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Generation and Derivation of Practical Optimization-Oriented Models of Inductors

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Abstract—Magnetic components contribute significantly to the volume and losses of power electronic systems. Their optimal design is therefore crucial for the overall optimization of power density and efficiency of power converters. Recently it has been shown that converters can accurately be modeled using posynomial functions, thus allowing for the use of Geometric Programming, a type of convex optimization problem, to be used to quickly produce globally optimum designs of entire converter families. Existing studies have however treated magnetic components in a case-specific way, deriving posynomial models suited for the given converter or range of operating points. This paper demonstrates and validates methods for the derivation and generation of posynomial models of inductors for an entire catalogue of standard components, allowing a set of models to be generated once and then re-used subsequently, or “plugged into” an overall converter optimization, repeatedly, without the need for re-derivation.

I. INTRODUCTION

Recently it has been shown [1], [2], [3], [4], [5], that geometric programming [6], a class of convex optimization problems, can be used to accurately and quickly optimize power converters for multiple objectives. There are two main advantages of using this approach. First, in a feasible convex optimization problem, a local minimum is guaranteed to be a global minimum. Second, many efficient solvers for convex optimization problems exist: in [2], [5] it was shown that complex converter topologies can be optimized for efficiency and volume in under one minute. A Geometric Program (GP) has the following form

$$\begin{aligned} \text{minimize } f_o(x) &= \sum_{l=1}^n (a_{ol} x_1^{k_{ol1}} x_2^{k_{ol2}} \dots x_n^{k_{oln}}) \\ \text{subject to } f_i(x) &= \sum_{l=1}^n (a_{il} x_1^{k_{il1}} \dots x_n^{k_{iln}}) \leq 1, i = 1, \dots, m \\ h_j(x) &= a_{jl} x_1^{k_{j1}} x_2^{k_{j2}} \dots x_n^{k_{jn}} = 1, j = 1, \dots, p \end{aligned} \quad (1)$$

where $x = (x_1, x_2, \dots, x_n)$ is the vector of design (input) variables, $f_o(x)$ is the objective function to be minimized, and $f_i(x)$ and $h_j(x)$ are the inequality and equality constraints, respectively, that must be satisfied by the solution. In order for $f_o(x)$, $f_i(x)$ and $h_j(x)$ to be convex functions, x must be a vector of positive non-zero real numbers, as must be the coefficients (a_o, a_i, a_j) , while the exponents (k_o, k_i, k_j) can be any real number. Such functions are called *posynomials* (“positive polynomials”) [6].

Therefore, in order to model a power converter as a GP, each component in the system must be modeled using posynomial functions. As shown in [2], [4], [5], semiconductor models are often already in posynomial form or can be manipulated to be in posynomial form in a relatively straightforward manner. Accurate models of magnetic models, such as those in [7], [8], are more difficult to put into posynomial form. In [1], [4], comprehensive posynomial models of inductors were not considered. In [2], [5], low-power SMD inductors were considered and modeled in an application-specific way, with a posynomial model that is appropriate to the converters considered for optimization within the operating range envisioned for the studied application. Therefore, these models cannot be easily transferred to a different converter optimization problem, and in most cases, the models would need to be re-derived. In [3], it was demonstrated that deriving posynomial models of nonSMD custom-built inductors (constructed from standard cores and wires), such as those shown in Fig. 1 was feasible, and a model derivation procedure, based on [9], was presented. However, as [3] was more general in nature and considered other approaches to model derivation as well, the presented example was only a limited proof of concept that also is not easily transferrable to other applications.

This goal of this paper therefore is to present a method for the derivation of re-usable, “generate-once use-multiple-times” posynomial models for higher-power, custom-built inductors made from standard cores, wires, and bobbins. Such a method allows, for a given catalogue of standard parts (cores of a single type of geometry and from the same magnetic material, and

wires of standard gauge from the same conductor material), a set of posynomial functions modeling inductance, peak flux density, total losses, boxed volume, and core and winding temperature to be generated over the entire set of feasible inductors that can be constructed with the considered standard parts. These models can then be reused repeatedly within different converter optimization problems.

In Section II it is shown how such a goal may be attempted to be achieved using analytical equations of inductor geometry based on [10]. While useful results may be obtained with this approach, it is difficult to accurately model winding losses in

Fig. 1. Photographs of the built Inductor 1 (EPCOS E55 core, N27 ferrite, 18 turns) and Inductor 2 (EPCOS E32 core, N87 ferrite, 18 turns) used for loss measurements.

TABLE I

QUANTITIES OF THE ANALYTICAL POSYNOMIAL INDUCTOR MODEL

f_s	frequency of the applied square voltage
I_{DC}	DC component of the inductor current
ΔI	normalized inductor current ripple
	saturation flux density
ΔT_{max}	maximum allowed temperature rise
K_w	winding factor (inverse of fill factor)
$Kc1, Kc2, \alpha1, \alpha2, \beta$	Core loss model parameters
ρ	winding resistivity
S	thermal exchange area proportion factor
Z, Y	winding and core volume proportion factors
J_{RMS}	current density, including harmonics of current ripple

gapped cores over a wide range of core geometries and winding arrangements in this manner. Therefore, in Section III, an approach which is a significant extension of the method from [3], using simulation data from a dedicated power electronics inductor simulation tool, is derived and demonstrated. The accuracy of the simulation tool, and therefore its suitability for use as a source of data for the derivation of posynomial models, is confirmed using experimental measurements. The accuracy of the derived models is evaluated using a set of sample optimization problems. The paper is concluded with Section IV.

II. MODEL DERIVATION BASED ON ANALYTICAL EQUATIONS

Starting from the well-known concept of area product ($A_e W_a$ —where A_e is the core cross-sectional area and W_a the core winding window area), in [10], [11], functions describing several different core shapes are derived, with derived geometry-specific correction factors. The resulting models are used in [10] in an iterative transformer design procedure, and in [11] in an iterative inductor design procedure. Most models presented in [10], [11] are in fact in, or close to, posynomial form. Key quantities of these models are summarized in Table I. Each considered core shape is characterized such that the core lengths are proportional to $A_e W_a^{\frac{1}{4}}$, the core areas to $A_e W_a^{\frac{1}{2}}$, and the core volumes and weights to $A_e W_a^{\frac{3}{4}}$. Additionally, a thermal exchange coefficient H_{therm} is specified as a constant value to be typical either of natural convection, forced convection, or liquid cooling. For details on the expressions for and values of the quantities in Table I, the reader is directed to [10], [11].

Therefore, a GP can be formulated where the main free variable is the size of the core, i.e. $A_e W_a$, along with the induction at nominal current B_{DC} and the current density at

Fig. 2. A Pareto front of optimum inductor designs for a given L, f_s , core material and shape for different values of H_{therm} . nominal current J_{DC} :

minimize $A_e W_a$ (2)

$$L I_{DC}^2 < A_e B_{DC} \frac{W_a}{K_w} J_{DC}$$

$$B_{DC} \left(1 + \frac{\Delta I_n}{2} \right) < B_{sat}$$

$$\left[\frac{Z}{K_w} \rho J_{RMS}^2 + Y (K_{c1} f^{\alpha_1} + K_{c2} f^{\alpha_2}) \right] A_e W_a^{\frac{3}{4}}$$

subject to $\leq \Delta T_{max} H_{therm} S A_e W_a^{\frac{1}{2}}$

where L is the desired inductance. The first constraint in (2) is the inductance requirement, the second constraint is the saturation limit, and the third the thermal limit, i.e. the total amount of allowed losses. Therefore, (2) minimizes the area product of an inductor built from a specified core material and core shape, for a given upper loss limit and desired inductance. The objective function in (2) could be changed from $A_e W_a$ to the $A_e W_a$ -dependent function for losses, weight or volume given in [10], [11], or to a weighted sum of two or more of these functions, giving a multi-objective problem.

A loss-volume Pareto front can be constructed by repeatedly solving (2) for different values of H_{therm} , as shown in Fig. 2. This immediately shows the feasible (theoretically realizable) and non-feasible regions of the inductor design space, showing the minimum volume achievable for different types of cooling. The same can be repeated with different core materials and at different frequencies (as in Fig. 3) or with different core shapes, etc. Since (2) can be solved in a few seconds, it is a powerful tool for the exploration of large design spaces, and can be used to quickly identify which cooling methods, core shapes, core materials, and so on, are appropriate to a particular application or design task.

Unfortunately, this approach is limited when applied to gapped cores. As shown in [7], a general closed-form analytical solution does not exist for proximity losses due to an air gap's fringing flux, and at least a 2-D summative calculation method must be used instead. The reluctance of an air gap is

Fig. 3. Pareto fronts for a given L for different core material and f_s pairs.

Fig. 4. The general, structured, successive fitting procedure for deriving posynomial models from data characterizing a family of inductors [9], [3].

also difficult to encapsulate in a simple analytical equation. Therefore an extension of the approach in [10], [11] was developed in [12], using finite element method (FEM) simulations to derive correction factors for ρ that include the effects the air gap on the winding losses. Such correction factors are, however, extremely geometry-dependent and require a large number of FEM simulations to refine. Considering the complexity of the problem, it is simpler to then characterize the entire inductor using a proven, comprehensive power electronics-oriented simulation tool. This different approach is described in detail in the following Section.

III. SIMULATION DATA-BASED MODEL GENERATION

Following [9], [3] presents a procedure, shown in Fig. 4, to fit posynomial functions to data that has been generated by simulations, measurement, or from the evaluation of nonposynomial analytical models. First, the data is partitioned. The number of partitions determines the number of terms in the fitted posynomial functions. Then, to each partition, a monomial function, shown in (3), is fitted separately. The results of the fits for each partition are used as an initial guess for fitting a max-monomial function, shown in (4), to the entire data set. The results of the max-monomial fit are then used as an initial guess for fitting the final posynomial function, shown in (5), to the data.

$$h(x) = e^{a_l} x_1^{k_{l1}} \dots x_n^{k_{ln}} \quad (3)$$

$$h_m(x) = \max_{1 \dots l} [e^{a_l} x_1^{k_{l1}} \dots x_n^{k_{ln}}] \quad (4)$$

$$f(x) = e^{a_l} x_1^{k_{l1}} \dots x_n^{k_{ln}} \quad (5)$$

It should be noted that monomial and posynomial functions must be converted to their logarithmic form in order to become convex. Taking the logarithm of both sides in (3) transforms the monomial into a linear (affine) function, the max-monomial in (4) into a max-affine function, and the posynomial in (5) into a softmax-affine function. In the procedure of Fig. 4, the logarithm of the data to be fitted is also taken and fitting is performed to the logarithmic forms of the functions. For more details, the reader is directed to [3] and [9].

In this section, the method presented in [3] is extended to generate posynomial models covering an entire family of potential inductor components, that can be built from cores of the same shape and material but different dimensions, and a standard range of copper wires.

A. Experimental Verification

To generate data for fitting, the magnetic simulation software tool GeckoMAGNETICS [13], version 1.5.0, was selected. It is based on advanced, published models that have been experimentally verified [7], [8], [14], [15]. It contains measurements of core losses with rectangular voltage excitations and DC bias current, allowing for accurate core loss and temperature calculations, and a detailed air gap reluctance model allowing for accurate inductance, winding loss and temperature calculations. To confirm the accuracy of the version of the software used, a measurement setup, shown in Fig. 5, was constructed. The inductor being measured is the device under test (DUT) in Fig. 5 and it is placed between two switching cells. *Cell 2* operates in open loop and generates a quasi-constant voltage V_f across the LC filter. *Cell 1*, switching at switching period T_s , generates a square wave voltage v_{out} ranging from 0 to V_{in} . It operates in closed loop in order to ensure a given current I_{DC} through the DUT. Therefore by adjusting the operation of the two cells, a current waveform with a peak to peak ripple Δi_L , as shown in Fig. 6 can be imposed on the DUT. In this way the measurement setup can mimic, from the point of view of the DUT, the behaviour of several different buck- and boost-type converter topologies.

The loss measurement was performed in a manner similar

Fig. 5. Schematic of the measurement setup used to measure inductor losses to verify the accuracy of the GeckoMAGNETICS software.

Fig. 6. The waveform imposed on the DUT by the measurement setup of Fig. 5, with a duty ratio of D and a switching frequency $f_s = 1/T_s$.

to that presented in [7], [8]. Total DUT losses were measured using a Yokogawa WT3000 Precision Power Analyzer. The total power P_{total} is given as

$$P_{total} = \frac{1}{T_s} \int_0^{T_s} v_L(t) i_L(t) dt \quad (6)$$

and was measured across the main inductor winding N_1 . Additionally, a second sense winding N_2 was added in order to measure core losses P_{Core} only, given as

$$P_{total} = \frac{1}{T_s} \int_0^{T_s} \frac{N_1}{N_2} v_2(t) i_L(t) dt \quad (7)$$

where v_2 is the voltage across N_2 . Winding losses were determined by subtracting P_{Core} from P_{Total} , and additionally by measuring the DC and AC resistance of the winding using a Hioki IM3533 LCR meter. Measurements were performed on Inductor 1, the characteristics of which are shown in Fig. 7 and the built prototype in Fig. 1(a), and Inductor 2, the characteristics of which are shown in Fig. 8 and the built prototype in Fig. 1(b). A selection of the performed measurements and their comparison to GeckoMAGNETICS simulations for the two inductors is shown in Table II and Table III, respectively. It can be seen that the agreement between measurements and simulations is very good. Therefore, GeckoMAGNETICS is an appropriate tool to use to generate data for the derivation of posynomial inductor models.

B. Data Generation

In order to generate posynomial models that can be used over a wide variety of applications, the entire catalogue of TDK EPCOS EE cores (ranging from E13 to E55) made from

Fig. 7. Specifications of the Inductor 1 DUT, built from an EPCOS E55 core and N27 ferrite material.

Fig. 8. Specifications of the Inductor 2 DUT, built from an EPCOS E32 core and N87 ferrite material.

the N87 ferrite was selected for simulation. The cores were not stacked, and only cores with a built-in centre air gap, ranging from 0.04 mm to 2 mm, were selected. To construct the windings, standard American wire gauge (AWG) solid copper wires were used, ranging from AWG10 to AWG44. The dimensions shown in Fig. 9 and Table IV were used to characterize an inductor.

An algorithm was designed to generate a set of feasible inductors over the entire range of the parts catalogue. A maximum allowable current density of $J_{max} = 4 \text{ A/mm}^2$ was assumed. Fill factor was kept between 0.1 and 0.6. For each core (including air gap), the core reluctance R_{tot} was calculated using GeckoMAGNETICS. Assuming first $N = 1$, and using the core material's saturation flux density B_{sat} , the peak allowable current for the inductor was calculated using

$$I_{pk,max} = \frac{B_{sat} R_{tot} A_e}{N}. \quad (8)$$

From this, the smallest wire diameter d_{min} that can accommodate $I_{pk,max}$ while respecting the limit on J_{max} was calculated as

$$d_{min} = 2 \sqrt{\frac{I_{pk,max}}{J_{max} \pi}}. \quad (9)$$

To construct the inductor, the next largest AWG wire was

Fig. 9. The physical dimensions used to characterize an inductor.

TABLE II

MEASUREMENT RESULTS FOR INDUCTOR 1

Waveform			Experimental measurement			GeckoMAGNETICS simulation			Error (Total loss)	
ΔB_{pk-pk} (T)	f_s (kHz)	IDC (A)	Core loss (W)	Winding loss (W)	Total loss (W)	Core loss (W)	Winding loss (W)	Total loss (W)	%	(W)
0.2	10	0	0.45	0.28	0.73	0.483	0.336	0.818	12.05	0.088
0.3	10	0	1.14	0.65	1.79	1.071	0.751	1.822	1.79	0.032
0.5	5	0	1.56	1.11	2.67	1.271	0.961	2.232	16.4	0.438

TABLE III

MEASUREMENT RESULTS FOR INDUCTOR 2

Waveform			Experimental measurement			GeckoMAGNETICS simulation			Error (Total loss)	
ΔB_{pk-pk} (T)	f_s (kHz)	IDC (A)	Core loss (W)	Winding loss (W)	Total loss (W)	Core loss (W)	Winding loss (W)	Total loss (W)	%	(W)
0.3	10	0	0.17	0.18	0.35	0.163	0.140	0.303	13.43	0.047
0.3	20	0	0.36	0.24	0.60	0.358	0.197	0.555	7.5	0.045
0.35	20	0	0.53	0.32	0.85	0.489	0.269	0.758	10.8	0.092

TABLE IV

QUANTITIES OF THE SIMULATION-DERIVED POSYNOMIAL INDUCTOR MODEL

f_s	frequency of the inductor current
I_{DC}	DC component of the inductor current
Δ	peak-to-peak inductor current ripple
A_e	core cross-sectional area (mm)
	core magnetic path length (mm)
g	air gap length (mm)
MLT	mean length of a turn (mm)
	number of turns
	wire copper diameter (mm)
s	wire isolation thickness (mm)
W_{au}	usable winding window area (mm ²)
hb, wb	bobbin height and width (mm)

selected. A series of inductors was then constructed with the same number of turns but using every AWG wire size that respected the limits on the fill factor. When the number of possible designs was exhausted, N was incremented to 2 and then the entire procedure was repeated, and so on, until $N = 50$. For each core in the catalogue, this then created the entire set of feasible designs that could be constructed with AWG solid copper wires with up to 50 turns.

At the first attempt this generated a very large number of very similar inductor designs. In order to reduce the amount of designs to simulate, designs using adjacent AWG wire sizes were discarded (for example, designs using AWG10, AWG12, and AWG14 were selected for simulation, but not those with AWG11 and AWG13). This was acceptable since the difference between adjacent AWG wire dimeters is only about 11%. Similarly, an additional turn produces a much larger change inductance when added to a low number of turns than when added to a large number of turns. For this reason, the number of turns was incremented by 1 until $N = 10$, then by 2 until $N = 20$, and by 5 thereafter. This finally produced a total of 880 inductor designs, with inductance L ranging from 1 μ H to 4 mH. The generated designs therefore spanned the entire set of applications appropriate to this catalogue of cores.

For all simulations, an ambient temperature of 25°C was assumed. Each inductor was simulated using triangular current waveforms, such as those shown in Fig. 6, with 50% duty ratio and different average current I_{DC} and peak-to-peak current ripple Δi_L . For each inductor, I_{DC} was selected to be 0.1, 0.3, 0.6, and 0.8 of the $I_{pk,max}$ for that inductor. Δi_L was then selected to be 0.1, 0.2, 0.4, 0.6, and 0.9 of the selected values of I_{DC} . Waveform frequencies f_s were selected to be 20, 50, 100, and 200 kHz, as that was the range within which the GeckoMAGNETICS tool contained accurate core loss data. Taking together all of the generated inductors, this produced 63360 valid simulations. For each simulated inductor, besides the geometry characteristics shown in Fig. 9 and the previously detailed waveform characteristics, the peak flux density B_{pk} , total losses P_{Total} , core temperature T_{core} , winding temperature T_{wind} , and boxed volume $V_{ol_{box}}$ were recorded. The simulations took about 15 hours to complete on a low to mid-range laptop computer using a single thread of execution.

C. Posynomial Fitting

Following the approach of [3], data was divided into four partitions based on waveform frequency f_s . Each partition had the same frequency value. The successive fitting method was used to produce a four-term posynomial for the total losses P_{total} (in mW) and the core and winding temperature T_{core} and T_{wind} , respectively (in °C):

4

$$P_{Total} = (e a_{lp} A e_{k_{lp1}} l e_{k_{lp2}} W a u_{k_{lp3}} d_{k_{lp4}} s_{k_{lp5}} M L T_{k_{lp6}} N_{k_{lp7}} I_{p=1} \cdot g_{k_{lp8}} f_s k_{lp9} \Delta i_L k_{lp10} I_{DC} k_{lp11}) \quad (10)$$

$$T_{core} = (e_{alc} A_{eklc1} l_{eklc2} W_{auklc3} d_{klc4} S_{klc5} M L T_{klc6} N_{klc7} l_c = 1 \\ \cdot g_{klc8} f_{sklc9} \Delta i_{Lklc10} I_{DCklc11}) \quad (11)$$

$$T_{wind} = (e_{alw} A_{eklw1} l_{eklw2} W_{auklw3} d_{klw4} S_{klw5} M L T_{klw6} l_w = 1 \\ \cdot N_{klw7} g_{klw8} f_{sklw9} \Delta i_{Lklw10} I_{DCklw11}) \quad (12)$$

The exponent values for (10) - (12) are shown in Table V.

However, for the inductance L (μH), peak flux density B_{pk} (mT), and boxed volume $V_{ol_{box}}$ (cm^3) it was sufficient to fit just a monomial function to the entire data set:

$$L = e^{-5.82} A_e^{0.839} l_e^{0.062} N^2 g^{-0.722} \quad (13)$$

$$B_{pk} = e^{1.083} l_e^{-0.269} N^{0.997} g^{-0.785} \Delta i_L^{1.0026} I_{DC}^{-0.00486}$$

(14)

$$V_{ol_{box}} = e^{-22.42} A_e^{-3.08} l_e^{2.14} W_{au}^{-1.28} d^{0.528} S^{-0.0156} \\ \cdot M L T^{8.97} N^{0.243} g^{-0.00294}. \quad (15)$$

L must be a monomial in order to be included into a GP as an equality constraint. Since inductance is physically a function of the number of turns and the core geometry, a monomial function of A_e , l_e , g , and N can be fitted with high accuracy. Similarly, $V_{ol_{box}}$ is also a function only of the inductor's dimensions, so the variables related to the applied waveform can be omitted. The form of the function for B_{pk} was determined by trial and error based on the intuition that it should depend only on the inductor geometry and the peak current.

The fitting was performed using least-squares functions in MATLAB and took about 30 minutes to execute on the same computer which was used to produce the simulation data. As can be seen in Table VI, the produced posynomial functions fit quite well to the simulation data. The highest error, but still less than 10%, is present when fitting P_{tot} , which is understandable taking into consideration the complexity of accurate inductor loss modeling.

D. Alternative Partitioning Approaches

In addition to the approach of the previous subsection, different partitioning schemes were considered. First, the frequency-sorted data was divided into two partitions, with each partition containing two adjacent values of frequency. Then the data was sorted according to B_{pk} ascending, and divided into four partitions. The data contained nine different values of A_e , so three core geometry-based partitions were created, each containing three adjacent values of A_e . Finally, the successive fitting approach of Fig. 4 was discarded and a posynomial with 2, 3, and 4 terms was fitted directly to the simulation data without any partitioning (the data was sorted by f_s ascending). The results of the different approaches are compared in Table VII using the fitting error for the total losses. The fitting errors for T_{core} and T_{wind} follow broadly the same trend. The sorting of the data did not have any effect on the fitting results of the monomial functions, as identical outcomes were achieved for them in each case.

Interestingly, the 4-term B_{pk} partition produces an identical error as the 4-term f_s partition and the 4-term non-partitioned data, although with different exponent values. The same relationship holds between the 2-term f_s partition and 2-term nonpartitioned results, as well as between the 3-term A_e partition and the 3-term non-partitioned results. This suggests that the number of terms, and not the sorting and the partitioning of the data, is most important, with a higher number of terms producing a better fit.

E. Optimization Example

To demonstrate the use of the derived posynomial models, an inductor with $L = 100 \mu\text{H}$, operating at $f_s = 36 \text{ kHz}$, $\Delta i_L =$

TABLE V

EXPONENT VALUES FOR P_{Total} , T_{core} , AND T_{wind} FITS

	P_{Total}	T_{core}	T_{wind}
α_1	-25.9732	248.0399	27.0296

k_{11}	-2.9014	66.7886	7.7489
k_{12}	2.1662	-12.7809	-1.9070
k_{13}	-1.7791	26.8207	3.3015
k_{14}	0.5600	-0.3203	-0.7791
k_{15}	0.3944	0.6059	1.3365
k_{16}	7.2951	-	-19.3796
		174.6128	
k_{17}	2.5932	11.7956	1.2532
k_{18}	0.2473	-9.0848	-0.9883
k_{19}	0.6617	0.9039	0.6758
k_{110}	2.0082	3.4054	1.1878
k_{111}	-0.0435	8.3293	0.1596
a_2	171.9199	12.1514	-6.3509
k_{21}	52.8727	5.5936	2.5028
k_{22}	-6.8350	-1.3211	2.1783
k_{23}	20.8084	2.2491	-0.0768
k_{24}	-0.0674	-0.1623	1.3027
k_{25}	-0.0355	0.1870	-2.0040
k_{26}	-138.7475	-13.4502	-8.0003
k_{27}	11.9716	1.3202	2.0799
k_{28}	-9.1608	-1.1549	0.2497
k_{29}	1.0854	0.7037	0.4624
k_{210}	3.5435	1.3332	1.4654
k_{211}	8.3084	0.0262	0.0447
a_3	-10.3364	19.6335	2.4875
k_{31}	3.4004	8.6238	-0.1779
k_{32}	-0.0813	1.0635	0.0316
k_{33}	0.8497	1.8095	-0.0751
k_{34}	-0.0999	2.3180	0.0072
k_{35}	0.2385	-2.9849	-0.0039
k_{36}	-6.0294	-23.6927	0.4638
k_{37}	2.2013	2.0271	-0.0164
k_{38}	-1.7766	0.4142	0.0083
k_{39}	1.0828	0.3862	-0.0063
k_{310}	2.2052	1.0735	-0.0121
k_{311}	0.0031	0.2321	-0.0049
a_4	-6.5900	2.7913	16.1347
k_{41}	-0.8898	-0.1711	3.7509
k_{42}	-0.6872	0.0319	-0.6237
k_{43}	-0.3636	-0.0751	1.4514
k_{44}	-1.4864	-0.0105	-1.6475
k_{45}	-0.2330	0.0072	-0.7248
k_{46}	3.6423	0.4345	-10.2363
k_{47}	1.1712	-0.0359	0.5925
k_{48}	0.0047	0.0283	0.0505
k_{49}	0.0125	-0.0186	-0.1289
k_{410}	0.0615	-0.0325	-0.1782
k_4	1.9857	0.0020	1.8727

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TABLE VI
ERROR OF FITTED POSYNOMIAL FUNCTIONS WITH RESPECT TO THE

Function	SIMULATION DATA	
	Average Error	R_2
Total losses P_{Total}	9.37%	0.8661
Core temperature T_{core}	2.34%	0.9507
T_{wind}	1.66%	0.978
Winding temperature	0.62%	0.9998
Inductance		
Peak flux density		
B_{pk}	3.5%	0.9965
Boxed volume $V_{ol_{box}}$	4.5%	0.9965

1.67 A, and $I_{DC} = 3$ A was optimized for losses and volume. Core and winding temperature were limited to be at most 100°C. As in [5], the two conflicting objectives were weighted by a factor γ and added into a single objective function. Since loss and volume are calculated in different units, they were normalized to a range between 0 and 1. To achieve this,

TABLE VII

ERRORS OF DIFFERENT PARTITIONING APPROACHES (P_{Total})

Partitioned by	# Terms	Average Error	R_2
f_s	4	9.37%	0.8661
f_s	2	17.78%	0.7571
B_{pk}	4	9.37%	0.8661
A_e	3	10.25%	0.8588
None	2	17.78%	0.7571
None	3	10.25%	0.8588
None	4	9.37%	0.8661

first the volume was minimized in a single-objective problem, and due to the inverse relationship of the volume and losses of Pareto-optimal solutions [16], the losses of the minimum volume design, $P_{tot,max}$, were used to normalize the losses in the multi-objective problem. Similarly, loss was minimized in a single-objective problem as well, and the volume of the minimum-loss solution, $Vol_{box,max}$, was used to normalize the volume in the multi-objective problem. This gave the final multi-objective GP formulation:

$$\begin{aligned} &\text{minimize} \quad \gamma \frac{P_{Total}}{P_{tot,max}} + \frac{(1-\gamma)Vol_{box}}{Vol_{box,max}} \\ &\text{subject to} \quad L = e^{-5.82A_e 0.839l_e 0.062N^2g - 0.722} \end{aligned} \quad (16)$$

$$T_{core} \leq 100^\circ C$$

$$T_{wind} \leq 100^\circ C$$

$$B_{pk} \leq B_{sat} A_{e,min} \leq A_e \leq A_{e,max} l_{e,min} \leq l_e \leq l_{e,max}$$

$$W_{au,min} \leq W_{au} \leq W_{au,max} d_{min} \leq d \leq d_{max} S_{min} \leq s \leq S_{max} MLT_{min} \leq MLT \leq MLT_{max} N_{min} \leq N \leq N_{max} g_{min} \leq$$

$$g \leq g_{max}$$

$$d \geq 2(((I_{DC} + 0.5\Delta I_L)/J_{max})/\pi)^{0.5}$$

$$0.1 \leq N\pi((0.5d)^2)/W_{au} \leq 0.6 \quad MLT = 5.793A_e^{0.4723} l_e = 7.196(W_{au}^{0.5184})$$

$A_e = 0.0006963(l_e^{2.683})$, where the design variables – the geometric parameters – were constrained to be within the minimum and maximum bounds found in the generated simulation data. Additional constraints were added to keep the current density and fill factor of the optimized designs within the limits of the simulated data as well. This however was not sufficient to force the GP to produce inductor designs using the desired core catalogue, as in the absence of further constraints, A_e , l_e , W_{au} , and MLT are treated as independent variables by the GP solver, potentially producing unrealizable core geometries.

Fig. 10. The resulting Pareto-optimal set of inductors for the example optimization problem.

TABLE VIII

REALIZABLE OPTIMUM INDUCTORS

γ	0.0	0.5	1.0
Core	E20	E32	E55
Air gap (mm)	0.5	0.5	0.5
Wire	AWG18	AWG14	AWG10
N	28	19	10

Since the winding is wound around the centre core leg in this case, MLT depends on A_e , which is defined by the centre leg. Similarly, l_e depends on the size of W_{au} . Therefore two monomial expressions, giving MLT as a function of A_e , and l_e as a function of W_{au} , were derived by fitting to the simulation data, and added to the GP as equality constraints. A third monomial constraint relating A_e to l_e was derived in the same manner in order to force the GP to produce cores with roughly the same proportions as those available in the actual core catalogue.

The GP (16) was solved in MATLAB using CVX [17] and the SeDuMi solver. 21 GPs total were solved, with each corresponding to a value of γ ranging from 0 to 1, incremented by 0.05. Execution time was under one minute. The resulting loss-volume Pareto front is shown in Fig. 10. Three points are indicated on the Pareto front, $\gamma = 0$, corresponding to the most compact (minimum volume) design, $\gamma = 1$, corresponding to the most efficient (minimum loss) design, and $\gamma = 0.5$, corresponding to the halfway compromise between the two. Based on the characteristics of these designs, realizable inductors, shown in Table VIII, were built using the cores and wires of the initially simulated catalogue, and then re-simulated in GeckoMAGNETICS. The results of those simulations are also shown in Fig. 10. Note that the plotted values for the realizable designs are directly from GeckoMAGNETICS simulations, and not from the posynomial models, therefore the difference between them and the optimized curve includes the model error from Table VI. The total losses and boxed volumes given by the GP results, the realizable inductors evaluated using the fitted posynomial functions, and the GeckoMAGNETICS simulations are compared in Table IX. It can be seen that for $\gamma = 0.5$ and $\gamma = 1.0$, the realizable designs are extremely close to the theoretical optima resulting from the GP. For $\gamma = 0$, the volume of the realizable design is very close to the theoretical one, which is most important in this case, as this is the minimum volume design. However, there is a large discrepancy in the losses, with the losses of the realizable design being significantly lower than that of the theoretical design. As can be seen in Table IX, this large discrepancy shows up between the posynomial loss model's calculation and the GeckoMAGNETICS simulation. This is due to a larger fitting error in the area of the data around the selected core. The error reported in Table VI is an average value and there are sections of the data where it is both significantly larger and smaller than the average.

TABLE IX

COMPARISON OF OPTIMIZED INDUCTOR DESIGNS: GP SOLUTIONS VS. REALIZABLE INDUCTORS

γ	0.0		0.5		1.0	
	P_{Total} (mW)	Vol_{box} (cm ³)	P_{Total} (mW)	Vol_{box} (cm ³)	P_{Total} (mW)	Vol_{box} (cm ³)
GP result (ideal inductor)	1328	6.522	326	17.924	167	89.38
Realizable inductor, posynomial function evaluation	1154	5.508	338	19.943	185	91.48
Realizable inductor, GeckoMAGNETICS simulation	619	5.807	324	19.321	143	87.93

F. Future Improvements

As noted, the error distribution of the fitted functions is not uniform throughout the data set. Therefore, it would be desirable to add a third metric evaluating the goodness of the fit with respect to the error distribution, and to compare the different fits in Table VII accordingly. Also, all of the presented models were derived from simulations using a fixed ambient temperature, and as a next step the ambient temperature should be added to the posynomial functions for

P_{Total} , T_{core} , and T_{wind} .

Finally, as noted, all of the simulations were performed using waveforms with a 50% duty ratio. Preliminary investigations performed on the results of the optimization example show that changing the duty ratio changes the loss simulation results noticeably but not significantly. It would therefore be necessary to rigorously quantify the effect of the duty ratio and determine whether is necessary to include it as a variable in the above-mentioned posynomial functions.

IV. CONCLUSIONS

Geometric programming, which operates on functions in posynomial form, is a powerful framework for the multiobjective optimization of power electronics. Previous studies, while showing that converters can be modeled as GPs, have not produced “portable” models of inductors, meaning that a re-derivation of posynomial inductor models was required for each new optimization problem. In this paper, an approach has been presented to mitigate this shortcoming, allowing comprehensive posynomial models of inductors to be generated once from simulation data and re-used in different applications.

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