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This work draws on the Programme for the International Assessment of Adult Competencies (PIAAC) survey. Last year a first review was conducted on the PIAAC Numeracy Framework (Tout. et al., 2017). In 2018 and 2019 the framework for the second cycle of PIAAC will be developed. This second cycle of the PIAAC survey aims to update the data about the numeracy skills of adults in different countries around the World (Hoogland, Díez-Palomar, Maguire, 2019). The objective of this paper is to highlight some relevant findings from literature on the concept numeracy in order to discuss a potential enrichment of the PIAAC Numeracy Assessment Framework (NAF).

Keywords: Numeracy, assessment, adult learning.

Introduction

We are now well into the 21st century, and lifelong learning is becoming a crucial feature in adults' lives, especially in terms of numeracy, because of the need to live in an increasingly globalized world characterized by rapid technological and economic change. There are major societal and policy pressures on education to prepare citizens for a complex and technologized society (Hoogland, Díez-Palomar, & Vliegenthart, 2018; Voogt & Roblin, 2012). What is expected from numeracy education in the current situation? New means of communication and types of services have changed the way individuals interact with governments, institutions, services and each other, and social and economic transformations have, in turn, changed the nature of the demand for skills as well. Globally, too many adults and young people lack the necessary numeracy competencies to participate autonomously and effectively in our technologized and number-drenched society. As a consequence, many people are disadvantaged in terms of employment and face preventable challenges in relation to social well-being and financial security. The results of the last PIAAC survey (OECD, 2016) show that a quarter of the participating countries in PIAAC have numeracy outcomes below level 2 of the 6-point scale. These outcomes give rise to serious cause for concern for the future economic development for many nations. This is an even more pressing issue since the amount of mathematical data that needs to be interpreted and used is increasing rapidly due to technological developments and the emergence of (big) data. However, numeracy is a complex notion that entails different components (Geiger, Goos & Forgasz, 2015). More than a decade ago (in 2003), several scholars meeting in Strobl discussed, during an ALM (Adults Learning Mathematics) conference, the meaning of numeracy versus mathematical literacy (Maasz & Schloeglmann, 2003). It was not clear whether numeracy was just referring to the ability to use mathematics in different situations, or something wider that we can call "mathematics literacy." In this paper we adopt the definition of numeracy developed for the first cycle of the PIAAC survey as starting point, to discuss the concept of "numeracy" seeking to identify what numeracy concepts are, so a meaningful assessment framework can be developed to recognize these concepts. We will discuss the notion numeracy in order to enrich the basis to develop the NAF that will be used in the second cycle of the PIAAC survey. We draw in a literature review exploring different concepts of numeracy, putting the emphasis on its components. In the next section we present the methodology used to conduct the review discussed in this paper. Then, we introduce the main findings.

Methodology

In order to answer the research question mentioned above, we conducted a literature review on the concept numeracy. Typically, a literature review is used to frame the problem presented in the introduction of the study (Creswell, 2003). In our case, we are using the literature review as a research instrument to identify relevant literature in order to discuss the different definitions about numeracy. According to Creswell (2003), the first step of doing a literature review is identifying "key works useful in locating materials" (p. 34) that may be relevant for the purposes of the study. In our review we elaborated on past finding by cross-referencing the concept numeracy with a number of reoccurring concepts in the discussions on numeracy. These concepts are: big mathematics ideas, number sense, embeddedness, and authenticity. We decided using those categories since they emerge from the first cycle of PIAAC. We have carried out a documental analysis of articles published in scientific journals included in the main databases and repositories, including the Web of Science, Scopus, for relevant journal articles, and ERIC Database for scholar documents and reports. Additionally, Google Scholar was used to identify potentially relevant literature, that we have subsequently contrasted with the Web of Science and, especially, with the Journal Citations Report database. We used the mentioned categories above as keywords to conduct the searches.

Results

Following Creswell (2003) recommendation, we present the global results of the review for each of the selected concepts. We also use what Creswell calls a literature map to illustrate the different components regarding to numeracy found in the literature review. Drawing mainly on the documents produced by the OECD (PIAAC, etc.), we created a first approach to the concept of numeracy, using a web-based software to count the number to times (frequency) that each word is used. This provided a first view of the concept of numeracy and how is it being used (Figure 1).

Figure 1: Word cloud of terms related to numeracy (based on OECD documents)

We can see that the word “numeracy” is usually connected to concepts such as mathematics, skills, literacy, education, teaching, workplace, use, work, and data (see Figure 1). However, this first approach is based in a limited exploration. For this reason, we conducted a more fine-grained analysis, using a purposeful procedure (Creswell, 2003) using the three key words, as cited above, in the methodological section. As we can see in Figure 2, those three keywords provided a conceptual network in which aspects such as competence, access, use and interpret mathematics, embeddedness and social practice, invisibility, authenticity, workplace, powerful mathematical ideas, appears interconnected.

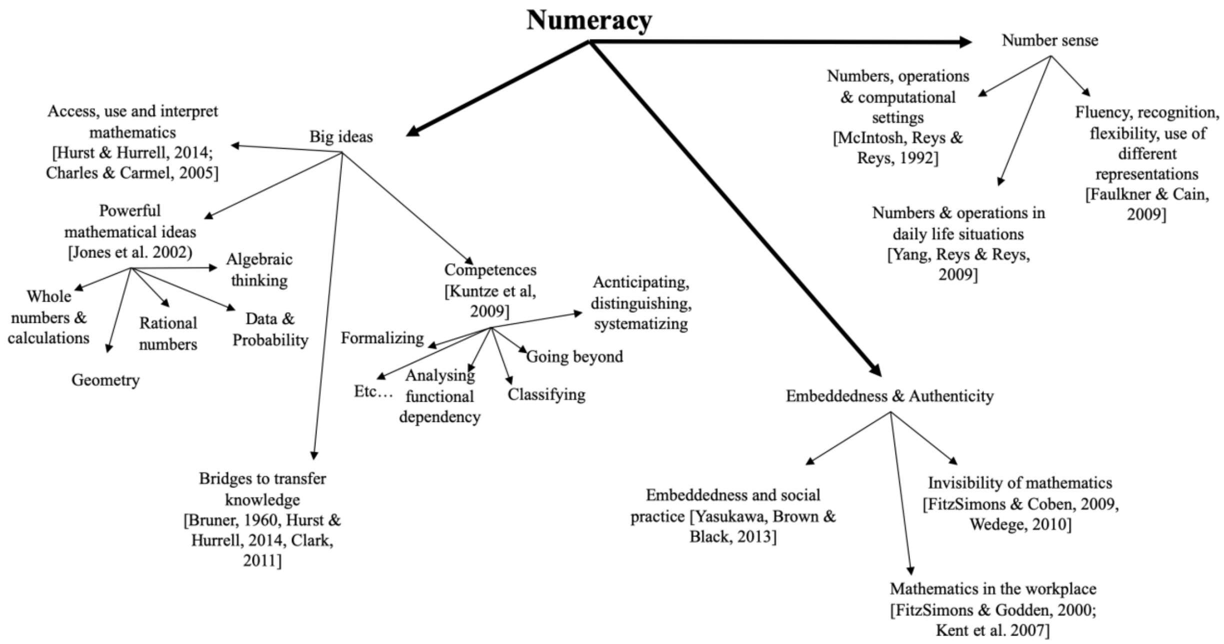

Figure 2: Literature map about the notion of numeracy

Numeracy and big ideas in mathematics

An entry point to identify relevant schemes, models or instruments to establish the basis of the new numeracy components is the so-called “big ideas in mathematics.” This is a well-known research domain in mathematics education (Jones et al. 2002). There is a general agreement that mathematics proficiency means noticing connections among different mathematics’ concepts and competence in using them. After many decades of mathematics education as a sort of utilitarian discipline looking for ways and strategies for children to perform calculations and solve problems, the vast majority of the mathematics education researchers defend a more relational focused approach to teach and learn mathematics. Jones and his colleagues (2002) provide a great summary of the main contributions of the research to what they call “powerful mathematical ideas”, including the following domains: whole number and operations, rational numbers, geometry, probability, data exploration and algebraic thinking and other underrepresented domains. It could be argued that being numerated means using the contents of all these domains not just as procedures (instrumental understanding in Skemp's terms) but in a critical / meaningful manner.

In a more recent article, Hurst and Hurrell (2014), quoting Charles and Carmel (2005), state that "big ideas" allow us to see mathematics as a coherent set of ideas, encouraging a deep understanding of mathematics. It could be suggested that being numerate as defined within the PIAAC NAF may link to the idea of being able to access, use, interpret and communicate mathematical information around what the international scientific community calls "big ideas in mathematics." Although it seems that everyone might understand what "big ideas in mathematics" encompasses, the reality is that the construct remains contentious. Kuntze and his colleagues (2011) mention a plethora of different terms referring to the area of big ideas, e.g. fundamental ideas (Schweiger, 2006), central ideas or universal ideas (Schreiber, 1983), core ideas (Gallin & Ruf, 1993), leading ideas (Vollrath, 1978), basic ideas and basic conceptions (Hofe, 1995).

Charles and Carmel (2005) define "big idea" as "a statement of an idea that is central to the learning of mathematics, one that links numerous mathematical understandings into a coherent whole." This definition is also shared by other authors such as Hurst and Hurrell (2014). In their article, they track the notion of "big idea" back to the work of Bruner (1960), who inspired Clark's (2011) definition of big idea as a "cognitive file folder" that we can file with "an almost limitless amount of information." (Clark, 2011, p. 32). Big ideas became conceptual structures (schema in Skemp's terms) that we can use to provide a NAF where content might be characterized by multiple connections. As Bruner (1960), Hurst and Hurrell (2014), Clark (2011) and other authors claim, big ideas may become bridges for the transfer of learning. Drawing on their thoughts, we suggest here using big ideas in mathematics as a skeleton for developing PIAAC NAF.

Numeracy and number sense

Number sense appears to be one of the main components of "numeracy." Being numerate means having a certain sense of numbers and how we use them to represent, inform, predict, estimate the reality.

McIntosh, Reys and Reys (1992) develop a framework for number sense including three components: numbers, operations and computational settings, which are interconnected. According to them, number sense involves being able to use numbers, operations and their applications in different computational settings. They talk about the meaningful understanding the Hindu-Arabic number system, the development of a sense of orderliness of the number, the multiple representations for numbers (including the idea of composition / decomposition), the understanding of mathematical properties, and the relationship between operations. For them, having "number sense" means being able to solve problems in the real world, providing suitable answers, using (or creating) effective strategies to compute, count, etc. It is not just reproducing instrumentally a certain algorithm but being able to use the mathematical knowledge and components in a flexible manner.

Yang, Reys and Reys (2009) define number sense as "a person's general understanding of numbers and operations and the ability to handle daily life situations that include numbers. This ability is used to develop flexible and efficient strategies (including mental computation and estimation) to handle numerical problems.” (Yang, Reys & Reys, 2009, p. 384). Regarding the components of number sense, these authors argue “Number sense is a complex process involving many different components of numbers, operations, and their relationships.” (Yang, Reys & Reys, 2009, p. 384). Among these processes, they highlight two aspects, (1) the use of benchmarks in recognizing the magnitude of numbers, and (2) the knowledge on the relative effects of an operation on various numbers.

Faulkner and Cain (2009) claim that “the characteristics of good number sense include: (a) fluency in estimating and judging magnitude, (b) ability to recognize unreasonable results, (c) flexibility when mentally computing, (d) ability to move among different representations and to use the most appropriate representations” (p. 25). The main components of their approach to number sense are: quantity and magnitude, numeration, equality, base ten, form of a number, proportional reasoning, algebraic and geometric thinking.

Numeracy, embeddedness and authenticity

The concept of embedded mathematics emerges primarily from studies related to mathematics in the workplace but also has significance in the broader notion of numeracy. The embeddedness of mathematics refers to a deep connection to the context in which it is utilized. This can mean that the way mathematics is used to operate on a task is fundamentally shaped by the context in which it is employed. This includes socio-cultural influences that afford or constrain action in school, civic, personal or workplace environments. In this view there is a clear separation between school mathematical knowledge, how it is taught, learnt and practiced, and the use of this knowledge outside of schooling. As Harris (1991) notes:

> In work [. . .] mathematical activity arises from within practical tasks, often from the spoken instruction of a supervisor and always for an obvious purpose which has nothing to do with the numbers working out well. Thus, students taught to react to isolated, abstract and written commands in the specialist language and carefully controlled figures of a school mathematics class, find themselves confronted with the urgent spoken, if not shouted, instructions in a completely different context and code (p. 138).

Yasukawa, Brown and Black (2013) make a clear connection between embeddedness and social practice arguing that numeracy practices cannot be understood independently of the social, cultural, historical and political contexts. They illustrate this point, they compare students completing calculations individually, using paper and pen and perhaps a calculator against the use of mathematics in the supermarket, in which the same calculations completed at a checkout counter by the shop assistant using a cash register. In this situation the shopper might perform an estimation to avoid being overcharged. However, the shop assistant is equally concerned with charging the customer the correct price and recording accurate record of the items sold via the cash register. The calculations are the same but the purpose – which is related to context - is different.

Embeddedness has led some researchers to talk about the invisibility of mathematics within work or social contexts. This means that mathematics can be fundamental to activities that are not obviously mathematical (FitzSimons & Coben, 2009). This is most clearly apparent in the use of technology in the workplace where digital tools used to complete tasks often obscure underpinning mathematical activity. As Kent, Noss, Guile, Hoyles and Bakker (2007) argue, within techno-mathematical situations in workplaces there is a shift from ‘fluency in doing explicit pen and paper mathematical procedures to a fluency with using and interpreting output from IT systems and software, and the mathematical models deployed within them’ (p. 2-3).

Building on this point, Wedege (2010) defines two forms of invisible mathematics as (a) subjectively invisible mathematics where people do not recognize the mathematics that they do as mathematics and (b) objectively invisible mathematics in which mathematics is hidden in technology.

Discussion

Drawing on the contributions coming from the literature review, some considerations emerge.

First, big ideas in mathematics, number sense, embeddedness, and authenticity are important concepts in trying to define the notion of numeracy in the context of the 21st century. The *Numeracy Expert Group* working at the PIAAC survey defined numeracy as “ability to access, use, interpret and communicate mathematical information and ideas” (PIAAC Numeracy Expert Group, 2009, p. 21). However, this definition does not provide clues about what “mathematical information and ideas” means. This is a relevant topic, since in the 21st century the “mathematics” that may be relevant for adults probably include some of the components highlighted by the scholars working around the notion of big ideas of mathematics. This is also interlinked to the notion of authenticity, since relevant mathematics, perhaps, must to be also authentic (or they are relevant because they are also authentic). If we want to create a new framework for numeracy assessment, then probably we need to look for the mathematics embedded in real situations and draw on them in order to be able to measure adults’ numeracy.

Second, more research is required to identify other important elements of numeracy, especially in the current context of the 21st century and the “new” skills that adults must to develop. In fact, numeracy capability is increasingly vital in a world characterized by rapid technological and economic change.

Third, numeracy is vital for social well-being, financial security and informed citizenship. Hence, a framework to assess numeracy must explore how this notion is embedded in authentic practices related with those societal dimensions mentioned above.

Fourth, the critical aspect of numeracy, (not discussed here) related to making evidenced based judgements and decisions, is an aspect of numeracy that has often been underplayed for adults but is an essential element for informed participation in personal, civic and work life. More work is needed in identifying how to promote this critical capability.

Fifth, evidence suggest that adult numeracy has been limited to studies in context (workplace, personal settings or activities, such as shopping, etc.). However, there is a lack of research in terms of cognitive, epistemological considerations of how adults learn/ use mathematics (numeracy skills), which means that additional research is needed to cover those aspects.

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