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ANALYSIS OF METHODOLOGIES FOR THE COUPLING OF MULTI-PHYSICS PHENOMENA IN THE QUASI-STATIC APPROACH TO NUCLEAR REACTOR DYNAMICS

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1. INTRODUCTION

In the analysis of nuclear reactor dynamics, a variety of time-scales, possibly differing by orders of magnitude, may be encountered. The diversity of these time-scales originates both from purely neutronic phenomena, namely the prompt and the delayed emission of neutrons in fission, as well as from external and multi-physics phenomena that influence the neutronic properties of the system. The variable nature of these time-scales often leads to situations where the evolution of the neutron flux can be decomposed into that of two, weakly-coupled components: an amplitude (proportional to the integral power) and a shape (a distribution on the phase-space). In such situations, the quasi-static method [1] is an acceptable and computationally appealing alternative to the direct numerical integration of the time- and phase-space-dependent balance equations for the neutron flux and the delayed neutron precursor concentrations.

When considered in the context of multi-physics analyses, the quasi-static modelling of neutron kinetics leads to diverse possibilities for the multi-physics coupling scheme. Indeed, the feedback effects that arise from multi-physics phenomena may intervene on either or both the amplitude and/or the shape. This characteristic motivates the traditional multi-scale algorithmic approach [2, 3] by which the multi-physics coupling for medium time-scale phenomena is performed on a time-step that is inferior to that utilised to update shape component but greater than that utilised to update the amplitude component. Yet, additional considerations are relevant to the coupling algorithm.

A current research activity of the Commissariat à l’énergie atomique et aux énergies alternatives (CEA) involves the study of methods by which to solve the time- and phase-space-dependent neutronics equations in the context of multi-physics analyses, together with appropriate coupling schemes [4]. For this reason, quasi-static capabilities are being developed [5] for the neutronics code APOLLO3® [6, 7] to be utilised in the multi-physics simulation environment provided by CORPUS [8]. The present work develops and assesses a coupling strategy for the quasi-static method when utilised in the context of multi-physics analyses of nuclear reactor dynamics. In particular, the algorithmic aspects of the scheme by which to update the operators of the neutron and delayed neutron precursor balance equations is considered, examining aspects such as the

*APOLLO3® is a registered trademark of CEA.
potential benefits of implicit coupling schemes with respect to explicit coupling schemes at the various time-scales.

2. PHYSICAL-MATHEMATICAL MODELS

The neutron balance is described by the time-dependent neutron transport equation with delayed neutron precursors, which may be written as [9]

\[
\begin{aligned}
\frac{1}{v(E)} \frac{\partial}{\partial t} \phi(r, E, \Omega, t) &= \left( \mathcal{H} - \sum_{r=1}^{R} \frac{X_r}{4\pi} \mathcal{F}_r \right) \phi(r, E, \Omega, t) + \sum_{r=1}^{R} \frac{X_r(r, E)}{4\pi} \lambda_r \bar{c}_r(r, t), \\
\frac{\partial}{\partial t} \bar{c}_r(r, t) &= \mathcal{F}_r \phi(r, t) - \lambda_r \bar{c}_r(r, t), \quad r = 1, \ldots, R,
\end{aligned}
\]

(1)

with \( \phi \) the neutron flux, \( c_r \) the concentration of delayed neutron precursors in decay family \( r \), \( v \) the neutron velocity, \( \mathcal{H} \) the superposition of the operators that describe neutron loss, scattering and production in fission and \( \mathcal{F}_r, X_r, \lambda_r \) representing the operator for delayed neutron generation, the emission spectrum and the decay constant, respectively, of delayed neutron precursor family \( r \). The presence of multi-physics phenomena is understood through the time-dependence of the operators.

Equations (1) are discretised in time according to the quasi-static approach [1], which proposes to factorise the flux into the product of a time-dependent amplitude function, \( T \), and a time- and phase-space-dependent shape function, \( \psi \), as

\[
\phi(r, E, \Omega, t) = T(t) \psi(r, E, \Omega, t).
\]

(2)

The factorised form of the flux is then projected onto an appropriate weight function, ultimately leading to separate systems of balance equations for the two new unknowns. The amplitude is obtained as the solution of the system of ordinary differential equations

\[
\begin{aligned}
\frac{d}{dt} T(t) &= \left[ \frac{\rho(t)}{\Lambda(t)} - \sum_{r=1}^{R} \frac{\bar{\beta}_r(t)}{\Lambda(t)} \right] T(t) + \sum_{r=1}^{R} \lambda_r \bar{c}_r(t), \\
\frac{d}{dt} \bar{c}_r(t) &= \frac{\bar{\beta}_r(t)}{\Lambda(t)} T(t) - \lambda_r \bar{c}_r(t), \quad r = 1, \ldots, R,
\end{aligned}
\]

(3)

with the effective generation time \( \Lambda \), the dynamic reactivity \( \rho \), the effective delayed neutron fractions \( \bar{\beta}_r \) and the effective delayed neutron precursor concentrations \( \bar{c}_r \) given by their usual definitions [1]. Likewise, the shape can be obtained according to a predictor-corrector approach by solving Eqs. (1) and then extracting the shape by enforcing a normalisation condition [10]. Details of the mathematical methods utilised to solve the resulting systems of quasi-static equations in APOLLO3® are provided elsewhere [5].

A flow diagram of the general multi-physics coupling scheme for the analysis of a transient according to the predictor-corrector quasi-static approach is shown in Fig. 1. The flux is advanced in time by integrating first the shape on the shape time-step (\( \Delta t_\psi \)), which allows to parametrise the integral kinetic parameters (\( \Lambda, \rho \) and \( \bar{\beta}_r \)). The amplitude and the “other” physics are integrated next on the reactivity time-step (\( \Delta t_\rho \leq \Delta t_\psi \)) in order to update the relevant state variables and, consequently,
the parametrisation of the integral kinetic parameters; this process is repeated to the end of the shape time-step. At either or both the shape time-step and/or the reactivity time-step, it is possible to perform the coupling implicitly (Fig. 1) or explicitly (Fig. 1, without considering the convergence criteria). A more complete explanation of the coupling strategies will be provided in the manuscript.

![Diagram of coupling algorithm](image)

**Figure 1:** Coupling algorithm for the shape-reactivity-amplitude ($\psi$-$\rho$-$T$) problem on a single shape time-step ($\Delta t_\psi$) by the predictor-corrector quasi-static method (left) and the reactivity-amplitude ($\rho$-$T$) problem on a single reactivity time-step ($\Delta t_\rho$) (right).

### 3. APPLICATIONS AND RESULTS

Results are presented for a representative multi-physics problem, namely a reactivity-initiated transient applied to a light-water reactor motif in reduced three-dimensional geometry, for which
a cross-sectional, quarter-core view is shown in Fig. 2. The transient is initiated by extracting completely the control element located at the centre of the array on the interval 0–0.1 s. Multi-physics feedback effects are incorporated by introducing an adiabatic model for the fuel temperature and using a library of cross-sections that is parametrised in terms of this physical quantity.

![Figure 2: Planar view of the geometry of the system considered for the transient.](image)

The behaviour of the integral power (indicative of the amplitude), the total power factor (indicative of the shape) and the maximum value of the fuel temperature are shown in in Fig. 3. It is clear that the degree of coupling among the amplitude, the shape and the thermal-hydraulics undergoes a significant variation as a function of time, rendering this scenario relevant for quasi-static neutronics modelling and furthermore relevant to the coupling scheme in consideration.

![Figure 3: Evolution of the integral power (left), the power factor (centre) and the fuel temperature (right) for the transient.](image)

Indicative results are presented in Table 1 for the integral power as a function of the time-step and as a function of the coupling scheme. It is observed that an implicit coupling scheme presents benefits at larger time-steps at times where the multi-physics feedback is relevant ($t > 0.15$ s), yet can be penalising in terms of computational time when this is not the case ($t < 0.15$ s). In the full paper, additional results will be provided that demonstrate a more in-depth analysis of the coupling schemes, including a complete spectrum of explicit/implicit approaches to the coupling on the shape time-step and on the reactivity time-step.

**REFERENCES**


Table 1: Integral power [W] as a function of the time-step and the coupling scheme
\((\Delta t = \Delta t_r = \Delta t_\psi)\), with the shape time-step \(\Delta t_\psi\) and the reactivity time-step \(\Delta t_r\).

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