DIFFERENTIAL FORMS ON STRATIFIED SPACES II

Serap Gürer, Patrick Iglesias-Zemmour

► **To cite this version:**

Serap Gürer, Patrick Iglesias-Zemmour. DIFFERENTIAL FORMS ON STRATIFIED SPACES II. Bulletin of the Australian Mathematical Society, 2018. hal-02401460

HAL Id: hal-02401460

<https://hal.science/hal-02401460>

Submitted on 10 Dec 2019

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

DIFFERENTIAL FORMS ON STRATIFIED SPACES II

SERAP GÜNER AND PATRICK IGLESIAS-ZEMMOUR

ABSTRACT. We prove that, for a conelike stratified diffeological spaces, a zero-
perverse form is the restriction of a global differential form if and only if its index
is equal to 1 for every stratum.

INTRODUCTION

In the previous paper “Differential Forms On Stratified Spaces” [GIZ18], for each zero-
perverse form defined on the regular part of a diffeological stratified space, we
introduced an index for every stratum, counting the number of different forms gen-
erated by the zero-
perverse form around the stratum. We showed that under some
natural condition on the stratification — if two points could be connected by a path
cutting the singular subset in a finite number of points — the zero-
perverse form is the restriction of a differential form, defined globally on the space, if and only if the
index is equal to 1 for every stratum.

In this paper, we prove this statement for locally conelike diffeological stratified spaces,
without accessory conditions.

THANKS. — We are thankful to the referee who, by his relevant remarks, allowed us
to improve the readability and the content of the paper.

LOCALLY CONELIKE DIFFEOLOGICAL STRATIFIED SPACES

We suggest Benoît Kloeckner’s survey [Klo07] for the classical topological approach
of stratified spaces. About diffeology, we refer to the textbook [PIZ13], and about
conelike stratified spaces, Larry Siebenmann’s paper [Sie72].

Date: December 22, 2018.

1991 Mathematics Subject Classification. 58A35, 58A10.

Key words and phrases. Stratification, Diffeology, Differential Forms.

This research is partially supported by Tübitak, Career Grant No. 115F410, and the French-Turkish
Research Fellowships Program, Embassy of France in Turkey 2017. The authors thank the «Espace
Forbin» of the Faculty of Economy from Aix-Marseille University, and the people there, for their con-
stant hospitality.

Let us begin by specifying what we understand by *Stratified Diffeological Spaces*. Note that the topology of a diffeological space we referring to in the following is its natural D-Topology, defined in [PIZ13, §2.8].

1. STRATIFIED DIFFEOLOGICAL SPACES. A *stratification* of a diffeological space X by manifolds is a finite family of subspaces $\mathcal{S} = \{S_i\}_{i \in \mathcal{I}}$, called *strata*, such that:

- (1) The strata form a partition of X .
- (2) Each stratum is a manifold for the subset diffeology¹.
- (3) The strata satisfy the frontier condition:

$$S_i \cap \overline{S_j} \neq \emptyset \Rightarrow S_i \subset \overline{S_j}.$$

- (4) Let $m = \max\{\dim(S_i)\}_{i \in \mathcal{I}}$. Then,

$$X_{\text{reg}} = \bigcup_{i \in \mathcal{I} \mid \dim S_i = m} S_i$$

is an open dense subset, called the *regular part* of the stratification.

- (5) Let $\mathcal{I}'$ be the subset of indices $i \in \mathcal{I}$ such that $\dim(S_i) < m$. If $\mathcal{I}'$ is not empty, then let $\mathcal{S}' = \{S_i\}_{i \in \mathcal{I}'}$ and $X' = \bigcup_{i \in \mathcal{I}'} S_i$, equipped with the subset diffeology. Then $\mathcal{S}'$ is a stratification of X' .

The space X is assumed to be Hausdorff, metrisable and connected for the D-topology. The subset X' which is equal to $X - X_{\text{reg}}$ is called the *singular part* of the stratification, and can be sometimes denoted by X_{sing} . The elements of $\mathcal{S}'$ are called the singular strata. The first four axioms can be called the *structure axioms* and the fifth, the *recursion axiom*. For topological stratified spaces, the definition can be found in Pflaum survey [Pfl01]. By this recursion we get a filtration of diffeological subspaces:

$$X_0 \subset X_1 \subset \cdots \subset X_\ell \subset \cdots \subset X_{k-1} \subset X_k.$$

where $X_k = X$, $X_{k-1} = X'$ etc. Each term X_ℓ of the filtration is a stratified space with the according subset $\mathcal{S}_\ell$ of strata, and the main strata has some dimension n_ℓ . Each term being the singular part of the term above.

The next step consists in specializing the structure near the singular strata. We said that the stratification is *locally fibered* [GIZ18] — or the space is a *locally fibered stratified spaces* — if there exists a *tube system* $\{\pi_S: TS \rightarrow S\}_{S \in \mathcal{S}'}$ such that:

- (A) TS is an open neighborhood of S , called a *tube* over/around S .
- (B) The map $\pi_S: TS \rightarrow S$ is a smooth retraction which is a diffeological fibration.
- (C) For all $x \in TS \cap TS' \cap \pi_S^{-1}(TS)$, one has $\pi_S(\pi_{S'}(x)) = \pi_S(x)$.

The fiber $F_x = \pi_S^{-1}(x)$, with $x \in S$, is a diffeological space itself which is stratified inherited by the ambient stratification. Its intersection with the stratum is reduced to one point: $F_x \cap S = \{x\}$. We call it the *apex* of the fiber.

¹See [PIZ13, §4.1] for *manifolds as diffeologies*.

FIGURE 1. — A formal cone (left) and a geometric cone (right).

NOTE. — To be locally fibered is not *a priori* recursive. Do we need a natural recursion on the local fibration? This is a legitimate question, but we do not need this property for the case treated in this paper.

2. DIFFEOLOGICAL CONES. Let Σ be a diffeological space. We call a *cone over Σ* the quotient space $\text{Cone}(\Sigma) = \Sigma \times [0, \infty[/ \Sigma \times \{0\}$, equipped with the quotient diffeology. We recall that the quotient diffeology is the finest diffeology on the quotient that makes the projection smooth [PIZ13, §1.50]. Consider now a diffeological space F with a smooth bijection $\varphi: \text{Cone}(\Sigma) \rightarrow F$ such that $\varphi \upharpoonright [\text{Cone}(\Sigma) - \star]$ is a diffeomorphism onto its image, where $\star = \text{class}(\Sigma \times \{0\})$ is the *apex* of the cone. Thus, without their apex, these spaces are equivalent to $\Sigma \times]0, \infty[$. Then, F will be called a *diffeological cone*, or simply a *cone*, Σ is called the *base* of the cone. The cone F differs from the cone over Σ only by the germ of the diffeology at the apex.

$$\begin{array}{ccc}
 \Sigma \times [0, \infty[& & \\
 \text{class} \downarrow & \searrow \Phi & \\
 \text{Cone}(\Sigma) & \xrightarrow{\varphi} & F
 \end{array}$$

This consideration is necessary because diffeology discriminates between cones, as shows for example the positive cone in $\mathbf{R}^3$ defined by $x^2 + y^2 = z^2$ & $z \geq 0$, equipped with the subset diffeology, compared with the cone over S^1 , $\text{Cone}(S^1) = S^1 \times [0, \infty[/ S^1 \times \{0\}$ see [GIZ18, §3].

NOTE. — An equivocal situation arises with this definition, related to the different types of stratified spaces [GIZ18, Introduction]. We have called *geometric stratified spaces*, the stratified spaces for which the strata are the (connected components of the) orbits of the pseudogroup of local diffeomorphisms. The *formal stratified spaces* are the other ones. The same situation happens with diffeological cones when we compare the two examples sketched in Fig. 1 above: the hemisphere (left cone) defined by $x^2 + y^2 + (z-1)^2 = 1$ with $0 \leq z < 1$, and the embedded cone (right cone) defined by $x^2 + y^2 - z^2 = 0$, both equipped with the subset diffeology of $\mathbf{R}^3$. What differentiates these two cones is clearly the behavior of the apex $\star$. On the left cone it is interchangeable

with any other point. Precisely, the pseudogroup of local diffeomorphisms does not distinguish the apex from the other points. On the other hand, the apex of the right cone is alone in its orbit. That observation leads us to distinguish at least two classes of cones:

DEFINITION *We shall say that a cone is a geometric cone if its apex is alone in its orbit by the pseudogroup of local diffeomorphisms. Otherwise, we shall say that it is a formal cone.*

This remark has a real content, related to the idea about what is a cone. Intuitively, the apex of a cone should be distinguished. In diffeology it is the role of local diffeomorphisms to discriminate between points.

3. DIFFERENTIAL FORMS ON CONELIKE STRATIFIED DIFFEOLOGICAL SPACES. We have introduced the general notion of locally fibered stratified space in [GIZ18]. They are diffeological spaces, together with a stratification, such that the space looks like a diffeological fiber bundle [PIZ13, §8.8, 8.9] on the neighborhood of each stratum. This is *a priori* a two step generalization of the standard situation in the classical theory of stratified spaces, where the fiber is assumed to be conelike over a stratified base, as it is stated in [Sie72]. Hence, we have a diffeological intermediary to consider, the analogue of the usual conelike stratified spaces:

DEFINITION — *We call a Locally Conelike Diffeological Stratified Spaces any locally fibered stratified diffeological space with fibers some diffeological cones.*

Note first that, in a locally conelike diffeological stratified space, the bases of the cones are stratified spaces as well. Secondly, the apex of the fibers is their intersection with the stratum.

Then, let $\alpha \in \Omega_0^k[X]$ be a zero-perverse form defined on the regular part X_{reg} of a diffeological stratified space X , and let v the index function on zero-perverse forms, defined in [GIZ18, §5].

PROPOSITION — *If X is a locally conelike diffeological stratified space, then there exists a (unique) differential form $\underline{\alpha} \in \Omega^k(X)$ such that $\alpha = \underline{\alpha} \upharpoonright X_{\text{reg}}$, if and only if $v_S(\alpha) = 1$ for all strata S .*

NOTE 1. Actually, the proposition applies to every stratum which is in the closure of the regular part. But since, by definition, X_{reg} is dense in X then the property applies for all strata.

NOTE 2. The proof of the proposition could be easily extended to the case of locally fibered diffeological stratified spaces with contractible fibers, maybe with some condition on the retraction with respect to the connected components of the regular part. But that would be a minor generalisation.

Proof. If $v_S(\alpha) = 1$ for all $S \in \mathcal{S}$, then there exists a (unique) differential form $\underline{\alpha} \in \Omega^k(X)$ such that $\alpha = \underline{\alpha} \upharpoonright X_{\text{reg}}$. That has been proven in a general context in [GIZ18].

Let then $\underline{\alpha} \in \Omega^k(X)$. For all strata S , let $\pi_S: TS \rightarrow S$ be the retraction of the tube around a singular stratum S , which is by hypothesis a local fibration with fiber a cone F with base Σ . Recall that $S \subset TS$ and for all $x \in S$, $\pi_S(x) = x$.

Let $\alpha = \underline{\alpha} \upharpoonright X_{\text{reg}}$, and assume that α is zero-perverse with respect to these local structures. To follow the construction of the index ν we consider the pullback of the tube TS by the projection of the universal covering $\text{pr}: \tilde{S} \rightarrow S$, even if it is not completely necessary, since our considerations will be local. Let us denote by $T\hat{S}$ the pullback $\text{pr}^*(TS)$, that is,

$$T\hat{S} = \{(\tilde{x}, y) \in \tilde{S} \times TS \mid \text{pr}(\tilde{x}) = \pi_S(y)\}.$$

This pullback is actually a tube around the embedding $\hat{S}$ of the universal covering $\tilde{S}$ by the smooth section $\sigma: \tilde{x} \rightarrow (\tilde{x}, \text{pr}(\tilde{x}))$.

$$\hat{S} = \{(\tilde{x}, \text{pr}(\tilde{x})) \mid \tilde{x} \in \tilde{S}\} \subset T\hat{S},$$

which justifies the notation. Indeed, since $\text{pr}(\tilde{x}) \in S$, $\text{pr}(\tilde{x}) = \pi_S(\text{pr}(\tilde{x}))$ which is the condition for $(\tilde{x}, \text{pr}(\tilde{x}))$ to belong to $T\hat{S}$, and obviously $\text{pr}_1 \circ \sigma = 1_{\tilde{S}}$. Now,

$$\pi_{\hat{S}}: (\tilde{x}, y) \rightarrow (\tilde{x}, \pi_S(y)),$$

is a retraction from $T\hat{S}$ to $\hat{S}$. The tube $T\hat{S}$ is also a fibration with the same fiber F , it is the deployment of the tube TS along the embedded universal covering $\hat{S}$.

Hence, $T\hat{S}$ is everywhere locally diffeomorphic to a product $\mathcal{O} \times F$, where $\mathcal{O} \subset \hat{S}$ is some open subset, and the projection $\pi_{\hat{S}}: (\tilde{x}, y) \rightarrow (\tilde{x}, \pi_S(y))$ locally equivalent to the first projection $\text{pr}_1: \mathcal{O} \times F \rightarrow \mathcal{O}$. That means that there exists a local diffeomorphism ψ , defined on $\mathcal{O} \times F$, into $T\hat{S}$ such that $\pi_{\hat{S}} \circ \psi = \text{pr}_1$. We choose always $\mathcal{O}$ to be connected and simply connected. Now, the regular part of $\pi_{\hat{S}}^{-1}(\mathcal{O})$ is the image by ψ of the regular part of the product $\mathcal{O} \times F$, that is, $\mathcal{O} \times F_{\text{reg}}$. But F_{reg} is naturally identified to $]0, \infty[\times \Sigma_{\text{reg}}$, by construction. Therefore we have the following chain of square commutative diagrams.

$$\begin{array}{ccccccc}
 \mathcal{O} \times \Sigma_{\text{reg}} \times]0, \infty[& \xlongequal{\quad} & \mathcal{O} \times F_{\text{reg}} & \hookrightarrow & \mathcal{O} \times F & \xrightarrow{\psi} & T\hat{S} \xrightarrow{\text{pr}_2} TS \\
 \downarrow \text{pr}_1 & & \downarrow \text{pr}_1 & & \downarrow \pi_{\star} & & \downarrow \pi_{\hat{S}} \\
 \mathcal{O} & \xrightarrow{1_{\mathcal{O}}} & \mathcal{O} & \xrightarrow{1_{\mathcal{O}}} & \mathcal{O} \times \{\star\} & \xrightarrow{\psi|_{\mathcal{O} \times \{\star\}}} & \hat{S} \xrightarrow{\text{pr}_2} S \\
 & & & & \downarrow \text{pr}_1 & & \downarrow \text{pr}_1 \\
 & & & & \mathcal{O} & \hookrightarrow & \tilde{S} \xrightarrow{\text{pr}} S \\
 & & & & & & \parallel \\
 & & & & & & S
 \end{array}$$

We need to make a remark here about the arrow $\psi|_{\mathcal{O} \times \{\star\}}: \mathcal{O} \times \{\star\} \rightarrow \hat{S}$ that associates $(\tilde{x}, \star)$ with $(\tilde{x}, x = \text{pr}(\tilde{x}))$. For geometric cones, this arrow exists by construction since the apex is alone in its orbit by the pseudogroup of local diffeomorphisms.

FIGURE 2. — The smashing function λ .

For formal cones only, the local triviality should be understood in the category of stratified spaces. Let us now introduce the following notations:

$$\hat{\alpha} = \text{pr}_2^*(\underline{\alpha}) \in \Omega^k(\text{T}\hat{\mathbb{S}}), \quad \text{and} \quad \tilde{\alpha} = \hat{\alpha} \upharpoonright \text{T}\hat{\mathbb{S}}_{\text{reg}} = \text{pr}_2^*(\alpha) \in \Omega^k(\text{T}\hat{\mathbb{S}}_{\text{reg}}).$$

Next, we decompose F_{reg} in terms of connected components and have:

$$\mathcal{O} \times F_{\text{reg}} = \mathcal{O} \times \coprod_a F_{\text{reg}}^a = \mathcal{O} \times \coprod_a \Sigma_{\text{reg}}^a \times]0, \infty[\quad \text{with} \quad a \in \pi_0(\Sigma_{\text{reg}}).$$

And we get a two-squares diagram summarizing the situation we will deal with:

$$\begin{array}{ccccc} \mathcal{O} \times \coprod_a \Sigma_{\text{reg}}^a \times]0, \infty[& \hookrightarrow & \mathcal{O} \times F & \xrightarrow{\psi} & \text{T}\hat{\mathbb{S}} \\ \text{pr}_1 \downarrow & & \downarrow \text{pr}_1 & & \downarrow \text{pr}_1 \\ \mathcal{O} & \xrightarrow{1_{\mathcal{O}}} & \mathcal{O} & \hookrightarrow & \tilde{\mathbb{S}} \end{array}$$

Now, consider a n -plot P in $\tilde{\mathbb{S}}$ defined on some open set U , and $r_0 \in U$. There exists an open neighborhood $V \subset U$ of r_0 such that $Q = P \upharpoonright V$ takes its values in some trivialization open set $\mathcal{O}$, which we assume to be connected and simply connected. Now let $y \in F_{\text{reg}} = \coprod_a \Sigma_{\text{reg}}^a \times]0, \infty[$, then $y = \Phi(z, t)$ for some $z \in \Sigma_{\text{reg}}^a$ and $t \in]0, \infty[$, where Φ has been introduced in (art. 2). Consider then the (stationary) smooth path

$$\gamma: s \mapsto \Phi(z, \lambda(s)t),$$

where λ is the smashing function described by Fig. 2.

Now:

- (1) For all $s \in]-\infty, \varepsilon]$, $\gamma(s) = \star$ is the apex of F .
- (2) For all $s \in]\varepsilon, \infty[$, $\gamma(s) \in F_{\text{reg}}^a$.
- (3) In particular $\gamma(0) = \star$ and $\gamma(1) = y$.

Next, let $Q \times \gamma$ be the plot $(r, s) \mapsto (Q(r), \gamma(s))$, into $\mathcal{O} \times F$, when (r, s) runs over $V \times \mathbf{R}$. Let us fix $(u_i)_{i=1}^k$ with $u_i \in \mathbf{R}^n$, and $(\tau_i)_{i=1}^k$ with $\tau_i \in \mathbf{R}$. We consider then the real smooth function defined on $V \times \mathbf{R}$,

$$f(r, s) = \hat{\alpha}(\psi \circ (Q \times \gamma))_{(r, s)}(u_i, \tau_i)_{i=1}^k$$

Indeed: $\psi \circ (Q \times \gamma)$ is a plot in $\mathcal{O} \times Y$, ψ is a local diffeomorphism into $T\hat{S}$, $\hat{\alpha}$ is a k -form on $T\hat{S}$, and the $(u_i, \tau_i)_{i=1}^k$ have been fixed.

(1) Consider firstly $s \in]-\infty, \varepsilon[$, thus $\gamma(s) = \star$. Let us denote $\tilde{x}_r = Q(r)$, then $\psi(Q(r), \gamma(s)) = \psi(\tilde{x}_r, \star) = (\tilde{x}_r, x_r)$, with $x_r = \text{pr}(\tilde{x}_r)$. Hence, we have:

$$\begin{aligned} f(r, s) &= \hat{\alpha}((r, s) \mapsto \psi(\tilde{x}_r, \star))_{(r, s)}(u_i, \tau_i)_{i=1}^k \\ f(r, s) &= \hat{\alpha}((r, s) \mapsto (\tilde{x}_r, x_r))_{(r, s)}(u_i, \tau_i)_{i=1}^k \\ &= \hat{\alpha}((r, s) \mapsto r \mapsto (\tilde{x}_r, x_r))_{(r, s)}(u_i, \tau_i)_{i=1}^k \\ &= \text{pr}_1^*(\hat{\alpha}(r \mapsto (\tilde{x}_r, x_r)))_{(r, s)}(u_i, \tau_i)_{i=1}^k \\ &= \hat{\alpha}(r \mapsto (\tilde{x}_r, x_r))_r(u_i)_{i=1}^k \\ &= \underline{\alpha}(r \mapsto x_r)_r(u_i)_{i=1}^k, \text{ because } \hat{\alpha} = \text{pr}_2^*(\underline{\alpha}) \\ &= \underline{\alpha}(\text{pr} \circ Q)_r(u_i)_{i=1}^k. \end{aligned}$$

Therefore

$$f \upharpoonright V \times]-\infty, \varepsilon[= \text{pr}^*(\underline{\alpha})(Q)_r(u_i)_{i=1}^k.$$

(2) Consider secondly $s \in]\varepsilon, +\infty[$. Then, $\gamma(s) \in F_{\text{reg}}^a$, $Q \times \gamma$ is a plot in $\mathcal{O} \times F_{\text{reg}}^a$ and $\psi \circ (Q \times \gamma)$ is a plot of $\{T\hat{S}\}_a$, the connected component of $T\hat{S} = \text{pr}^*(TS)$ associated with the component $F_{\text{reg}}^a = \Sigma_{\text{reg}}^a \times]0, \infty[$, see [GIZ18, §5, Step 1]. Hence, according to [GIZ18, (♠)], $\hat{\alpha} \upharpoonright \{T\hat{S}\}_a = \text{pr}_1^*(\bar{\alpha}_a)$, where $\bar{\alpha}_a \in \Omega^k(\tilde{S})$. Thus,

$$\begin{aligned} f(r, s) &= \hat{\alpha}((r, s) \mapsto \psi(\tilde{x}_r, \gamma(s)))_{(r, s)}(u_i, \tau_i)_{i=1}^k \\ &= \text{pr}_1^*(\bar{\alpha}_a)((r, s) \mapsto \psi(\tilde{x}_r, \gamma(s)))_{(r, s)}(u_i, \tau_i)_{i=1}^k \\ &= \bar{\alpha}_a(\text{pr}_1 \circ [(r, s) \mapsto \psi(\tilde{x}_r, \gamma(s))])_{(r, s)}(u_i, \tau_i)_{i=1}^k \\ &= \bar{\alpha}_a((r, s) \mapsto \text{pr}_1 \circ \psi(\tilde{x}_r, \gamma(s)))_{(r, s)}(u_i, \tau_i)_{i=1}^k \\ &= \bar{\alpha}_a((r, s) \mapsto \tilde{x}_r)_{(r, s)}(u_i, \tau_i)_{i=1}^k, \text{ since } \text{pr}_1 \circ \psi = \text{pr}_1 \\ &= \bar{\alpha}_a(r \mapsto \tilde{x}_r)_r(u_i)_{i=1}^k. \end{aligned}$$

Therefore,

$$f \upharpoonright V \times]\varepsilon, \infty[= \bar{\alpha}_a(Q)_r(u_i)_{i=1}^k.$$

(3) In conclusion — For all $r \in \mathcal{O}$ $f_r: s \mapsto f(r, s)$ is a smooth function defined on $\mathbf{R}$. On the interval $]-\infty, \varepsilon[$, f_r is constant and equal to $\text{pr}^*(\underline{\alpha})(Q)_r(u_i)_{i=1}^k$, and on $]\varepsilon, +\infty[$, f_r is constant and equal to $\bar{\alpha}_a(Q)_r(u_i)_{i=1}^k$. Since f_r is smooth, then

continuous, these two constants must be equal. Thus, since that is true locally for any r and any vectors u_i in $\mathbf{R}^n$, we get

$$\bar{\alpha}_a = \text{pr}^*(\underline{\alpha} \upharpoonright S),$$

for all indices a . That is, $v(S) = 1$. Now, that applies to every stratum which is in the closure of X_{reg} . Since by definition X_{reg} is dense in X , $v(S) = 1$ for all strata. $\square$

REFERENCES

- [GIZ18] Serap Güler and Patrick Iglesias-Zemmour. *Differential Forms On Stratified Spaces*. To appear in the Bulletin of the Australian Mathematical Society
<https://doi.org/10.1017/S0004972718000436>
- [PIZ13] Patrick Iglesias-Zemmour *Diffeology*. Mathematical Surveys and Monographs. The American Mathematical Society, vol. 185, USA R.I. 2013.
- [Klo07] Benoît Kloeckner. *Quelques Notions d'Espaces Stratifiés*. Séminaire de Théorie Spectrale et Géométrie. Vol. 26, pp. 13–28, Grenoble 2007-08.
- [Pfl01] Markus Pflaum. *Analytic and geometric study of stratified spaces*. Lecture notes in Mathematics, vol.1768, Springer verlag, 2001.
- [Sie72] Larry Siebenmann. *Deformation of Homeomorphisms on Stratified Sets*. Comment. Math. Helv. vol. 47, pp. 123–163, 1972.

SERAP GÜRER — GALATASARAY UNIVERSITY, ORTAKÖY, ÇIRAĞAN CD. NO:36, 34349 BEŞİKTAŞ / İSTANBUL, TURKEY.

E-mail address: `sgurer@gsu.edu.tr`

PATRICK IGLESIAS-ZEMMOUR — AIX MARSEILLE UNIV, CNRS, CENTRALE MARSEILLE, I2M, MARSEILLE, FRANCE — & — EINSTEIN INSTITUTE OF MATHEMATICS EDMOND J. SAFRA CAMPUS, THE HEBREW UNIVERSITY OF JERUSALEM GIVAT RAM. JERUSALEM, 9190401, ISRAEL.

E-mail address: `piz@math.huji.ac.il`