Professional competencies for teaching mathematical modelling – supporting the modelling-specific task competency of prospective teachers in the teaching laboratory

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Mathematical modelling has been a mandatory component of the German educational standards in mathematics for the last 15 years. Teachers need a range of cognitive abilities and skills to adequately teach modelling to their students. Within the MiRA+ teaching laboratory, we focused on supporting the development of the professional competencies associated with mathematical modelling in prospective teachers. Encouraging teachers to design own modelling tasks and apply them in practice successfully aided the development of modelling-specific task competencies with a crucial role in the acquisition of competency by their students. This article presents a promising array of initial results on the development of task competency in teacher education.

Keywords: Mathematical modelling, modelling task, task competency, teacher education.

Introduction

As part of the teacher training project "Qualitätsoffensive Lehrerbildung", the University of Muenster has created learning opportunities in the form of practical sessions with structured reflection to equip prospective teachers with productive ways of managing heterogeneous learning groups. Teaching-learning laboratories provide invaluable opportunities to incorporate practical elements that specifically facilitate the professionalisation of prospective teachers at an early stage of their studies by hosting interactive reflection on teaching-learning processes. In the spirit of a potential-driven approach to managing heterogeneity in the classroom, this allows teachers to gain experience working with groups of students who display different levels of individual performance, e.g. by testing and discussing differentiated learning materials and varying the instructions given to specific students.

From the perspective of mathematics education, mathematical modelling processes are inherently good at organically differentiating between students. Open modelling tasks represent a constructive heterogeneity management strategy that allows students to work individually and in a differentiated manner according to their own prior knowledge, interests, and performance levels. However, despite its extensive potential, mathematical modelling is found to be challenging by both the students themselves and their (prospective) teachers (Blum, 2015). As a result, a detailed study of the teaching competencies needed to support mathematical modelling is urgently needed by the current quality development initiatives in teacher education.

This paper studies the development of modelling-specific task competency, which is viewed as one of the aspects of professional competency for teaching mathematical modelling, in the teaching laboratory MiRA^{+} (mathematics in real applications) attended by university candidates during the first phase of teacher education.

Mathematical modelling

Mathematical modelling is the basis of the teaching-learning processes facilitated by the MiRA$^+$ teaching laboratory. Realistic settings provide an abundant source of authentic problems that can be solved mathematically and then reapplied to the original context. Modelling is defined as “the process leading from a[n] [authentic] problem situation to a mathematical model” (Blum, 2002, p. 153). The following section presents some of the theoretical and pedagogical background of modelling, introducing a few ideas that will be essential for the subsequent discussion.

Modelling as a competency

Mathematical modelling is one of the six general-purpose mathematical competencies encountered at every level of the German educational standards. Students are expected to be capable of translating problems from reality to mathematics and vice versa, as well as being able to work mathematically within the model itself. Blum (2015) defines modelling competency as the ability to construct, exploit, or adapt mathematical models by executing each step of the modelling process adequately and appropriately for the given problem, together with the ability to analyse and critically compare specific models. The student’s ability to carry out specific subprocesses can be viewed as the subcompetencies of mathematical modelling (Niss, 2003). Table 1 characterises selected subcompetencies in accordance with the modelling cycle by Blum and Leiss (2007).

Subcompetency	Indicator
Constructing	Students construct their own mental model from a given problem and thus formulate an understanding of their problem
Simplifying	Students identify relevant and irrelevant information from a real problem
Mathematising	Students translate specific, simplified real situations into mathematical models (e.g., terms, equations, figures, diagrams, and functions)
Interpreting	Students relate results obtained from manipulation within the model to the real situation and thus obtain real results
Validating	Students judge the real results obtained in terms of plausibility

Table 1: Selected subcompetencies of modelling (Greefrath & Vorhölter, 2016, p. 19)

Working mathematically itself is not explicitly listed as a modelling subcompetency, since it is not specific to modelling processes and is therefore not specifically emphasised when teaching mathematical modelling.

Modelling tasks

Tasks represent a strongly dominant component of mathematical teaching. The communication of mathematics to students during the teaching-learning process has always heavily relied on setting and solving tasks (Neubrand et al., 2011). More concretely, modelling processes can be stimulated in the classroom by setting suitable tasks. Various categories of tasks can be defined to analyse and classify modelling tasks. For example, Maaß (2010) proposes a comprehensive classification of modelling tasks that considers the nature of their reference to reality, their openness, and their focus on modelling activities, among other criteria. Greefrath, Siller, and Ludwig (2017) refine the criterion of “reference to reality” by considering the authenticity and the contextual relevance of each task in further detail. For the criterion of “focus on modelling activities”, they adopt the modelling subcompetencies listed above. Building upon this theoretical background, we compiled the list of criteria (see Table 2) reproduced for the development and evaluation of modelling tasks.

Criteria	**Concretisation**
Reference to reality	Does the problem refer to an extra-mathematical factual context?
Relevance	Is the factual context relevant to the students (factual problem)?
Authenticity	Is the factual context authentically related to the actual situation? Does the factual context apply mathematics in an authentic manner?
Openness	Is there more than one possible way to solve the problem (solution variety)? Are there different levels of solutions?
Modelling subcompetencies	Which modelling subcompetencies are required to work on the problem?

Table 2: Criteria for the creation and evaluation of modelling tasks

Professional competency for teaching mathematical modelling

To achieve quality development during the first phase of teacher education, a detailed analysis of the professional competencies associated with teaching mathematical modelling needs to be performed. Models that accurately characterize the requirements placed upon teachers must also be formulated to measure these competencies. Research on mathematical teaching is not only concerned with competence models that characterize the general areas of responsibility of educators, for example by establishing a global description of teaching knowledge, but also with professional competence models that focus on specific parts of the educational standards. A collaborative project by the Universities of Koblenz-Landau and Muenster sought to provide some initial structural answers for mathematical modelling by developing an instrument that measures certain key aspects of the professional competency for teaching mathematical modelling (Klock, Wess, Greefrath, & Siller, 2019).

Task-related dimension	a) Ability and knowledge to solve a modelling task in multiple ways b) Ability and knowledge to analyse modelling tasks c) Ability and knowledge to develop modelling tasks

Table 3: Task-related dimension for teaching modelling competency (Borromeo Ferri & Blum, 2010)

This conceptualisation is primarily oriented towards the COACTIV model, which gives a description of the professional competency of teachers (Baumert et al., 2010). Formulating modelling-specific interpretations of the competency facets of teaching knowledge identified by this model along the competency dimensions proposed by Borromeo Ferri and Blum (2010) gives facets of modelling-specific teaching knowledge that serve as the basis for the proposed conceptualisation of professional competency for teaching mathematical modelling (Klock et al., 2019).

Modelling-specific task competency

At a theoretical level, the modelling-specific task competency is directly derived from the task-related dimension in Table 3 and represented in terms of the knowledge facets of modelling tasks specified by the professional competence model for teaching mathematical modelling. The task competency characterising mathematical lesson planning (Neubrand et al., 2011; Borromeo Ferri, 2018) was found to be significant, not just in relation to the model cited above, but also for the professional competency review of prospective teachers performed as part of the quality development initiative. Thus it became clear that a positive development of this aspect of professional competency in particular helps to overcome obstacles to the use of modelling tasks in the classroom and to exploit their organically differentiating potential in the sense of a constructive heterogeneity management strategy. The general task competency of a (prospective) teacher may be defined as:

> "the ability to design and use tasks to cognitively stimulate students and evaluate their learning performance, as well as the ability to analyse the work performed by students on these tasks" (Sjuts, 2010, p. 807).

These abilities, together with an understanding of the diverse pedagogical potential of tasks, represent a key dimension of teaching knowledge. The task competency aspects of teaching mathematical modelling are just as crucial. Properly understanding the multiplicity of possible solutions, cognitive analysis, and the process of developing modelling tasks results in high lesson flexibility and serves as a modelling-specific interpretation of the task competency (Borromeo Ferri, 2018).

Research question

In light of the theoretical discussion given above, the global research objective is to determine the extent to which the professional competencies of prospective teachers in mathematical modelling can be supported by a teaching laboratory. In particular, we are interested in studying any task-related abilities of (prospective) teachers that are relevant to the acquisition of competency by students. The research objective can therefore be formulated more concretely as follows: *To what extent do self-prepared modelling tasks in a mathematical teaching laboratory environment improve aspects of the modelling-specific task competency of prospective teachers relative to well-established modelling tasks?*

Methodological approach

Sample and design

To answer this research question, a paper-and-pencil questionnaire with a pre-post comparison group design was used to collect data from 107 candidates at the Universities of Koblenz-Landau and Muenster who were studying for a teaching position at secondary schools. In addition to the treatment group (TG) in Muenster (N=35) and the comparison group (CG) in Koblenz (N=43), data were collected from a baseline group in Muenster (N=29) to control for any test repetition effects.

Teaching laboratory concept

For this study, based on the concept of a teaching-learning laboratory (Lengnink & Roth, 2016), a series of teaching seminars on mathematical modelling featuring practical components were developed at the two participating institutions. The contents of the seminars were coordinated but emphasised different facets of modelling-specific competency. The seminars in Muenster focused on developing the task-related skills of prospective teachers by asking them to develop and apply their own modelling tasks. By contrast, the seminars in Koblenz focused on developing modelling-specific intervention competency by applying well-established modelling tasks without analyzing and adapting them. The teaching laboratory MiRA$^+$ discussed in this paper consists of a theory-based preparation phase, a practical phase, and a reflection phase (see Figure 1).

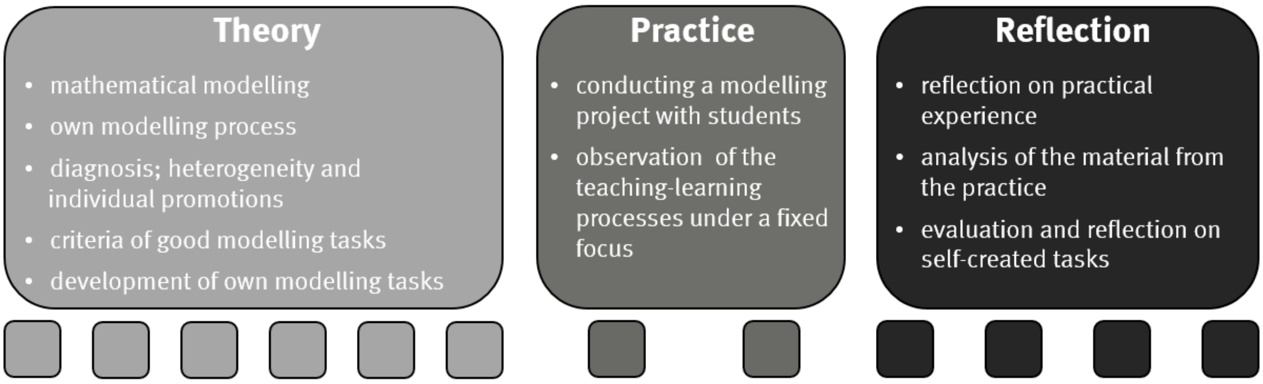

Figure 1: Concept of the MiRA$^+$ teaching laboratory

Modelling processes represent the core content of every phase, together with sensitisation to potential-driven methods of managing heterogeneity in the classroom. The term "teaching laboratory" reflects the emphasis on teacher education. However, the teaching-learning processes of the university candidates naturally unfold in parallel to the learning processes of the high school students. The theory phase of the seminar serves to impart the theoretical foundations of mathematical modelling and pedagogical diagnostics. The dimensions of heterogeneity are discussed as well as the basics of individual promotion. Finally, modelling tasks for use in the practical phase are developed on the basis of the criteria mentioned (see Table 2). During the practice phase, a small team of prospective teachers look after a small group of high school students to work on the designed modelling tasks. Each team focuses its observations of the teaching-learning processes on subcompetencies of mathematical modelling (see Table 1). After the sessions, implications will be drawn from the insights for the upcoming teaching laboratory appointments, for example addressing a variation of the instructions. The reflection phase serves to reflect practical experiences from the observed teaching-learning processes. The focus is on the managing of heterogeneity in the classroom, the insights gained from it for the professionalisation of the prospective teachers own teaching activity as well as the consequences for the conception and the execution of the own modelling tasks. The aforementioned conception of a teaching laboratory as a seminar with practice integration should especially promote the acquisition of professional knowledge and competency. In addition, the testing of theory-based practical didactic action with regard to the participating prospective teachers is a central element of teaching laboratories. Thus, the acquired didactic and professional knowledge in the development of tasks as well as in the supervision of the students is interlinked and implemented.

Test for measuring aspects of professional competency

An instrument was developed and piloted in the summer semester of 2017 to measure specific aspects of the professional competency for teaching mathematical modelling (Klock & Wess, 2018). The modelling-specific task competency was measured using dichotomous combined-singlechoice items targeting cognitive analysis, solution multiplicity, and the development of modelling tasks. The scale values are based on the standardised sum scores of the dichotomous coding and are examined in a group comparison using a repeated measures analysis of variance for developmental differences. An example of a test item is given in Table 4.

Scale	**Number of items**	**Item example**		
Task-related competency	12	A. Modelling tasks can be underdetermined.	☐right	☐wrong
		B. Modelling tasks can be overdetermined.	☐right	☐wrong
		C. Modelling tasks should be as closed as possible.	☐right	☐wrong

Table 4: Example test item measuring the modelling-specific task competency (Klock & Wess, 2018)

Results

This section presents the progression of the competency aspects cited above observed in the study participants. There were no significant differences between groups in the facets of modelling-specific task competency at the time of first measurement.

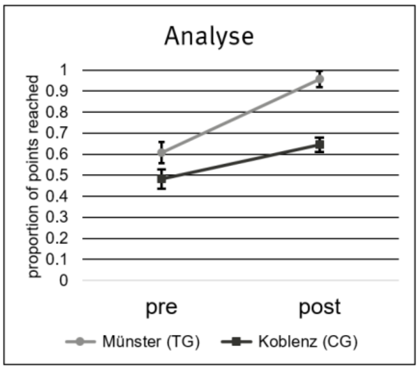

Figure 2: Analyse modelling tasks

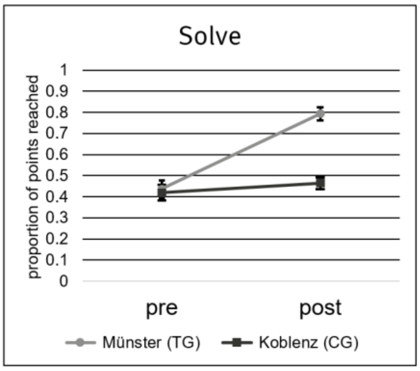

Figure 3: Solve modelling tasks

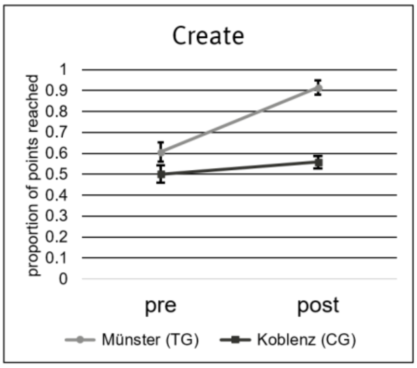

Figure 4: Create modelling tasks

Repeated measures analysis of variance reveals that the cognitive analysis aspect of modelling tasks ($F(1,76) = 12.109$, $p = .001$, $\eta^2 = .185$; Figure 2) in the treatment group (TG) at Muenster progressed significantly more favourably with a greater effect size than the comparison group (CG) at Koblenz. For the solution multiplicity aspect of modelling tasks, a significantly higher increase with a larger effect size ($F(1,76) = 24.322$, $p = .00$, $\eta^2 = .242$; Figure 3) was observed in the TG relative to the CG. Similarly, for the development aspect of modelling tasks, the TG displayed a significantly stronger progression with a larger effect size ($F(1,76) = 12.558$, $p = .001$, $\eta^2 = .182$; Figure 4) when measured against the progression of the CG.

No significant changes in the aspects of modelling-specific task competencies considered by this study were observed in the baseline group (Analyse: $t(28) = 1.154$, $p = .258$; Solve: $t(28) = 1.410$, $p = .169$; Create: $t(28) = 1.797$, $p = .093$; see Figure 4). Consequently, there were no test repetition effects.

Summary and discussion

On the basis of a widely accepted competency concept and a well-established structural model of professional competency, we were able to constructively apply a survey instrument measuring the teaching of mathematical modelling to prospective teachers in a teaching laboratory. Our study of the modelling-specific task competency, which has a strong effect on the acquisition of mathematical modelling competency by students, found clear signs that the professional competency for teaching mathematical modelling can be successfully enhanced within the teaching laboratory MiRA^{+} by encouraging teachers to apply criteria-led self-prepared modelling tasks. Significant contrasts with large effect sizes were observed relative to the progression of the comparison group, which used well-established modelling tasks. These contrasts were concretely measured by the aspects of development, cognitive analysis, and multiplicity of solutions.

Limitations and outlook of the study

Despite the large effect sizes and the significant differences, the number of participants in the experimental group (N = 35) does not provide a sufficiently robust basis for any definitive statements regarding the professional competency for teaching mathematical modelling. The conclusiveness of these findings can be increased in future by cumulating over multiple seminar cycles – provided of course that the treatment is controlled appropriately.

Acknowledgment

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