Introduction to the work of TWG 2: Arithmetic and number systems

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Introduction

Working Group 2 was formed in 2011 and has developed as a forum for discussing theoretical and empirical research on the teaching and learning of arithmetic and number systems. Our common work targets to gain and enhance knowledge about students' understanding and meaningful learning in this content area regarding different age and achievement levels. The scope of the TWG comprises kindergarten to 12th grade, and emphasizes research-based specifications of domain specific frameworks, concepts and goals, analysis of learning processes and learning outcomes in different classroom cultures, as well as innovative teaching and diagnostic approaches that balance procedural and conceptual knowledge.

Learning arithmetic and number systems from Kindergarten to High School is a very broad field with a great variety of themes and research approaches. This year, the group intensively discussed twenty papers and four posters that reflected the richness of the field. According to the path we pursued at CERME 10, we had the chance to broaden our discussion by regarding the same topics from different perspectives; cognitive phycologists and mathematics education researchers. In comparison to the TWG work of the last years, there was also a change regarding the research approaches. Whereas the last two CERME conferences were dominated by design based research, this year was characterized by a good balance of qualitative and quantitative approaches.

Overview on discussed papers and posters

The various presented and discussed papers can be pooled in four groups of themes: The role of manipulatives in learning processes in early arithmetic, learning and teaching numbers and operations in kindergarten to first grade, learning and teaching arithmetic in second to sixth grade, as well as different foci on learning rational numbers. Additionally, there was one paper on metacognition.

Role of manipulatives

Natividad Adamuz-Povedano, Elvira Fernández-Ahumada, Teresa García-Pérez and *Rafael Bracho-López* reported on an exploratory study of 25 students using manipulative materials to develop number sense understanding in 6 year olds by presenting two activities, jumping frogs and skip kangaroo. The findings illustrated how effective active behaviors from the students had enhanced.

Doris Jeannotte and *Claudia Corriveau* explored the use of manipulatives by children, in particular based-ten blocks and abacus, for solving arithmetical tasks. Considering the concept of affordance,

as related to interactions between an individual and the environment, they analyzed both, the use of manipulatives and the affordance in two grade 3 classes, as inseparable units of teaching and learning.

Numbers and operations in early arithmetic

Camila P. Nogues and *Beatriz V. Dorneles* explored deeper into the skills of number estimation and quantitative reasoning through the discussion of the relationship between these two important concepts of understanding. They found indeed a correlation between grades 3 and 4 students' performance in the quantitative reasoning task and number line estimation tasks.

Évelin Assis and *Luciana Vellinho Corso* presented an intervention study in Brazil which involved 136 first-grade students in the teaching and understanding of counting principles. A key product of this experiment was to analyze the efficiency of the intervention in order that it could be used in any first grade classroom.

Pernille Sunde and *Peter Sunde* investigated the development of mental strategies in single-digit addition in 83 first grade students. The used strategies were assessed by two interviews that were conducted in a period of five month. Surprisingly, quantitative analysis revealed no influence of instruction for strategy change.

Michael Gaidoschik discussed the "structure-genetic didactical analysis" paradigm, which emphasizes the need of children explore and internalize part-whole relations between numbers. He questioned the role of counting in early math classrooms, and offered a didactic alternative in line with the needs of the children as well as the subject matter.

Luciana V. Corso and Évelin Assis analyzed teacher's perceptions of their first grade students' profiles including students' behavior, attention, interaction and knowledge by area of mathematics writing and reading. The findings presented teachers perceptions related very closely to student achievement in a counting principle test, but only for students with a good performance.

Learning arithmetic in second to sixth grade

Joana Brocardo, Catarina Delgado, Fátima Mendes and *Jean M. Kraemer* presented a teaching experiment with grade 3 students in the domain of partitive division. It aimed to explore how students adapt personal knowledge to varying task conditions. Data analysis confirmed that variation influences students' strategies, and allowed to designate three critical aspect of the learning process.

Ems Lord and *Andreas J. Stylianides* presented results of a mixed methods study that focused on calculation flexibility and written algorithm on Year 6 students. Results revealed that formal algorithms were the most frequently selected strategy, and gender, prior attainment and confidence were all significant predictors of the use of formal algorithms.

Robert Gunnarson and *Ioannis Papadopoulos* analyzed written solutions for arithmetic expressions of Swedish and Greece students (grades 5 and 6) focusing on the order of operation (addition, subtraction and multiplication) and the precedence rules. Data showed that paring is as frequently

used as sequential calculations. Thereby, three qualitatively different types of pairing were identified.

An analysis of four popular German textbooks (grades 2 and 3) with a focus on developing and consolidation of understanding of multiplication was presented by *Sandra Gleißberg* and *Klaus-Peter Eichler*. Using their own conceptual framework based on the work of Bruner, the analysis showed that the vast majority of tasks require working on the non-verbal-symbolic level alone.

Different foci on learning rational numbers

Lalina Coulange and *Grégory Trains'* study aimed to understand the role of register of numeration units (tenths, hundredths) for teaching and learning decimal numbers at primary level in France. Their findings revealed students' difficulties in unit-conversions and in associating the numeration units with the decimal notation. A new perspective on teaching is suggested to prevent those difficulties.

Ioannis Papadopoulos, Styliani Panagiotopoulou and *Michail Karakostas* investigated strategies used by primary, secondary and tertiary education students to calculate mentally operations involving rational numbers. The findings revealed the mental form of the written algorithm as dominant strategy across all educational levels.

Pernille L. Pedersen and Peter Sunde investigated the relationship between 99 fourth-grade students' ability to compare fractions and to solve tasks all four operations. Data analyzes showed that comparative rational number tasks are correlated with division and multiplicative items, and therefore suggested that understanding of fractions is closely connected to multiplicative reasoning.

A summary of fours studies on secondary students' conceptual and procedural knowledge of fractions was presented by *Xenia Vamvakoussi, Maria Bempeni, Stavroula Poulopoulou* and *Ioanna Tsiplaki*. The researchers developed and evaluated an instrument to measure both knowledge types, and identified students' individual differences in combining procedural and conceptual knowledge.

Carlos Valenzuela Garcia and *Olimpia Figueras* designed a teaching model for fractions based on applets aiming to support the development of fraction mental objects in elementary students. The performance of two students, described in the paper, showed that working with the applet lead to an enhanced notion of fractions, e.g., from fractions as fracturer to fractions as number and as measurer.

Sofia I. Graça, João P. Ponte and *António Guerreiro* developed a study that aimed to identify grade 5 students' knowledge of rational number operations and number sense before and after a teaching experiment. Results indicated that students' performance changed; at the beginning it based on their whole number knowledge, later they exhibited conceptual knowledge, number and operations sense.

Carvalho, Renata and *João P. Ponte* surveyed grade 6 students' strategies in mental computation in rational numbers within a qualitative design research approach including three phases. Data analysis allowed to describe typical strategies regarding tasks with different cognitive demands, and revealed that, e.g., memorized rules are mostly applied without meaning.

Maria T. Sanz Garcia, Olimpia Figueras and *Bernardo Gómez* focused on students' performance (15 and 16 years old) in solving particular fraction word problems, and measured their difficulties by accuracy. Results suggested that the increase of steps in a problem leads to the increase of students' difficulties caused by insufficient basic fraction knowledge.

Metacognition in problem solving

Aikaterini Vissariou and *Despina Desli* investigated metacognitive strategies used by sixth graders solving a non-routine mathematics problem. Despite the fact that students reported the use of metacognition, their demonstration of the used strategies was quite low. Additionally, their successful solution to a problem was not necessarily connected to the employment of metacognitive strategies.

Discussed posters

Évelin Assis and *Luciana V. Corsos'* poster depicted well their analysis of the intervention pilot study discussing the adaptations that needed to occur on route. *Alix Boissière* presented a board game around the notion of fractions for grades 4 to 6. It was developed based on the theory of didactical situations and pretested in several classrooms. *Kazuhiro Kuriharas'* poster introduced a theoretical framework for analyzing the development of students' understanding the algebraic structure. Hereby, the focus is on the extension of number sets. *Charlotte Rechtsteiner* analyzed flexible mental calculation skills of freshmen and graduates. Results indicated that the flexibility scores are not adequate for prospective primary teachers.

Summary and outcomes

The great variety of papers was challenging, but also the strength of this TWG since it ensured interesting and often animated discussions in a supportive quizzical environment. Thereby, we went far beyond the specific topics of the single papers, worked out differences and commonalities of used terms, concepts, theoretical frameworks and methodological approaches. We realized once more different conventions in different counties of naming numbers and operations, as well as partly conflicting views of quantitative and qualitative approaches. Altogether, we broadened our own culturally influenced perspectives and drew new inspiration for further research. The aspects we discussed reflected the whole spectrum of arithmetic in primary and secondary level and comprised

- the importance of counting in different cultures and curricula, as well as a critical reflection on counting for developing strategies in mental calculation.
- the influence of instruction and the need to design learning environments based on our knowledge of students' strategies, conceptions and misconceptions.
- the importance of supporting students' conceptual knowledge regarding numbers and operations in different number systems.
- the clarification of methodology in terms of what is measured, why is a measure used, what does a result mean, and how does it contribute to better understanding of teaching and learning.
- the meaning of 'number estimation' as positioning natural numbers in a segment, and its connection to relational thinking.

- the challenges of transition between number systems, between representations, and from intervention research to students understanding (long term)

Reflecting our work in a conclusive discussion, we agreed that our various research is driven by a common goal: Supporting all student's understanding and meaningful learning to develop conceptual knowledge, number sense and structure sense in different number systems, as well as flexible and adaptive expertise in calculation strategies in all operations. Our further work is directed towards specifying terms and work on common understanding of number sense, structure sense, flexibility, manipulatives and mental calculation. Furthermore, we want to advance our methodological approaches and bridge the gap between quantitative and qualitative by triangulation.