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TWG26 (*Mathematics in the Context of STEM Education*) was introduced in the CERME 11 in recognition of its importance in mathematics education. Science, Technology, Engineering and Mathematics (STEM) education merges a variety of subjects in the fields of science, technology, engineering and mathematics to solve real world problems (Sanders, 2012). Since the discrete subjects of science, technology, engineering, and mathematics were conjoined into the ubiquitous STEM acronym teachers, researchers, policy makers, and funders have worked to integrate the discrete subjects. The process of integration begins with the introduction of new instructional materials and practices, typically through curriculum development, publication of supporting materials, and professional development (Sanders, 2012). The challenge of integrating STEM lies at the nexus of teacher preparation and in-service teacher professional development (deMora & Wood 2014). Teaching STEM in a more connected manner, especially in the context of real-world issues, can make the STEM subjects more relevant to students and teachers (Honey, Pearson, & Schweingruber, 2014). This in turn can enhance motivation for learning and improve student interest, achievement, and persistence (Honey, Pearson, & Schweingruber, 2014).

What level of integration should be strived for, however, is not undisputed. Especially, if and how mathematics fits such an integrated approach is debatable. A specific point of attention concerns the fact that mathematics education asks for careful vertical planning, which might be compromised in an integrated approach. Mathematics encompasses more fields of application than science, technology, and engineering alone. So, real-life applications involving modeling, engineering design and the integration of various disciplines has to be taken into account in mathematics as it prepares students for the future.

This working group particularly aimed at addressing mathematics in the context of STEM education. Mathematics encompasses more fields of application than science, technology, and engineering alone. So, the integration of a broader variety of disciplines has to be taken into account in mathematics to prepare students for the future. Papers and posters submitted to the group focused on tasks, courses, curriculum, students, and in-service teachers mainly using qualitative methodology and interpretative approach. The issues and ideas emerged are discussed in reference to seven central themes. The mathematical contents concerned various topics (e.g., Numbers and Operations, Geometry (i.e., angles, parallel lines), and Data Organisation and Processing, second-order differential equations, matrices, linear equations).

Thematic areas

Here we present the issues and ideas that emerged in reference to the seven central themes.

Mathematical preparedness and value

Within this theme two papers were presented. First paper by Lave and Walshe concerned the preparedness of high-school students for tackling the mathematical aspects of STEM courses in tertiary education. The second paper by Nelleke den Braber and et al centered on the value of mathematics for an inter-disciplinary STEM course.

The first paper within this theme concerned the high non-completion/dropout rates in STEM courses at tertiary level in Ireland. It was noticed that even students with good marks might struggle with the mathematical aspects of third-level science and engineering courses. Lave and Walshe therefore searched for other factors that might affect the preparedness of the students. They investigated the perceptions of three stakeholders, teachers, students and lecturers with questionnaires. They further aimed to clarify what teachers and students themselves currently understand by interdisciplinary STEM education.

Factors that were taken into account are,

- students have to know how to use math knowledge in other subjects;

- mathematics has to be taught in an integrated way;

- high-school teachers should teach with students' future third-level degrees in mind;

- teachers are unaware of the benefits of integration.

In the reaction, it was argued that an alternative explanation might be in how mathematics is taught. In this respect a reference was made to Freudenthal, who argued that mathematics should be taught as to be useful. He argued that, "In an objective sense the most abstract mathematics is without doubt also the most flexible. But not subjectively, since it is wasted on individuals who are not able to avail themselves of this flexibility" (Freudenthal, 1968, p.5). Another remark was that next to benefits of integration of STEM subjects, there are also costs. These concern sustained facilities at school level (time, funding, schedule, room), teacher commitment, professional development and teacher support (Gresnigt, Taconis, van Keulen, Gravemeijer, & Baartman, 2014).

The second paper elaborated on the value of mathematics in an interdisciplinary STEM course. The key concept in this paper was that of disciplinary perspectives. It was argued the problems we face in today's world in the context of STEM call for perspectives and knowledge from many different areas. Against this background the researchers tried to get a handle on the value of mathematics in the context of a successful interdisciplinary STEM course in the Netherlands. With open questions Nelleke den Braber and et al investigated how students and teachers think about the value of mathematics. They introduced a model for the value of mathematics for interdisciplinary STEM courses. This encompassed, an overall sense of what mathematics is, ways of working and thinking, modeling, reasoning and problem solving. This model was used to analyze answers of students and teachers on an open questionnaire. It showed that both teachers and students had little awareness of the value of mathematics as a discipline. If the students mentioned mathematics, it was mainly that it was used. It seemed that the role of mathematics was not really noticed by them.

In the reaction in was noticed that the value of mathematics appeared to be equated with its usefulness. Which appeared not to be the intent. It was further argued that in practice the use of mathematics by workers was limited. As the actual mathematical work would be delegated to experts or machines. Therefore, being able to communicate about the mathematics might be more important. Following Kaput (1997) it was further argued that one should understand the key underlying ideas and mechanisms. Freudenthal was quoted again to make a distinction between mathematical activity as "organizing subject matter in reality and organizing mathematical subject matter". The latter seemed to be missing in the model of the value of mathematics. It was further noticed that Freudenthal would not be surprised about the lack of awareness of the role of mathematics. In 1977 he predicted that, "In 2000 mathematics will have been gone as a separate subject in education. Mathematics is there to be experienced, and lived, just like reading, writing, tinkering, drawing, singing, breathing, in integrated education."

Product and STEM

In this theme two papers were presented. The first one by Wohak and Frank revolved around the value of inverse problems in making students aware of the role of mathematics in applications. The second one by Bock, Bracke and Capraro aimed at such awareness but tried to realize this with product-oriented tasks.

The first paper made a case for the development of interactive, problem-oriented material for high school students, in order to make students see the potential of mathematics for solving real life problems. The example in the paper concerned so-called inverse problems. The concrete case being the problem of computer tomography. Computer tomography is used to establish the internal structure of an object—such as a human body—on the basis of the entry and exit intensity of the X-ray radiation. The objects are irradiated with parallel beams at different fixed angle settings, and the challenge is to infer the structure of the object from the patterns of the imprint the X-ray make after passing through the object. This is an inverse problem as it is the inverse of predicting what the imprint would be when the structure of the object is known. Where the latter is rather straightforward, the former is complex and not always possible as small variations may cause big errors. Wohak and Frank show that the mathematics for solving a simplified version of this type of problem is doable for high school students. They argue that by working on this problem, students will come to see the potential of mathematics to solve real-life problems.

One of the points of critique was on the strong focus on the mathematical solution of the problem. It was argued that today mathematics is done by computers and that the remaining mathematical work resides in the translation of the practical problem into a mathematical problem. It was further noted that there was little attention for the capabilities that are needed for mathematical modeling. In this respect the detailed list of capabilities that the PISA Mathematics framework offers (OECD, 2017) was mentioned.

The second paper focused on product-oriented modeling. The starting point being the task of making a product brings with it unique characteristics—which are lacking in school problems. For, the success criteria are dictated by whether the solution is actually working and feasible, often decisions have to be made, and restrictions apply. Two product-oriented modeling tasks were discussed. Both were aiming at showing the usefulness of Fourier analysis. The first concerned the task of making a musical fountain. Two sub-groups decided not to solve the original task but focused on loudness & rhythm instead. So, there was no need for Fourier analysis. One sub-group did find mathematical techniques to smooth the highly irregular amplitude-time-function. The other sub-group worked on an algorithm that could find an underlying rhythm in a song. The second task involved building a light organ. Here also two sub-groups emerged. One worked with electronic frequency filters, and thus did not need Fourier analysis. The other group wanted to use mathematics and was supported in using Fourier analysis (in a piece-by-piece manner).

One of the questions that was brought up by the discussing was whether the students of the last group really understood the Fourier analysis in terms of the underlying concepts and mechanisms. Further similar remarks could be made about the intended use of Fourier analysis as were made on the reverse-problems paper. In this case, however, part of the students was involved in translating practical problems into mathematical problems. As was the case with the loudness and rhythm-problems. Doubts were expressed about the feasibility of upscaling such projects.

Engineering design and mathematical modelling processes

Three papers were presented in this thematic area. The first paper by Abou-Hayt, Dahl and Rump concerned with the exemplification of the integration of the engineering design process (EDP) and the mathematical modelling process in two university students' projects. Abou-Hayt, Dahl and Rump used the characteristics of engineering design process (Tayal, 2013) and mathematical modelling process (Blomhøj & Jensen, 2003) to analyze the projects. Their conclusion was that the both processes were visible in the projects. This paper led to raising some further questions: What is the sign for showing student learn best when they are actively participate and apply theory?; What is students learning in this context?; How is the interaction between the lecturer and students?; How can we compare traditional and this form of teaching outcome?; and What is the meaning of mathematics in abstract and real world?

The second paper by Costa and Domingos elaborated on the development and the implementation of mathematical interdisciplinary task related to STEM integration, in the context of a collaborative Continuing Professional Development (PD) Program targeted at primary school teachers. Costa and Domingos concluded that a PD program can only be successful if teachers can implement the task developed with their students. The following questions led the discussion regarding to this paper: What is the meaning of STEM integration in the context of a collaborative Continuing Professional Development Program targeted to primary school teachers?; How can teacher educators help teachers to develop mathematical interdisciplinary tasks related to STEM integration? Are they different for prospective teachers´ education and in-service teachers?; What are the expectations from the teachers while developing mathematical interdisciplinary tasks related to STEM integration?; What are the expectations from the teachers while implementing mathematical interdisciplinary tasks related to STEM integration?; How could different types of knowledge and skills regarding the integration of STEM be promoted in pre- and in-service teacher education?

In the third paper, Ubuz used the characteristics of EDP outlined by Berland, Steingut, and Ko (2014) to analyze the curriculum learning outcome of a Technology and Design course offered to 7th grade students in Turkey. By identifying the most common verbs and objects contained within the learning outcomes of the curricular documents guiding the 7th grade course, Ubuz found that 23 of the learning outcomes contained EDP knowledge and 29 of them concerned EDP skills. The conclusion is that the EDP knowledge and skills recommended by Berland et al. (2014) are largely present in the 7th grade curriculum. Ubuz suggests that these findings can help teachers integrate similar EDP knowledge and skills in science and mathematics curriculum in middle school.

Design research

A paper by Pugalenthi, Stephan, and Pugalee was presented in this thematic area. This paper sought to capture the students' conception of angles and parallel lines in order to design engineering context based instructional sequences for middle grades (7th grade) mathematics classroom. This research was comprised of a pre-interview to assess the existing conceptions of angles and parallel lines, design and implementation of engineering-based instructional sequences and a post-interview to assess the changes in conceptions of angles and parallel lines. This research paper, however, focused mainly on the analysis of the pre-interview questions dealing with angles and parallel lines along with preliminary discussion of the design of the instructional sequence regarding designing a residential community. The students' conceptions of angles and parallel lines were traced under three themes: Prototypes or reference images, tracing of the lines, and decoupling versus decomposing. This paper leads to raising some further questions: What is mathematical instructional task with an engineering problem?; How the difficulties identified could be prevented through mathematical instructional tasks with an engineering problem?; How mathematical instructional tasks with an engineering problem let to dive deeply into a topic and use mathematics to create a solution?; How the difficulties identified do lead us to develop mathematical instructional tasks with an engineering problem such as designing a residential community?; How could the mathematical difficulties identified be prevented through designing a residential community?; What is effective STEM integration?; How could we provide training or prior skills regarding engineering content and principles to the teachers?

Statistics in STEM

Under this theme one paper was presented. The paper written by Oliveira, Henriques, and Batista dealt with the preservice teachers (PT) perspectives about the role of statistics in a learning scenario with one 8^{th} grade class, fostering the integration of physics and statistics. It was claimed that successful STEM integration depends on teachers' perspectives and their competence to choose learning materials which are suitable in this context. Previous studies showed that teachers had a negative perception of STEM education based on the feeling that they were unprepared to teach within an interdisciplinary curriculum.

The data (group lesson plans and PT's reflections regarding planning and enacting learning scenario) were analysed with a focus on the dimensions of the model of *authentic integration* (Treacy & O'Donoghue, 2014): Application to real-world scenarios, high order thinking processes, and knowledge development, synthesis and application. According to the paper the PTs concluded that the mathematical content (in this case statistics) was not only incidental to the STEM context, but had a great centrality in the learning scenario. Mathematics did not only appear in the sense of calculation methods, since high order thinking processes took place that were supported by statistics. Furthermore, the role of technology, in this case with statistical software, was pointed out. The collaboration with colleagues of other subjects was perceived as beneficial for the PTs.

Questions raised during the discussion were: If teachers gain experience with interdisciplinary classroom activities, do they perform better in a future approach, even if the topic is different? Is it a lack of knowledge from other disciplines or rather a lack of experience that makes teachers feel underprepared?

Approaches to introduce STEM

In this theme two papers were presented. Both had in common that they used game-like structures to enhance the students' motivation in their particular learning environment. One paper written by Viamonte and Figueiredo was about gamification in an e-learning tool for university students. In that paper dropout rates as well as the personal opinion of the students who used the e-learning tool were examined. A significantly lower dropout rate compared to earlier years could be measured, which supports the claim that gamification can be used to stimulate motivation. Also, the students' responses concerning the competitive elements seemed to confirm the expected influence, although not all reactions of the students had been positive. The issues raised to be discussed were:

- Although the statistical data concerning dropout rates show a significant impact, it could be possible that there are effects involved that can't be measured easily (e.g. a new teaching concept increases motivation, but the effect decreases over a longer period of time). One could think about examining further iterations, maybe with slight variations.
- Similarly, could the effect be decreasing if gamification was used in a broader extent (i.e. not only one lecture, but all lectures the students attend during the same semester)?
- The data are probably not representative for the whole university population. The participants were engineering students only, and furthermore most of them (93%) were male. Is it possible to transfer the conclusions to a broader range of students?

In the other paper written by Abboud, Hoppenot, and Rollinde a classroom situation was analysed, where students experienced the orbits of astronomical objects from our solar system in a role-playing activity. The aim was to enhance activity theory (Abboud et al. 2018; Vandebrouck 2012) by the notion of bodily perception. Several implementations of the Human Orrery project were analysed qualitatively with questionnaires and interviews. It was suggested that participating students experience STEM-related concepts from an emotional perspective and describe them as "more real". With regard to this paper the issues raised to be discussed were:

- How can we measure the impact of bodily perception on the motivation at a quantitative level?
- How is it possible, after a certain STEM-related concept was experienced in this framework, to have a smooth transition to a more theoretical approach to the topic? This might be interesting when working with older students.

STEM teacher preparation

In this theme, one collaborative team consisting of mathematics and physics educators from Vietnam and Germany aimed to create a teacher preparation program that invited mathematics and physics teachers to participate in classes that utilize both theoretical and practical aspects of teaching these two disciplines. The theoretical part would provide opportunities for physics and mathematics teachers to compare and contrast epistemological foundations of each discipline, thereby illuminating the differences and similarities in the mathematics and physics. In particular, teachers would explore the meaning and practices of *modelling* from mathematics and physics perspectives. These theoretical foundations would then support teachers to create lessons that potentially integrate physics and mathematics that they would then use in their classrooms. Although the project is still in development, Krause and et al intend to videotape the implementation of these lessons by project teachers and determine to what extent physics teachers focused on mathematics in their class and how much mathematics teachers integrated physics into their class. The critical issues raised by this paper harken back to earlier papers in which questions about the extent to which modelling from different disciplines can be mutually supportive. It also calls to the forefront what it means to genuinely integrate science and mathematics. Finally, the crucial problem that Krause and et al attempt to solve is how to design appropriate instruction for teachers who are knowledgeable about physics and mathematics separately so that they are knowledgeable about the means of integrating the two.

Critical issues

We would like to close our introduction by including some critical issues that the participants expressed during the sessions.

- Experts within and across the STEM disciplines model for different purposes and, thus, in different ways; given this diversity of modeling, how do we characterize modeling in STEM?
- Teachers are not experts in all STEM disciplines. In the real world, there is also no such thing as a STEM expert. Rather, discipline-specific experts are invited into a project when there is a need for their expertise. This implies that teachers should work collaboratively to design and implement STEM units. Does this change the goal of STEM education from STEM *integration* to STEM *collaboration*?
- How do we design curriculum so that mathematics is not just a tool for doing the other three content areas? And what is really meant by technology use in curriculum?
- What kinds of research experiments are needed to understand STEM education? While creating nice STEM lessons has potential, there is a need to go beyond studies that make claims about the success of these lessons that are based upon one student. Rather, future research on effective STEM curricula should involve collective student learning.

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