focal plane, the wavefront error induced by the perturbations, ϕ , may be written as:

$$\phi(\alpha) = \mathcal{M}_\alpha \psi, \quad (1)$$

where α denotes the direction in the field. $\phi(\alpha)$ does not include the residual aberrations when the telescope is perfectly aligned. They are assumed to be known and consequently removed. In the following, $\mathcal{M}_\alpha^c$ denotes the restriction of $\mathcal{M}_\alpha$ to the vector of the controlled perturbations. The set of sources available for wavefront sensing being indexed by m , the set of wavefront measurements is expressed as:

Fig. 1 Optical layout of the telescope

$$\varphi(\alpha_m) = \mathcal{A}\phi(\alpha_m) + n_m, \quad (2)$$

where α_m is the angular position of source m . $\mathcal{A}$ describes the wavefront sensor measure and n_m the measurement noise. Finally:

$$\varphi = \mathcal{M}_{\text{wfs}}\psi + n, \quad (3)$$

with $\varphi = (\varphi(\alpha_m))$, $n = (n_m)$, and $\mathcal{M}_{\text{wfs}} = (\mathcal{A}\mathcal{M}_{\alpha_m})$.

3.2 Linear MMSE estimation

As initially proposed for multi-conjugated adaptive optics [7, 16, 20], the minimum mean square error (MMSE) estimator $\hat{\psi}$ of ψ is defined as the one that minimizes the quadratic distance:

$$\epsilon^2 = \left\langle \int_{\text{FoV}} \|\hat{\phi}(\alpha) - \phi(\alpha)\|^2 d\alpha \right\rangle_{\psi, n}; \quad (4)$$

FoV denotes the domain of directions defining the telescope field of view. The estimator sought is linear and, therefore, written as:

$$\hat{\psi} = W\varphi. \quad (5)$$

Statistics of measurement noise and of perturbations are assumed to be independent and Gaussian. Their covariance matrices are denoted C_n and C_ψ , respectively. The linear MMSE matrix, W , minimizes ϵ^2 with respect to W . By combining Eqs. (1), (3), (4), and (5), it comes:

$$W = \left[\int_{\text{FoV}} (\mathcal{M}_\alpha^c)^T \mathcal{M}_\alpha^c d\alpha \right]^\dagger \left[\int_{\text{FoV}} (\mathcal{M}_\alpha^c)^T \mathcal{M}_\alpha^c d\alpha \right] C_\psi \mathcal{M}_{\text{wfs}}^T \left[\mathcal{M}_{\text{wfs}} C_\psi \mathcal{M}_{\text{wfs}}^T + C_n \right]^{-1}, \quad (6)$$

where $\dagger$ denotes the generalized inverse. The solution given in Eq. (6) consists of an estimation step, described in Eq. (7), followed by a projection onto the effective solution space, i.e., the controlled perturbations. Both steps correspond to two issues which may be treated separately without loss of information [10]. When all perturbations are controlled, $\mathcal{M}_\alpha^c = \mathcal{M}_\alpha$, the estimation matrix writes:

$$W_{\mathcal{M}_\alpha^c = \mathcal{M}_\alpha} = C_\psi \mathcal{M}_{\text{wfs}}^T \left[\mathcal{M}_{\text{wfs}} C_\psi \mathcal{M}_{\text{wfs}}^T + C_n \right]^{-1}, \quad (7)$$

which compares to the least-square matrix:

$$W_{\text{LS}} = \left[\mathcal{M}_{\text{wfs}}^T \mathcal{M}_{\text{wfs}} \right]^{-1} \mathcal{M}_{\text{wfs}}^T. \quad (8)$$

3.3 MMSE estimation applied to the active telescope

The MMSE estimation introduced in Sect. 3.2 is applied to the active telescope defined in Sect. 2. At this point, the system is described by its linear model (see Eq. 3). All the perturbations are assumed to be controlled. The deformation of M_1 is described by the first hundred Zernike modes [19], the misalignment of M_2 by its tip and tilt angles, linear decenters, and linear despace. We assume that two sources in the field of view are used for wavefront sensing. For simplicity, the wavefront measurement, in each direction of the field, is the set of coefficients of its decomposition on the one hundred first Zernike modes (see [2] for normalization). The wavefront spatial sampling effects are not taken into account. Noise is modeled by adding an uncorrelated random term to each coefficient following the noise propagation encountered with a Shack–Hartmann wavefront sensor [23]. Hence, C_n is a diagonal matrix which diagonal represents the variances of the Zernike coefficient noise terms. In the following, the noise variance denotes the total of these noise term variances. The mechanical perturbations of the telescope being assumed uncorrelated, C_ψ is diagonal as well. Each term of its diagonal represents the variances of the mechanical displacements. $\mathcal{M}_{\text{wfs}}$ is processed from Eq. (7) with Zemax and in-house algorithms. Figure 2 presents the criterion, ϵ^2 , function of noise variance for different distances between both sources. At high noise, ϵ^2 saturates at the level of the initial wavefront error, 10^7 nm^2 . Otherwise, ϵ^2 decreases linearly with the noise variance. ϵ^2 equals the noise variance for

Fig. 2 ϵ^2 function of wavefront measurement noise variance for different angular distances between sources. The first source is at the edge of the field: $+0.60^\circ$. The position of the second source varies from -0.60° to $+0.55^\circ$. The misalignment aberrations are simulated with the linear model (see Eq. 3)

well-separated sources (0.6 or 1.2°), as expected in case of a well-conditioned problem and MMSE estimation.

Above noise levels of 100 nm rms, the mean square error is already close to the MMSE limit with one source only. An additional source does not improve the estimation error. With low noise, the performance depends on the distance between the sources, with significant deviations from the MMSE limit if the distance is below 0.60° for the chosen telescope configuration. In this case, the misalignment and M_1 deformations cannot be disentangled.

In conclusion, two sources provide enough information to estimate the perturbations of the active telescope, provided that the distance is greater than half its field of view. In the following, the most favorable configuration is chosen, *i.e.*, two wavefront sensors installed symmetrically at $\pm 0.6^\circ$.

4 Linear telescope model validity

In the previous section, the MMSE estimation has been implemented and tested, assuming that the system behaves linearly. In fact, The response of the active telescope to misalignment is far from linear. In the following, the limit of this approximation, that is to say the performance of the MMSE estimation, is characterized. The system description—perturbations and wavefront sensing—as well as the estimation matrix are unchanged, with respect to the previous section, but the behavior of the telescope is described with the model provided by Zemax. The performance is characterized considering two different levels of perturbations. The moderate perturbation case has already been introduced in an optical deformation of the M_1 surface with 140 nm rms, M_2 misalignment aberrations with $2\ \mu\text{m}$ rms. A low perturbation case is added: an M_1 deformation still with 140 nm rms, but M_2 misalignment aberrations with 150 nm rms.

Figure 3 presents ϵ^2 function of the noise level, for both perturbation cases. The result obtained with the linear model is plotted for comparison. The non-linearity of the telescope behavior introduces, for low noise levels, a floor in ϵ^2 . Its level is 26 nm rms ($680\ \text{nm}^2$) for moderate perturbations, and 3 nm rms ($10\ \text{nm}^2$) for low perturbations. To highlight the effect of the field shift, the estimation has been performed from, either the wavefronts in the real directions, or the wavefronts in the directions that would have been used in the absence of the field shift calculated with Zemax. In the moderate perturbation case, the floor level which is $680\ \text{nm}^2$ with real directions falls down to $40\ \text{nm}^2$ when the shift is removed. Hence, the field shift effect appears to be the major contributor of the estimation error induced by non-linearity.

Therefore, the model non-linearity error appears quite small. If this error is not acceptable, it may be removed, either by minimizing numerically ϵ^2 in an iterative way, determining the residual perturbations at each iteration with

Fig. 3 ϵ^2 function of wavefront measurement noise level for two levels of perturbation amplitude. The misalignment aberrations are simulated with Zemax model. The estimation is performed either taking into account the field shift or not

$\mathcal{W}$, or, if the correction device is available, by performing several correction loops.

5 Estimation of the perturbations to be controlled

Practically, the control of all perturbations is either not possible or unnecessary. Nevertheless, the uncontrolled perturbations, and uncorrected aberrations, may induce aliasing effects, *i.e.*, errors in the estimation of the controlled perturbations. This section is dedicated to the characterization of such aliasing effects. The position of M_2 is controlled using tip, tilt, and despace only, and the deformable mirror controls Z_5 to Z_{25} first Zernike polynomials. The complexity of the telescope and that of the wavefront sensing are gradually introduced. The first part deals with the effect of additional aberrations introduced by M_3 and the second part with the spatial sampling of the wavefront sensor. In both cases, the performance of the MMSE estimation is compared with the one obtained by the Least-Square estimation (LS) with truncated SVD, which is classically used in that case [1].

As a reference, Fig. 4 presents the performance obtained with the LS and MMSE estimators when the surface of M_3 is perfect. The same model of wavefront sensor as the one introduced in Sect. 3.3, is used to put apart the aliasing effect induced by the wavefront sensor. The noise is better managed by MMSE than LS. However, for low noise, MMSE and LS errors are similar and correspond to the uncontrolled part of M_1 deformations and M_2 misalignment. Such cases of low noise are given in space telescope, where, because of

Fig. 4 ϵ^2 function of measurement noise variance. Estimation of controlled perturbations only, performed with MMSE and LS. Perfect wavefront sensing. Perfect M_3

the absence of atmospheric turbulence, the noise is mainly generated by intrinsic errors of the sensor.

Figure 5 presents the performance obtained with the LS and MMSE estimators when M_3 polishing errors modeled using a classical decreasing power law for its PSD introduce additional field varying aberrations. The wavefront sensor is unchanged. At low noise, ϵ^2_{MMSE} corresponds to the uncontrolled part of M_1 and M_3 deformations as well as the M_2 misalignment. The wavefront rms error in the field is plotted in Fig. 6. The LS error is approximately 10 nm higher than the MMSE one. The difference between the two errors is due

Fig. 5 ϵ^2 function of the measurement noise variance. Estimation of controlled perturbations only, performed with MMSE and LS. Perfect wavefront sensing. Polishing errors on M_3 (10 nm rms)

Fig. 6 Residual aberration in the field. Measurement noise : 1 nm rms. Estimation of controlled perturbations only, performed with MMSE and LS. Perfect wavefront sensing. Polishing errors on M_3 (10 nm rms)

to the aliasing of uncontrolled, ill-analyzed M_3 deformation onto the estimate of M_1 deformation and M_2 misalignment.

The aliasing of high spatial frequencies onto low frequencies, due to a finite sampling in the wavefront sensing, is another source of error. The MMSE approach has already been proposed to minimize this effect in the wavefront reconstruction process [15, 31]. In the following, the mitigation of the aliasing in wavefront sensing, with MMSE estimation for active telescope, is illustrated considering a Shack–Hartmann [22]. The pupil is segmented in subapertures defined by the lenslet array. The wavefront measurements taken into account in the MMSE estimation are now the wavefront slopes, averaged on the subapertures, plus random uncorrelated noise terms. To put in evidence the aliasing error, two lenslet arrays are considered. The number of subapertures in the 6×6 array is close to the number of controlled perturbations, the number of subapertures in the 10×10 , close to the whole number of perturbations. Note that, since the Shack–Hartmann noise measurement decreases, because of the higher number of photons, with the subaperture surface [13, 23], the measurement noise variance is 3 times lower with 6×6 subaperture array than with the 10×10 . Figure 7 presents the simulation results obtained with these Shack–Hartmann models. The error ϵ^2 is presented as a function of the slope noise variance. The MMSE error obtained with 6×6 array is satisfying, even if a gain is obtained with the 10×10 array. On the contrary, the performance obtained with the LS estimator depends strongly on the Shack–Hartmann configuration, although the result is given for a truncation optimized for low noise level. With the 6×6 array, the LS estimation cannot manage the measurement aliasing. In fact, the high-order aberrations of

Fig. 7 ϵ^2 function of the measurement noise variance. Estimation of controlled perturbations only, performed with MMSE and LS. Shack-Hartmann wavefront sensing. Perfect M_3

M_1 are aliased by the wavefront sensor: M_2 perturbations and low-order aberrations of M_1 are badly estimated. A 10×10 array is mandatory to obtain an acceptable error.

Finally, MMSE manages the aliasing of the uncontrolled perturbations much better than LS, whatever the source of perturbations.

6 Conclusion

An approach dedicated to the estimation of the perturbations of the optical elements of a active space telescope, based on an MMSE estimation, is proposed. It minimizes the aberrations in the field of view of the telescope, given a few wavefront measurements in the focal plane.

The method has been implemented in the case of a active space telescope with two wavefront sensors at the edge of its field of view. Its performance has been characterized and compared to the classical Least-Square approach by numerical modeling. The MMSE estimation brings a significant gain in terms of robustness and accuracy. No tricky filtering, such as that required in the SVD inversion for Least-Square estimation, has to be performed to implement the MMSE estimation. In the low noise regime, all perturbations are handled by the MMSE, even those that are not in the space of the controlled perturbations. In the high noise regime, the MMSE approach provides optimal management of measurement noise.

Acknowledgements This work has been performed with the support of a Ph.D. grant from CNES. The authors would like to thank F. Cassaing, J. M. Conan, and L. Mugnier for fruitful discussion.

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