Rainfall-Runoff Analysis by Bayesian Inverse Methods

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Abstract:

Over the past century, hydrologists have developed sophisticated methods for the correlation and prediction of streamflow from rainfall data, including protocols for hyetograph and hydrograph separation, and deconvolution to determine the unit hydrograph. Very recently, some researchers have advocated an alternative approach in which the hydrological system is treated as a simple dynamical system, for example [1; 2; 3]:

$$y_t = \alpha + \beta u_t + \epsilon_t \tag{1}$$

where y_t is the streamflow (possibly in transformed form), u_t is the rainfall (or other climate proxies), ϵ_t is the error or stochastic component, α and β are parameters and t is a time index. The parameters are then calculated by linear regression [2; 3]. Recently, this approach was used to interpolate a 1 km resolution daily streamflow and hydrological metrics dataset for all of Germany [3]. Eq. (1) has been subject to the obvious criticism that it does not include other variables, in particular the catchment characteristics [2]. In addition, reported studies have required *ad hoc* data processing, for example to eliminate inferred negative streamflows [3]. To address these criticisms, more complicated models have been proposed, for example [2]:

$$\mathbf{x_{t+1}} = \mathbf{A x_t} + \mathbf{B u_t} + \boldsymbol{\epsilon}_\mathbf{t}, \qquad \mathbf{y_t} = \mathbf{C x_t} + \mathbf{D u_t} + \boldsymbol{\eta}_\mathbf{t} \tag{2}$$

where $\mathbf{x_t}$ is the system vector, $\mathbf{y_t}$ is the streamflow vector, $\mathbf{u_t}$ is the rainfall or climate variable vector, $\boldsymbol{\epsilon}_t$ and $\boldsymbol{\eta}_t$ are noise vectors, and $\mathbf{A}$ to $\mathbf{D}$ are parameter matrices. Eqs. (2) can be extended further to include derivatives, multiple time steps, nonlinear functions or mechanistic insights [e.g., 1, 4]. While such models are empirical, they are no more empirical in practice than many physical hydrological models based on assumed or lumped parameter values, and also make no *a priori* assumptions on the baseflow or infiltration characteristics, or uniqueness of the unit hydrograph.

In this study, we consider the problem of parameter identification for linear or nonlinear hydrological dynamical system models (1)-(2), in general based on a regularisation or sparse regression method. We demonstrate that such methods fall within the framework of Bayesian inverse methods. In their simplest form, the Bayesian maximum *a posteriori* method can be shown to be equivalent to Tikhonov regularisation based on Euclidean norms [e.g., 5]. This viewpoint provides a Bayesian rationale for the choice of residual and regularisation terms, respectively from the Bayesian likelihood and prior. For hydrological variables such as rainfall and runoff – which cannot be negative – this also provides a rigorous approach to implement alternative distributions, such as lognormal distributions. At the next level, the Bayesian framework enables the estimation of uncertainties in the inferred parameters and the model, the ranking of models by posterior Bayes factors, and the estimation or elimination of intermediate variables. At the highest level, Bayesian algorithms such as Markov chain Monte Carlo methods or nested sampling can be used to explore the posterior probability distribution, should this be desired. We demonstrate these features of Bayesian rainfall-runoff analysis using data from several sources, including Australia and Germany.

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