Variance-reduction methods for Monte Carlo kinetic simulations

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VARIANCE-REDUCTION METHODS FOR Monte Carlo KINETIC SIMULATIONS

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1. INTRODUCTION

Time-dependent Monte Carlo neutron transport is key to addressing several applications, including reactor start-up and shut-down, operational transients, and accidental excursions. The use of Monte Carlo simulations for kinetic (i.e., time-dependent) transport problems should be regarded as the reference methodology for faster but approximate deterministic solvers, similarly to what is currently done for stationary transport problems.

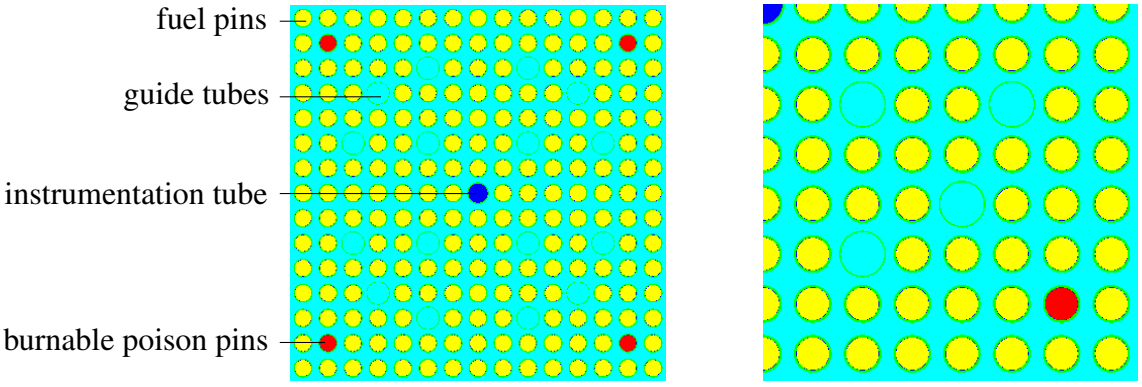
Solving non-stationary transport problems in reactor physics requires to simultaneously take into account neutrons and precursors. The typical time scales of neutrons and precursors are widely separated: the lifetime of the neutron-induced fission chain is of the order of $10\,\mathrm{ms}$, while the precursor β^- decay (which is responsible for delayed neutron emission) takes a few seconds [1]. A straightforward implementation of precursors in a Monte Carlo neutron transport code, with analog sampling of the precursor creation and decay, would thus lead to severe statistical undersampling when the time scale of the transient is comparable to the time scale of the precursors [2]. Generally speaking, analog Monte Carlo simulations are not a viable approach to kinetic Monte Carlo simulations in the context of reactor physics. A natural remedy in order to overcome the lack of statistics would be to increase the number of simulated particles, but this approach would be doomed to fail. For a reactor operating close to the equilibrium (i.e., critical) regime, the neutron-to-precursor ratio at any given time is of the order of 10^4 in water-moderated systems, which would lead to an unreasonable memory footprint if a large number of neutrons is to be tracked in order to collect sufficient statistics. The combination of the separation of the time scales and of the population ratio between neutrons and precursors in analog simulation has been shown to induce sharp statistical fluctuations [3].

Kinetic capabilities have recently been implemented in TRIPOLI-4®, the production Monte Carlo code developed at CEA [4]: the main features and the preliminary simulation results for a few significant benchmark configurations have been detailed in [5]. We have shown that special variance reduction and population control techniques conceived in order to deal with the time variable are mandatory for kinetic simulations [5], which is consistent with previous findings [3]. In this work, we explore all the kinetic variance-reduction methods that have been implemented in TRIPOLI-4®, including forced precursor decay, branchless collisions, Russian roulette and splitting, neutron and precursor combing, and relative neutron/precursor importance. We will assess the performances of each method (or combination of methods) in terms of figure of merit, for a benchmark configuration based on a TMI-1 fuel assembly, which is representative of a reactor physics problem.

The impact of variance reduction techniques for critical, sub- and super-critical regimes will be separately examined. Finally, we will examine the scaling of the variance of the scores in kinetics simulations with respect to the spatial discretization.

2. TEST CASE

As a benchmark configuration for our analysis, we have selected a simplified version of an un-rodded assembly from the TMI-1 reactor core, whose specifications can be found in [6]. The configuration is illustrated in Fig. 1. The width of the assembly is 21.641 cm. For the sake of simplicity, we consider a quarter of assembly with a reduced active height of 1 m, and we add radial and axial reflections to compensate. The full assembly contains 15x15 rods, including 204 4.12%-enriched UOX fuel pins, 4 Gd_2O_3+UO_2 burnable poison pins, 16 guide tubes filled with water and one instrumentation tube located in the center of the assembly. A radial view of the TRIPOLI-4® model is illustrated in Fig. 1. Material compositions were taken from [6]. The configuration is made critical, sub- or super-critical by adjusting the Boron concentration in the water moderator. The critical configuration is determined by using a critical Boron search functionality available in TRIPOLI-4®.



a) TRIPOLI-4® model for the assembly. b) 1/4 of the assembly.

Table 1: Radial views of TMI un-rodded assembly.

3. VARIANCE REDUCTION AND POPULATION CONTROL TECHNIQUES

The tested variance reduction and population control methods for kinetic simulations are the following.

3.1. Forced precursor decay

This method aims at reducing the variance of time-dependent scores by forcing the decay of the precursors into delayed neutrons [2,3]. The decay probability of the precursor particles is adjusted; to keep the Monte Carlo game unbiased, the statistical weight of the delayed neutrons emitted

upon decay is also consequently modified. This ensures a statistically stable production of delayed neutrons during the fission chains and thus a reduced variance in the total population. For this purpose, a new time mesh is introduced for the simulation, and the forced decay is applied to each time interval of this mesh [2,3,5]. For our analysis, we will systematically adopt forced precursor decay.

3.2. Population control

The number of neutrons and precursors is very sensitive to the reactivity of the system. If the reactor is sub-critical, the total population size may shrink to zero and the simulation will stop; if it is super-critical, the population may grow without bound, eventually exhausting all the memory and bringing the simulation to a halt. For this reason, TRIPOLI-4® applies some control mechanisms to the population size in order to keep it constant during the simulation. For the sake of simplicity, population control is applied at the end of the time intervals selected for forced decay. Hence, the size of these time bins should be chosen according to the system reactivity. In TRIPOLI-4®, the user can choose between two population-control mechanisms: *Russian roulette and splitting* (which ensure that the average particle weight lies within a given window) or *combing* (which ensures that the population size stays constant, while preserving the particle weight) [3,5]. The two methods will be compared in this work.

3.3. Branchless collisions

The fission chains in Monte Carlo transport are responsible for an increased variance in multiplying media with respect to the case of purely diffusing and absorbing configurations [7–9]. These fluctuations, whose origin lies in the fission-induced spatial correlations between successive generations, might globally hinder the convergence of the kinetic Monte Carlo calculations. In TRIPOLI-4® we have chosen to cope with the variance associated to fission chains by implementing the *branchless* transport method [3]. The idea behind this algorithm is to suppress the variability due to the simultaneous propagation of the several branches associated to a fission event (i.e., the histories of all the neutrons descending from a common ancestor) and to collapse all the contributions into a single history carrying the average weight of all the descendants. In principle, the branchless transport has been shown to be quite effective in reducing the variance in multiplying systems, although in practice some care must be taken [3]: a single collapsed history will have on average a larger statistical weight, so that sooner or later some particle splitting must be adopted to prevent an anomalous statistical behaviour. If particle splitting occurs too often, the benefits of the branchless method will be entirely outdone by the necessity of artificially reintroducing several particle histories. In this work we will examine the effectiveness of the branchless method in combination with other methods.

3.4. Relative neutron/precursor importance

Forced decay may not be sufficient to fill the time intervals with scoring particles. The statistical weight of (forced) delayed neutrons is proportional to the time interval: if the simulation time steps are too small, forced decay will produce delayed neutrons with very small weights, which will subsequently be killed by the ordinary Russian roulette. The simulation will then suffer from severe under-sampling issues. Yet, a finely discretized time mesh might be necessary for the simulation of

fast transients, so as to accurately capture the time variations of the population. In order to increase the number of neutrons in such simulations, we have implemented a time-dependent importance scheme in TRIPOLI-4®. For each time step of the simulation time grid, the user provides a relative neutron/precursor importance ratio. At the beginning of a time step, TRIPOLI-4® uses the current particle weights to define a neutron and a precursor importance, in such a way that the "physical" weights are preserved [5].

The choice of the importance ratio depends on a set of parameters: the simulation time grid and the physics of the simulated system (the inserted reactivity, the delayed neutron fraction and the typical time scales of neutrons and precursors). In practice, we can perform several short simulations with different values for the importance ratio and eventually select the one leading to the best figure of merit. The choice of an optimal importance ratio will be examined in this work.

4. SPATIAL DISCRETIZATION

The simulation scores (neutron flux and power) are computed on a spatial mesh superimposed to the geometry. The statistics collected by each mesh cell is proportional to the number of neutrons reaching it, and also roughly proportional to the volume of the cell (whose size is adjusted depending on the level of spatial detail needed for the simulation). We will investigate the scaling of the variance as a function of the spatial mesh size for kinetic simulations and we will compare our results to the case of static eigenvalue calculations (for the same configuration).

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