Advanced simulations of turbulent boundary layers under pressure-gradient conditions

Jaime Vaquero, Nicolas Renard, Sébastien Deck

► To cite this version:

Jaime Vaquero, Nicolas Renard, Sébastien Deck. Advanced simulations of turbulent boundary layers under pressure-gradient conditions. *Physics of Fluids*, 2019, 31, pp.115111-1 - 115111-31. 10.1063/1.5126932 . hal-02413211

HAL Id: hal-02413211

<https://hal.science/hal-02413211>

Submitted on 16 Dec 2019

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Advanced simulations of turbulent boundary layers under pressure-gradient conditions

J. Vaquero,¹ N. Renard,^{1, a)} and S. Deck¹

ONERA The French Aerospace Lab

8 Rue des Vertugadins, F-92190 Meudon, France

(Dated: 29 October 2019)

A high-Reynolds-number turbulent boundary layer experiencing pressure gradients is simulated with Reynolds-averaged Navier-Stokes (RANS) and hybrid RANS/LES (Large Eddy Simulation) advanced turbulence modeling approaches; namely two eddy viscosity models, two Reynolds Stress models (RSM) and Zonal Detached Eddy Simulation (ZDES) mode 3 which corresponds to a Wall-Modeled Large Eddy Simulation (WMLES) approach. Such a study is a first to the authors' best knowledge. The test-case considered is the experimental work of Cuvier *et al.* ["Extensive characterisation of a high Reynolds number decelerating boundary layer using advanced optical metrology", *Journal of Turbulence*, **18**, 929-972 (2017)]. Some modifications of the top wall geometry have been proposed to take into account the blockage effect of the boundary layers developing over the wind tunnel side walls so that statistically two-dimensional simulations are possible. Comparisons have shown that there are some difficulties to properly predict the mean skin friction and the Reynolds stresses in the adverse-pressure-gradient region for the ZDES and RSM models. The mean velocity profiles in this region are however poorly reproduced by all models. The atypical profiles experimentally observed at the beginning of the favorable-pressure-gradient region are well reproduced by RSM models, one eddy viscosity model and ZDES for the mean velocity, however only ZDES is able to satisfactorily predict the Reynolds stresses at this station. A spectral analysis of streamwise velocity fluctuations and Reynolds shear stress by means of ZDES has allowed to identify external energetic turbulent structures at $y \approx 0.5\delta$ and of size $\lambda_x \approx 3\delta$ which are probably responsible for these atypical profiles. The present numerical test-case may constitute a development base for turbulence modeling under pressure gradient effects.

^{a)}Electronic mail: nicolas.renard@onera.fr.

I. INTRODUCTION

Many research works have been devoted to the turbulent boundary layer with a zero pressure gradient (ZPG). However, in most engineering applications, turbulent boundary layers are under pressure-gradient effects which may significantly modify the dynamics of turbulence as well as the dynamics of the entire flow (for instance in the case of boundary layer separation induced by a pressure gradient). Therefore the interest of the effects of pressure gradients on turbulent boundary layers may be considered twofold. From an application point of view, pressure gradients are directly linked to performance limits of engineering devices and from a more fundamental point of view, the dynamics of turbulence is altered by the effects of a pressure gradient.

Numerous studies have analyzed pressure gradient effects on turbulent boundary layers experimentally. In Ref. 1 the boundary layer is subjected to a moderate pressure gradient at low Reynolds numbers which are kept low enough to perform a Direct Numerical Simulation (DNS) for comparison. Boundary layer parameters as well as boundary layer profiles and momentum balances are given showing the effects of an adverse pressure gradient (APG). In APG conditions, the shape factor tends to increase since the wake region of the boundary layer profile is enlarged. The inner-scaled profiles of Reynolds stresses show greater values in the outer part compared to profiles of ZPG turbulent boundary layer, which suggests higher turbulent energy in the outer part of the boundary layer. Similar results are presented in Ref. 2 for boundary layer at high Reynolds numbers (Re_θ from 2.5×10^4 to 5.4×10^4) and stronger pressure gradient in near-equilibrium conditions (as defined in Clauser³). The production of $\langle u'^2 \rangle$ (notation $\langle \bullet \rangle$ refers to Reynolds-averaged quantities) shows an outer peak near the middle of the boundary layer associated to the higher mean velocity gradient in this region (the wake region is enlarged by the APG) and the higher levels of $\langle u'v' \rangle$ in the outer part compared to ZPG, where u' and v' are the velocity fluctuations in the streamwise and wall-normal directions respectively. Higher order statistics are also given with momentum statistics of order 3 for the mentioned study. Identification of the turbulent structures contributing to the mean streamwise fluctuating velocity is presented in Ref. 4 in a parametric study of turbulent boundary layers under moderate pressure gradients. Again high order statistics are presented and the effect of the Reynolds number and the pressure gradient on the shape factor are separated concluding that the first one tends to decrease the shape factor as in ZPG conditions and the second one tends to increase it for APG (for the range of Reynolds number and pressure gradient parameter values considered in the study). Boundary-layer turbulent structures are also

Advanced simulations of turbulent boundary layers under pressure-gradient conditions

studied in Ref. 5 and 6 in which the impact of pressure gradients on these is evidenced in their size and spatial distribution.

One of the main difficulties in experimental studies is to ensure that the mean flow is two-dimensional in the symmetry plane which is important for both turbulence modeling and numerical simulation of the experimental problem. At the corners of wind tunnel walls, secondary flows of the second kind may develop⁷ and impact the mean flow. Such effect can lead to a mean flow that develops differently than expected^{2,8–10}. In Ref. 11 those corner flows are analyzed and it is shown that they do not significantly impact the mean flow in the center plane of the wind tunnel thanks in part to its long span and therefore the two-dimensionality of the mean flow is preserved in the symmetry plane. From a setting-up point of view, the experiment performed in Ref. 11 is of particular interest and it has been chosen to be reproduced in the present study by means of numerical simulations. Moreover, the range of Reynolds numbers considered is very interesting as well since it is not as great as for instance in Ref. 2 but it is high enough to make it appealing for studies concerning numerical simulations by means of turbulence resolving methods (especially for Wall-Modeled Large Eddy Simulation, WMLES). In addition, the boundary layer develops over a long distance (of about 20 *m*) which, together with the APG, results in boundary layers whose thicknesses reach almost 30 *cm* thus allowing for easier near wall measurements. Finally, another key point of Ref. 11 is the exhaustive database of the boundary layer available from the experiment.

On the numerical side, most of the works regarding pressure gradients in turbulent boundary layers have been carried out by means of DNS. Comparisons between an experiment and a DNS are made in Ref. 1 for boundary layer parameters and profiles as well as momentum balance. The same kind of analysis for near-equilibrium pressure-gradient turbulent boundary layers is made in Ref. 12 and 13, the latter giving also budgets of Reynolds stresses. Further analysis of turbulent structures in APG turbulent boundary layers in near-equilibrium conditions can be found in Ref. 14–19 by means of correlation techniques or turbulence spectral analysis. These studies focus on physics of turbulence in pressure gradient conditions at moderate Reynolds numbers because of the use of DNS. However, specificities of turbulence at high Reynolds numbers are expected as it is the case in more canonical wall-bounded flows²⁰. Besides, in most engineering applications, the Reynolds number is considerably greater than that of the DNS mentioned ($Re_\theta \sim 1000$ for most of the mentioned references).

Solving all the turbulent structures of a turbulent flow by means of DNS is possible for flows

Advanced simulations of turbulent boundary layers under pressure-gradient conditions

whose characteristic Reynolds number is low or moderate but it becomes unaffordable for high-Reynolds-number flows. The same problem applies for the Wall-Resolved Large Eddy Simulation approach (WRLES). The cost of numerical simulations of turbulent flows increases with the Reynolds number. For wall bounded turbulent flows, there are several estimations of the computational effort (in number of grid points) as a function of the Reynolds number based on the momentum thickness θ defined as $Re_\theta = \frac{U_\infty \theta}{\nu}$ where ν is the kinematic viscosity (ratio of dynamic viscosity μ and density ρ). As estimated in Ref. 21, the number of grid points N_{xyz} depends on Re_θ as $N_{xyz} \sim Re_\theta^{2.40}$ for DNS, $N_{xyz} \sim Re_\theta^{2.17}$ for WRLES and $N_{xyz} \sim Re_\theta^{1.17}$ for WMLES. The trends for DNS and WRLES are quite close whereas the increase of computational cost with the Reynolds number is significantly slower in the case of WMLES. In Ref. 22, the cost of numerical simulation of turbulent boundary layers is evaluated for a WRLES as a function of Re_δ (the Reynolds number based on the boundary layer thickness δ). It is stated that for $Re_\delta \sim 10^4$ more than 50% of the computational effort is employed for the inner layer of the boundary layer (defined in Ref. 22 as $y < 0.1\delta$). That fraction increases up to 99% for $Re_\delta \sim 10^5 - 10^6$. It is thus evidenced the interest of the WMLES approach for turbulence resolving simulations at high Reynolds number, since the computational effort is significantly reduced when using this approach due to the reduction of the number of points in the inner part of the boundary layer.

For the test-case of the present paper, the mesh criteria are those corresponding to a WMLES^{21,23} (Sec. II C 1). The mesh for the WMLES type simulation is made of almost 46 million points. According to typical DNS criteria²⁴, a DNS of the same case would require about 5×10^{10} points which is more than a thousand times greater. For a WRLES, the number of grid points would be around 3×10^9 , which is just about 10 times lower than DNS in accordance with Ref. 25 in which WRLES is referred to as QDNS (*Quasi-Direct Numerical Simulation*). This is an example of the huge difference of computational effort between DNS and WRLES with respect to WMLES, confirming again the interest of WMLES for high Reynolds number computations. In addition, to the authors' best knowledge, the present study is a first since there have not been previous studies of attached boundary layers in pressure gradient conditions simulated by means of WMLES using hybrid RANS/LES methods (RANS stands for Reynolds-averaged Navier Stokes and LES for Large Eddy Simulation.). Besides, these methods allow to solve part of the boundary layer in RANS which is very appealing for the industry and they are relatively simple compared to more complex methods as the one presented in Ref. 26. Hybrid RANS/LES methods have already been used as WMLES approach in boundary layer separation studies²².

Advanced simulations of turbulent boundary layers under pressure-gradient conditions

Solving the Reynolds-averaged Navier Stokes (RANS) equations numerically allows to avoid solving turbulence structures, which results in much coarser meshes and therefore in a huge reduction of the computational effort even compared to WMLES. There are however some important shortcomings to be considered when using the RANS approach. The first one is the lack of information from the fluctuating field, which for some applications is essential for instance in aeroacoustics or for unsteady loading prediction. Also, in the work presented in Ref. 27 or 28 the dynamics of the shear layer in the separated region is strongly influenced by the structures of the upstream boundary layer. This kind of information is unavailable in RANS simulations and this may be one of the reasons why RANS simulations struggle to properly predict boundary layer separation^{29,30}. The second shortcoming is related to the Reynolds stress in RANS equations. This term appears when applying the Reynolds average to the Navier-Stokes equations and it requires turbulence models^{29,31} (either for eddy viscosity approaches or Reynolds Stress Models (RSM) approaches). When simulating turbulent boundary layers in pressure gradient conditions, the mentioned turbulence models give less satisfactory results²⁹ (as it will be confirmed in the present work) than in the case of more canonical flows as for instance in a ZPG turbulent boundary layer. Indeed, according to Ref. 32 a good prediction of some kind of flows would require tuning the model especially for these, and it is not clear that there will be a generally-applicable way of improving RANS models. Hence, although the RANS approach allows for a strong reduction of the computational effort, its lack of information on fluctuating quantities and the difficulty to improve its accuracy in the prediction of pressure gradient effects on turbulent boundary layers makes turbulence resolving approaches of huge interest.

The present paper is structured as follows. The experimental facility of the chosen test-case, together with the different turbulence modeling approaches considered and the numerical solvers and setup are described in Sec. II. The test-case requires some geometry modifications for a proper statistically two-dimensional simulation of the problem, which are presented in Sec. III. Results from the numerical simulations are discussed in Sec. IV including instantaneous fields, Reynolds-averaged data and spectral analysis of turbulence. Finally, conclusions are given in Sec. V.

Advanced simulations of turbulent boundary layers under pressure-gradient conditions

II. COMPUTATIONAL DESCRIPTION

A. The LMFL wind tunnel configuration

The experiment of Ref. 11 will be reproduced by means of numerical simulations. A turbulent boundary layer has been studied under both favorable and adverse pressure-gradient conditions in the wind tunnel of the Laboratoire de Mécanique de Fluides de Lille (LMFL), whose test section is 20.6 *m* long, $L = 1$ *m* high and 2 *m* wide. The height of the test section will be called L and it will be used in the present work as a reference length. An extensive characterization of the boundary layer is made in Ref. 11 in the pressure gradient regions by means of the PIV technique (Particle Image Velocimetry) for two flow configurations, one for $U_\infty = 5$ *m/s* and another for $U_\infty = 9$ *m/s*. Due to the length of the LMFL wind tunnel, high Reynolds numbers are reached, and thick boundary layers develop thus allowing for near-wall measurements in the viscous sub-layer, which are in general complicated to perform. Two working configurations are possible for the facility: a closed loop configuration with temperature regulation or an open loop one with outside temperature. The first configuration has been used for the experimental study of Ref. 11, with an upstream temperature $T_\infty = 293$ *K* for both flow configurations.

Besides the extensive database and the high Reynolds numbers reached by this experiment, other aspects have also been considered by Ref. 11 in order to make their experimental work very suitable for its reproduction in numerical simulations. Indeed, one of the main difficulties is associated with streamwise corner vortices and the two-dimensional character of the flow. There are some experimental works for which it is not completely clear whether the flow can be considered as two-dimensional in the symmetry plane due to the mentioned corner flows. Such vortices have been measured in Ref. 11 and their size compared to the span size of the LMFL wind tunnel allows for a two-dimensional flow in the center plane (and in most of the span of the wind tunnel), as shown in their work.

The pressure gradient results from a specific geometry of the bottom wall which includes two angled flat plates at 1.5° and -5.0° for the favorable pressure gradient (FPG) and adverse pressure gradient (APG) regions respectively (see Fig. 1 for a sketch of the wind tunnel test section). In order to trigger turbulence in the boundary layer a rod is placed at the entry of the test section.

Of the two configurations studied by Ref. 11, the one with $U_\infty = 5$ *m/s* is reproduced in the present work. Under this configuration the flow reaches Reynolds numbers up to $Re_\theta \sim 1.2 \times 10^4$

FIG. 1: Sketch of the LMFL test section. Courtesy of C. Cuvier.

and a boundary layer thickness close to $0.3L$ (it is recalled that L is the height of the test section at its entry which is $L = 1\text{ m}$).

B. Advanced turbulence modeling

In this work, different approaches for turbulence modeling are considered. Those may be classified in two groups that are briefly described in the following: the RANS approach and the ZDES approach.

1. RANS approach

The RANS approach solves the Reynolds-averaged equations with a closure for the Reynolds stress $\langle u'_i u'_j \rangle$ for which different strategies exist. In the present paper, two eddy viscosity models and two RSM models (Reynolds Stress Model) are considered. The idea of the eddy viscosity models is to find a relation between the Reynolds stress and the mean field through turbulence variables^{29,31}. Such relation is frequently given by the Boussinesq's hypothesis and the introduction of the eddy viscosity μ_t :

$$-\rho \langle u'_i u'_j \rangle = 2\mu_t \langle S_{ij} \rangle - \frac{2}{3} \rho k \delta_{ij}, \quad \langle S_{ij} \rangle = \frac{1}{2} \left(\frac{\partial \langle u_i \rangle}{\partial x_j} + \frac{\partial \langle u_j \rangle}{\partial x_i} \right) \quad (1)$$

where k is the turbulent kinetic energy and δ_{ij} the Kronecker delta (tensor). The closure in equation (1) is given in incompressible form since in the present work the flow may be considered incompressible. More precisely, in the present study the eddy viscosity models considered are the

Advanced simulations of turbulent boundary layers under pressure-gradient conditions

Spalart-Allmaras model³³ and the $k - \omega$ SST from Ref. 34. The first one includes a transport equation for the eddy viscosity itself whereas the second one uses two transport equations, one for the turbulent kinetic energy k and another for ω (which is the specific dissipation rate of k) and a relation between k , ω and μ_t is also used.

In the case of RSM models, the closure for the Reynolds stresses consists in solving the transport equations for the Reynolds stresses. An additional transport equation is required for a turbulent quantity which can be the turbulent kinetic energy dissipation ε or the specific dissipation rate ω for instance³¹. The transport equation for the Reynolds stresses is³¹:

$$\frac{\partial}{\partial t} \langle u'_i u'_j \rangle + \langle u_l \rangle \frac{\partial}{\partial x_l} \langle u'_i u'_j \rangle = \mathcal{P}_{ij} + \mathcal{R}_{ij} - \frac{\partial}{\partial x_l} \mathcal{T}_{lij} - \varepsilon_{ij}. \quad (2)$$

where $\mathcal{P}_{ij}$ is the production tensor, $\mathcal{R}_{ij}$ the pressure-rate-of-strain tensor, $\mathcal{T}_{lij}$ the Reynolds-stress flux and ε_{ij} the dissipation tensor. Due to the symmetry of the Reynolds stress tensor, 7 additional transport equations are solved when using this closure (including the equation for the turbulent variable). The known variables are the mean flow variables as well as the Reynolds stresses and the chosen turbulent quantity for the seventh transport equation. Hence, the only term on the right-hand side of equation (2) in closed form is $\mathcal{P}_{ij}$, and so modeling is required for the other terms. Regarding this closure, in the present work two models have been considered: the SSG-LRR- ω ^{35–37} and the SSG- ω modified version by Ref. 38, which will be called SSG- ω Aup. in the following.

2. ZDES approach as WMLES

The other approach for turbulence modeling used in the present work is the Zonal Detached Eddy Simulation³⁹ (ZDES) which is a hybrid RANS/LES method that allows the user to choose between three different operating modes depending on the flow studied as illustrated in Fig 2. To briefly present the different modes, when using the first two modes, the whole attached boundary layer is treated in RANS approach, whereas in mode 3 the inner part of the boundary layer is treated in RANS and the outer part in LES whereas the separation between both parts is given by a RANS/LES interface defined by the user. This mode is the one used in the present work and corresponds to a WMLES approach^{21,39,40}. The Spalart-Allmaras turbulence model is both used as a turbulence model for the RANS branch and as a sub-grid scale model (SGS model) for the LES branch^{39,41,42}. It is important to mention that ZDES also allows to perform WRLES as done in Ref. 43.

FIG. 2: Classification of typical flow types for the ZDES approach. I: separation fixed by the geometry, II: separation induced by a pressure gradient on a curved surface, and III: separation strongly influenced by the dynamics of the incoming boundary layer.

The ZDES mode 3 allows the user to define the positioning of the RANS/LES interface $d_w^{\text{interface}}$. The modified wall distance $\tilde{d}_{\text{ZDES}}^{\text{III}}$ used in the Spalart-Allmaras equation for this mode is given as follows³⁹:

$$\tilde{d}_{\text{ZDES}}^{\text{III}} = \begin{cases} d_w & \text{if } d_w < d_w^{\text{interface}} \\ \min(d_w, C_{\text{DES}} \Delta_{\text{vol}}) & \text{if } d_w \geq d_w^{\text{interface}} \end{cases} \quad (3)$$

where $C_{\text{DES}} = 0.65$, d_w is the wall distance, $\Delta_{\text{vol}} = \mathcal{V}^{\frac{1}{3}}$ and $\mathcal{V}$ is the cell volume. Further modifications are made in the case where $d_w \geq d_w^{\text{interface}}$ for some of the functions used in the Spalart-Allmaras model³³:

$$f_{v1} = 1, \quad f_{v2} = 0, \quad f_w = 1. \quad (4)$$

These allow for a better RANS to LES interfacing (see Ref. 39). A recent improvement of the method, proposed in Ref. 40, regularizes the binary switch from RANS to LES in (3) by means of a smooth function f_δ . The new expression for $\tilde{d}_{\text{ZDES}}^{\text{III}}$ is then:

$$\tilde{d}_{\text{ZDES}}^{\text{III}} = (1 - f_\delta) \cdot d_w + f_\delta \cdot \min(d_w, C_{\text{DES}} \Delta_{\text{vol}}) \quad (5)$$

and the functions modified in (4):

$$\begin{aligned} f_{v1}^{\text{III}} &= (1 - f_\delta) \cdot f_{v1} + f_\delta \\ f_{v2}^{\text{III}} &= (1 - f_\delta) \cdot f_{v2} \\ f_w^{\text{III}} &= (1 - f_\delta) \cdot f_w + f_\delta \end{aligned} \quad (6)$$

Advanced simulations of turbulent boundary layers under pressure-gradient conditions

The main advantages of the ZDES mode 3 is its cost reduction of turbulence simulation compared to other turbulence resolving approaches as DNS or WRLES and the capability to simultaneously use the RANS approach in other regions of the flow where resolved turbulence is not needed (contrary to non zonal WMLES approaches). This has allowed to perform numerical computations at high Reynolds numbers (as in Ref. 21) that are usually not affordable with DNS or WRLES. Besides, the ZDES mode 3 has been validated for ZPG turbulent boundary layers at high Reynolds numbers²¹.

C. Numerical methods

1. Numerical solvers and setup

The simulations that are presented have been obtained using two different solvers. In particular, the RANS simulations with the Spalart-Allmaras model (2D and 3D with Quadratic Constitutive Relation (QCR) for the closure as presented in Sec. III C) and the ZDES simulation have been performed with the solver FLU3M developed at ONERA, which has been used in other studies concerning ZDES^{21,39,43–48}. For the other computations (RANS simulations with the $k - \omega$ SST model and the two RSM models), the solver *elsA*⁴⁹ has been used. The numerics employed in both solvers are quite similar.

The finite volume method is used in all the simulations of the present work. The numerical schemes considered are the same for all RANS computations, and comparisons of solutions with the Spalart-Allmaras model between both solvers have shown an excellent agreement thus eliminating any solver dependence in the results. Roe's scheme is used for spatial discretization together with a MUSCL (Monotonic Upstream Scheme for Conservation Laws) approach for fluxes reconstruction at the cell faces and the pseudo temporal integration is done by means of an implicit Euler's scheme.

Regarding the ZDES computation, the definition of the numerical schemes used is more complex. In fact, the boundary layer of interest is the one located on the bottom wall since it is the one for which experimental measurements have been made in Ref. 11. In order to analyze the modeling effect on that boundary layer, for the ZDES simulation the upper boundary layer is completely solved in RANS approach (with the Spalart-Allmaras turbulence model) and the lower boundary layer is solved in ZDES mode 3 (WMLES) as shown in Fig. 3. This explains why fluctuations

Advanced simulations of turbulent boundary layers under pressure-gradient conditions

FIG. 3: Computational domain for the ZDES mode 3 simulation.

in Fig. 13 are only observable in the lower boundary layer. For the ZDES mode 3 region, the spatial scheme is the AUSM+(P) proposed in Ref. 50 and modified as in Ref. 51 to significantly reduce the numerical simulation. For the upper boundary layer, the AUSM+(P) scheme (without the modification in Ref. 51) is used instead of Roe's scheme as in the pure RANS simulations in order to avoid numerical issues in the ZDES simulation at the interface between the upper region (where full RANS approach is used) and the lower region (where the mode 3 of ZDES is used), shown in Fig. 3 in red dots.

It is important to mention that the top wall geometry is modified with respect to that of the experimental facility so that the effect of the side walls boundary layers is considered in a statistically-two dimensional simulation. The reason as well as the procedure for such a modification and its validation are thoroughly presented in Sec. III.

In order to quantify the effect of the numerical scheme in RANS simulations, comparisons between full RANS simulations using either Roe's scheme or the AUSM+(P) and the same turbulence model have been made and they have shown an excellent agreement so the use of the AUSM+(P) in the upper boundary layer is not expected to be a source of error.

Regarding the temporal integration in the ZDES simulation, a second order implicit Gear's scheme is applied. The time step is $\Delta t = 3.08 \times 10^{-6} s$ which is chosen such that the acoustic CFL is not greater than 13 at any point of the domain at any time. Besides, the value of the time step gives $\Delta t^+ < 0.4$ (where $\Delta t^+ = \frac{\Delta t u_\tau^2}{\nu}$) thus satisfying the requirements from Ref. 52 for a proper time step in turbulence resolving simulations.

The mesh used for the ZDES mode 3 computation is shown in Fig. 4 in the (x,y) plane. For all the computations (RANS 2D, RANS 3D and ZDES) the mesh parameters in the (x,y) plane are the same: $\Delta x^+ = 200$ and $\Delta y^+|_w = 1$ (cell center). The superscript '+' denotes wall units and the subscript 'w' refers to the first cell at the walls. In the case of the three-dimensional

Advanced simulations of turbulent boundary layers under pressure-gradient conditions

FIG. 4: Mesh of the ZDES mode 3 computation. One point over eight is shown in both x and y directions. The number of points in each direction is $N_x = 1050$, $N_y = 273$ and $N_z = 160$.

RANS computation, the same criterion for the side wall is used so $\Delta z^+|_w = 1$. The ZDES mode 3 computation is three-dimensional (although statistically two-dimensional) because part of the turbulence is directly resolved. For this one it is taken $\Delta z^+ = 100$ and $L_z \approx 4\delta_{max}$ (and at the inlet $L_z \approx 8\delta_0$, with $\delta_0 = 0.121L$) which are classical criteria for simulations using this method²¹ and in particular in the present case we have $L_z = 0.96L$ since $\delta_{max} = 0.231L^{11}$. The mesh for the ZDES simulations has 46 million cells ($N_x = 1050$, $N_y = 273$ and $N_z = 160$), much less than the 3000 million cells that a WRLES would require. This is an important point since it is already a significant mesh size and the span is not even half of the real span of the wind tunnel. Besides, if the lateral boundary layers were simulated, the spanwise distribution of mesh points would not be uniform, but it would concentrate points inside the boundary layers. This justifies all the effort made to avoid solving the lateral boundary layers by modifying the shape of the top wall.

The computational domain shown in Fig. 3 and 4 corresponds to that of the ZDES simulation which starts at $x/L = 6.6$ and ends at $x/L = 20.11$. A longer computational domain is considered for the RANS simulations which starts at $x/L = -2.01$ from a stagnation point for all of them (the outlet location is maintained). The shorter domain is chosen in Fig. 3 and 4 for clarity. The reason why the ZDES computational domain is shorter in the streamwise direction is the need to set the inlet in a fully turbulent region at a sufficient Reynolds number, which also reduces computational effort. As mentioned, the simulation is three-dimensional since most of the turbulent scales in the outer layer are resolved. However, for the turbulent inflow condition required for this simulation (see Sec. II C 2) a boundary layer profile of a RANS solution is needed. This one is obtained from the RANS simulation with the Spalart-Allmaras turbulence model, since it is the underlying model in the ZDES approach, and therefore RANS simulations require an inlet farther upstream

Advanced simulations of turbulent boundary layers under pressure-gradient conditions

for the boundary layer development. In addition, experimental data of the boundary layer at two stations $x_A = 3.2L$ and $x_B = 6.8L$ are given in Ref. 11 and presented in Tab. I and II. As shown in those tables, the Spalart-Allmaras model (in both the 2D and 3D cases) is very accurate. The ZDES mode 3 simulation uses this model (as RANS and SGS models) and therefore such an agreement is very satisfactory in order to reproduce the boundary layer as accurately as possible upstream of the region of interest. Furthermore, the boundary layer profile used for the turbulent inlet conditions of the ZDES mode 3 simulation is extracted from the RANS simulation with the Spalart-Allmaras model.

Another important point concerning the numerical setup is a modification of the flow conditions with respect to the experiment that has been made for numerical purposes. The experimental Mach number (for the $U_\infty = 5 \text{ m/s}$ case) is about 0.015 which is not appropriate for a compressible solver without preconditioner as it is the case of the solver FLU3M. The Mach number in the simulations has been increased keeping the same temperature, streamwise momentum ($\rho_\infty U_\infty$) and spatial dimensions as in the experiment so that the Reynolds number is unchanged. Besides, the maximum Mach number in the simulations has been kept below 0.2 in order to maintain the quasi-incompressible feature of the flow. Finally, the inflow uniform conditions for all the RANS simulations (for which the inlet is at $x_0/L = -2.01$) are $p_0 = 11716 \text{ Pa}$, $T_0 = 293 \text{ K}$ and $U_0 = 43.5 \text{ m/s}$. For the ZDES simulation, the inlet being at $x_0/L = 6.6$, the inflow conditions are slightly different due to the blockage effects from the boundary layers development. The inflow conditions are $p_0 = 11700 \text{ Pa}$, $T_0 = 293 \text{ K}$ and $U_0 = 45.9 \text{ m/s}$ with an inlet Reynolds number of $Re_\theta = 4616$.

For the ZDES simulation, a transient phase is required after which the collection of statistics starts, and it is performed over a duration of about $362\delta_0/U_0$. The total simulation (including the transient phase) requires 1.5×10^5 CPU hours in 196 processors.

2. Turbulent inflow condition for ZDES

Since the ZDES mode 3 corresponds to a WMLES approach, turbulent structures are resolved in the LES region (defined by the RANS/LES interface). Inflow turbulent conditions are thus necessary for this kind of simulations. Among the different techniques for dealing with inflow turbulent conditions⁴⁷, the Synthetic Eddy Method (SEM) is used in the present work. The main idea is to introduce fluctuations superimposed to the mean field that are supposed to mimic the

Advanced simulations of turbulent boundary layers under pressure-gradient conditions

turbulent structures in a turbulent boundary layer. The characteristic time, length and velocity of the structures are obtained from a precursor RANS boundary layer profile and are dependent on the wall distance. Besides, the anisotropy of the Reynolds stress is taken into account. It is important that the profile given from the RANS simulation reproduces the desired boundary layer characteristics as accurately as possible, so that the turbulent boundary layer in the ZDES simulation develops with the proper flow conditions (mainly the proper Reynolds number). A thorough explanation of the method can be found in the works of Ref. 53–55.

3. *RANS/LES interface for the ZDES mode 3 computation*

In the WMLES approach of the ZDES (mode 3), the separation between the RANS and the LES regions is given by an interface whose law is specified by the user. Several works have dealt with the interface positioning and the effect of different interface laws^{21,40,46,48,56,57}. Very satisfactory results have been obtained in the case of a high-Reynolds-number turbulent boundary layer at zero pressure gradient in quasi-incompressible conditions^{21,40,56} for the interface law $d_{int} = 0.1\delta$. Therefore in the present computation the same law is used.

It is important to detail how the boundary layer thickness is computed. Indeed, in the present case, due to the curvature and pressure gradient effects, velocity profiles outside of the boundary layer are not always uniform. For that reason, following the suggestion of Ref. 1 the spanwise vorticity is used to define the edge of the boundary layer. However, contrary to Ref. 1, the whole spanwise vorticity is considered instead of using the boundary layer approximation $\langle \omega_z \rangle \approx -\partial \langle u \rangle / \partial y$ (where $\langle \omega_z \rangle$ denotes the mean spanwise vorticity). The idea is to define the edge of the boundary layer when the local mean spanwise vorticity (in the wall-normal direction) is below a given threshold (about 10^{-4}) of the mean spanwise vorticity at the wall. The velocity at that point is taken as the external velocity $U_e(x)$ and then the boundary layer thickness is obtained from its classical definition: $\langle u \rangle|_{y=\delta(x)}(x) = 0.99U_e(x)$.

III. DESIGN OF A TWO-DIMENSIONAL TEST-CASE

A. Lateral contraction

Although an important effort has been made in Ref. 11 to ensure the two-dimensional character of the mean flow in the symmetry plane, it is not possible to properly reproduce the experiment by

Advanced simulations of turbulent boundary layers under pressure-gradient conditions

performing a purely two-dimensional simulation using the wind tunnel geometry. Experimental measurements show that the boundary layer in the bottom wall is quite thick (reaching almost $0.3L$) and so it is expected that similar thicknesses are reached by the boundary layers developing over the side walls of the LMFL wind tunnel. The development of boundary layers over the walls results in a blockage effect that tends to accelerate the flow. This effect is better quantified by the displacement thickness δ_1 of the boundary layers, which in the experiment reaches $0.0587L$ in the bottom wall. If one assumes to have the same displacement thickness for the lateral boundary layers, these would represent more than 5% of the wind tunnel span and therefore the blockage effect will be non-negligible. This has been verified in the evolution of the pressure coefficient C_p at both the top and bottom walls when performing a pure 2D simulation where a slower pressure drop in FPG regions was observed together with a faster pressure rise in APG regions compared to the experiment.

The simplest and most direct way to deal with this issue would be to perform three-dimensional simulations taking into account the full three-dimensional geometry of the LMFL wind tunnel. However, ZDES simulations with the mode 3 (Ref. 39) are carried out in the present study. Solving the three-dimensional problem with this method would result in a huge computational effort even if this method is known to significantly reduce it compared to other turbulence resolving methods such as WRLES and DNS²¹. Moreover a statistically 2D test case is more relevant for the efficient analysis of the flow statistics. The alternative proposed consists in modifying the top wall geometry in order to take into account the blockage effect due to the boundary layers developing on the side walls of the wind tunnel without simulating these boundary layers. Only the top wall is modified since the measurements in Ref. 11 are made for the boundary layer on the bottom wall.

B. Top wall geometry modification

The objective is to modify the geometry of the top wall such that the potential flow seen by the bottom-wall boundary layer in the symmetry plane (according to the boundary layer hypothesis⁵⁸) remains close between the three-dimensional case and the two-dimensional case with geometry modification. This idea is illustrated in Fig. 5 where the (transverse) area outside the displacement thicknesses is called the effective area and it is supposed to be crossed by a potential flow (under the boundary layer hypothesis). The (incompressible) displacement thickness, δ_1 , of the boundary

Advanced simulations of turbulent boundary layers under pressure-gradient conditions

layer is defined as follows:

$$\delta_1 = \int_0^\infty \left(1 - \frac{\langle u \rangle}{U_e}\right) dy \quad (7)$$

being y the wall normal direction increasing away from the wall and U_e the external velocity.

In addition, two extra hypothesis are considered. It is first assumed that the displacement thicknesses of the lateral boundary layers are similar to that of the top wall boundary layer (for the same streamwise position). The law sought for the top wall geometry should give the same effective area keeping the same span as in the case where the lateral boundary layers are considered and the top wall is flat. A scheme of the distances used is given in Fig. 5 and 6. By calling L_{ye} and L_{ze} the effective lengths of the potential flow region at a given position x in y and z directions respectively and according to the mentioned hypothesis, it is obtained:

$$L_{ye2} \times L_{ze2} = L_{ye1} \times L_{ze1} \simeq L_{ye1} \times \underbrace{(L_{zt} - 2\delta_{1top1})}_{\approx L_{ze1}}. \quad (8)$$

The subscript '1' corresponds to the variables for the case of flat top wall, the subscript '2' corresponds to variables in the case where the top wall is changed and the subscript t to the total lengths. For the second hypothesis, it is considered that at a given position x the displacement thicknesses of the top and bottom wall boundary layers are not modified between the old and the new geometry of the top wall, so in the following development it is used $\delta_{1top2} = \delta_{1top1} = \delta_{1top}$ and $\delta_{1bot2} = \delta_{1bot1} = \delta_{1bot}$. This hypothesis is strong, however it is expected that variations are not very important and thus the law sought can be expressed only as a function of known parameters. It would however require some iteration in the law obtained as it will be shown later. Besides, it is demanded that $L_{ze2} = L_{zt}$ which gives from (8):

$$L_{ye2} = \left(1 - \frac{2\delta_{1top}}{L_{zt}}\right) L_{ye1}. \quad (9)$$

In addition, one may write:

$$L_{yei} = L_i - y_w - \delta_{1top} - \delta_{1bot} \quad (10)$$

for $i = 1, 2$, so (with $L_{yt1} = L_{ye1} + \delta_{1top} + \delta_{1bot}$)

$$L_2 = y_w + L_{yt1} - \frac{2\delta_{1top}}{L_{zt}} L_{ye1}. \quad (11)$$

Knowing that

$$L_1 = y_w + L_{yt1} = L \quad (12)$$

Advanced simulations of turbulent boundary layers under pressure-gradient conditions

in the displacement thicknesses between computations with a flat top wall and a top wall modified by the given law. Although these differences are small, an iterative process should be done in which the results from numerical simulations are used in (13) in order to obtain a new geometry, and then perform a new simulation for this new geometry whose results are again used in (13) for another simulation and so on until the differences observed between simulations are negligible. Such a procedure would require to regenerate the mesh and perform new simulations several times. Therefore, instead of carrying out such a tedious procedure, another option has been chosen. This one consists in considering again that the variations of the displacement thickness are small between the computations so that the final evolutions of the displacement thicknesses can be obtained by linear interpolation. Let us consider two simulations (simulation details are given in section II C). The first one (F) is made with a flat top wall and it is used to obtain a top wall geometry according to (13) and this geometry is used in the second one (S). Then, at each streamwise position x , the interpolations of the top and bottom displacement thicknesses from the mentioned computations are:

$$\begin{cases} \delta_{1top2} = \frac{\delta_{1topF} - \delta_{1topS}}{L_F - L_S} (L_2 - L_F) + \delta_{1topF} \\ \delta_{1bot2} = \frac{\delta_{1botF} - \delta_{1botS}}{L_F - L_S} (L_2 - L_F) + \delta_{1botF} \end{cases} . \quad (14)$$

In the present study, the first simulation has been performed with a flat top and so $L_F = L$. It is reminded that from equation (13) $L_S = L_S(x, \delta_{1topF}, \delta_{1botF})$. The results from these two simulations F and S allow to obtain interpolated values (denoted with the subscript '2') of the displacement thicknesses thanks to equation (14). Those interpolated thicknesses are then used in (13) to obtain a new geometry of the top wall $L_2 = L_2(x, \delta_{1top2}, \delta_{1bot2})$ which is then used for the final computation. It is important to point out that excellent agreement has been observed when comparing the interpolated displacement thicknesses with the final displacement thicknesses as observed in Fig. 7 for the top-wall boundary layer (even better agreement has been obtained for bottom boundary layer). Such an agreement indicates that the interpolation strategy has given a converged geometry which is presented in Fig. 8 since the δ_1 extracted from the final computation, and that would be used for a further iteration, are almost identical to the δ_1 used to define the top wall for the final computation. It should be clarified that the curve in Fig. 8 is not at scale and that variations of the top wall position are below 5% of L (which corresponds to less than 5 cm).

FIG. 7: Comparison of the top-wall boundary layer displacement thicknesses obtained from interpolation and from the simulation using the latest top-wall geometry.

FIG. 8: Modified geometry of the top wall for the two-dimensional simulations.

C. Validation of new geometry against a three-dimensional computation

In the previous section, a geometry for the top wall has been obtained in order to take into account the blockage effect from boundary layers developing on the LMFL wind tunnel side walls without simulating them. In order to validate the geometry, comparisons with a three-dimensional case should be made.

A three-dimensional RANS computation of the whole geometry with side walls is performed (with a flat top wall geometry). The mesh details and the computational domain are as described in section II C. Due to the symmetry of the LMFL wind tunnel in the spanwise direction a symmetric RANS field is expected. Therefore, the computational domain in the spanwise direction is taken as one half of the span of the wind tunnel (from $z = 0$ to $z = L$, the symmetry plane is located at $z = 0$) and only the side wall boundary layer at $z = L$ is solved.

Attention must be paid to the turbulence modeling. Due to the side wall at $z = L$, there will be secondary flows of the second kind at the corners between top/bottom walls and the side wall^{7,11}. Using the classical linear closure for the Reynolds stresses in the RANS equations (Boussinesq's hypothesis), those secondary flows are not reproduced and therefore a different closure is chosen instead in order to properly reproduce the flow structure. Ref. 25 suggests to link the Reynolds stress tensor to the mean strain-rate tensor $\langle S_{ij} \rangle$ through a non-linear relation called Quadratic Constitutive Relation (QCR) so that the Reynolds stress is given by $R_{ij,QCR}$:

$$R_{ij,QCR} = R_{ij,B} - C_{cr1} [O_{ik}R_{jk,B} + O_{jk}R_{ik,B}] \quad (15)$$

with $C_{cr1} = 0.3$ and

$$O_{ik} = 2\langle \Omega_{ik} \rangle / \sqrt{\frac{\partial \langle u_m \rangle}{\partial x_n} \frac{\partial \langle u_m \rangle}{\partial x_n}}, \quad (16)$$

$$\langle \Omega_{ij} \rangle = \frac{1}{2} \left(\frac{\partial \langle u_i \rangle}{\partial x_j} - \frac{\partial \langle u_j \rangle}{\partial x_i} \right).$$

$R_{ij,B}$ corresponds to the Reynolds stress given by the Boussinesq's hypothesis, and $\langle \Omega_{ij} \rangle$ is the mean rotation tensor. The turbulence model used in the simulation is the Spalart-Allmaras model³³.

An example of the difference between the use of the Boussinesq's hypothesis and the QCR relation is given in Fig. 9 and 10, in which the streamlines in the (y, z) plane are also shown. The non dimensional transverse mean kinetic energy is given by:

$$\frac{K_{vw}}{U_\infty^2} = \frac{1}{2} \frac{\langle v \rangle^2 + \langle w \rangle^2}{U_\infty^2} \quad (17)$$

FIG. 9: Transverse mean kinetic energy field in the plane $x = 6.8L$. Turbulence closure: Boussinesq's hypothesis. Streamlines obtained from $\langle v \rangle$ and $\langle w \rangle$.

It is evidenced that the Spalart-Allmaras model with the Boussinesq's hypothesis is not able to reproduce the secondary flows of the second kind as already mentioned, and therefore the mean kinetic energy is not well reproduced either. Since the secondary flows are confined in a relatively small region of the transverse domain, it should be expected that these do not influence significantly the flow in the center plane ($z = 0$) of the wind tunnel. The influence of the side-walls boundary layers in the mean streamwise velocity is negligible beyond $0.2L$ from the side wall, which is in keeping with the experimental results from Ref. 11. Besides, corner vortices are observed in Ref. 11 which justifies the use of the QCR relation instead of the Boussinesq's hypothesis as a closure for the Reynolds stress in the three-dimensional simulation.

A comparison of the pressure coefficient in both the top and bottom walls between the three-dimensional simulation (QCR closure) and the two-dimensional computation with the top wall modification is given in Fig. 11 and 12. In both cases the Spalart-Allmaras turbulence model is used. It is important to notice that the C_p is not close to 0 upstream of the region of interest be-

Advanced simulations of turbulent boundary layers under pressure-gradient conditions

FIG. 10: Transverse mean kinetic energy field in the plane $x = 6.8L$. Turbulence closure: QCR. Streamlines obtained from $\langle v \rangle$ and $\langle w \rangle$.

cause the reference pressure has been chosen as the lowest one in Ref. 11. Very good agreement between both computations is observed for the whole domain as well as between computations and experimental results, with a good reproduction of the pressure drop and rise in the FPG and APG regions respectively. Therefore the geometry modification proposed in the two-dimensional computation (Fig. 8) is able to reproduce the blockage effect from the side wall boundary layers in the mean field. This validates the two-dimensional geometry also used for other computations with different turbulence models or different turbulence modeling approaches in the present work. Hence, the geometry used in the present study is slightly modified from the reference one, including additional minor modifications which allow to satisfactorily reproduce the pressure coefficient of the experiment.

Moreover, Tab. I and II compare some of the bottom boundary layer characteristics at stations $x_A = 3.2L$ and $x_B = 6.8L$ respectively. Those stations are upstream the region of interest (upstream the pressure gradient region) and allow to verify whether the boundary layer develops properly

FIG. 11: Pressure coefficient on the bottom wall for the two-dimensional case with top wall geometry modification and the three-dimensional case with flat top wall. Experimental measurements from Ref. 11.

TABLE I: Comparison of some boundary layer parameters at $x_A = 3.2L$ for the Spalart-Allmaras turbulence model. For the 3D case the QCR closure is used. Errors to experimental measurements are given in bracket.

Case	Top wall	u_τ/U_∞	$(v/u_\tau)/L \times 10^6$
Exp. ¹¹	Flat wall	0.0424 (0.00%)	71.7 (0.00%)
S-A 2D	Curved wall	0.04118 (−2.87%)	71.96 (0.36%)
S-A 3D	Flat wall	0.04115 (−2.95%)	72.01 (0.43%)

before the region of interest. This is especially important for the ZDES computation. Indeed, as it is explained in Sec. IIC 1 and IIC 2, the inlet for the ZDES simulation is located farther downstream and it requires turbulent inflow conditions for which a boundary layer profile at that

FIG. 12: Pressure coefficient on the top wall for the two-dimensional case with top wall geometry modification and the three-dimensional case with flat top wall. Experimental measurements from Ref. 11.

TABLE II: Comparison of some boundary layer parameters at $x_B = 6.8L$ for the Spalart-Allmaras turbulence model. For the 3D case the QCR closure is used. Errors to experimental measurements are given in bracket.

Case	Top wall	u_τ/U_∞	$(v/u_\tau)/L \times 10^6$	$\delta_1/L \times 10^3$	$\theta/L \times 10^3$	Re_θ
Exp. ¹¹	Flat wall	0.0406 (0.00%)	73.53 (0.00%)	18 (0.00%)	13.1 (0.00%)	4700 (0.00%)
S-A 2D	Curved wall	0.04039 (−0.52%)	73.41 (−0.17%)	18.10 (0.56%)	13.4 (2.29%)	4736 (0.77%)
S-A 3D	Flat wall	0.04037 (−0.56%)	73.43 (−0.14%)	18.17 (0.94%)	13.5 (3.05%)	4755 (1.17%)

location is required. It is therefore important to have a satisfactory development of the boundary layer at that position with the Spalart-Allmaras model, which is the case as observed in Tab. I and II, because it is the underlying RANS model used in the ZDES mode 3 and it is the model from

Advanced simulations of turbulent boundary layers under pressure-gradient conditions

FIG. 13: Numerical schlieren $\left(\sqrt{\frac{\partial \rho}{\partial x_i} \frac{\partial \rho}{\partial x_i}}\right)$ in the symmetry plane of the ZDES simulation using mode 3.

which the profile for inflow conditions is extracted. Besides, from Tab. I and II the results from the Spalart-Allmaras simulations in 2D and 3D are again very close to each other.

IV. RESULTS DISCUSSION

A. Instantaneous field

An overview of the instantaneous flow is presented for the bottom boundary layer thanks to the ZDES mode 3 simulation. A two-dimensional numerical schlieren and an iso-surface of the Q -criterion are shown in Fig. 13 and 14 respectively. A slightly shorter domain is shown in Fig. 14 for clarity. These figures show that an important part of the turbulence in the boundary layer is resolved. Besides, it is interesting to see that turbulent structures are not observed in Fig. 13 in the upper wall because the corresponding boundary layer is solved in full RANS approach (see Fig. 3). A smooth gradient is nonetheless visible resulting from temperature variation across the boundary layer as a consequence of friction losses. In the APG region, a fast growth of the boundary layer is observed, as expected from the effects of adverse pressure gradients on the boundary layer thickness. Some structures go beyond the boundary layer thickness which, for the region where the boundary layer is the thickest, is close to $0.3L$ (30 cm). In Fig. 13, close to the outlet, the turbulent structures are dissipated intentionally for numerical purposes by mesh stretching in the streamwise direction but analysis are not made in that zone. Moreover, the hairpin type structure is observable in Fig. 14 which is a typical structure in the outer part of the turbulent boundary layer⁵⁹.

FIG. 14: Iso-surface of Q -criterion for $Q = 0.14 \left(\frac{U_0}{\delta_0} \right)^2$ together with numerical schlieren contours.

B. Reynolds-averaged data

The pressure coefficient evolution in both the bottom and top walls is presented in Fig. 15 and 16 respectively for all turbulence models studied compared to experimental measurements. It is recalled that all RANS simulations presented are two-dimensional, and the ZDES simulation is three-dimensional (since turbulence is partly resolved) but statistically two-dimensional in the spanwise direction. For all cases, the pressure coefficient is in good agreement with the experimental data which suggests that the dynamics of the mean flow are well reproduced.

The evolution of the friction velocity u_τ in the bottom wall is presented in Fig. 18. It is recalled that $u_\tau = \sqrt{\tau_w / \rho}$ where $\tau_w = (\partial \langle u \rangle / \partial y)|_w$ is the wall friction. A greater discrepancy between different models is observed compared to the evolutions of C_p and such differences are present in the whole domain. An interesting point is the initial evolution of the friction velocity for the ZDES computation which shows a decrease and then recovers values close to those at the inlet. This is not surprising since it has already been observed in previous works^{21,40,56} and this behavior is identified to be caused by the turbulent inflow condition (SEM). Indeed, some works are devoted to the improvement of the SEM in order to decrease the extent of this transitory region^{47,60}. The turbulent structures introduced by the SEM are reconstructed from a boundary layer profile, and they require some time to completely adapt to the turbulent flow studied. The present results

FIG. 15: Pressure coefficient on the bottom wall. Experimental measurements from Ref. 11.

suggest that the resolved turbulence is adapted slightly before the end of the ZPG region, hence before the region of interest.

All the RANS models except for the SSG- ω Aup. are in fairly good agreement with the experimental points in the APG region. For the ZDES however a different trend than the RANS models is observable, initially underestimating and then overestimating the friction velocity in the APG region. This shows the potential of the present test case to contribute to the development of hybrid RANS/LES methods for the prediction of turbulent boundary layers in pressure-gradient conditions.

Profiles of the mean streamwise velocity and the Reynolds shear stress are given at three stations (Fig. 17): one at the beginning of the FPG flat plate region ($x/L = 10.73$), another one at the beginning of the APG flat plate ($x/L = 12.95$) and the last one close to the end of the APG flat plate ($x/L = 16.27$). The mean velocity and the Reynolds shear stress profiles at those stations are shown in Fig. 19, 20 and 21 respectively. At the first station, the profiles of both $\langle u \rangle$ and $\langle u'v' \rangle$ are quite different from what is observed in ZPG turbulent boundary layers or even in pressure

FIG. 16: Pressure coefficient on the top wall. Experimental measurements from Ref. 11.

FIG. 17: Sketch of the three positions where boundary layer profiles are studied.

gradient turbulent boundary layers (see for instance Ref. 13, 17, and 61). Indeed, the experimental measurements show a local mean velocity excess near $y/\delta = 0.15$ and an outer peak is also observed for $\langle u'v' \rangle$.

The Spalart-Allmaras model is not able to properly capture the mean velocity excess and important differences are observed especially below $y/\delta = 0.1$. Results from the $k - \omega$ SST model are significantly more accurate which is also the case for the two RSM models and the ZDES. Concerning the profile of $\langle u'v' \rangle$, none of the eddy viscosity models are able to reproduce the outer peak. The SSG-LRR- ω model is quite accurate in the inner peak and reproduces the outer peak

FIG. 18: Friction velocity evolution with different turbulence models. Experimental measurements from Ref. 11.

FIG. 19: Profiles of mean velocity (a) and Reynolds shear stress (b) at $x/L = 10.73$. Experimental measurements from Ref. 11. Blue dashed line: RANS/LES interface.

FIG. 20: Profiles of mean velocity (a) and Reynolds shear stress (b) at $x/L = 12.95$. Experimental measurements from Ref. 11. Blue dashed line: RANS/LES interface.

FIG. 21: Profiles of mean velocity (a) and Reynolds shear stress (b) at $x/L = 16.27$. Experimental measurements from Ref. 11. Blue dashed line: RANS/LES interface.

although with lower levels. The SSG- ω Aup. model instead is not able to reproduce the outer peak and overpredicts significantly the inner one. Lastly, the ZDES profile is the most satisfying since it shows quite accurate predictions of both the inner and outer peaks, although they are slightly underestimated and overestimated respectively. As it will be seen in Sec. IV C, the outer peak is well reproduced by the ZDES mode 3 probably because it is located in the LES region and it may result from energetic turbulent structures resolved in the outer part of the boundary layer.

At the beginning of the adverse-pressure-gradient flat plate, the mean velocity profile (Fig.

Advanced simulations of turbulent boundary layers under pressure-gradient conditions

20a) suggests that RANS models are able to properly reproduce the boundary layer. Indeed, the external region of the mean velocity profile is very well reproduced by all RANS models, with very small differences between them and very accurate agreement with experimental results. However, there is an important discrepancy in the inner layer with a significant scatter of the predictions of different RANS models. Such differences are also observed in the Reynolds shear stress profiles (Fig. 20b) where an important overestimation is obtained for all RANS models except for the SSG-LRR- ω RSM model which is quite accurate. In the case of the ZDES mean velocity profile, less accurate results are obtained in the outer layer above the RANS/LES interface with some mean velocity overestimation, but it gives quite satisfactory results (among all the models studied) regarding the Reynolds shear stress profile.

Farther in the APG flat plate, at station $x/L = 16.27$, none of the models is able to accurately reproduce the mean velocity profile. Best results are obtained for the Spalart-Allmaras, the $k - \omega$ SST and the SSG-LRR- ω models, but significant discrepancies are observable near $y/\delta \approx 0.2$. The RSM SSG- ω Aup. overestimates the mean velocity in the wake region of the boundary layer and the ZDES overestimates the profile as well in a larger extent of the boundary layer. In the Reynolds shear stress profile, both eddy viscosity models show accurate profiles. The RSM models are less satisfactory with an overestimation and underestimation respectively for the SSG- ω Aup. and the SSG-LRR- ω in the inner part of the boundary layer. Contrary to the latter model, the SSG- ω Aup. gives good levels of the Reynolds shear stress in the outer part of the boundary layer. In the case of the ZDES the inner part is well reproduced, which is not surprising since a RANS approach is used in this region with the Spalart-Allmaras model, and this model in full RANS approach shows satisfactory results. However in the LES region, the Reynolds shear stress is underestimated suggesting a lack of resolved turbulence.

Contrary to the Spalart-Allmaras and the $k - \omega$ SST models, the RSM models and the ZDES predict u_{rms} and v_{rms} profiles ('rms' refers to *Root Mean Square*) which are presented in Fig. 22 and 23 for the first station ($x/L = 10.73$) and in Fig. 24 and 25 for the last one ($x/L = 16.27$). It is important to mention that profiles of u_{rms} and v_{rms} for the ZDES computation in the RANS region (below the RANS/LES interface) are not representative since turbulence is essentially resolved in the LES region and so only the LES region should be considered for ZDES profiles. This explains the strong underestimations of the resolved u_{rms} and v_{rms} below the RANS/LES interface.

At station $x/L = 10.73$ where the mean velocity excess and the outer peak in the Reynolds shear stress were observed, another outer peak is identified for both u_{rms} and v_{rms} and very satis-

FIG. 22: Profile of u_{rms} at $x/L = 10.73$. Experimental measurements from Ref. 11. Blue dashed line: RANS/LES interface.

factory profiles are obtained for the ZDES (in the LES region). The SSG-LRR- ω model is able to reproduce both peaks but levels are not well reproduced. Indeed, the u_{rms} profile is in general underestimated and the inner peak and outer peak of v_{rms} are over- and underestimated respectively. More accurate results are obtained for the outer peak of both u_{rms} and v_{rms} in the case of the SSG- ω Aup. model, but the inner peak of the v_{rms} profile is overestimated.

In the $x/L = 16.27$ station, all of the models fail to properly predict profiles for both u_{rms} and v_{rms} . Both RSM models underestimate significantly the levels of u_{rms} in the inner region. Beyond $y/\delta = 0.1$ quite satisfactory levels are obtained only for the SSG- ω Aup. The levels of v_{rms} in the inner region are better reproduced than for u_{rms} in the case of RSM models (especially for the SSG-LRR- ω model), but the outer part is underestimated. The ZDES method also underestimates v_{rms} in the outer region, but this is not a surprising result bearing in mind v_{rms} profiles for zero pressure gradient turbulent boundary layers in ZDES mode 3²¹. However, the underestimation of the u_{rms} profile in the LES region suggests again a lack of resolved turbulence in this region.

FIG. 23: Profile of v_{rms} at $x/L = 10.73$. Experimental measurements from Ref. 11. Blue dashed line: RANS/LES interface.

C. Spectral analysis of the Reynolds stresses

The main advantage of using ZDES mode 3 with respect to RANS models is to get direct information about turbulence through resolved fluctuating quantities. This allows to perform spectral analysis of turbulence and thus to gain a better insight on the contribution of different turbulent spatial scales to the streamwise velocity, as it is presented in this section.

The streamwise spectrum of the streamwise velocity fluctuation and the streamwise co-spectrum of $\langle u'v' \rangle$ are presented in Fig. 26 et 27 respectively. The spectra are obtained from time signals at a given location, which is $x/L = 10.76$, and then the Taylor's frozen flow hypothesis is used to link the temporal fluctuations with spatial fluctuations, by means of a duality between the frequency f and the wave number k_x : $\lambda_x = 2\pi/k_x = U_c/f$, where λ_x is the streamwise wave length and $U_c(y/\delta)$ the local advection velocity. This velocity is computed as described in Ref. 21 based on a space and time correlation of the fluctuating streamwise velocity. The power spectral

FIG. 24: Profile of u_{rms} at $x/L = 16.27$. Experimental measurements from Ref. 11. Blue dashed line: RANS/LES interface.

density (PSD) for the streamwise velocity fluctuation $G_{uu}(k_x)$ is defined such that⁶²:

$$\langle u'^2 \rangle = \int_0^{+\infty} G_{uu}(k_x) dk_x = \int_{-\infty}^{+\infty} k_x G_{uu}(k_x) d(\ln(k_x)). \quad (18)$$

An equivalent notation has been taken for the co-spectrum of the Reynolds shear stress which gives:

$$\langle u'v' \rangle = \int_0^{+\infty} G_{uv}(k_x) dk_x = \int_{-\infty}^{+\infty} k_x G_{uv}(k_x) d(\ln(k_x)). \quad (19)$$

The premultiplied PSD for $\langle u'^2 \rangle$ ($k_x G_{uu}(k_x)$) and the premultiplied co-spectrum for $\langle u'v' \rangle$ ($k_x G_{uv}(k_x)$) are presented in Fig. 26 and 27 respectively. Several similarities to spectral analysis of zero pressure gradient turbulent boundary layers simulated with the ZDES mode 3 are identified^{21,57}. The peak of the PSD for $\langle u'^2 \rangle$ is located below the RANS/LES interface near $d_w/\delta \approx 0.05$ for structures of size $\lambda_x \approx 8\delta$. In the LES region, smaller structures of size $\lambda_x \approx 2\delta$ with low energetic content are found but they are not visible in the RANS region due to the dissipation of the RANS approach. If turbulence were resolved down to the wall (WRLES or DNS)

FIG. 25: Profile of u_{rms} at $x/L = 16.27$. Experimental measurements from Ref. 11. Blue dashed line: RANS/LES interface.

significant energy levels associated to the inner site (modeled in WMLES) and to the footprint of outer very large scale would also be observed near the wall for λ_x from about 0.2δ to almost 10δ ^{43,63–65}, although this does not mean that the longest structures are located that close to the wall^{63,64,66}. In the present flow location, another energy site is observed at about the middle of the boundary layer with lower energetic content. These structures of size $\lambda_x \approx 3\delta$ are the main contribution to the outer peak observed in the profiles of u_{rms} (Fig. 22) at the same location. In the case of the co-spectrum of the Reynolds shear stress, the very low energy content in the RANS region in Fig. 27 is due to the low levels of wall-normal velocity fluctuations in that region as shown in the v_{rms} profiles. The spectral site associated to the outer peak in the profiles of $\langle u'v' \rangle$ is clearly observed in Fig. 27. Its location and the size of the corresponding turbulent structures match those of the outer peak of the PSD of $\langle u'^2 \rangle$. Therefore, the outer peak observed at this station in the different profiles shown seems to be caused by energetic turbulent structures whose size in the streamwise direction reaches more than three times the local boundary layer thickness.

FIG. 26: Premultiplied Power Spectral Density of the streamwise velocity fluctuations $k_x G_{uu}/u_\tau^2$ at $x/L = 10.76$. $Re_\theta = 3583$. Black dashed line: RANS/LES interface.

V. CONCLUSION

An experimental study of a pressure gradient turbulent boundary layer has been reproduced numerically with different approaches for turbulence modeling. A procedure has been presented to consider the problem as purely two-dimensional in terms of the mean flow dynamics which consists in modifying the top wall geometry in order to take into account the blockage effect resulting from the boundary layers developing in the experiment on the side walls of the wind tunnel without simulating them. The comparisons of RANS simulations with the modified geometry and three-dimensional simulation have shown an excellent agreement meaning that the modified ge-

FIG. 27: Premultiplied co-spectrum of the Reynolds shear stress $k_x G_{uv}/u_\tau^2$ at $x/L = 10.76$. $Re_\theta = 3583$. PSD of streamwise velocity fluctuations, $k_x G_{uu}/u_\tau^2$ with same levels as in Fig. 26.

Black dashed line: RANS/LES interface.

ometry allows to take into account the mentioned blockage effect. Besides, a QCR closure of the Reynolds stress has been used for the three-dimensional simulation so that secondary flows of the second kind are obtained in the corners between the wind tunnel walls, whose structure has shown to satisfactorily match that of the experiment in Ref. 11.

This modified geometry has then been used to compare the results from different turbulence models (first and second order models: Spalart-Allmaras, $k - \omega$ SST, SSG-LRR- ω and SSG- ω Aup.) and from ZDES mode 3 (which corresponds to a WMLES approach). In general, very

Advanced simulations of turbulent boundary layers under pressure-gradient conditions

satisfactory results have been obtained for the pressure coefficient evolution over both bottom and top walls, but more discrepancies between the turbulence models have been observed when looking at the mean skin friction velocity as well as profiles of mean velocity and Reynolds shear stress. The mean velocity excess at the beginning of the FPG flat plate is satisfactorily reproduced by all turbulence models except for the Spalart-Allmaras and the outer peak observed in profiles of Reynolds stresses from curvature effects at the same station has not been well reproduced by any model except for the ZDES. However, this method has shown to lack resolved turbulence in the LES region for the profiles shown near the end of the APG region. For that station, none of the models has shown to properly reproduce the mean velocity profile, and the Reynolds shear stress is only well predicted by the Spalart-Allmaras and the $k - \omega$ SST models.

The profiles of u_{rms} and v_{rms} are in general not very well reproduced by the two RSM models at any of the two stations. The outer peak at the first station is instead better reproduced by the ZDES mode 3. In the APG region however, these profiles also show a lack of resolved turbulence in the LES region of the ZDES simulation.

A spectral analysis of the streamwise velocity fluctuation and of the Reynolds shear stress has been made thanks to the ZDES mode 3. This analysis has allowed to identify the turbulent structures that are responsible for the mean velocity excess and the secondary peak on the Reynolds stresses near $x/L = 10.76$. Those structures are located close to the middle of the boundary layer, with streamwise sizes that are about three times the local boundary layer thickness.

The present work has shown that pressure-gradient conditions in turbulent boundary layers are still a challenge from a turbulence modeling point of view since even advanced modeling techniques have presented some difficulties. Besides, the experimental study of Ref. 11 is very suitable for further developments in turbulence modeling in order to properly reproduce the effects of pressure gradients on turbulent boundary layers thanks to the exhaustive measurements database and the two-dimensional structure of the mean flow near the symmetry plane. Further work on the present test case may consist of a deeper spectral analysis of the resolved scales by means of ZDES mode 3.

ACKNOWLEDGMENTS

The authors are greatly thankful to researchers from LMFL for fruitful discussions, especially with C. Cuvier and J.-M. Foucaut. All the people involved in the evolution of the FLU3M and *elsA*

Advanced simulations of turbulent boundary layers under pressure-gradient conditions

codes are warmly thanked. The thesis of J. Vaquero is partly funded by DGA (French defense procurement agency). Support from the framework of the ONERA research project FROTTEMENT is also acknowledged.

REFERENCES

- ¹P. R. Spalart and J. H. Watmuff, “Experimental and numerical study of a turbulent boundary layer with pressure gradients,” *Journal of Fluid Mechanics* **249**, 337 (1993).
- ²P. E. Skåre and P.-A. Krogstad, “A turbulent equilibrium boundary layer near separation,” *Journal of Fluid Mechanics* **272**, 319 (1994).
- ³F. H. Clauser, “Turbulent boundary layers in adverse pressure gradients,” *Journal of the Aeronautical Sciences* **21**, 91–108 (1954).
- ⁴J. Monty, Z. Harun, and I. Marusic, “A parametric study of adverse pressure gradient turbulent boundary layers,” *International Journal of Heat and Fluid Flow* **32**, 575–585 (2011).
- ⁵S. Rahgozar and Y. Maciel, “Low- and high-speed structures in the outer region of an adverse-pressure-gradient turbulent boundary layer,” *Experimental Thermal and Fluid Science* **35**, 1575–1587 (2011).
- ⁶Z. Harun, J. P. Monty, R. Mathis, and I. Marusic, “Pressure gradient effects on the large-scale structure of turbulent boundary layers,” *Journal of Fluid Mechanics* **715**, 477–498 (2013).
- ⁷P. Bradshaw, *Turbulence* (Springer-Verlag, Berlin New York, 1976).
- ⁸Y. Maciel, K.-S. Rossignol, and J. Lemay, “A study of a separated turbulent boundary layer in stalled-airfoil-type flow conditions,” in *35th AIAA Fluid Dynamics Conference and Exhibit* (American Institute of Aeronautics and Astronautics, 2005).
- ⁹S. Gorton, L. Jenkins, and S. Anders, “Flow control device evaluation for an internal flow with an adverse pressure gradient,” in *40th AIAA Aerospace Sciences Meeting & Exhibit* (American Institute of Aeronautics and Astronautics, 2002).
- ¹⁰D. Simmons, F. Thomas, and T. Corke, “Smooth body flow separation experiments and their surface flow topology characterization,” in *AIAA Aviation 2019 Forum* (American Institute of Aeronautics and Astronautics, 2019).
- ¹¹C. Cuvier, S. Srinath, M. Stanislas, J. M. Foucaut, J. P. Laval, C. J. Kähler, R. Hain, S. Scharnowski, A. Schröder, R. Geisler, J. Agocs, A. Röse, C. Willert, J. Klinner, O. Amili, C. Atkinson, and J. Soria, “Extensive characterisation of a high Reynolds number decelerating

Advanced simulations of turbulent boundary layers under pressure-gradient conditions

- boundary layer using advanced optical metrology,” *Journal of Turbulence* **18**, 929–972 (2017).
- ¹²M. Skote, D. Henningson, and R. Henkes, “Direct numerical simulation of self-similar turbulent boundary layers in adverse pressure gradient conditions,” *Flow, Turbulence and Combustion* **60**, 47–85 (1998).
- ¹³J.-H. Lee and H. J. Sung, “Effects of an adverse pressure gradient on a turbulent boundary layer,” *International Journal of Heat and Fluid Flow* **29**, 568–578 (2008).
- ¹⁴J.-H. Lee and H. J. Sung, “Structures in turbulent boundary layers subjected to adverse pressure gradients,” *Journal of Fluid Mechanics* **639**, 101 (2009).
- ¹⁵A. G. Gungor, Y. Maciel, M. P. Simens, and J. Soria, “Analysis of a turbulent boundary layer subjected to a strong adverse pressure gradient,” *Journal of Physics: Conference Series* **506**, 012007 (2014).
- ¹⁶V. Kitsios, A. Sekimoto, C. Atkinson, J. A. Sillero, G. Borrell, A. G. Gungor, J. Jiménez, and J. Soria, “Direct numerical simulation of a self-similar adverse pressure gradient turbulent boundary layer at the verge of separation,” *Journal of Fluid Mechanics* **829**, 392–419 (2017).
- ¹⁷J. H. Lee, “Large-scale motions in turbulent boundary layers subjected to adverse pressure gradients,” *Journal of Fluid Mechanics* **810**, 323–361 (2017).
- ¹⁸Y. Maciel, M. P. Simens, and A. G. Gungor, “Coherent structures in a non-equilibrium large-velocity-defect turbulent boundary layer,” *Flow, Turbulence and Combustion* **98**, 1–20 (2017).
- ¹⁹C. Atkinson, A. Sekimoto, J. Jiménez, and J. Soria, “Reynolds stress structures in a self-similar adverse pressure gradient turbulent boundary layer at the verge of separation,” *Journal of Physics: Conference Series* **1001**, 012001 (2018).
- ²⁰A. J. Smits, B. J. McKeon, and I. Marusic, “High Reynolds number wall turbulence,” *Annual Review of Fluid Mechanics* **43**, 353–375 (2011), <https://doi.org/10.1146/annurev-fluid-122109-160753>.
- ²¹S. Deck, N. Renard, R. Laraufie, and P. Sagaut, “Zonal Detached Eddy Simulation (ZDES) of a spatially developing flat plate turbulent boundary layer over the Reynolds number range $3150 \leq Re_\theta \leq 14000$,” *Physics of Fluids* **26**, 025116 (2014).
- ²²U. Piomelli, “Wall-layer models for Large-Eddy Simulations,” *Progress in Aerospace Sciences* **44**, 437–446 (2008).
- ²³N. Renard and S. Deck, “Recent improvements in the formulation of mode III of ZDES (Zonal Detached Eddy Simulation) for WMLES use at $Re_\theta > 10^4$,” in *53rd AIAA Aerospace Sciences Meeting* (American Institute of Aeronautics and Astronautics, 2015).

Advanced simulations of turbulent boundary layers under pressure-gradient conditions

- ²⁴A. Bannier, É. Garnier, and P. Sagaut, “Riblet flow model based on an extended FIK identity,” *Flow, Turbulence and Combustion* **95**, 351–376 (2015).
- ²⁵P. Spalart, “Strategies for turbulence modelling and simulations,” *International Journal of Heat and Fluid Flow* **21**, 252–263 (2000).
- ²⁶G. I. Park and P. Moin, “An improved dynamic non-equilibrium wall-model for large eddy simulation,” *Physics of Fluids* **26**, 015108 (2014).
- ²⁷Y. Na and P. Moin, “Direct numerical simulation of a separated turbulent boundary layer,” *Journal of Fluid Mechanics* **374**, 379–405 (1998).
- ²⁸M. Chong, J. Soria, A. Perry, J. Chacin, B. Cantwell, and Y. Na, “Turbulence structures of wall-bounded shear flows found using DNS data,” *Journal of Fluid Mechanics* **357**, 225–247 (1998).
- ²⁹D. Wilcox, *Turbulence modeling for CFD* (DCW Industries, La Cañada, Calif, 2006).
- ³⁰S. Radhakrishnan, U. Piomelli, A. Keating, and A. S. Lopes, “Reynolds-averaged and large-eddy simulations of turbulent non-equilibrium flows,” *Journal of Turbulence* **7**, N63 (2006).
- ³¹S. B. Pope, *Turbulent Flows* (Cambridge University Press, 2000).
- ³²C. Rumsey and P. Spalart, “Initial efforts to improve Reynolds Stress Models for separated flows,” (NASA Symposium on Advances in Turbulence Modeling, 2017).
- ³³P. Spalart and S. Allmaras, “A one-equation turbulence model for aerodynamic flows,” *La Recherche Aérospatiale* (1994).
- ³⁴F. R. Menter, “Two-equation eddy-viscosity turbulence models for engineering applications,” *AIAA Journal* **32**, 1598–1605 (1994).
- ³⁵B. Launder, G. Reece, and W. Rodi, “Progress in the development of a reynolds-stress turbulence closure,” *Journal of Fluid Mechanics* **68**, 537–566 (1975).
- ³⁶C. Speziale, S. Sarkar, and T. Gatski, “Modelling the pressure–strain correlation of turbulence: an invariant dynamical systems approach,” *Journal of Fluid Mechanics* **227**, 245–272 (1991).
- ³⁷R. Cécora, R. Radespiel, B. Eisfeld, and A. Probst, “Differential Reynolds-Stress Modeling for Aeronautics,” *AIAA Journal* **53**, 739–755 (2015).
- ³⁸B. Aupoix, “Improving the coupling of the Speziale, Sarkar and Gatski differential Reynolds stress model with an ω -scale equation.” Tech. Rep. (RT 1/13929, ONERA, 2012).
- ³⁹S. Deck, “Recent improvements in the Zonal Detached Eddy Simulation (ZDES) formulation,” *Theoretical and Computational Fluid Dynamics* **26**, 523–550 (2012).
- ⁴⁰N. Renard and S. Deck, “Improvements in Zonal Detached Eddy Simulation for Wall Modeled

Advanced simulations of turbulent boundary layers under pressure-gradient conditions

Large Eddy Simulation,” AIAA Journal **53**, 3499–3504 (2015).

- ⁴¹P. R. Spalart, W.-H. Jou, M. Strelets, and S. R. Allmaras, “Comments on the feasibility of LES for wings, and on a hybrid RANS/LES approach,” in *Proceedings of the First AFOSR International Conference on DNS/LES* (Greyden Press, 1997) pp. 137–147.
- ⁴²P. Sagaut, S. Deck, and M. Terracol, *Multiscale and Multiresolution Approaches in Turbulence*, 2nd ed. (Imperial College Press, 2013).
- ⁴³S. Deck, N. Renard, R. Laraufie, and P.-É. Weiss, “Large-scale contribution to mean wall shear stress in high-Reynolds-number flat-plate boundary layers up to $Re_\theta = 13650$,” *Journal of Fluid Mechanics* **743**, 202–248 (2014).
- ⁴⁴S. Deck, “Numerical simulation of transonic buffet over a supercritical airfoil,” AIAA Journal **43**, 1556–1566 (2005).
- ⁴⁵S. Deck, “Zonal-detached-eddy simulation of the flow around a high-lift configuration,” AIAA Journal **43**, 2372–2384 (2005).
- ⁴⁶S. Deck, P.-É. Weiss, M. Pamiès, and E. Garnier, “Zonal Detached Eddy Simulation of a spatially developing flat plate turbulent boundary layer,” *Computers & Fluids* **48**, 1–15 (2011).
- ⁴⁷S. Deck, P.-E. Weiss, and N. Renard, “A rapid and low noise switch from RANS to WMLES on curvilinear grids with compressible flow solvers,” *Journal of Computational Physics* **363**, 231–255 (2018).
- ⁴⁸N. Renard, S. Deck, and P.-E. Weiss, “On the WMLES use of ZDES in compressible flows,” in *AIAA Scitech 2019 Forum* (American Institute of Aeronautics and Astronautics, 2019).
- ⁴⁹L. Cambier, S. Heib, and S. Plot, “The Onera elsA CFD software: input from research and feedback from industry,” *Mechanics & Industry* **14**, 159–174 (2013).
- ⁵⁰M.-S. Liou, “A sequel to AUSM: AUSM+,” *Journal of Computational Physics* **129**, 364–382 (1996).
- ⁵¹I. Mary and P. Sagaut, “Large eddy simulation of flow around an airfoil near stall,” AIAA Journal **40**, 1139–1145 (2002).
- ⁵²H. Choi and P. Moin, “Effects of the computational time step on numerical solutions of turbulent flow,” *Journal of Computational Physics* **113**, 1–4 (1994).
- ⁵³N. Jarrin, S. Benhamadouche, D. Laurence, and R. Prosser, “A synthetic-eddy-method for generating inflow conditions for large-eddy simulations,” *International Journal of Heat and Fluid Flow* **27**, 585–593 (2006).
- ⁵⁴M. Pamiès, P.-E. Weiss, E. Garnier, S. Deck, and P. Sagaut, “Generation of synthetic turbu-

Advanced simulations of turbulent boundary layers under pressure-gradient conditions

lent inflow data for large eddy simulation of spatially evolving wall-bounded flows,” *Physics of Fluids* **21**, 045103 (2009).

- ⁵⁵R. Laroche and S. Deck, “Assessment of Reynolds stresses tensor reconstruction methods for synthetic turbulent inflow conditions. Application to hybrid RANS/LES methods,” *International Journal of Heat and Fluid Flow* **42**, 68–78 (2013).
- ⁵⁶N. Renard and S. Deck, “On the interface positioning in a Zonal Detached Eddy Simulation (ZDES) of a spatially developing flat plate turbulent boundary layer,” in *Progress in Hybrid RANS-LES Modelling* (Springer International Publishing, 2015) pp. 203–213.
- ⁵⁷N. Renard and S. Deck, “On the convection velocity of wall-bounded turbulence resolved by ZDES mode III at $Re_\theta = 13000$,” in *Progress in Hybrid RANS-LES Modelling* (Springer International Publishing, 2018) pp. 325–336.
- ⁵⁸J. Cousteix, *Turbulence et couche limite* (Cepadues-éditions, Toulouse, 1989).
- ⁵⁹M. R. Head and P. Bandyopadhyay, “New aspects of turbulent boundary-layer structure,” *Journal of Fluid Mechanics* **107**, 297 (1981).
- ⁶⁰R. Laroche, S. Deck, and P. Sagaut, “A dynamic forcing method for unsteady turbulent inflow conditions,” *Journal of Computational Physics* **230**, 8647–8663 (2011).
- ⁶¹J. A. Sillero, J. Jiménez, and R. D. Moser, “One-point statistics for turbulent wall-bounded flows at Reynolds numbers up to $\delta^+ \approx 2000$,” *Physics of Fluids* **25**, 105102 (2013).
- ⁶²J. S. Bendat and A. G. Piersol, *Random Data: Analysis and Measurement Procedures*, 4th ed. (Wiley, 2010).
- ⁶³I. Marusic, R. Mathis, and N. Hutchins, “High Reynolds number effects in wall turbulence,” *International Journal of Heat and Fluid Flow* **31**, 418–428 (2010).
- ⁶⁴R. Mathis, N. Hutchins, and I. Marusic, “Large-scale amplitude modulation of the small-scale structures in turbulent boundary layers,” *Journal of Fluid Mechanics* **628**, 311 (2009).
- ⁶⁵N. Renard and S. Deck, “On the scale-dependent turbulent convection velocity in a spatially developing flat plate turbulent boundary layer at Reynolds number,” *Journal of Fluid Mechanics* **775**, 105–148 (2015).
- ⁶⁶N. Hutchins and I. Marusic, “Large-scale influences in near-wall turbulence,” *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences* **365**, 647–664 (2007).

Front view
Scale : 1:30

(I)

$$\frac{\delta}{H} \ll 1$$

(II)

$$\frac{\delta}{H} \ll 1$$

(III)

$$\frac{\delta}{H} = O(1) \quad \text{or} \quad \frac{\delta}{H} > 1$$

$K_{vw}/U_\infty^2:$ 1.00E-09 1.50E-06 3.00E-06 4.50E-06

$\langle u \rangle / U_e :$

0.6 0.7 0.8 0.9 1 1.1